Laser Doppler Velocimetry (LDV)
Part - 02

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• By using a laser beam of wavelength $\lambda=488\text{nm}$ (Argon-Ion laser), the maximum Doppler shift from a particle moving with a velocity of $V$ would be:
  - $V=1.0\text{m/s}$ $\Delta f \approx 4.1\text{ MHz}$
  - $V=10.0\text{m/s}$ $\Delta f \approx 41\text{ MHz}$
  - $V=100.0\text{m/s}$ $\Delta f \approx 410\text{ MHz}$
  - $V=1000\text{m/s}$ $\Delta f \approx 4100\text{ MHz}$

• However, since $C = 2.998 \times 10^8 \text{ m/s}$, $\lambda=488\text{nm}$, then, $f=c/\lambda=1.4 \times 10^9\text{ MHz}$. the Doppler shift in frequency is very small compared with the frequency of the source laser light.

• In practice, it is always quite difficult to measure the Doppler shift of frequency accurately for low-speed flows by measuring the received total frequency directly.

• Dual-beam LDV technique was developed to measure the relative frequency change due to the Doppler shift other than the total frequency.
If the intensity of each scattered beam collected by the photo detector varies sinusoidal,

\[ A_i \sin 2\pi(f + \Delta f_i)t, \quad i = 1, 2. \]

Then, the optical mixing of these beams on the photodetector (heterodyning process) produces an output voltage \( E \) that is proportional to the square of the combined light intensity.

\[
E \sim \{A_1 \sin 2\pi(f + \Delta f_1)t + A_2 \sin 2\pi(f + \Delta f_2)t\}
\]

\[
= A_1^2 \sin^2 2\pi(f + \Delta f_1)t + A_2^2 \sin^2 2\pi(f + \Delta f_2)t + 2A_1A_2[\sin 2\pi(f + \Delta f_1)t][\sin 2\pi(f + \Delta f_2)t]
\]

\[
= A_1^2 \sin^2 2\pi(f + \Delta f_1)t + A_2^2 \sin^2 2\pi(f + \Delta f_2)t + A_1A_2[\cos 2\pi(\Delta f_1 - \Delta f_2)t] - \cos 2\pi(2f + \Delta f_1 + \Delta f_2)t
\]

If we define, \( \Delta f_1 - \Delta f_2 = f' \)

then : \( E \sim a + b \sin 2\pi f't \)

\[ f' = \Delta f_1 - \Delta f_2 = \frac{\vec{V} \cdot (\vec{e}_{\parallel_1} - \vec{e}_{\parallel_2})}{\lambda} = \frac{\vec{V} \cdot (\vec{e}_{\parallel_2} - \vec{e}_{\parallel_1})}{\lambda} \]

Since \( \vec{e}_{\parallel_1} = \vec{e}_{\parallel_2} \),

then

\[ f' = \frac{\vec{V} \cdot (\vec{e}_{\parallel_2} - \vec{e}_{\parallel_1})}{\lambda} = \frac{2\sin(\theta)}{\lambda} V_0 \]

\[ \Rightarrow V_0 = \frac{\lambda}{2 \sin(\theta/2)} f' \]

The above equation is independent of observation angle!
**Fundamentals of Dual-Beam LDV**

Width: \[ d_{fe} = \frac{4f_T \lambda}{\pi d_e} \]

Height: \[ h = \frac{d_{fe}}{\cos(\theta / 2)} \]

Length: \[ l = \frac{d_{fe}}{\sin(\theta / 2)} \]

Volume: \[ Volume = \frac{\pi d_{3}^{3}_{fe}}{6 \sin(\theta / 2) \cdot \cos(\theta / 2)} \]
Generated Fringes for the Dual-Beam LDV

Fring spacing: \[ \delta = \frac{\lambda}{2 \sin(\theta/2)} \]

Fring number: \[ N = \frac{4 D_T}{\pi d_e} \]
\[ D_T = 2 f_t \sin(\theta/2) \]

Frequency of the scattering light:
\[ f = \frac{V_\perp}{\delta} = \frac{2 \sin(\theta/2)}{\lambda} V_\perp \]

Frequency shift according to Doppler shift theory:
\[ f = \frac{2 \sin(\theta/2)}{\lambda} V_\perp \]

A burst
A beam expander is a commonly used LDV accessory, whose function is to reduce the size of the measuring volume.

