Evaluation of the Cross Correlation Method by Using PIV Standard Images

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Abstract: Effects of various parameters for PIV image acquiring and processing on the final velocity field is studied by using PIV standard images (Okamoto et al., 1997) to evaluate the cross correlation method. The studied parameters include the size of interrogation window, the size of search window, the number of tracer particles, the diameter of tracer particles, out-of-plane velocity and average image velocity or the time interval between two images. In order to improve the PIV sub-pixel accuracy, the validity of the "sub-pixel interpolation" process also is discussed in the paper. Some useful conclusions are suggested for the optimal parameter selection for a final PIV result with high accuracy.

Keywords: PIV accuracy, cross correlation method, PIV parameter optimism, PIV standard image.

1. Introduction

Compared with other velocity measurement techniques such as Laser Doppler Anemometry and hot-wire anemometry, Particle Imaging Velocimetry (PIV) can offer many advantages for the study of fluid mechanics. It can measure the velocity of whole two dimensional or three dimensional flow fields instantaneously without disturbing the flows and reveal the global structure of complicated and/or unsteady flow field quantitatively (Adrian, 1991), so it has been studied intensely and developed rapidly in the past two decades.

Image processing is a very important stage in the process to obtain the velocity field from PIV images with high accuracy. The cross correlation method is one of the most common methods for the PIV image process, which had been used successfully in many fluid mechanics researches (Kobayashi et al., 1991; Willert and Gharib, 1991; Lee, 1997). However, few systematical researches were performed to evaluate the effectiveness and accuracy of final PIV results obtained by using the cross correlation method. Hence, users of the cross correlation method had to spend a lot of time and cost to optimize various parameters for PIV image acquiring and processing to get an accurate velocity field. So, there is still a lot of work need to be done to provide enough information for the optimal selection of parameters. This is the object of the present research.

In the present research, effects the size of interrogation window, the size of the search window, the number of tracer particles, the diameter of tracer particles, out-of-plane velocity and average image velocity or the time interval between two images, on the final PIV velocity field obtained by using the cross correlation method are studied. In order to improve PIV sub-grid accuracy, the validity of the "sub-pixel interpolation" process and the comparison of two "sub-pixel interpolation" functions also are discussed in the paper. In the summary of the paper, some useful conclusions for the optimal parameter selection to get a final PIV result with high accuracy are suggested for users of the cross correlation method.

2. Cross Correlation Method and Standard Images

2.1 Cross Correlation Method

For two successive PIV images acquired at time T and $T+\Delta t$ (Fig. 1), it is assumed that the interrogation windows of the first and the second images are denoted as $f(x,y)$ and $g(x,y)$. For the point $p(x,y)$ in the first image, its corresponding point $q(x,y)$ in the second image is determined with the maximum cross correlation coefficient:

$$R(p,q) = \frac{\int (f(x,y)-\overline{f})(g(x,y)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2 dv \int (g(x,y)-\overline{g})^2 dv}}$$

and the velocity (U,V) at point $p(x,y)$ is calculated from the displacement of $p(x,y)$ and $q(x,y)$.

By using the cross correlation method, two parameters must first be decided for the image processing code development. One is the size of interrogation window (SA) around the point $p(x,y)$, and the other is the size of the search window (SB) for searching the point $q(x,y)$ in the second image. The effect of these two parameters on the final PIV result will be discussed first.
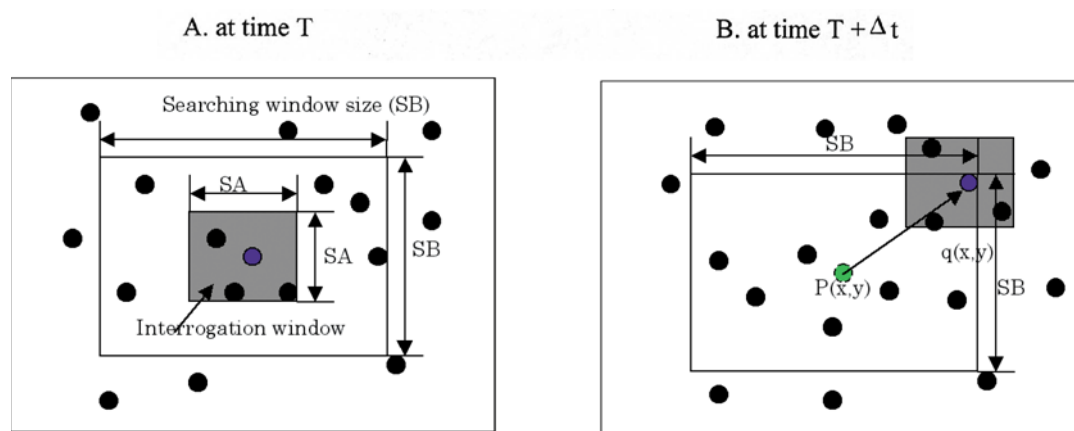

