Optimal power generation and load management for off-grid hybrid power systems with renewable sources via mixed-integer programming

Ran Dai a,⁎, Mehran Mesbahi b

aAerospace Engineering Department, Iowa State University, Ames, IA 50011-2271, USA
bDepartment of Aeronautics and Astronautics, University of Washington, Seattle, WA 98195-2400, USA

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A B S T R A C T

This paper addresses the optimal power generation and load management problems in off-grid hybrid electric systems with renewable sources based on appropriately constructed optimization problems. In this venue, the capacity and operating constraints for generating, storage and load units are first formulated as mixed-integer linear programming (MILP) models. In addition, we integrate the power curtailment strategies, such as temporary pause and multiple power supplies, for the load units in the MILP models to alleviate the peak power demands and power shortage without an adverse effect on the overall operation of the system. We subsequently consider the application of this framework for representative scenarios in the context of a residential power system with solar sources. Simulation results for the case of predetermined schedules with preset power requests, as well as for the case of varying schedules with updated power requests, are presented using the proposed optimization-based approach.

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1. Introduction

Technologies for efficient utilization of green and renewable energy sources have seen a surge of renewed research activities in recent years. Renewable energy – especially solar and wind power – however, are intermittent energy sources that are affected by environmental factors, e.g., weather. Therefore, efficient scheduling for power generation, storage, and usage are crucial when considering renewable energy sources. Renewable sources are commonly seen in self-contained energy systems, referred as off-grid power systems, disconnected from the central grid [1–3]. Examples of such systems include all-electric shipboards [4], residential homes in rural areas [5], and more-electric airplanes [6]. Improving the energy management in off-grid applications will benefit the renewable energy providers as well as the consumers. By accurately tracking and estimation of consumers’ power demands within a certain time period, the energy management will be able to coordinate on efficiently satisfying users’ energy demands. This coordination will also lead to a robust operation with fast and reasonable response to peak power demands as well as the dynamic nature of energy delivery capacity of renewable sources. In this paper, we examine an optimization-based formalism for efficiently satisfying off-grid energy demands while reducing the overall cost of energy generation and distribution.

The implementation and operation of the smart grid technology include many facets, such as smart power generation [7], storage [8,9], and load management (LM) [10]. Optimal power generation – often referred to as the unit commitment (UC) problem – involves economic generation of power for a given power demand, while satisfying constraints such as production limits, ramping limits, and minimum on–off times. Another facet of the smart grid operation is efficient load management. In this case the aim is controlling the loads rather than the power generation in order to minimize the peak load requests and production cost, which in turn may be affected by the constraints on the shut-off times, the operation priority for the units, and certain specific tariffs.

Previous work in this area generally focuses on one aspect of smart grid operation, i.e., power generation [11], storage [12] or LM [10]. Our work revolves along the thesis that as cooperation between generators and loads is strengthened in the context of energy management, simultaneously optimizing the schedules for power generation and load management becomes feasible. In fact, the cooperation between the two technologies will also yield additional benefits [13]. Furthermore, inclusion of storage devices becomes necessary as the generation of power from renewable sources becomes less predictable due to factors such as weather. These devices can generally have multiple modes of operation. In this case, during a power shortage or peak demand period, these devices can work as sources. On the other hand, while excess power is generated than required, these devices can “buy in” energy and operate as loads in the system [14]. As components such as generators, loads, and storage devices are included in the hybrid
system with their specific capacities and operational modalities, it becomes imperative to manage them efficiently.

The objective of this paper is to propose a framework for optimal coordination among multiple types of components in the off-grid system during various operational modes, while satisfying the underlying capacity and operational constraints. The MILP approach, generally applicable to small and medium size optimization problems with both continuous and integer variables, is applied here to model the operational cost and constraints of individual electric units in the system, aiming to find the optimum continuous and integer variables. Distinct from the Lagrangian relaxation method which is applicable to large scale problems [15,16] by sacrificing global optimality, the MILP approach, on the other hand, obtains the global optimal solution via a branch and cut algorithm [17].

In order to estimate the cost of schedules in the power system, it is imperative to set up an appropriate criteria for evaluation purposes. The price based strategy is commonly used in the evaluation of cost functions [18,19]. The electricity prices vary from different generators, as well as temporally in terms of months, days and even hours. In the residential power supply, most electricity companies charge “real-time” prices during different hours of the day in order to moderate peak hour demands [20,21]. In this direction, the work presented in [22,23] has proposed strategies for scheduling household appliances according to the fluctuating prices issued by the electricity company. When solar power is included in the energy sources, it has traditionally been estimated as an expensive source of energy at the cost of $0.15–0.30 per kWh. However, with the development of solar photovoltaic technology and the federal and state incentives on renewable energy, the actual capital cost on solar system has greatly been reduced. For example, a solar energy system for a medium size home has a list price of $11,700, with an expected monthly output of 529 kWh in the California area, and a free life-time maintenance [24]. Considering the 30% federal tax return and $1000–$2000 state rebate, the solar energy system with a 30 year usage has an electricity cost around $0.04 per kWh.

