Introduction-1

- **Index of refraction:**
  \[ n = \frac{c}{v} = \frac{\lambda_0}{\lambda} > 1 \]

- **Depend on variation of index of refraction in a transparent medium and the resulting effect on a light beam passing through the test section**

- **Shadowgraph systems:** are used to indicate the variation of the second derivatives (normal to the light beam) of the index of refraction.

- **Schlieren Systems:** are used to indicate the variation of the first directive of the index of refraction.

- **Interferometry systems:** response directly the difference of optical path length, especially giving the index of reflection field within the flow field.

  - Holographic interferometry image of shock-vortex interaction
  - Shadowgraph depicting the flow generated by a bullet at supersonic speeds. (by Andrew Davidhazy)
  - Schlieren images of the muzzle blast and supersonic bullet from firing a .30-06 caliber high-powered rifle (by Gary S. Settles)
Shadowgraph and Schlieren Systems are often used in shock waves and flame phenomena, in which density gradient is quite big.

Interfereometry are often used to study a flow in which density gradient are small.

While these techniques are mostly used for qualitative flow visualization, they can be used to determine pressure, density or temperature measurements theoretically.

These techniques are often used to determine the integrated quantity over the length of light beam.

These techniques are usually used for 2-D flow without index of refraction or density variations along the beam.
Index of refraction is a function of thermodynamic state (density) for homogeneous medium:

Lorenz-Lorentz relationship:

\[
\frac{1}{\rho} \frac{n^2 - 1}{n^2 + 2} = \text{const}
\]

When \( n \approx 1 \), for gaseous flow:

\[
\frac{n - 1}{\rho} = \text{const} \implies \rho = \frac{n - 1}{\text{const}}
\]

at standard condition, with \( n_0 \) and \( \rho_0 \):

\[
\frac{n_0 - 1}{\rho_0} = \text{const} \implies n - 1 = \frac{\rho}{\rho_0} (n_0 - 1)
\]

\[
\rho = \rho_0 \frac{n-1}{n_0-1}
\]

When first and second derivative is determined as in Schlieren and shadowgraph apparatus:

\[
\frac{\partial \rho}{\partial y} = \frac{1}{\text{const}} \frac{\partial n}{\partial y} \implies \frac{\partial \rho}{\partial y} = \frac{\rho_0}{n_0 - 1} \frac{\partial n}{\partial y}
\]

\[
\frac{\partial^2 \rho}{\partial y^2} = \frac{1}{\text{const}} \frac{\partial^2 n}{\partial y^2} \implies \frac{\partial^2 \rho}{\partial y^2} = \frac{\rho_0}{n_0 - 1} \frac{\partial^2 n}{\partial y^2}
\]
• Application of the Schlieren and shadowgraph techniques:
  
  – Compressible flow with shock waves \( \Rightarrow \) density changes
  – Natural convective flow \( \Rightarrow \) density changes
  – Flame and combustion system: \( \Rightarrow \) density changes

• Temperature changes inside flows:
  
  – For low speed flow with heat transfer:
  – \( P = \) constant

\[
\rho = \frac{P}{RT} \Rightarrow \frac{\partial \rho}{\partial y} = \frac{P}{RT^2} \frac{\partial T}{\partial y} = \frac{\rho}{T} \frac{\partial T}{\partial y}
\]

\[
\Rightarrow \frac{\partial n}{\partial y} = \frac{n_0 - 1}{\rho_0} \frac{\partial \rho}{\partial y} = \frac{n_0 - 1}{\rho_0} \frac{\rho}{T} \frac{\partial T}{\partial y}
\]

\[
\Rightarrow \frac{\partial T}{\partial y} = \frac{T}{n_0 - 1} \frac{\rho_0}{\rho} \frac{\partial n}{\partial y}
\]

\[
\Rightarrow \frac{\partial^2 n}{\partial y^2} = \frac{n_0 - 1}{\rho_0} \left[- \frac{\rho}{T} \frac{\partial^2 T}{\partial y^2} + \frac{2\rho}{T^2} \left(\frac{\partial T}{\partial y}\right)^2 \right]
\]
• **For reversible, adiabatic process:**

\[
\frac{P}{\rho^k} = \text{const} \Rightarrow \frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^k
\]

\(k\) is the ratio of specific heat; \(k = \frac{C_p}{C_v}\)

\[
\rho = \frac{n - 1}{\text{const}} \Rightarrow \frac{P}{P_0} = \left(\frac{n - 1}{n_0 - 1}\right)^k
\]

\[
\frac{\partial P}{\partial y} = P_0 \left(\frac{n - 1}{n_0 - 1}\right)^{k-1} \frac{\partial n}{\partial y} = \frac{P}{n - 1} \frac{\partial n}{\partial y}
\]

\[
\Rightarrow \frac{\partial n}{\partial y} = \frac{1}{P} \frac{n - 1}{k} \frac{\partial (n - 1)}{\partial y}
\]
Fundamentals of Schlieren System

- According to definition of index of refraction, the light velocity will be $V = \frac{c}{n}$.