Fig. 1.   A Pair of PIV image frames.

2.2 The PIV Standard Images and Accurate Solution

Although lots of PIV techniques had been developed and many kinds of PIV systems had been set up in the past, PIV is still far from being a popular or general technique. To popularize the PIV technique for practical use, the "PIV-STD" Project was proposed to develop PIV standards and PIV guide tools to popularize PIV technique. In that project, a "custom made PIV image" tool is provided to be able to generate standard images with parameters selected by users. The PIV standard images and accurate solution (Fig.2) used in this study are just generated by this "custom made PIV image" tool and downloaded from "PIV-STD" Project homepage INTERNET address: http://sap.gen.u-tokyo.ac.jp. The main parameters of these standard images are listed in Table 1. Details about the "STD-PIV" Project and accurate solution can be obtained in the paper of Okamoto et al. (1997) and the homepage of the "PIV-STD " Project.
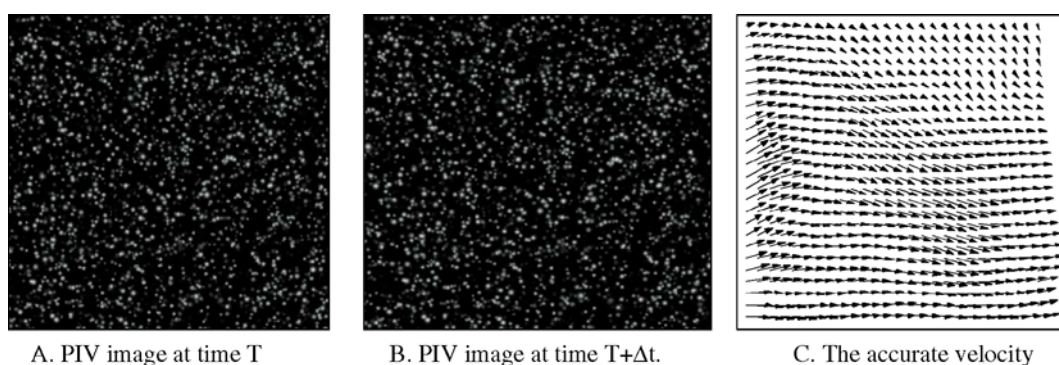

A. PIV image at time T   B. PIV image at time T+Δt.   C. The accurate velocity
Fig. 2.   Standard images and corresponding accurate velocity field in "PIV-STD" Project.

Table 1 PIV standard images used in the present study.

CASE	V_M	V_X	W_M	N	D_M	CR-U	CR-V	Ave−Err
A	7.5	15.0	0.017	4,000	5.0	0.991	0.980	7.12%
B	7.5	15.0	0.017	10,000	5.0	0.993	0.986	6.37%
C	7.5	15.0	0.017	1,000	5.0	0.899	0.870	10.92%
D	7.5	15.0	0.017	4,000	2.5	0.916	0.871	9.05%
E	7.5	15.0	0.017	4,000	10.0	0.992	0.982	6.98%
F	7.5	15.0	0.17	4,000	5.0	0.698	0.748	13.75%
G	7.5	15.0	0.34	4,000	5.0	0.363	0.394	41.30%
H	7.5	15.0	1.70	4,000	5.0	-0.06	-0.05	132.10%
I	22.5	45.0	0.063	4,000	5.0	0.288	0.471	55.41%
J	2.5	5.0	0.006	4,000	5.0	0.951	0.917	18.94%
A*	7.5	15.0	0.017	4,000	5.0	0.996	0.993	4.13%
J *	2.5	5.0	0.006	4,000	5.0	0.993	0.988	4.23%
A**	7.5	15.0	0.017	4,000	5.0	0.996	0.992	4.30%
J **	2.5	5.0	0.006	4,000	5.0	0.991	0.982	6.39%