The loads in these scenarios display on–off modes based on the designed optimal schedules. However, actual load power usage curtailment could possess more strategies than temporarily pausing respective operations. For example, we may use lower level power supplied for illumination by dimming out the lights. The cost of power generation per kWh for the off-grid system can be evaluated by the expenses on capital, fuel and maintenance, together with the expected annual hours run of renewable sources [25]. If the “disutility” for loads are also expressed by a price profile, the overall problem leads to a bargaining criteria between the sources and loads. In this case, assigning reasonable price profiles for the operational modes of each component will directly affect the resulting optimal schedules.

The main contribution of the present paper is to combine the power generation and LM problems for power systems with heterogeneous electric components as an MILP. In this direction, we first introduce a cost profile for all components associated with their operational modes in order to setup an accurate and flexible evaluation criteria. We also propose power curtailment strategies according to the characteristics of the specific components for a streamlined implementation in the context of an off-grid system.

The organization of the paper is as follows. First, we formulate the combined problem of generation schedules and load management for two distinct scenarios in Section 2. Subsequently, the solution methodology for this combined problem is detailed in Section 3, followed by the MILP models of generators, storage units and loads in Section 4. Simulation examples demonstrating the applicability of the proposed approach are detailed in Section 5, followed by concluding remarks in Section 6.

2. Problem formulation

In the currently envisioned off-grid systems, such as the systems shown in Fig. 1, consumers submit power requests for the loads to a centralized control processor, which in turn generates optimal schedules for loads, as well as for generators and storage devices, during the designated time interval. On the energy consumption side, each load in the network could consist of different appliances. For example, an off-grid residential system can consist of lights, a washing machine, a drier, and multiple ovens. In the another setting, an electric boat may include heating units, chillers, exhaust fans, among other loads. It is assumed that the centralized processor has access to the cost and capacity limitations for each energy unit, such as the generators and storage devices.

From the above description, we now set up our objective function in order to minimize the overall system cost while satisfying the units’ capacity and operational constraints and power balance constraint:

\[
\min_{p, p', \beta, s, x, \beta', u, w} f(p, p', \beta, s, x, \beta', u, w), \quad (2.1)
\]

subject to:

\[
\begin{align*}
(2.2) & \quad (p_i(t), x_i^k(t), x_i^l(t)) \in Q_i^k(t), \quad i \in G, \quad t \in T, \\
(2.3) & \quad (p_i(t), x_i^k(t), x_i^l(t)) \in Q_i^k(t), \quad j \in L, \quad t \in T, \\
(2.4) & \quad x_i^k(t+1) = f_i^k(p_i(t), x_i^k(t), x_i^l(t)), \quad i \in G, \quad t \in T, \\
(2.5) & \quad x_i^l(t+1) = f_i^l(p_i(t), x_i^k(t), x_i^l(t)), \quad j \in L, \quad t \in T, \\
(2.6) & \quad \sum_{t=1}^{T} p_i(t) + \sum_{t=1}^{T} p_i(t) + \sum_{t=1}^{T} p_i(t) = 0, \quad t \in T. \\
\end{align*}
\]

The above objective function is optimized over a given period T which consists of a set of consecutive time horizons indexed by the variable t. The superscripts g, l, s in (2.1) represent, respectively, generators, loads, and storage devices. For each component in the system, we use the state x(t), control u(t), and power output p(t), to represent its operational characteristics over the time horizon t. These variables can be continuous or discrete, i.e., the control variable u(t) can represent on–off switches, while the power output p(t) from a generator can be considered as a continuous variable. For each type of component, the sets

\[
\begin{align*}
Q_i^k(t) &= \mathbb{P}_i^k(t) \times X_i^k(t) \times L_i^k(t), \\
Q_i^l(t) &= \mathbb{P}_i^l(t) \times X_i^l(t) \times L_i^l(t),
\end{align*}
\]

specify the constraint sets for states, control, and power output for the unit; we note that the subscripts l, j and k are the indices for each type of component. The operational constraint, from current horizon t to the next horizon t + 1 is expressed by the state transition map f(p, x, u). The objective function in (2.1) is composed by two items summarized as

\[
J = C(p, p', \beta, s, x, \beta', u, u') + U(p, x, u, u'),
\]

