Particle Image Velocimetry
Part - 3

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PIV System Setup

**Particle tracers:** to track the fluid movement.

**Illumination system:** to illuminate the flow field in the interest region.

**Camera:** to capture the images of the particle tracers.

**Synchronizer:** to control the timing of the laser illumination and camera acquisition.

**Host computer:** to store the particle images and conduct image processing.
Particle Tracking Velocimetry (PTV)-2

1. Find position of the particles at each images
2. Find corresponding particle image pair in the different image frame
3. Find the displacements between the particle pairs.
4. Velocity of particle equates the displacement divided by the time interval between the frames.

Four-frame-particle tracking algorithm

(a) Original image (8 bit grayscale)
Overlap of four consecutive frames

PTV results
Correlation-based PIV methods

Corresponding flow velocity field

$t = t_0$

$t = t_0 + \Delta t$

high particle-image density
Correlation-based PIV methods

Correlation coefficient function

\[ R(p,q) = \frac{\int (f(x,y) - \bar{f})(g(x,y) - \bar{g}) \, dv}{\sqrt{\int (f(x,y) - \bar{f})^2 \, dv \int (g(x,y) - \bar{g})^2 \, dv}} \]
Correlation coefficient distribution

\[ R(p,q) = \frac{\int (f(x,y) - \overline{f})(g(x+p,y+q) - \overline{g}) \, dv}{\sqrt{\int (f(x,y) - \overline{f})^2 \, dv \int (g(x+p,y+q) - \overline{g})^2 \, dv}} \]

Peak location


**FFT-based Cross Correlation and Direct Cross Correlation**

\[
R(p,q) = \frac{\int (f(x,y) - \bar{f})(g(x+p,y+q) - \bar{g})dy}{\sqrt{\int (f(x,y) - \bar{f})^2 dy} \sqrt{\int (g(x+p,y+q) - \bar{g})^2 dy}}
\]

**FFT-based Cross-correlation method**
- **advantage:** Fast
- **disadvantage:** additional error.

**Direct cross-correlation method**
- **advantage:** accurate
- **disadvantage:** time consuming

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**Fig. 5.16.** Implementation of cross-correlation using fast Fourier transforms.
Sub-pixel interpolation for Digital PIV

\[
R(p,q) = \frac{\int (f(x,y)-\overline{f})(g(x+p,y+q)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2dv \int (g(x+p,y+q)-\overline{g})^2dv}}
\]

- With sub-pixel interpolation processing, the accuracy of the PIV measurement could be about 0.1 pixel

<table>
<thead>
<tr>
<th>Pixel</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>0.75</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>0.90</td>
</tr>
<tr>
<td>6</td>
<td>0.65</td>
</tr>
<tr>
<td>7</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 5.1. Three-point estimators for determining the displacement from the correlation data at the subpixel level

<table>
<thead>
<tr>
<th>Fitting function</th>
<th>Estimators</th>
</tr>
</thead>
</table>
| Peak centroid    | \[\begin{align*}
  x_0 &= \frac{(i-1)R_{(i-1,j)} + iR_{i,j} + (i+1)R_{(i+1,j)}}{R_{(i-1,j)} + R_{i,j} + R_{(i+1,j)}} \\
  y_0 &= \frac{(j-1)R_{(i,j-1)} + jR_{i,j} + (j+1)R_{(i,j+1)}}{R_{(i,j-1)} + R_{i,j} + R_{(i,j+1)}}
\end{align*}\] |
| Parabolic peak fit | \[\begin{align*}
  x_0 &= i + \frac{R_{(i-1,j)} - R_{(i+1,j)}}{2R_{(i-1,j)} - 4R_{(i,j)} + 2R_{(i+1,j)}} \\
  y_0 &= j + \frac{R_{(i,j-1)} - R_{(i,j+1)}}{2R_{(i-1,j)} - 4R_{(i,j)} + 2R_{(i+1,j)}}
\end{align*}\] |
| Gaussian peak fit | \[\begin{align*}
  x_0 &= i + \frac{\ln R_{(i-1,j)} - \ln R_{(i+1,j)}}{2\ln R_{(i-1,j)} - 4\ln R_{(i,j)} + 2\ln R_{(i+1,j)}} \\
  y_0 &= j + \frac{\ln R_{(i,j-1)} - \ln R_{(i,j+1)}}{2\ln R_{(i-1,j)} - 4\ln R_{(i,j)} + 2\ln R_{(i,j+1)}}
\end{align*}\] |

\[
\begin{align*}
R(p,q) &= a \exp(-b(x-c) \exp(-d(y-e))) \\
R(p,q) &= ax^2 + by^2 + cxy + dx + ey + f
\end{align*}
\]
Overlapping rate for PIV image processing

Overlapping rate = 1.0 - D / S

50% overlapping is usually used for PIV image processing!!
Effect of interrogation window size

