Advanced Experimental Aerodynamics and Flow Diagnostic Techniques

Lecture # 02: Technical Basis for Optical Experimentation

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Measurement Techniques for Thermal-Fluids Studies

Thermal-Fluids measurement techniques

Intrusive techniques
- Pitot probe
- hotwire, hot film
- thermocouples
- etc...

Non-intrusive techniques
- Laser Doppler Velocimetry (LDV)
- Planar Doppler Velocimetry (PDV)
- Particle Image Velocimetry (PIV)
- etc...

Velocity, temperature, density (concentration), etc...

particle-based techniques
- Laser Induced Fluorescence (LIF)
- Molecular Tagging Velocimetry (MTV)
- Molecular Tagging Thermometry (MTT)
- Pressure Sensitive Paint (PSP)
- Temperature Sensitive Paint (TSP)
- Quantum Dot Imaging
- etc...

molecule-based techniques

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The nature of light

• According to classical electromagnetic theory, light is considered to be radiation that propagates through vacuum in free spaced in the form of electromagnetic waves, both oscillating transversely to the direction of wave propagation and normal to each other.

\[ E_y(x, t) = E_{y0} \sin 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \]

\[ B_z(x, t) = B_{z0} \sin 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \]

\( \lambda \): is wavelength  
\( T \): is the period of the oscillation  
\( \nu \): The reciprocal of the period, is called frequency, \( \nu = 1/T \)
The nature of light-1

- Light propagation velocity, \( V = f \lambda \)
- Light propagation velocity in Vacuum, \( C = 2.998 \times 10^8 \text{ m/s} \)
- Wave front: the locus of all points along the different paths that have the same phase.
- If all the wave fronts are plane, then, the light is considered to be a plane wave.
- If all the wave fronts are spherical or cylindrical, then, the light is considered to be a spherical or cylindrical wave.
- Light propagation is associated with electric and magnetic fields. They are in phase and their amplitudes are related as: \( E_y^0 = c B_z^0 \)
- It is usually sufficiently to analyze electromagnetic waves by considering only electric field.
- The polarization is associate with the orientation of the plane of the plane of oscillation of the electric field.
- Concepts of linearly polarized light, elliptically polarized light and circularly polarized light, unpolarized or randomly polarized light.
### The nature of light -2

<table>
<thead>
<tr>
<th>RADIATION TYPE</th>
<th>WAVELENGTH RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cosmic rays</td>
<td>$\lambda &lt; 10^{-4}$ nm</td>
</tr>
<tr>
<td>Gamma rays</td>
<td>$10^{-4}$ nm &lt; $\lambda &lt; 10^{-1}$ nm</td>
</tr>
<tr>
<td>X-rays</td>
<td>$10^{-2}$ nm &lt; $\lambda &lt; 10^{2}$ nm</td>
</tr>
<tr>
<td>Disinfecting radiation</td>
<td>$10$ nm &lt; $\lambda &lt; 380$ nm</td>
</tr>
<tr>
<td>Visible light</td>
<td>$380$ nm &lt; $\lambda &lt; 750$ nm</td>
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<tr>
<td>Space heating</td>
<td>$750$ nm &lt; $\lambda &lt; 10^{7}$ nm</td>
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<tr>
<td>Microwaves</td>
<td>$10^{6}$ nm &lt; $\lambda &lt; 10^{9}$ nm</td>
</tr>
<tr>
<td>Radar</td>
<td>$10^{7}$ nm &lt; $\lambda &lt; 10^{9}$ nm</td>
</tr>
<tr>
<td>Radio and Television</td>
<td>$10^{8}$ nm &lt; $\lambda &lt; 10^{13}$ nm</td>
</tr>
<tr>
<td>Electrical power waves</td>
<td>$10^{14}$ nm &lt; $\lambda &lt; 10^{17}$ nm</td>
</tr>
</tbody>
</table>
The nature of light -2

• The colors: visible light consists of radiation with wavelength in the range of 380~750nm (1nm=10^{-9}m) which corresponds to the frequency range between 4.0 \times 10^{15} to 7.9 \times 10^{15} Hz.