where C represents the cost from generator and storage units and U represents the “disutility” from loads. The disutility term can be explained as the negative impact on comfort by delayed power delivery, temporarily shutting down of loads or other abnormal operations due to the power shortage or peak demands. Minimizing the cost and disutility simultaneously seems are often conflicting requirements. For example, during a peak demand time, by the power balance constraint in (2.8), either sufficient power is generated to supply the requested loads and reduce the disutility with increased cost of C, or power curtailment strategy is utilized to reduce
cost of $C$ with increased disutility $U$. Therefore, the optimal schedule is expected to search for the ideal equilibrium point between these objectives. Now we consider two types of scenarios, namely the off-line schedule and on-line schedule. For the off-line schedule, power requests from every load unit $j \in L$ have been priorly specified. In this case, each request has a specific starting time $t_0$, ending time $t_f$, and operation time $d_j$. Such assumptions are common in real-life scenarios. For example, a hotel may have specific power requests for water heating, ventilation, cooking, etc., during special time intervals of a day. These requests can be scheduled ahead of time in order to avoid an undesirable peak in the power demand. Such an assumption might also be applicable to household users. The solution for such a scenario, shown in Fig. 2, has an open loop structure as it does not include unpredicted changes in the power request. All power requests are recorded off-line and sent directly to the centralized control processor to generate optimal control signals. However, the open loop scheduling strategy is not robust to unforeseen power requests and disturbances for real-time operations. An unpredicted change could interrupt the pre-scheduled operations. Thus, we also consider a closed loop on-line scheduling approach as shown in Fig. 3. In this case, we have an additional operational module to detect the updated power requests at the beginning of each time horizon $t$. The new generated control signals incorporate the detected new requests, unprocessed requests, as well as the current state for each unit. In the following, we will discuss the detailed MILP-based modeling for each control structure.

### 3. Energy management via MILPs

Mixed-integer linear programming (MILP) is the optimization problem of minimizing an objective function expressed by a linear combination of integral and real-valued state variables, subject to linear equality and inequality constraints. A special case of MILPs is when the variables are binary, representing logical states or logical relationships in the optimization problem. Linear programs can be solved efficiently using the simplex or interior point algorithms. Mixed-integer linear programs on the other hand are often solved using the branch and bound, branch and cut, or branch and price algorithms [26]. There are numerous applications for MILPs in many areas of operations research, including network flow [27], path planning [28], and scheduling [29].

The MILP problem can be expressed in the standard form as:

$$\text{minimize} \quad c^T x,$$

subject to

$$b_L \leq Ax \leq b_U,$$

$$x_L \leq x \leq x_U, \quad x_j \in \mathbb{Z},$$

where $x$ of dimension $n$ is the unknown state variables to be determined, $c$ is the vector of coefficients with the same dimension as $x$ and $j$ is the index of the state variables belong to the integer set $\mathbb{Z}$. In (3.10), the vectors $b_i$ and $b_j$ are $m \times 1$ that specify, respectively, the upper and lower bounds for the linear constraints; when $b_i = b_j$, the corresponding constraints reduce to equality constraints. The matrix $A$ in (3.10) is an $m \times n$ matrix which specifies the $m$ linear (composite) equality or inequality constraints associated with the objective function to be minimized. The matrix $A$ can be decomposed into $K$ sub-matrices $A_1, \ldots, A_k$ divided by the columns of $A$. If the dimension of each sub-matrix is denoted by $m_k \times n_k$, then we have

$$A_k x_k \leq b_k, \quad \sum_{k=1}^{K} m_k = m, \quad \sum_{k=1}^{K} n_k = n.$$  

$$ (3.11)$$
where $x_k$ is an $n_k \times 1$ vector. Thus, when solving MILP problem with constraints for multiple units, one can decompose the MILP problem into $K$ subproblems with the corresponding constraints and states.

An important advantage of applying the MILP formulation in the context optimal load management is the availability of extensive commercial software for solving this class of problems. Although there is a wide variety of choices for MILP solvers, in this work, we use the CPLEX solver that implements a branch and cut algorithm for solving large-scale MILPs with practical computation time [17]. In the context of MILPs, the problem of optimal load management is transformed into modeling electric units costs and dynamics as linear objective and constraints in the form of (3.10).

In this direction, assume we have a set of $\mathcal{N}$ electric units in the power system and that the time horizon has been specified as the discrete set $T$. Thereby, we include the control and state variables for unit $i, i \in \mathcal{N}$ in the MILP over the time horizon $t, t \in T$. Each unit is assigned to have a cost vector $C_i(t)$, linear constraints matrix $A_i(t)$, states and controls boundary constraints, $x_{\text{if}}(t)$ and $x_{\text{of}}(t)$, respectively. The linear constraints for each unit are uncoupled from each other except through the power balance constraint which can be stated as $A_{\text{in}}x(t) = 0$, where $A_{\text{in}}(t)$ is a row vector with dimension that is the sum of dimensions for the state and control variables. The states of each unit at current time $t$ may be coupled with its states at $t + 1$ by the transition function. But we could assemble all the states constraints except power balance for unit $i$ over $T$ in one block $A_i$. In order to formulate the optimal power management problem in the form of (3.10), the uncoupled constraint matrices are included as block diagonal matrix $A_i$ in the overall constraint matrix over $T$ intervals is expressed in matrix $A_{\text{in}}$ with dimension $T \times n$. Finally, the objective function and constraints over the time horizon $T$ are summarized as,

$$
\begin{align*}
\min \quad & \sum_{t=1}^{T} \sum_{i=1}^{\mathcal{N}} C_i(t)^T x_i(t), \\
\text{s.t.} \quad & b_i \leq \begin{cases} A_i & x \leq b_i, \\
A_{\text{in}} & x \leq b_i, \\
A_i & A_{\text{in}} \\
\end{cases} \\
& x_i \leq x \leq x_{\text{of}}, \quad x_f \in \mathbb{Z}. \\
\end{align*}
$$