- The slope of the wave front of the light: $\frac{dy}{dz}$

- If the angle $\Delta \alpha'$ is quite small.

\[ \Delta Z = \frac{C_0}{n} \Delta \tau \]

\[ \Delta^2 Z = \Delta Z - \Delta Z_{y+\Delta y} = -C_0 (\frac{1}{n}) \Delta \tau \Delta y \]

\[ \Delta \alpha' = \frac{\Delta^2 Z}{\Delta y} = -n \left( \frac{1}{n} \right) \Delta Z \]

\[ \frac{dy}{dz} = d\alpha' = -n \left[ -\frac{n}{dy} \right] dz = n \left[ -\frac{dn}{dy} \right] dz = \frac{1}{n} \left( \frac{dn}{dy} \right) dz = \frac{d(ln n)}{dy} dz \]

\[ \frac{d^2 y}{dz^2} = \frac{d(ln n)}{dy} \]

\[ d\alpha' = -n \left[ -\frac{n}{dy} \right] dz = n \left[ -\frac{dn}{dy} \right] dz = \frac{1}{n} \left( \frac{dn}{dy} \right) dz = \frac{d(ln n)}{dy} dz \]

\[ \Rightarrow \alpha' = \int \frac{dn}{dy} dz \quad \Rightarrow \quad \alpha' = \int \frac{dn}{dy} dz \]
**Fundamentals of Schlieren System**

- **The intensity after the shape razor blade (knife edge) before the experiment**
  \[
  I_k = \frac{a_K}{a_0} I_0
  \]

- **The intensity after the deformation due to the variation of the index of refraction**
  \[
  I_d = I_k + \frac{\Delta a}{a_K} I_k = (1 + \frac{\Delta a}{a_K}) I_k
  \]
  
  contrast = \[ \frac{\Delta I}{I_k} = \frac{I_d - I_k}{I_k} = \frac{\Delta a}{a_K} = \pm \alpha \frac{f_2}{a_K} \]
  
  sensitivity: \[ \frac{d(\text{contrast})}{d\alpha} = \frac{f_2}{a_K} \]

- **Sensitivity is proportional to \( f_2 \) and inversely to \( a_K \).**
Fundamentals of Schlieren System

- The intensity after the shape razor blade (knife edge) before the experiment
  \[ I_k = \frac{a_K}{a_0} I_0 \]
- The intensity after the deformation due to the variation of the index of refraction
  \[ I_d = I_k + \frac{\Delta a}{a_K} I_k = (1 + \frac{\Delta a}{a_K}) I_k \]
  \[ \text{contrast} = \frac{\Delta I}{I_k} = \frac{I_d - I_k}{I_k} = \frac{\Delta a}{a_K} = \pm \frac{\alpha f_2}{a_K} \]
  \[ \text{sensitivity}: \quad \frac{d(\text{contrast})}{d\alpha} = \frac{f_2}{a_K} \]
- Sensitivity is proportional to \( f_2 \) and inversely to \( a_K \).
Fundamentals of Schlieren System

Figure 7.4 Ray displacement at knife-edge for a given angular deflection

\[ \begin{align*}
\alpha &= \Delta y / p \\
\beta &= \Delta y / f_2 \\
\gamma &= \Delta y / q \\
\alpha' &= \beta - \gamma \\
\Delta y &= \Delta y (1 / f_2 - 1 / q) = \Delta y / p \cdot \alpha \\
\Delta \alpha &= \alpha f_2
\end{align*} \]
Figure 7.7  Schlieren images of a helium jet entering an atmosphere of air: The effect of knife-edge orientation (Re = 630)
Figure 7.8  Schlieren images of the flow structure of a helium jet entering air at different numbers.
Fundamentals of Schlieren System

- For a gas flow with density change:

\[
\frac{\Delta I}{I_k} = \pm \frac{\alpha f_2}{a_k}
\]

\[
\alpha' = \int \frac{dn}{dy} \, dz \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_k} \int \frac{dn}{dy} \, dz
\]

\[
\frac{\partial \rho}{\partial y} = \frac{\rho_0}{n_0 - 1} \frac{\partial n}{\partial y} \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_k} \frac{n_0 - 1}{\rho_0} \int \frac{d\rho}{dy} \, dz
\]

\[
n \approx 1 \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_k} \frac{n_0 - 1}{\rho_0} \int \frac{d\rho}{dy} \, L
\]
Fundamentals of Schlieren System

- For a gas flow with constant pressure distribution:

\[
\frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \left(\frac{n_0 - 1}{\rho_0}\right) \int \frac{dn}{dy} dy \\
\frac{\partial T}{\partial y} = \frac{T}{\rho_0} \frac{\partial n}{\partial y} \\
\Rightarrow \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \left(\frac{n_0 - 1}{\rho_0}\right) \int \frac{T}{\rho} \frac{dT}{dy} dy \\
\Rightarrow \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \left(\frac{n_0 - 1}{\rho_0}\right) \int \frac{P}{RT^2} \frac{dT}{dy} dy \\
\Rightarrow \frac{\Delta I}{I_k} \approx \pm \frac{f_2}{a_K} \left(\frac{n_0 - 1}{\rho_0}\right) \frac{P}{RT^2} \frac{dT}{dy} L
\]
Fundamentals of Schlieren System

• **For a liquid flow:**
  – \( n \) is a function of temperature.

\[
\frac{dn}{dy} = \frac{\partial n}{\partial T} \frac{\partial T}{\partial y}
\]

\[
\frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \int \frac{dn}{dy} dz = \frac{f_2}{a_K} \int \frac{\partial n}{\partial T} \frac{\partial T}{\partial y} dz
\]

if \( n \to 1 \) \( \Rightarrow \) \( \frac{\partial n}{\partial T} \frac{\partial T}{\partial y} \approx \text{const} \)

\[
\Rightarrow \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{\partial n}{\partial T} \frac{\partial T}{\partial y} L
\]
Lab #3 - Visualization of shock wave in a transonic/supersonic nozzle using Schlieren technique
Alternative Schlieren system

A. Setup with one converging and one plane mirror

A. Setup with one converging mirror
Holographic Schlieren system

Figure 7.10  Continued