Note: V_M: average velocity (pixel/interval) V_X: maximum Velocity (pixel/interval)
W_M: out of plane velocity (laser width/interval) N: tracer number
D_M: tracer average diameter (pixel)   Aver-Err: average error of PIV results
CR-U, CR-V: correlation coefficients of obtained PIV result with accurate solution
*:   "sub-pixel interpolation" process by using interpolation function (1) (Fig.7)
** : "sub-pixel interpolation" process by using interpolation function (2) (Fig.7)

2.3 The Evaluation Parameters

Three kinds of precision indices are used in this study to evaluate the fitness of obtained PIV results to accurate solution velocities, they are:

(1)   The correlation coefficients of obtained PIV result with accurate solution (CR-U and CR-V);

$$\text{CR-U}= \frac{\sum_{i=1}^{M}(U_i-\bar{U})(U^*_i-\bar{U^*})}{\sqrt{\sum_{i=1}^{M}(U_i-\bar{U})^2\sum_{i=1}^{M}(U^*_i-\bar{U^*})^2}} \qquad \text{CR-V}= \frac{\sum_{i=1}^{M}(V_i-\bar{V})(V^*_i-\bar{V^*})}{\sqrt{\sum_{i=1}^{M}(V_i-\bar{V})^2\sum_{i=1}^{M}(V^*_i-\bar{V^*})^2}}$$

in which U and V are the velocities in X and Y directions respectively obtained by the cross correlation method, and U^* and V^* are the corresponding accurate solutions. $\bar{U}$, $\bar{U^*}$, $\bar{V}$ and $\bar{V^*}$ are ,the average values of U, U^*, V and V^* respectively. M is the number of total vectors, M=576 in the present study.

(2)   The error in the PIV result;

$$\text{Err}_i = \frac{\sqrt{(U_i-U^*_i)^2+(V_i-V^*_i)^2}}{\sqrt{U_i^{*2}+V_i^{*2}}}$$

(3)   The average error (Ave - Err) of the PIV results;

$$\text{Ave-Err}= \frac{\sum_{i=1}^{M}\text{Err}_i}{M}$$

3. Results and Discussions

3.1 The Effect of the Size of Interrogation Window

Figures 3 and 4 show the effect of the interrogation window size on the PIV result. From the figures, it can be seen that, when the size of the interrogation window is small (SA<9), there are lots of error vectors in the flow field, which results in a poor PIV result. However, when the size of interrogation window is too big (SA>27), many points $p(x,y)$ can not find their correspond points $q(x,y)$ (the maximum cross correlation coefficient is smaller that the threshold value 0.5 in the present study), so the vectors' spatial resolution becomes low, which also results in poor PIV results.
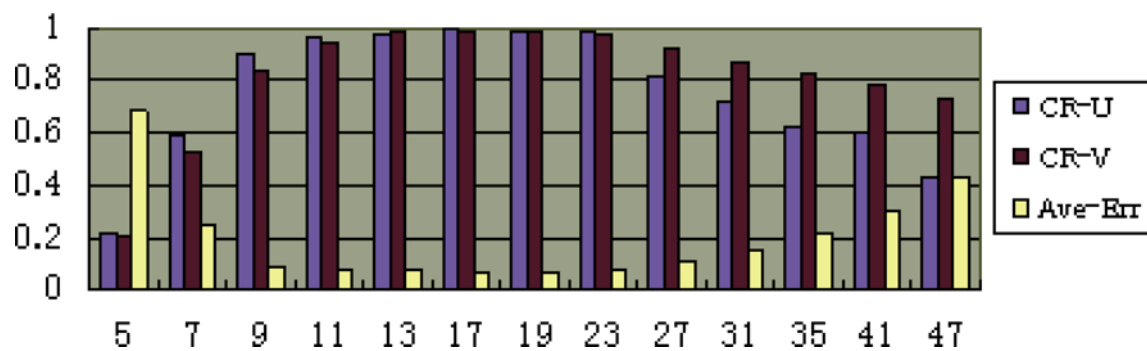