(3.12)

As structured in Fig. 4, each component leads to its own cost vector $C_i$ and constraint sub-matrix $A_i$, which comprises the entire system's cost vector and constraint matrix to be solved by the MILP solver. The structured formulation allows simple incorporation of cost and constraints when additional components are included in the system.

We classify the units to be modeled for inclusion in the MILP into three categories: generators, storage devices, and loads. In the following, we will discuss the detailed MILP-based modeling for each category.

### 4. Models for the MILP approach

In this section we focus on the MILP model for distinct classes of units included in the power system with explicit expressions for their cost structure, constraints, and power output during each time interval. As we stated previously, the time period considered here is represented by a set of consecutive time horizons $t \in T$. We assume that the lengths of the time horizons are non-overlapping,

<table>
<thead>
<tr>
<th>$G_i$ (cost, constraints)</th>
<th>$S_i$ (cost, constraints)</th>
<th>$L_i$ (cost, constraints)</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole system cost, constraints</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| power balance constraint |

**Fig. 4.** The architecture of the centralized MILP-based optimal scheduling algorithm.

have equal lengths $t$, whose union form the entire time horizon $T$ of interest. In subsequent sections, the integer variables are denoted by $z$.

### 4.1. Generators

Some electricity providers in the grid system set a threshold on the “preferred” total daily energy consumption and charge different rates for usage under and above this threshold. Others use real-time pricing which is fluctuated over time according to market demands. When different power sources, such as solar source and backup generator, are included in the system, we handle each source as individual power provider with corresponding cost rate. Under such assumptions, the electricity cost for all types of generators can be evaluated and the cost function can be represented by a set of piecewise-linear curves with respect to the output power [30,31], as illustrated in Fig. 5.

To make the model more general, we assume that the off-grid system is equipped with $n_g$ generators. Each generator $i, i \in 1, \ldots, n_g$ has a single mode of operation and associated cost coefficient $c_i$, within the power output limit $[g_{\text{lb}}, g_{\text{ub}}]$, where $g_{\text{lb}}$ and $g_{\text{ub}}$ are the lower and upper bounds for the power output, respectively. For the $n_g$ different generators, we assign $n_g$ binary variables $z_i$ to indicate whether generator $i$ is under working status. We also assign $n_k$ continuous variables $g_i$ to represent the power output in the corresponding generators. Obviously, for a specific time horizon $t$, the power output for any generator must fall between its limit under working status or has zero output when shut down. We can now specify the power output for generator $i$ as,

$$
g_iz_i \leq g_i(t) \leq g_iz_i, \quad t \in T. 
$$

(4.13)

**Fig. 5.** Piecewise-linear cost functions for generator cost.
Thereby the objective function over time period \( T \) is stated as

\[
C = \sum_{t=1}^{T} \sum_{i=1}^{n_0} c_i^t g_i(t). \tag{4.14}
\]

The above equations use combined binary and continuous variables to explicitly state the linear constraints and the overall cost of generators in the off-grid system. If we assume the unknown state variables for generator \( i \) are written as \( x_i = [g_i, z_i] \), then the respective constraint matrix \( A_i \) at time \( t \) is denoted by

\[
A_i = \begin{pmatrix}
1 & -g_{si} \\
-1 & g_i
\end{pmatrix}
\]

with power upper bound set as \( b_u = [0, 0]^T \). Another aspect of the unit’s operation to be specified is the power output at each time horizon; here the power output during the time horizon \( t \) is simply

\[
P(t) = \sum_{i=1}^{n_0} g_i(t). \tag{4.15}
\]

4.2. Storage units

The storage units can be in distinct operational modes as required by the power system with associated power outputs and costs. A storage unit acts as a load when the energy supply is sufficient and low cost and delivers power when there is a deficiency in power generation. An example of such a system is the plug-in hybrid electric vehicle (PHEV); it is normally in the charging mode overnight when the price of electricity is less.

