AerE 545 class notes #19

Shadowgraph, Schlieren and Interferometry
Part - 02

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For a gas flow with density change:

\[
\frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \int \frac{dn}{dy} \, dz
\]

\[
\Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \int \frac{d\rho}{dy} \, dz
\]

\[
\Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{\partial n}{\partial T} \frac{\partial T}{\partial y} L
\]
Shadowgraph technique

\[ I_{sc} = \frac{\Delta y}{\Delta y_{sc}} I_0 \]

\[ \Delta y_{sc} = \Delta y + Z_{sc} \cdot d\alpha \]

\[ \frac{\Delta I}{I_0} = \frac{I_{sc} - I_0}{I_0} = \frac{\Delta y}{\Delta y_{sc}} - 1 \]

\[ = -Z_{sc} \cdot \frac{d\alpha}{\Delta y_{sc}} \approx -Z_{sc} \cdot \frac{d\alpha}{dy} \]

\[ \Rightarrow \frac{\Delta I}{I_0} = \approx -Z_{sc} \cdot \frac{d\alpha}{dy} \]

since \[ \alpha = \frac{1}{n_a} \int \frac{dn}{dy} \]

\[ \Rightarrow \frac{\Delta I}{I_0} = \frac{-Z_{sc}}{n_a} \cdot \int \frac{d^2 n}{dy^2} \]

- Sensitive is proposal to \( Z_{sc} \)
Shadowgraph technique

Experimental setup with one converging mirror

Experimental setup without lens or mirror
Lab #3 - Visualization of shock wave in a transonic/supersonic nozzle using Schlieren technique

- Light bent by negative density gradient
- Light bent by positive density gradient
- Light stopped by knife edge
- Light passes knife edge
- Image brighter
- Image Darker
Schlieren vs. Shadowgraph

Shadowgraph

- Displays a mere shadow
- Shows light ray displacement
- Contrast level responds to $\frac{\partial^2 n}{\partial y^2}$
- No knife edge used

Schlieren

- Displays a focused image
- Shows ray refraction angle, $\varepsilon$
- Contrast level responds to $\frac{\partial n}{\partial y}$
- Knife edge used for cutoff
Examples

Figure 7.14  Shadowgraphs of a helium jet entering an atmosphere of a

Figure 7.7  Schlieren images of a helium jet entering an atmosphere of air: The effect of knife-edge orientation (Re = 630)

Figure 7.15  Shadowgraph of mixing of parallel-flowing streams of helium (above) and nitrogen
**Interferometers**

- Unlike the Schlieren and shadowgraph systems, an interferometer does not depend upon the deflection of a light beam to determine density or index of refraction variation.
- Interferometers are often used for quantitative measurements.
Inference of Waves from Two Sources

Constructive interference

In some places the water wavefronts are in phase (bright spots).

In other places the fronts overlap with peak and valley and interfere destructively (darker spots).
Coherent light Source

- Coherent sources...
- Two sources of light are said to be coherent if the waves emitted from them have the same frequency and are 'phase-linked'; that is, they have a zero or constant phase difference.
Interference of two coherence light waves

Amplitude of a plate light wave in a homogeneous medium can be expressed as:

\[ A = A_0 \sin \frac{2\pi}{\lambda} (ct - z) \]

Therefore:

wave 1: \[ A_1 = A_{01} \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 \right) \]

wave 2: \[ A_2 = A_{02} \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \Delta \right) \]

if \[ A_0 = A_{01} = A_{02} \]

then: \[ A = A_1 + A_2 \]

\[ = A_0 \left[ \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 \right) + \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \Delta \right) \right] \]

\[ = 2A_0 \cos \frac{\Delta}{2} \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \frac{\Delta}{2} \right) \]

Therefore, the intensity of the combined wave (which is proportional to the square of the peak amplitude) will be:

\[ I \sim 4A_0^2 \cos^2 \frac{\Delta}{2} \]
Interference of light waves

Thomas Young's Double Slit Experiment

Figure 1

Definitely not to scale

Thomas Young (1801)
The particle path length along a light beam is defined as:

\[ PL = \int ndz \]

or

\[ PL = \int \frac{C_0}{C} dz = \frac{1}{\lambda_0} \int \frac{dz}{\lambda} \]

Therefore, the difference between path 1 and path 2:

\[ \Delta PL = PL_1 - PL_2 = \int_{path-1} ndz - \int_{path-2} ndz \]

\[ = \frac{1}{\lambda_0} \left( \int_{path-1} \frac{dz}{\lambda} - \int_{path-1} \frac{dz}{\lambda} \right) \]

The phase difference between the two wave will be:

\[ \Delta = 2\pi \left( \int_{path-1} \frac{dz}{\lambda} - \int_{path-1} \frac{dz}{\lambda} \right) \]

or

\[ \frac{\Delta}{2\pi} = \frac{\Delta PL}{\lambda_0} \]

Figure 7.16  Mach-Zehnder interferometer
Interferometers

\[ \varepsilon = \frac{1}{\lambda_0} \int (n - n_{\text{ref}}) dz \]

According to the Glasdstone-Dale equation: \( \rho = \frac{n - 1}{\text{Const}} \)

\[ \Rightarrow \varepsilon = \frac{\text{const}}{\lambda_0} \int (\rho - \rho_{\text{ref}}) dz \]

If only varies over a length \( L \), then, the fringe shift will be:

\[ \varepsilon = \frac{n - n_{\text{ref}}}{\lambda_0} L \]

for gaseous flows

\[ \varepsilon = \frac{\text{const}}{\lambda_0} (\rho - \rho_{\text{ref}}) L \]

or

\[ \rho - \rho_{\text{ref}} = \frac{\lambda_0 \varepsilon}{\text{const} \cdot L} = \frac{\lambda_0 \varepsilon}{n - 1} \frac{\rho_0}{L} \]

for temperature measurements in gaseous flows

\[ T - T_{\text{ref}} = \frac{\lambda_0 \varepsilon}{L} \frac{1}{dn/DT} \]
Examples

Figure 7.20  Interferograms of a low-Reynolds-number helium jet entering a
Examples

Figure 7.24  Flow over sharp-tipped spike with conical flare: pressure 100 psi; Ma = 2
            (a) Finite-fringe interferogram. (b) Infinite-fringe interferogram. (From [40])

Figure 7.26  Wedge-fringe interferograms used for visualizing flow over a heated gas-turbine blade
            held in a cascade; oncoming flow direction is parallel to the visible wire carrying the heating current.
            (From [42])