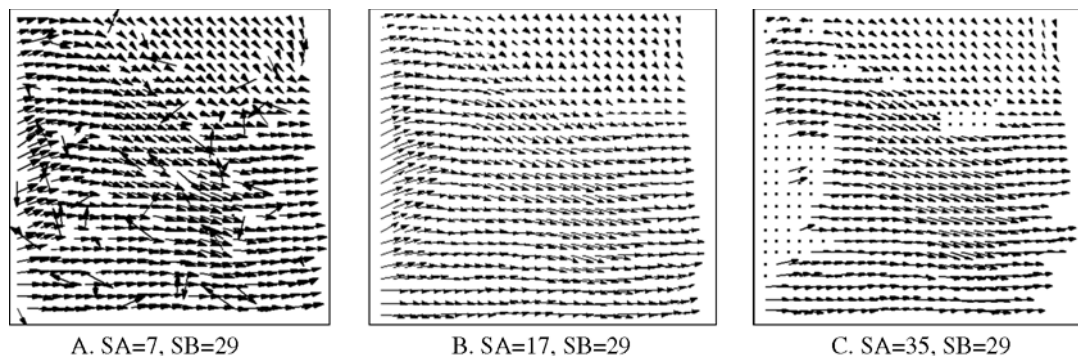Fig. 3. The effect of the size of interrogation window (SB=29).

A. SA=7, SB=29   B. SA=17, SB=29   C. SA=35, SB=29

Fig. 4. Velocity vector profile of case A for different interrogation window size.

The reason is that: as we know, for the point $p(x,y)$ at time T, its corresponding point $q(x,y)$ at time $T + \Delta t$ in the second image is determined through the position relationship of the particles included in the interrogation window. When the size of interrogation window is too small, the average number of particles included in the interrogation window is small and the position relationship of these particles is simple and easy to satisfy, but this will cause many error vectors to be determined. However, when the size of the interrogation window is too large, the average number of the particles in the interrogation window is also large, so the position relationship of these particles is too complex and hard to satisfy, hence many points can not find their corresponding position in the second image.

For the present research, the optimal size of interrogation window is between 13 and 23, and the corresponding average number of tracer particles in the interrogation window is about 10 to 34. The average error of the PIV results related to the accurate solution is 7.12% under these conditions, which correspond to the absolute error of 0.48 pixel.

It should be noticed that, the average error values mentioned above and listed in Table 1 are based on the difference between the PIV results obtained by using the cross correlation method and the accurate solution, which also include the errors due to the limited resolution of the digital images. For case A, the average error due to the limited resolution of the digital image is 5.92% (absolute error is 0.38 pixel), so the exact error due to the cross

correlation method is 1.20% (absolute error is about 0.1 pixel), which is much smaller than the error caused by limited resolution of the digital image. This verifies the effectiveness of the cross correlation method for PIV image processing, which also indicts that the error caused by the limited resolution of the digital image must be reduced for a final PIV result with high accuracy. To solve this problem, the sub-pixel accuracy improvement method which uses "sub-pixel interpolation" process will be discussed in the Section Seven of the present paper.

3.2 The Effect of the Search Window Size

Figure 5 shows the effect of the search window size on the PIV result. For the case A, its average velocity is 7.5 pixel and maximum velocity is about 15 pixel (the real maximum velocity is about 12 pixel in the researched grids). So, when the half size of the scanned region is less than the maximum velocity, some points with larger velocities cannot find their corresponding points in the search window. For example, when the half size of the search window is set as 7.5, the average error of the PIV result is about 50%. So, the quality of the PIV result is poor. When the half size of the search window is bigger than the maximum velocity, the size of the search window will not have any effect on the PIV result accuracy.