Our model considers a general storage unit with charging and discharging operational modes. In such a setting, we use \( c_c \) and \( c_d \) to represent the unit cost of charging and discharging modes per kWh, respectively. As an emergency power supply unit, the cost of storage is generally more expensive than the normal power generator when producing the same amount of electric energy. This assumption makes the normal power generator be the primary generating unit in minimal power system. Similarly, \( g_c \) and \( g_d \) denote the power output for the two modes. Correspondingly, we assign two binary variables \( z_c \) and \( z_d \) to indicate whether the storage unit is operating in the respective modes. As only one mode is selected for a specific time horizon \( t \), we thus have

\[
z_c(t) + z_d(t) = 1, \quad z_c, z_d \in \{0, 1\}, \quad t \in T. \tag{4.16}
\]

The two modes power outputs are bounded by

\[
z_c(t)g_c \leq g_c(t) \leq 0,
\]

\[
0 \leq g_d(t) \leq z_d(t)g_d, \quad t \in T,
\]

where \( g_c \) is the lower bound of the charge power output and \( g_d \) is the upper bound of the discharge power output. Since only one mode is allowed during each time interval, one of the power outputs is forced to be zero if the other mode is selected. The power output representing each mode can also be treated as control variables that determine the storage capacity transformation from current time horizon \( t \) to the next time horizon \( t + 1 \), in the update equation

\[
X(t+1) = X(t) + (g_c(t) + g_d(t))t, \quad t \in T,
\]

where \( X \) represents the storage capacity and is bounded by the maximal and minimal levels generalized as \( x_c \leq X(t) \leq x_d \). Distinct from the single mode units whose states are uncoupled across time horizons, the storage unit described here has a coupling across adjacent time horizons states. The objective function for time period \( T \) is stated as

\[
C = \sum_{t=1}^{T} \left(c_c g_c(t) + c_d g_d(t)\right). \tag{4.19}
\]

and the power output at horizon \( t \) is correspondingly

\[
P(t) = g_c(t) + g_d(t). \tag{4.20}
\]

4.3. Constant power load supply

When supplied power cannot meet the requested power, one can adopt several types of strategies for curtailing power requests. The first strategy described here is that of “interruptible” load. This type of loads can be briefly paused without bringing negative effect to the users, such as turning down the air conditioning temporarily to allow the temperature increase only by a few degrees. At the same time, it is desirable to avoid shutting down a unit too long and causing damage to the entire system. In order to accurately express the priority of each load unit with time, we introduce the terminology of “disutility” in Section 2. If the cost of normal operation of a load unit is reflected by its power usage (and the price charged by the generators and storage units), the cost of purposely shutting down the load is expressed by the term of disutility which directly causes the negative impact on comfort. By using the cost profile to describe the load disutility values, the load units with low priority/disutility costs will be shut down first and then the other units in sequence when there is no sufficient power supply. During each time horizon \( t \), we denote three binary variables \( z_p, z_o, z_d \) to represent the operation modes, pause, on, and off, respectively. A binary value “1” is assigned to the corresponding variable if the related mode is selected. For a power request constrained during the time horizon \([t_0, t_f]\) with operation time \( \delta \), the overall operation horizon equals to \( \delta \) over \([t_0, t_f]\) and no operation is allowed outside the interval \([t_0, t_f]\). Therefore, we have the following constraints

\[
\sum_{t \in \delta} z_p(t) = \delta, \quad z_p(t) \in \{0, 1\}, \quad t \in [t_0, t_f];
\]

\[
z_o(t) = 0, \quad t \in T \setminus [t_0, t_f].
\]

The above constraints will guarantee the required operation will occur during the specified interval. We also have the constraint that allows only one operational mode at time horizon \( t \),

\[
z_p(t) + z_o(t) + z_d(t) = 1, \quad z_p, z_o, z_d \in \{0, 1\}, \quad t \in T,
\]

and the “off” mode cannot be selected until the required operation time is finished,

\[
0 \leq z_o(t) \leq \sum_{t=0}^{T} z_p(t)/\delta.
\]

Therefore, unless the summation of the “on” mode counts equal to \( \delta/\tau \), the binary variable \( z_f \) for the “off” mode is always set to zero. In addition, to prevent frequent switches between “on” and “pause” modes for certain loads, we assume that once the unit is paused in current time horizon \( t \), it cannot be turned back in the next time horizon \( t + 1 \). This relationship can be expressed as

\[
z_o(t) + 1 - z_p(t + 1) + z_o(t + 2) < 3, \quad \forall t \in T, \quad t \leq t_f - 2.
\]

In order to make the model more general, we divide the interruptible loads into two types: those that are interruptible only before the start of their operation and those that are interruptible before and during their operation. Examples of loads that are interruptible only before the initiation of their operation are washing machines, ovens, ice makers, among others. Once the operation of such units has started, it is often desired that they are not interrupted. However, one has the flexibility of choosing the starting time for these units in order to reduce operational cost of the
overall system. For such type of loads, the disutility due to inter-
ruption after the initiation of the operation is much higher as com-
pared with the interruption disutility before the initiation of their
operation. Even for the loads that allow interruption before and
after the initiation of their operation, the disutility costs are
generally different between the two types of interruptions. Here, we
introduce another binary variable \( z_t \) to indicate whether the
operation has started and use a fractional inequality to express its
operational constraint as
\[
\sum_{i \in T} z_t(i) \gamma / \delta s \leq 1,
\]
0 \leq z_t(i) \leq \sum_{i \in T} z_t(i), \quad z_t(i) \in \{0, 1\}. \tag{4.25}

The above constraint indicates that as long as the operation starts
before the current horizon \( t \), the variable \( z_t(i) \) is set to “1”. In this
case, the pause command following the initiation of the operation is
decided by the difference \( z_t(i) - z_t(i) - z_t(i) \), for all \( t \in T \).