<table>
<thead>
<tr>
<th>COLOR</th>
<th>WAVELENGTH RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultraviolet (UV)</td>
<td>0.85 nm &lt; \lambda &lt; 380 nm</td>
</tr>
<tr>
<td>Violet</td>
<td>380 nm &lt; \lambda &lt; 424 nm</td>
</tr>
<tr>
<td>Blue</td>
<td>424 nm &lt; \lambda &lt; 491 nm</td>
</tr>
<tr>
<td>Green</td>
<td>491 nm &lt; \lambda &lt; 575 nm</td>
</tr>
<tr>
<td>Yellow</td>
<td>575 nm &lt; \lambda &lt; 585 nm</td>
</tr>
<tr>
<td>Orange</td>
<td>585 nm &lt; \lambda &lt; 647 nm</td>
</tr>
<tr>
<td>Red</td>
<td>647 nm &lt; \lambda &lt; 750 nm</td>
</tr>
<tr>
<td>infrared</td>
<td>750 nm &lt; \lambda &lt; 1000 nm</td>
</tr>
</tbody>
</table>
The nature of light – as photons

- Photon scattering.
  - one finds experimentally that the frequency of the scattered wave is changed, which does not come out of a wave picture of light. However, when the light is viewed as a photon with energy proportional to the associated light wave, excellent agreement with experiment is found.

- The photoelectric effect:
  - When light is shone at a metal plate, it is found that electrons are ejected. These electrons then get accelerated to a nearby plate by an external potential difference, and a photoelectric current is established, as below
  - The photons hit an electron in the metal, giving up its energy. This is enough to free the electron from the attractive forces holding it in the metal, and it is accelerated towards the other side, causing a flow of charges and hence a current.
  - It is found experimentally that the photoelectric current depends critically on the frequency of the light being used. This is a feature of the energy that the electrons gain when struck by the light, but in the wave picture the energy of the light depends on the amplitude, and not on the frequency.
  - However, in the photon picture of light the energy of the photon is proportional to the frequency of the associated wave, which therefore provides a natural explanation of the frequency dependence of the photoelectric current.
  - The explanation, which was first given by Einstein and which won him the Nobel Prize.

\[
\varepsilon = h\nu
\]

Planck constant \(= 6.624 \times 10^{-34} \text{ Js} \)
Light propagate through media

- **Refractive index:**\[ n = c / v = \frac{\lambda_0}{\lambda} > 1 \]

- **Index of refraction of a material generally increasing slightly with decreasing wavelength of the light. Such phenomena is called dispersion.**\[ n = \sqrt{\frac{1 + 2K_L\rho}{1 - K_L\rho}} \]

<table>
<thead>
<tr>
<th>Gas</th>
<th>n</th>
<th>Liquid</th>
<th>n</th>
<th>Solid</th>
<th>n</th>
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<tbody>
<tr>
<td>Air</td>
<td>1.00029</td>
<td>Water</td>
<td>1.333</td>
<td>Fused quartz</td>
<td>1.46</td>
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<td>He</td>
<td>1.00036</td>
<td>Ethyl alcohol</td>
<td>1.361</td>
<td>Pyrex glass</td>
<td>1.47</td>
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<td>CO₂</td>
<td>1.00045</td>
<td>Turpentine</td>
<td>1.472</td>
<td>Crown glass</td>
<td>1.52</td>
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<td>H₂</td>
<td>1.00013</td>
<td>Benzene</td>
<td>1.501</td>
<td>Flint glass</td>
<td>1.57~1.89</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Plexiglas</td>
<td>1.51</td>
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<td></td>
<td></td>
<td></td>
<td>Lexan</td>
<td>1.58</td>
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<td></td>
<td></td>
<td></td>
<td>Polystyrene</td>
<td>1.59</td>
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<tr>
<td>589nm</td>
<td></td>
<td></td>
<td></td>
<td>sapphire</td>
<td>1.77</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>zircon</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Diamond</td>
<td>2.42</td>
</tr>
</tbody>
</table>
Light propagate through media

- **Refraction:**
  - When light propagates through a homogenous medium, its path would be straight, whereas, if the medium is non-homogeneous or if the light across from one medium to another, the path may change direction gradually or abruptly. The change of light propagation direction is called refraction.
  