This improves the spatial resolution of velocity measurement while also improving the amplitude resolution as a result of increased light power density within the measuring volume.

A beam expander consists of a diverging lens and a converging lens, in addition to the transmitting lens.

The beam expansion ratio: \( E_x = d_{ex}/d_e = D_{Tx}/D_T \)

\[ \theta_x = 2 \sin^{-1} \left[ \frac{E_x f_{Tx}}{f_{Tx}} \sin(\theta/2) \right] \]

\( f_{Tx} \) is the focal length of the transmitting lens.

if \( f_{Tx} = f \)

then: \( d_{ex} = \frac{1}{E_x} d_e \approx \frac{4 f_T \lambda}{\pi d_e} \)

Fringe Numbers: \( N = \frac{4 D_T}{\pi d_e} = \frac{4 D_{Tx}}{\pi d_{ex}} \)

The fringe number within the measurement volume is remain unaffected.

The fringe spacing will reduced by a factor of \( E_x \).

The measurement volume reduced by a factor \( \sim E_x^4 \)

The SNR is also increased!
Doppler Signal

Frequency of the burst signal

\[ f = \frac{V_{\perp}}{\delta} = \frac{2\sin(\theta/2)}{\lambda} V_{\perp} \]

\[ V_{\perp} = \frac{f\lambda}{2\sin(\theta/2)} \]

- **A burst**
  - The pedestal is due to the intensity of the laser beam is usually has a Guess Distribution.
  - The scattering signal is depending on the size and reflective index of the particle and the its position in the measuring volume.
  - Burst intensity: the number of the particles crossing the measuring volume
Frequency Shift

Frequency of the burst signal

\[ f = \frac{V_\perp}{\delta} = \frac{2\sin(\theta/2)}{\lambda} V_\perp \]

\[ V_\perp = \frac{f\lambda}{2\sin(\theta/2)} \]

- The relationship between the frequency shift and the velocity is equally valid for both senses of direction of velocity.
- Bragg cell is used to remove this ambiguity.
- The frequency shift obtained by the Bragg cell makes the fringe pattern move at a constant velocity.
- Particles which are not moving will generate a signal of the shift frequency \( f_{\text{shift}} \). The velocities \( V_p \) and \( V_{\text{shift}} \) will generate signal frequencies \( f_p \) and \( V_{\text{shift}} \) respectively.

\[ f = f_{\text{particle}} + f_{\text{shift}} \]

\[ V_{\text{measured}} = V_p + V_{\text{shift}} \]

then:

\[ V_p = V_{\text{measured}} - V_{\text{shift}} \]
Doppler Signal Processing

- The signal is usually band pass filtered to remove the pedestal and high-frequency noise.
- A unit named as signal processor is used to determine the Doppler frequency.
- The signal processor includes:
  - **Burst analyzer**: FFT to get the power spectrum, then determine the Doppler frequency.
  - **Frequency counters**: count on the number of zero crossings of the filtered bursts. Determine the particle velocity as the ratio between the fringe space over the averaged time between the two zero crossing.
  - **Frequency trackers**: used for high particle density case. They contain an electric oscillator, which scans a frequency range and locks at the Doppler frequency, providing an analogue output proportional to it. Advantage is to provide analogue output, but has only limited dynamic range and need for heavy seeding.
  - **Photo correlators**: detect the emission of individual photons and correlated them with respect to their times of arrival to computer the time delay for peak correlation. Their advantage over other processor is that they can operate with very low light intensity and noisy signal. But their frequency range is limited, and quit time consuming.

\[ V_\perp = \frac{f \lambda}{2 \sin(\theta/2)} \]
Errors and uncertainty in LDV measurements

- **Fringe divergence uncertainty:**
  - if the beams do not intersect at their waists, the fringes will not be parallel planes.

- **Velocity bias:** due to many particles with different velocity passing the measuring volume.
  \[
  \frac{\bar{U}_m}{\bar{U}} \approx 1 + \frac{\bar{u}^2}{\bar{U}^2}
  \]

- **Directional bias:** due to the small angle between the particle velocity and fringe direction.