Assume that the coefficients of the disutility due to interruptions
before and during the operation for all units are, respectively,
defined by \( c_t(i) \) and \( c_t(i) \). Then the objective function for time pe-
riod \( T \) can be expressed as
\[
U = \sum_{i \in T} c_t(i) z_t(i) + (c_t(i) - c_t(i))(z_t(i) - z_t(i) - z_t(i)). \tag{4.26}
\]

The objective is composed of disutility due to interruptions before
and after the start of the operation. The first term in (4.26) accounts
for both types of interruption disutility with rate \( c_t(i) \). Therefore,
the extra cost accounted for an interruption during the operation
with rate \( c_t(i) \) in the first term is removed from the second term.

The predetermined disutility coefficients \( c_t(i) \) and \( c_t(i) \) can be
chosen according to the user’s preference. When these coefficients
are set as constants, the pause option scheduled in any time hori-
zon yields identical disutility values in that horizon. These coeffi-
cients can also be a function of time, in the form of \( c_t(i) = t^{-n} \),
where \( n > 1 \) is a constant parameter. For this type of coefficients,
the pause option is preferred to be scheduled at the beginning of
the specified time period in order to reduce the disutility cost.
On the contrary, we can set the coefficients as \( c_t(i) = t^{-n} \) in order
to make the preference of the pause option close to the end of the
time period. In addition, the disutility coefficients can be used as
a “bargaining” criteria when the cost of different schedules are com-
pared. For example, when comparing a single horizon schedule
 costs, if the disutility cost is smaller than the cost of normal power
supply from the generator or storage units, the optimization algo-


4.4. Multiple power supply load

Another strategy for power curtailment is to use lower power to
supply the unit instead of the normal level when there is an insuf-
cient or costly available power. For example, water heaters are
generally equipped with several heaters working simultaneously
in order to increase the water temperature to the desired setpoint.
When the power supply is insufficient, it is reasonable to initiate a

1 Note that the load has a positive power input; in order to generalize the
expression, we assume that loads have negative power output \( g \).
aggregated power for the entire system should be balanced during any time horizon as expressed by the power balance constraint (2.8). When a power request is issued by the load, normal operation of the load requires the sources generate sufficient power via the power balance constraint with a cost reflected by the electricity cost from the generator and/or discharging battery. Otherwise, the load is scheduled to temporarily shut down with no power output from sources and the cost is reflected by the disutility of the load during the corresponding time horizons. With this coupling constraint between the units in the system, the overall problem does not simply reduce to minimizing the objective function for each unit separately while satisfying the individual unit’s operational constraints. In fact, the power balance requires coordinating operational schedules for all components in the power system network for the efficient and feasible operation while also ensuring the power balance.

5. Simulation examples

In order to illustrate the feasibility of the proposed approach and models, an off-line and on-line simulation examples with multiple power requests and specifications for a solar powered off-grid community [32] with three households demonstrated in Fig. 6 are considered in this section. The major power supply of the off-grid community are three identical solar sources. In addition, this system is equipped with two sets of battery bank and a diesel backup generator.

5.1. Simulation for off-line scenario

In this scenario, the scheduler plans the optimal operation of all units in the three households for 1 day from 8 am to 24 pm (operation during 0–8 am is ignored due to negligible power usage). That is, all power requests have been specified in advance with explicit operation times, the type of load units, and possible power curtailment strategies. The generating capacity for this type of solar source is designed for medium sized home with average monthly output of 529 kW h [24]. As we discussed in Section 1, such type of solar source can have a net electricity cost of $0.04 per kW h. As the daily power output for the solar source is dynamic throughout the day, we allocate the maximum power output from 8 am to 24 pm for each solar source in Fig. 7.

It is furthermore assumed that each set of battery banks is charged when extra power is generated from energy sources and that it behaves as an energy source when power supply is insufficient [33]. Each battery set used in our scenario is assumed to have a capital and maintenance cost of $2870 and a life cycle of 3300 h. We assume that the battery set has a maximum power output/input of 3 kW for continuous 6 h; they are assumed to be half charged at 8 am in the considered example. If the battery set is 60% charged for each cycle on average, then the electricity cost from battery is around $0.08 per kW h. A diesel generator is included in the system as a backup which has a fixed power output of 8 kW for continuous 4 h. For every 1 kW h energy generated from the diesel generator, it will burn 0.51 fuel. Considering the current market price of diesel at $4 per gallon, the electricity cost from the backup generator is estimated as $0.5 per kW h. These units operate in unison as sources of power with multiple costs curves, depending on different levels of power output.