  - Optically denser medium ($n_1 < n_2$)

\[
\frac{\sin \varphi_1}{\sin \varphi_2} = \frac{n_2}{n_1}
\]
Light propagate through media

- **Total refraction**

\[
\frac{\sin \varphi_1}{\sin \varphi_2} = \frac{n_2}{n_1}
\]

\[n_1 > n_2 \Rightarrow \varphi_2 > \varphi_1\]

\[\varphi_{2\text{max}} > \pi / 2 \Rightarrow \varphi_{1\text{cri}} = \sin^{-1}(n_2 / n_1)\]

- **reflection**
Light propagate through media

- **Optical lens**
  - Convex lens
  - Concave lens
Light propagate through media

\[ n = \frac{c}{v} = \frac{\lambda_0}{\lambda} > 1 \]

*Figure 5.4. Interpretation of light refraction at the interface between two media as the rotation of wave fronts.*
How does refractive-index-matching help?

- Optical techniques avoid disturbing the flow to be measured
- Typical approaches are LDV, PIV, PTV, flow visualization, PLIF, etc.

 Unless the refractive indices are matched, the view may be distorted or impossible even with "transparent" materials and position measurements may be incorrect

Example of application of refractive-index-matching

(Marking is on the back of beaker)

Not matched

Matched

(Rod is resting on the bottom of the beaker)

Refractive index not matched
The Optical Index of Refection Matching Approach

Figure 3. Wool in air

Figure 4. Half submerged

Figure 5. Totally submerged
Flow Passing an Array of Circular Cylinders
Absorption

- Light is transmitted through a material, it will be absorbed by the molecules of the material

- Beer’s law: \[ I = I_0 \exp(-\alpha L) \]

  - \( \alpha \) is the absorption or attenuation coefficient
  - \( L_c = 1/\alpha \) is called penetration depth.
  - When \( L = L_c \), \( I/I_0 = 1/e = 37\% \), i.e., 63\% energy was absorbed
  - Metals have very small \( L_c = 1/\alpha \).
  - Copper, \( L_c = 0.6\text{nm} \) for 100 nm UV light
  - Copper, \( L_c = 6.0\text{nm} \) for 1000 nm infrared light.
  - 2nm copper plate as the high pass filter.
illumination

• **Light source**
  – **Thermal source:**
    • *Lamps: Continuous wave (CW)*

  – **Laser sources**
    • *Continuous wave (CW)*
    • *Pulsed laser*
    • *Single wavelength*

  – **Point source:**

  – **Plane source:**
The human eye

• **Human eye is enclosed with three membranes:**
  – **Starting from the outside are:** Cornea-sclera, choroid and retina.
  – **Retina is lined with a large number of receptors sensitive to the light.**
  – **Two receptors:**
    • **Coles (~ 7 million):** response only to bright light and color.
    • **Rods (~100 million):** only sensitive to dim light, and can not separate different colors.
  – **When light is bright:** photopic or bright-adapted vision.
  – **When light is dim:** scotopic or dark-adapted vision.
The human eye

- When light is bright: photopic or bright-adapted vision.
  - $\lambda_{\text{max}} = 555$ nm
- When light is dim: scotopic or dark-adapted vision.
  - $\lambda_{\text{max}} = 510$ nm

Figure 5.6. Luminous efficacies of the standard human eye for photopic (solid curve) and scotopic (dashed curve) visions. [3]
Color related terminology

- International Commission on Standardization (CIE – Comission Internationale de l’Eclairage)
- CIE defines colours as an attribute of visual perception consisting of any combination of chromatic and achromatic contents
- **Chromatic contents**: such as yellow, red, blue...
- **Achromatic contents**: such as white, black, gray ...
- **Hue** indicates whether an area appears to be similar to one of the received colors: red, yellow, green, and blue, or to a combination of two of them
- **Brightness** signifies whether an area appears to emit more or less light.
- **Lightness** is the brightness of an area judged relative to the brightness of a similarly illuminated area that appears to be white or highly transmitting.
- **Colorfulness** indicates whether the perceived color of an area to be more or less chromatic.
- **Saturation** is the ratio of the colorfulness to the brightness of any area
- **Vividness** of a color is indicated by its saturation, and intensity of a color is indicated by its lightness.
- Hue, Saturation and lightness (HSL) has been used as the parameters to determine color perception i.e, HSL model.
- **RGB model** (Red, green and blue is used as primary colors): – widely used for computer and TV monitor.
- **CMYK** (Cyan, Magenta, Yellow and Black) model: used for high-quality printers
Light source