- Interrogation size usually determines the spatial resolution of the PIV measurements.
- Smaller interrogation window size will give better spatial resolution of the PIV measurement.
- However, too small interrogation window size would result in many bad vectors.
- Usually to have about 10 ~ 20 particles inside an interrogation window would give a good PIV result!

$$R(p,q) = \frac{\int (f(x,y) - \bar{f})(g(x+p,y+q) - \bar{g}) \, dv}{\sqrt{\int (f(x,y) - \bar{f})^2 \, dv} \sqrt{\int (g(x+p,y+q) - \bar{g})^2 \, dh}}$$
Effect of search window size

- The size of the search window size would determine total time required for the cross correlation processing.

- Smaller search window size could save the computational time, however, would result in error vectors for the particles with larger displacement.

\[ R(p, q) = \frac{\int (f(x, y) - \bar{f})(g(x + p, y + q) - \bar{g}) \, dv}{\sqrt{\int (f(x, y) - \bar{f})^2 \, dv \int (g(x + p, y + q) - \bar{g})^2 \, dv}} \]

Fig. 5. The effect of the search window size (case A, SA=17).
Effect of the Displacements of the Tracer Particles or the Time Delay between the Two Laser Pulses

- Too large displacement of the particles (i.e., longer time delay) would result in bigger errors due to the relative movement of the particles inside the interrogation window.

- Too small displacement of the particles (i.e., smaller time delay) would also result in bigger errors due to the relative small displacement related to the limited resolution of the digital camera.

- It usually to have particle displacement about 4~6 pixels if the interrogation size is chosen to be 32 pixels.

![Image of particle displacement chart](image.png)

Fig. 6. The effect of the average image velocity or time interval of the two images.
Effect of the out-of-plan velocity for 2-D PIV measurements

<table>
<thead>
<tr>
<th>case</th>
<th>$V_M$</th>
<th>$V_X$</th>
<th>$W_M$</th>
<th>$N$</th>
<th>$D_M$</th>
<th>CR-U</th>
<th>CR-V</th>
<th>AVE-ERR</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7.5</td>
<td>15.0</td>
<td>0.017</td>
<td>4,000</td>
<td>5.0</td>
<td>0.991</td>
<td>0.980</td>
<td>7.12%</td>
</tr>
<tr>
<td>F</td>
<td>7.5</td>
<td>15.0</td>
<td>0.17</td>
<td>4,000</td>
<td>5.0</td>
<td>0.698</td>
<td>0.748</td>
<td>13.75%</td>
</tr>
<tr>
<td>G</td>
<td>7.5</td>
<td>15.0</td>
<td>0.34</td>
<td>4,000</td>
<td>5.0</td>
<td>0.363</td>
<td>0.394</td>
<td>41.30%</td>
</tr>
<tr>
<td>H</td>
<td>7.5</td>
<td>15.0</td>
<td>1.70</td>
<td>4,000</td>
<td>5.0</td>
<td>-0.06</td>
<td>-0.05</td>
<td>132.1%</td>
</tr>
</tbody>
</table>

$V_M$: average velocity (pixel/interval)
$V_X$: maximum Velocity (pixel/interval)
$W_M$: out of plane velocity (laser width/interval)
$N$: tracer number
$D_M$: tracer average diameter (pixel)
Aver-Err: average error of PIV results without sub-pixel interpolation.
**Effect of the diameter of particle images**


<table>
<thead>
<tr>
<th>CASE</th>
<th>$V_M$</th>
<th>$V_X$</th>
<th>$W_M$</th>
<th>$N$</th>
<th>$D_M$</th>
<th>CR-U</th>
<th>CR-V</th>
<th>AVE-ERR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.5</td>
<td>15.0</td>
<td>0.017</td>
<td>4,000</td>
<td>2.5</td>
<td>0.916</td>
<td>0.871</td>
<td>9.05%</td>
</tr>
<tr>
<td>2</td>
<td>7.5</td>
<td>15.0</td>
<td>0.017</td>
<td>4,000</td>
<td>5.0</td>
<td>0.991</td>
<td>0.980</td>
<td>7.12%</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
<td>15.0</td>
<td>0.017</td>
<td>4,000</td>
<td>10.0</td>
<td>0.992</td>
<td>0.982</td>
<td>6.98%</td>
</tr>
</tbody>
</table>

- $V_M$: average velocity (pixel/interval)
- $V_X$: maximum Velocity (pixel/interval)
- $W_M$: out of plane velocity (laser width/interval)
- $N$: tracer particle number
- $D_M$: tracer average diameter (pixel)
- Aver-Err: average error of PIV results without sub-pixel interpolation.

- Bigger particles maybe beneficial for PIV image processing
- Bigger particles would cause the problem for the unsteady flow tracking!