All types of loads described above are included in the system with the associated power curtailment strategies and parameters. The algorithm will estimate the cost of each strategy under insufficient power supply condition and then selects the minimum cost solution and the corresponding optimal schedule. Table 1 lists all requests during the 8 am to 24 pm period with earliest starting time \( t_0 \), latest ending time \( t_f \), requested power \( P \), and the corresponding disutilities before operation \( c_{di} \) and during operation \( c_{ci} \). The subscripts 1, 2 and 3 refer the index of the home. Each household has assigned priority factor \( \eta \) to prevent conflicting schedules and the corresponding disutility cost before and during operation are set as \( c_{di} = \eta c_{ci} \) and \( c_{ci} = \eta c_{ci} \), respectively. In this scenario, \( \eta \) for the households in the community is set as \( \eta_1 = 1, \eta_2 = 0.9, \) and \( \eta_3 = 0.8 \). The water heater below can be operated at lower power of level 1 kW, which requires twice as much time as that operated at normal power to reach the same setpoint temperature.

From the scenario description in Table 1, the overall daily energy demand request is 65.27 kW h which is near the solar daily output capability of 52.5 kW h. Considering that some appliances, i.e., the washing machine and spin drier, may not be used everyday, the proposed generation system of the photovoltaic arrays, battery sets, and backup generator can generally meet the demand requirements of the community. However, during certain peak request periods, i.e., in the mornings and evenings, conflicts can arise

![Diagram](a) A smart home. (b) An off-grid community.

Fig. 6. An off-grid solar powered community with three smart households.
when there are more load requests than available power; there will not be enough generating capability from the solar sources alone to supply the loads, especially when the maximum total solar power output is small. As such, either additional sources of power or a power curtailment strategy is required to serve the power request. All units included in this scenario are formulated as MILP models described in Section 4. We assume that all power requests and sources output capacity are specified before 8am and the simulation horizon $t$ is 0.5 h. The CPLEX solver is now employed to solve the 1089-variable MILP, corresponding to all the units in the system. The optimal schedule for the generated scenario is illustrated in Fig. 8 with sources power output histories and Table 2 with load’s operation histories.

From Fig. 8, we observe that solar sources, as the least expensive power supply units in this system, will provide as much power as required by the loads if the total request is under its maximum power output limit. When the request is above solar sources’ upper bound, the battery, as the second inexpensive energy supply unit, will begin to output power or some units are temporarily shut down until more power becomes available. For example, during the morning hours 8am to 10am, when the maximum power output of solar sources is low, the battery will supply the extra power required for certain important appliances, e.g. the microwave. However, when the solar power output is increased in the afternoon, the battery stores the net generated energy. At the 18 pm mark, the aggregate requested power from the loads exceeds the battery supply limit, initiating the backup generator to supply power. From the load operation history shown in Table 2, at the beginning, the clothes washer and vacuum robot in the household 3 are shut down temporarily to let the other units operate first with power supply coming from the solar source and the battery, which avoids using the backup generator which allows all units to operate simultaneously. This behaviour is due to the net benefit considered by both the cost of running the backup generator and the utility lost for turning off the lower priority units. Other units with lower assigned disutility values, i.e., dish washer from household 1 and 2, is briefly shut down and then operate continuously to obtain more power from solar source and at the same time catch up with the latest ending time. The water heater requests from households 1 and 2 at 19 pm are operated at lower power level for 1 h to avoid using the backup generator again. Some important units, e.g., the refrigerator, are never shut down due to the high disutility cost assigned to them. In order to better illustrate the mediation function of the optimization algorithm among all the units in the system, Fig. 9a shows how the aggregate available power is reduced during peak load request time periods. In this plot, the gray bar is the power supply and the black bar is the power request. When the power supply can meet the power request, the gray bar will cover the black bar. Otherwise, the gray bar is less than the black bar. Therefore, the black coloured bars indicate intervals when the load request could not be met and appropriate scheduling of the sources and loads are required.

The optimal schedule sufficiently takes advantage of all information provided in the system and at the same time satisfies all constraints for each unit. Fig. 9b shows the accumulative cost with respect to time from the results of optimal schedule and compared with the cost when requests are executed upon demand. As these plots show, the cost is greatly reduced from $11.12 to $8.03 when

![Fig. 7. Maximum single solar power during 1 day.](image-url)

**Table 1**

<table>
<thead>
<tr>
<th>Unit name</th>
<th>$P$ (kw)</th>
<th>$c_x$ ($\text{kw}$)</th>
<th>$c_r$ ($\text{kw}$)</th>
<th>$t_{01}$ (h)</th>
<th>$t_{02}$ (h)</th>
<th>$t_f$ (h)</th>
<th>$t_{01}$ (h)</th>
<th>$t_{02}$ (h)</th>
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<th>$t_{03}$ (h)</th>
<th>$t_f$ (h)</th>
<th>$r_3$ (h)</th>
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<td>8</td>
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<td>0.5</td>
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<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
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<td>0.002</td>
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<td>17</td>
<td>1</td>
<td>9.5</td>
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<td>1</td>
<td>11</td>
<td>17</td>
<td>1</td>
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</tr>
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<td>0.002</td>
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<td>24</td>
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</tr>
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</table>
the desired performance index is optimized with the scheduled operations. The daily savings may accumulate to $93 per month and $1112 per year. Although the scenario considered is a bit involved, it took a mere 0.499 s for the CPLEX to determine its solution on a Lenovo X201 laptop with intel i5 CPU and 4 GB RAM. The fast calculation performance makes the generation of the online schedule feasible for problems of a similar scale.