3.3 The Effect of the Number of Tracer Particles

From the correlation values (CR-U, CR-V) and average error of the PIV results listed in Table 1, it can be seen
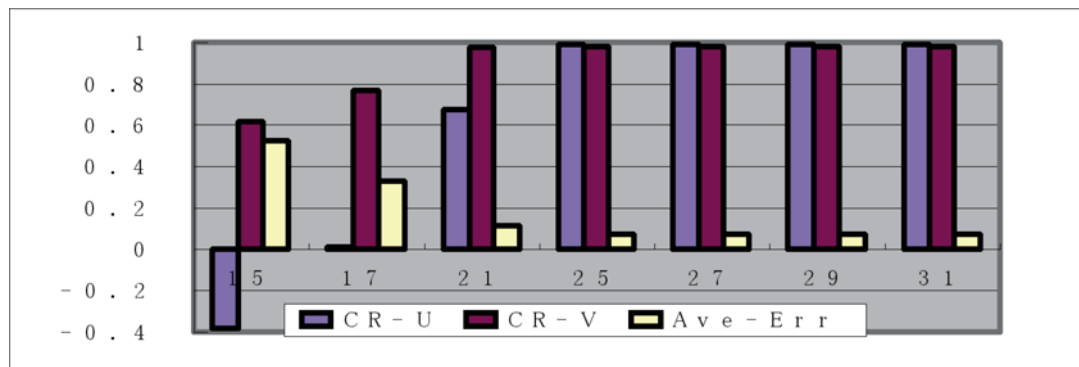

Fig. 5.   The effect of the search window size (case A, SA=17).

that, as the number of tracer particle increases (for case C, case A and case B, the number of tracer particles increase from 1,000 to 10,000), the accuracy of the PIV result also increases (average error decreases from 10.92% to 6.37%), which had also been proven by Okamoto et al., (1997). This is because, as the number of tracer particles increases, the tracer particle concentration increases, so the amount of information for the flow field velocity calculation increases, then the error from cross correlation method decreases.

3.4 The Effect of the Diameter of Tracer Particles

From the results listed in Table 1, it can also be seen that as the diameter of tracer particles increases (for the case D, case A and case E, the average diameter of tracer particles increase from 2.5 pixel to 10 pixel), the accuracy of the PIV result obtained by using cross correlation method also increases (average error decreases from 9.05% to 6.98%). So, increasing the tracer particle diameter is beneficial for PIV image processing. However, the bigger the diameter of tracer particles is, the worse would the trace ability of the particles be.

3.5 The Effect of the Out-of-Plane Velocity

For the cross correlation method, as the particle motion normal to the image plane increases, the accuracy of the PIV result decreases. The result of the present research listed in Table 1 also proved this. As the out-of-plane velocity increases (the out-of-plane velocity increases from 0.017 to 0.34 for case A, case F and case G), the accuracy of the PIV results decreases very rapidly (average error increases from 7.12% to 41.30%).

Because the out-of-plane velocity listed in Table 1 is normalized by the laser sheet thickness, when the average out-of-plane velocity (W_M) is greater than 1.0 (case H), which means that most of the particles in the first image will move out the laser sheet and disappear in the second image, the cross correlation method is completely invalid. (average error is about 132.1%)

