With increasing complexity and scale of numerical simulations of fluid flows, the problems we pursue and questions we ask become commensurably complicated. In the past, numerical simulations have been used to produce high-fidelity output to user-specified input. More recently, the inverse problem is being tackled: given a user-specified objective, we desire control strategies, design modifications or other manipulative measures to achieve this goal with a minimum amount of effort. Questions of this type require gradient or sensitivity information about the flow, which can be gained by formulating adjoint algorithms, equations or variables. We will introduce a general gradient-based framework for the analysis of compressible and incompressible fluid systems based on chained sparse matrix products (and their adjoints) and demonstrate the potential of this approach on two applications: (i) the analysis and control of tonal noise around an airfoil and (ii) the optimization of mixing by a blowing-and-suction strategy. Various extensions of this framework to related application areas will also be discussed.