- Thermal light source:
  - Emit electromagnetic radiation as a result of being heated to high-temperature
  - Line sources:
  - Continuum sources:
- Incandescent lamps: heated tungsten filament in a evacuated glass container.
- Electric discharge lamps: fluorescent lamps. Filled with mercury vapor at low pressure and utilize an electric discharge through it to produce light in ultraviolet (UV) range. Through fluorescent, it is convert to visible light.
- Flash lamps: tubes containing a noble gas such xenon, krypton or argon. For their operation, high voltage stored in a capacitor is discharged through the gas, producing a highly luminous corona discharge. Light pulse is about 1µs or 1 ms.
- Sparks: produced by the electric breakdown of a gas (helium, neo, argon or air) during an electric discharge between electrodes. The choice of different electrodes produces sparks of different shapes.
Laser: Light Amplification by Stimulated Emission of Radiation (LASER)

Advantages of laser light over thermal light source:
- Coherent light (with all light wave front in phase)
- Collimated and concentrated (parallel light with small cross area)
- Monochromic (energy concentrated in a very narrow wavelength band)

How a laser works:
- Radiation energy is produced by an activated medium (can be gas, crystal or semiconductor or liquid solution).
- The medium consists of particles (atom, ions or molecules).
- When a photo, having energy hv, approaching the particles, the photo may be absorbed cause an electron or atoms to be raised temporarily to high-energy level.
- When the excited electron or molecule to return ground level, spontaneous emission or stimulated emission would take place.
Laser

- **Spontaneous emission**: emit a photo with the same energy as that absorbed one, but in random direction.

- **Stimulated emission**: An electron or atom is already at a higher energy level could become excited by an incident photo, without absorb the photo, it will emission another photo with identical energy (frequency), phase, and direction as the incident photon.

- **External power source** is required to maintain the population of the atoms in higher energy level in order to make to stimulated emission taking place continuously.

- **Optical cavity**.

- **Q-switch**
Commonly used Lasers

- **Helium-neon (He-Ne) laser**
  - Active medium is helium neon atoms
  - Continuous wave laser
  - Power 0.3 ~15 mW
  - $\lambda$ =633nm (red)
Commonly used Lasers

- Argon-ion (Ar-ion) laser
  - Active medium is argon atoms maintained at the ion state.
  - Continuous wave laser
  - Power level: 100 mW ~10 W
  - Have seven wavelengths
    - $\lambda = 488$nm (blue)
    - $\lambda = 514.5$nm (green)
  - LDV application
  - LIF in liquid flows
Commonly used Lasers

- Nd-YAG laser
  - **Solid-state laser**
  - Active medium: neodymium (Nd³⁺) as active medium incorporated as an impurity into a crystal of Yttrium-Aluminium-Garnet (YAG) as a host
  - Flash lamp is used as external source
  - Pulsed laser: 10 - 400mJ/pulse or more
  - Pulse duration: 100ps ~ 10ns
  - Wavelength of tube $\lambda = 1064\text{nm}$ (infrared)
  - SHG: $\lambda = 532\text{nm}$ (green), THG: $\lambda = 355\text{nm}$ (UV), FHG: $\lambda = 266\text{nm}$ (deepUV)
  - PIV, MTV, PLIF
  - Repetition rate can be as high as 30 Hz.
Commonly used Lasers

- **Copper Vapor laser**
  - *Active medium: copper vapor*
  - *Pulsed laser: 10mJ/pulse or more*
  - *Pulse duration: 15 ~ 60ns*
  - *$\lambda = 510.6\text{nm} \text{ (green)}, \lambda = 578.2\text{nm} \text{ (yellow)}$*
  - *Repetition rate can be as high as $f=5,000 \sim 15,000 \text{ Hz.}$*
  - *High-speed PIV, LIF and others*
Commonly used Lasers

- **Dye laser**
  - Active medium: complex multi-atomic organic molecules
  - $\lambda = 200 \text{nm} \sim 1500 \text{nm}$

- **Excimer laser**
  - Gas laser KrF and XeCl
  - High-energy
  - UV wavelength
  - Pulsed laser
  - high repetition frequency
**Light Scattering**

- **Scattering**
  - Scattering is a general physical process whereby some forms of radiation, such as light, are forced to deviate from a straight trajectory by one or more localized non-uniformities in the medium through which it passes.

- **Elastic Scattering**
  - Excited electron or atoms emits a photo have exact the same frequency as the incident one.