5.2. Simulation for on-line scenario

In the on-line scenarios, some power requests are assumed to be unpredictable in advance. As such scheduling the operation for all units without updates for the entire day is not realistic. Considering the uncertainty in weather forecasts as an additional uncertain factor in the operation of solar-based energy sources, it is beneficial to reschedule the operation of the system when a new request has been detected or any parameter in the system has changed. In this case, as distinct from the off-line scenario, the weather forecast and power requests are updated every horizon in a closed loop fashion. As long as any changes are incorporated in the system, new schedules will be generated, generally slightly before the end of the current horizon, for the resuming hours until 24 pm.

In the simulation for the on-line scenario, we assume that the system is not notified of the power requests as listed in Table 1 until the corresponding earliest starting time $t_0$. The solar source performs consistent with Fig. 7 before 9 am. At 9 am, from weather forecast information, one can expect that the solar source output power keeps the same low level at 1 kW till 12 pm and then increases to 1.5 kW. After 13 pm, the solar source resumes normal operation as assumed in the off-line example. The detailed time history of the solar power output is shown in Fig. 10. The assumption of battery set and diesel generator is consistent with the off-line example. The special function block referred to as the power request detector, discussed in Section 2, will update the list of requests at every time horizon and other parameters related to the sources power output level and cost. The time histories for all sources power output for the on-line scenario are illustrated in Fig. 10 and the optimal schedule of all loads is listed in Table 3.

Compared to the simulation results for the off-line scenario, the operation schedules are quite different in the on-line scenario. Due to the updated information of solar power output, vacuum robots and dish washers get more interruption horizons to skip the period when solar source has a low power output. For example, the vacuum robot in household 1 starts operation at 9 am and is interrupted during the operation for 3.5 h to allow the units with higher disutility cost operate first. The vacuum robot resumes its operation from 13 pm until its task has been completed. The history of aggregate requested and supplied power and accumulative cost comparison plots are shown in Fig. 11a and b, respec-

Table 2
Optimal schedule of loads operation in off-line scenario.

<table>
<thead>
<tr>
<th>Unit name</th>
<th>$t_0$ (h)</th>
<th>$t_e$ (h)</th>
<th>$t_d$ (h)</th>
<th>$t_0$ (h)</th>
<th>$t_e$ (h)</th>
<th>$t_d$ (h)</th>
<th>$t_0$ (h)</th>
<th>$t_e$ (h)</th>
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<td>0</td>
<td>9</td>
<td>9.5</td>
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<td>n/a</td>
<td>n/a</td>
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<tr>
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</table>

Fig. 9. Time History of aggregate demand levels, delivered power and cumulative cost for the off-line scenario.
tively. Although all requests are unforeseen before the starting time and the solar power output is greatly reduced during some hori-
zons, the closed loop optimal scheduler can coordinate the units efficiently, concluding with similar cost as that calculated in the off-line example.

Both scenarios considered above are close to the real-life applications in the off-grid system. Optimal results are achieved with short computation time for the corresponding MILP problems, including requests of all appliances in one household. These favourable computational aspects make the implementation of the proposed optimal scheduling procedure promising for a small and medium sized off-grid system with considerable number of units. However, the successful implementation of the proposed framework depends on certain assumptions, including accessibility to the appliances’ operation, accurate power requests, and reliable weather forecasts.

6. Conclusions

This paper proposes an optimal energy efficient strategy for off-grid systems by unifying the power generation and optimal load management. In this direction, the paper describes an optimization-based modeling technique for a wide class of generating, storage, and load units via MILP, each with the least number of binary variables. Simulation results for both open loop and closed loop scenarios show the efficacy of optimal power curtailment strategies and feasibility of integrating the proposed algorithm in off-grid systems. At the same time, the flexibility in the cost assignment within the proposed framework allows the users to schedule operations according to their preferences without sacrificing efficiency. Future research will consider more complex systems including various classes of deterministic and stochastic

---

**Table 3**

<table>
<thead>
<tr>
<th>Unit name</th>
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<th>$t_3$ (h)</th>
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<td>11</td>
</tr>
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**Fig. 10.** Sources schedule and commitment levels in the on-line scenario.

**Fig. 11.** Time history of aggregate demand levels, delivered power, and cumulative cost for the on-line scenario.
parameters effecting the operation of various units in an off-grid energy system.

References