3.6 The Effect of Average Image Velocity or Time Interval of the Two Images

From Fig. 6 and the results listed in Table 1 (case I, case A and case J), it can be seen that: when the time interval Δt between two successive images is small (case J), the average movement distance of the tracer particles is also small. Although the PIV result has big correlation values (CR-U=0.951 and CR-V=0.917), the average error of the PIV result is still big (Ave-Err=18.94%, absolute error is 0.38 pixel) because of the limited resolution of digital image, so high accuracy of PIV results cannot be obtained. However, when the time interval Δt between two successive images is too large, the average movement distance of tracer particles is big, the deformation of the particle position relationships and the average out-of-plane movement at this large time interval are big, which will also cause a poor PIV result (Ave-Err=55.41% for case I). Therefore, during the PIV measurement, the optimum time interval Δt between two successive images or optimum average image velocity V_M should be selected for a high accuracy PIV result.
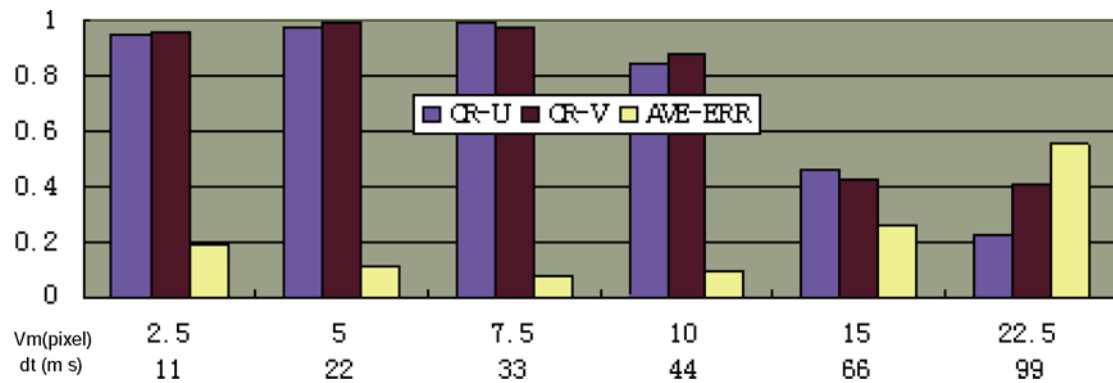

Fig. 6.   The effect of the average image velocity or time interval of the two images.

3.7 The Effect of Sub-grid Interpolation Process

As mentioned in the above, the error caused by the limited resolution of the digital image also must be reduced for getting a final PIV result with high accuracy. There are two ways to solve such a problem: one is to use a higher resolution digital image, which is not the topic of the present research; the other is to improve sub-pixel accuracy by using "sub-pixel interpolation" process (Fig. 7).

$$\begin{cases} R_{(p,q)} = a\exp(-b(x-c)).\exp(-d(y-e)) & (1) \\ R_{(p,q)} = ax^2 + by^2 + cxy + dx + ey + f & (2) \end{cases}$$
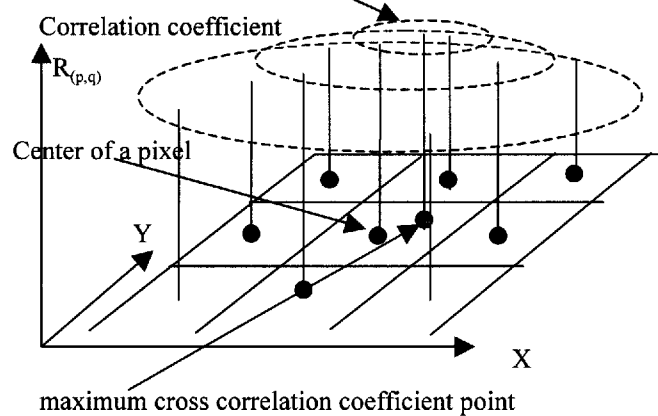

Fig. 7.   The "sub-pixel interpolation" process.

The error caused by the limited resolution of the digital image is due to only five digitized positions of the q(x,y) around point (0, 0) i.e., (1, 0), (0, 1), (–1, 0), (0, -1) and (0, 0). The actually meaning of the "sub-pixel interpolation" process is to do some kind of interpolation among the grid points to find the point with maximum cross correlation coefficient near the grid points. In the present research, two kinds of the "sub-pixel interpolation" functions are compared to study the effect of the "sub-pixel interpolation" process on the sub-pixel accuracy improvement by using cross correlation method. The first one is the same as used by Sakakibara et al., (1993), and the second one is suggested by Fijita and Komura(1992).