- **Inelastic scattering**
  - Excited electron or atoms emits a photo have a frequency different from the incident one.
Elastic scattering

- Rayleigh Scattering
  - Light scattering from particles that are smaller than 1/15 of the incident light wavelength (d < \(\lambda/15\)).
  - Efficiency of the scattering from a particle is expressed in terms of scattering cross section.

\[
\sigma_R = \sigma_T \left(\frac{\lambda_0}{\lambda}\right)^2
\]

\[
\sigma_T = 6.65 \times 10^{-29} \text{ m}^2
\]

\(\lambda_0\) is the characteristic wavelength of the atom.

- Mie Scattering
  - Light scattering from a particles with its size close on bigger than the incident light wavelength (d > \(\lambda\)).
  - Conservation of polarization direction
  - Angle dependent
    - Forward scattering
    - Back scattering
Inelastic Scattering

- Raman Scattering
  - Inelastic scattering from molecules.
  - Chance to occur is about $10^{-5} \text{ s} \times 10^{-2}$ of times lower than the Rayleigh scattering.
  - Scattering cross section is several orders smaller than the Rayleigh scattering.
  - Stoke transition: the energy of the emitted photon is higher than the absorbed photon.
  - Anti-stoke transition: the energy of the emitted photon is lower than the absorbed photon.
  - Time between the absorption and emission: $10^{-14} \text{ s}$.
  - Anti-stokes line will be stronger when the temperature is low.
Fluorescence and phosphorescence

- Rayleigh and Raman scattering occurs essentially instantaneously. Not allowing other energy conversion phenomena to occur.
- Fluorescence and phosphorescence: Photoluminescence with time delay
- Fluorescence
  - Emission when the excited from singlet state to ground,
  - lifetime is about $10^{-10} \sim 10^{-5}$ s.
Fluorescence and phosphorescence

- phosphorescence
  - Emission when the excited atom or molecule from triplet state to ground,
  - lifetime is about $10^{-4} \sim 10^{-5}$ s.

![Absorption and emission spectra](image)

![Spectrophotometer Output vs Wavelength](image)

MTV chemical: 1-BrNp•Mβ-CD•ROH complex
Light sensing and recording
Lenses

- **Focus length:**
- **Depth of focus:**
- *f-numbers or focal ratio:* is defined as the ratio of focal distance of the lens and its clear aperture diameter.

<table>
<thead>
<tr>
<th>T2 F2.8</th>
<th>T4 15.6 18 111</th>
<th>T16 T 22</th>
</tr>
</thead>
<tbody>
<tr>
<td>wide aperture</td>
<td>small aperture</td>
<td>small aperture</td>
</tr>
<tr>
<td>more light</td>
<td>less light</td>
<td>less light</td>
</tr>
<tr>
<td>small number</td>
<td>larger number</td>
<td>larger number</td>
</tr>
</tbody>
</table>
Photo detector

- Photo detector is a device to convert light to an electric current through photo electric effect.

- Quantum efficiency:
  \[ \eta_q = \frac{N_e}{N_p} \]
  
  - \( N_e \): Number of absorbed photons
  - \( N_p \): Number of emitted electrons

- Noise:
  - Shot noise: due to random fluctuation of the rate of photon collection and background illumination
  - Thermal noise: caused by amplification of current inside the photo detector and by external amplifier.

- Dark current: the current produced by the photo detector even in the absence of a desirable light source.

- Two kinds of photo detectors:
  - Photomultiplier tubes (PMT)
  - Photodiodes (PD) or photo electric cells
Photo detector

- **Photomultiplier tubes (PMT)**
  - **Photocathode**: absorbs photons and emits electrons.
  - **Dynodes**: increase number of photons
  - **Anodes**: output
Photo detector

- photodiodes (PD) or photo electric cells
  - P-n junctions of semiconductors, commonly silicon-silicon type.
  - High quantum efficiency
  - But not internal amplification
Linearity and Dynamic Range of a Digital Camera

- **Linearity:**
  - Intensified CCD cameras usually need to check its linearity

- **Dynamic Range:**
  - The ratio between the full-well capacity and the dark current noise.
  - For example, for a 8-bit CCD camera, maximum intensity is $2^8=256$, dark current noise is about 25, then Dynamic range is about 10.
  - Available bits number:
    - 8 bit, 16 bit, 24 bit
Interlaced Cameras

- The fastest response time of human being for images is about ~ 15Hz.
- Video format:
  - PAL (Phase Alternating Line) format with frame rate of $f=25$Hz (sometimes in 50Hz). Used by U.K., Germany, Spain, Portugal, Italy, China, India, most of Africa, and the Middle East.
  - NTSC format: established by National Television Standards Committee (NTSC) with frame rate of $f=30$Hz. Used by U.S., Canada, Mexico, some parts of Central and South America, Japan, Taiwan, and Korea.

![Diagram showing interlaced camera](image)

- 480 pixels by 640 pixels
- 1 frame $F=30$Hz
- Odd field
- Even field
- 16.6ms
- Time

Interlaced camera

One complete frame using interlaced scanning
Progressive scan camera

- All image systems produce a clear image of the background
- Jagged edges from motion with interlaced scan
- Motion blur caused by the lack of resolution in the 2CIF sample
- Only progressive scan makes it possible to identify the driver

Note: In these examples, the cameras have been using the same lens. The car has been driving at 20 km/h (15 mph) using cruise control.
Mystery of flying rods
Digital camera

• **CCD camera** and **Intensified CCD (ICCD) camera**:
  – **Spatial resolution**: 1K by 1k, 4K by 4K
  – **Frame rate**: 30 Hz, High speed camera 1khz ~ 10K hz
  – *In a CCD sensor, every pixel's charge is transferred through a very limited number of output nodes (often just one) to be converted to voltage, buffered, and sent off-chip as an analog signal. All of the pixel can be devoted to light capture, and the output's uniformity (a key factor in image quality) is high.*

• **CMOS (Complementary metal-oxide semiconductor) cameras**
  – *In a CMOS sensor, each pixel has its own charge-to-voltage conversion, and the sensor often also includes amplifiers, noise-correction, and digitization circuits, so that the chip outputs digital bits*
  – *These other functions increase the design complexity and reduce the area available for light capture. With each pixel doing its own conversion, uniformity is lower. But the chip can be built to require less off-chip circuitry.*
  – In summary for CMOS cameras
    – Low cost
    – Operation versatility
    – High speed
    – Quality is not as high as CCD cameras
Shadowgraph, Schlieren and Interferometry

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Subsonic, Transonic, Supersonic and Hypersonic Flows

**Subsonic flows:** $M < 1.0$

**Transonic flows:** $M \approx 1.0$

**Supersonic flows:** $M > 1.0$

**Hypersonic flows:** $M > 5.0$

Sonic boom
Subsonic and Supersonic Flow

- a. Stationary Sound Source
- b. Source moving with $V_{source} < V_{sound}$
- c. Source moving with $V_{source} = V_{sound}$
  (Mach 1 - breaking the sound barrier)
- d. Source moving with $V_{source} > V_{sound}$
  (Mach 1.4 - supersonic)
Shock Waves

Normal Shock Wave
(The airstream slows to subsonic)

Oblique Shock Wave
(The airstream slows down, but remains supersonic)

Expansion Wave
(The airstream accelerates, and the air behind the shock wave has higher supersonic)
Introduction-1

- **Index of refraction:** \( n = c / v = \frac{\lambda_0}{\lambda} > 1 \)

- **Depend on variation of index of refraction in a transparent medium and the resulting effect on a light beam passing through the test section**

- **Shadowgraph systems:** are used to indicate the variation of the second derivatives (normal to the light beam) of the index of refraction.

- **Schlieren Systems:** are used to indicate the variation of the first directive of the index of refraction

- **Interferometry systems:** response directly the difference of optical path length, especially giving the index of reflection field within the flow field.

Holographic interferometry image of shock-vortex interaction

Schlieren images of the muzzle blast and supersonic bullet from firing a .30-06 caliber high-powered rifle (by Gary S. Settles)

Shadowgraph depicting the flow generated by a bullet at supersonic speeds. (by Andrew Davidhazy)

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Shadowgraph and Schlieren Systems are often used in shock waves and flame phenomena, in which density gradient is quite big.

Interferometry are often used to study a flow in which density gradient are small.

While these techniques are mostly used for qualitative flow visualization, they can be used to determine pressure, density or temperature measurements theoretically.

These techniques are often used to determine the integrated quantity over the length of light beam.