From the results listed in Table 1 (comparison of case A with case A* and case J with case J*), it can be seen that, "sub-pixel interpolation" process can improve the PIV accuracy. This is especially effective for the case when the time interval of the two images is small. Such as for case J, "sub-pixel interpolation" process can improve PIV result accuracy by decreasing its average error from 18.94% to 4.23% (absolute error decrease from 0.38 pixel to 0.11 pixel), which is about the same average error level for case A*.

Figure 8 shows the error distribution of case A before and after the "sub-pixel interpolation" process. It can be seen that, the larger error region is located at the right upper corner where the average velocity is small (case A), which is also because of the limited resolution of the digital image. After the "sub-pixel interpolation process" (case A*), the average error decreases from 7.12% to 4.13% (absolute error decrease from 0.48 pixel to 0.29 pixel), the size of the region with large error also reduced very rapidly, especially for the low velocity region in the right upper corner.

A. before "sub-pixel interpolation"
process (case A)

B. after "sub-pixel interpolation"
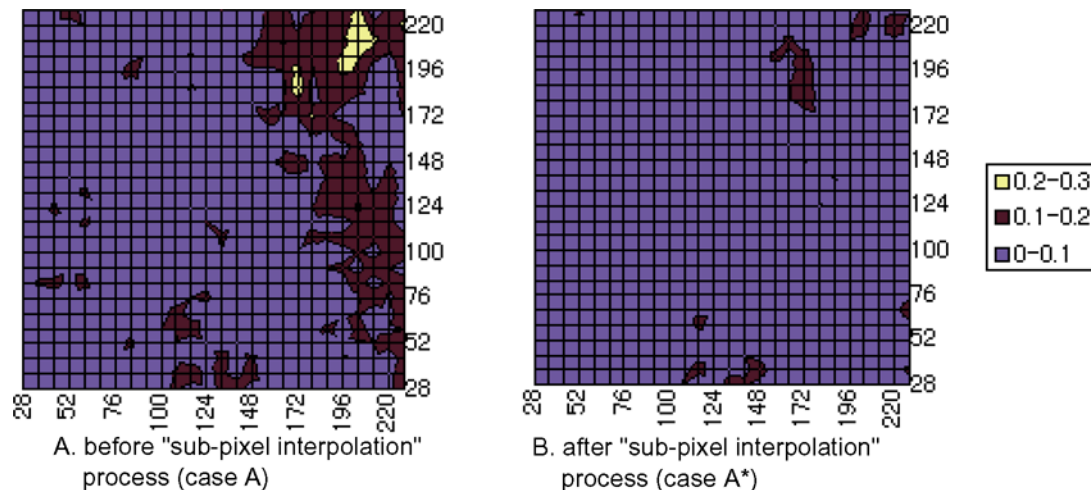process (case A*)

Fig. 8.   The effect of the "sub-pixel interpolation" process.

From the comparison of the results listed on Table 1 (case A* with case A** and the case J* with case J**), it can be suggested that the "sub-pixel interpolation" process using "sub-pixel interpolation" function (1) will be better than that using the "sub-pixel interpolation" function (2).

4. Conclusions

Through the present research, the following general conclusions can be stated:

1. In order to get a high accuracy PIV result by the using correlation method, the size of the interrogation window should be big enough to have a sufficiently large number of particles included in the interrogation window.
2. Increasing the tracer particle concentration is helpful for improving PIV accuracy by using cross correlation method.
3. Increasing the tracer particle diameter is helpful for PIV image processing, but poor for the tracer ability to follow the flow field.
4. The out-of-plane movement can affect the accuracy of PIV greatly. As the out-of-plane velocity increases, the accuracy of PIV result by using cross correlation method decreases rapidly.
5. The optimum time interval or the average image velocity must be selected for a high accuracy PIV result by using the cross correlation method.
6. "Sub-pixel interpolation" process is very useful for the improvement of PIV accuracy, and is especially effective for the case when the average displacement of the particles is small between two images.

Acknowledgments

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