These techniques are usually used for 2-D flow without index of refraction or density variations along the beam.

temperature fields and the heat transfer around a heated cylinder

shadowgraph image of plumes during solidification process (by Lum Chee)

Schlieren image
Introduction-3

- **Index of refraction is a function of thermodynamic state (density) for homogeneous medium:**

- **Lorenz-Lorentz relationship:**
  \[
  \frac{1}{\rho} \frac{n^2 - 1}{n^2 + 2} = \text{const}
  \]

- **When \( n=1 \), for gaseous flow:**
  \[
  \frac{n - 1}{\rho} = \text{const} \quad \Rightarrow \quad \rho = \frac{n - 1}{\text{const}}
  \]

- **at standard condition, with \( n_0 \) and \( \rho_0 \):**
  \[
  \frac{n_0 - 1}{\rho_0} = \text{const} \quad \Rightarrow \quad n - 1 = \frac{\rho}{\rho_0} (n_0 - 1)
  \]
  \[
  \Rightarrow \quad \rho = \rho_0 \frac{n - 1}{n_0 - 1}
  \]

- **When first and second derivative is determined as in Schlieren and shadowgraph apparatus:**
  \[
  \frac{\partial \rho}{\partial y} = \frac{1}{\text{const}} \frac{\partial n}{\partial y} \quad \Rightarrow \quad \frac{\partial \rho}{\partial y} = \frac{\rho_0}{n_0 - 1} \frac{\partial n}{\partial y}
  \]
  \[
  \frac{\partial^2 \rho}{\partial y^2} = \frac{1}{\text{const}} \frac{\partial^2 n}{\partial y^2} \quad \Rightarrow \quad \frac{\partial^2 \rho}{\partial y^2} = \frac{\rho_0}{n_0 - 1} \frac{\partial^2 n}{\partial y^2}
  \]
Application of the Schlieren and shadowgraph techniques:

- Compressible flow with shock waves ⇒ density changes
- Natural convective flow ⇒ density changes
- Flame and combustion system: ⇒ density changes

Temperature changes inside flows:
- For low speed flow with heat transfer:
  - $P = \text{constant}$

$$
\rho = P / RT \Rightarrow \frac{\partial \rho}{\partial y} = \frac{P}{RT^2} \frac{\partial T}{\partial y} = \frac{\rho}{T} \frac{\partial T}{\partial y}
$$

$$
\Rightarrow \frac{\partial n}{\partial y} = n_0 - 1 \frac{\partial \rho}{\partial y} = n_0 - 1 \frac{\rho}{T} \frac{\partial T}{\partial y}
$$

$$
\Rightarrow \frac{\partial T}{\partial y} = \frac{T}{n_0 - 1} \frac{\rho}{\rho_0} \frac{\partial n}{\partial y}
$$

$$
\Rightarrow \frac{\partial^2 n}{\partial y^2} = n_0 - 1 \frac{\rho}{\rho_0} [\frac{\partial^2 T}{T \partial y^2} + 2 \frac{\partial T}{T \partial y} \left(\frac{\partial T}{\partial y}\right)]
$$
• For reversible, adiabatic process:

\[
\frac{P}{\rho^k} = \text{const} \Rightarrow \frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^k
\]

\(k\) is the ratio of specific heat; \(k = \frac{C_p}{C_v}\)

\[
\rho = \frac{n-1}{\text{const}} \Rightarrow \frac{P}{P_0} = \left(\frac{n-1}{n_0-1}\right)^k
\]

\[
\Rightarrow \frac{\partial P}{\partial y} = P_0 \left(\frac{n-1}{n_0-1}\right)^{k-1} \frac{\partial n}{\partial y} = \frac{P}{n-1} \frac{\partial n}{\partial y}
\]

\[
\Rightarrow \frac{\partial n}{\partial y} = \frac{1}{P} \left(\frac{n-1}{k}\right) \partial(n-1)
\]
Schlieren technique

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• According to definition of index of refraction, the light velocity will be \( V = \frac{C}{n} \).

• The slope of the wave front of the light:
  \[
  \frac{dy}{dz}
  \]

• If the angle \( \Delta \alpha \) is quite small,
  \[
  \Delta Z = n \frac{C_0}{n} \Delta \tau
  \]
  \[
  \Delta^2 Z = \Delta Z - \Delta Z_{y+\Delta y} = -C_0 \left( \frac{1}{n} \frac{\Delta}{\Delta y} \right) \Delta \tau \Delta y
  \]
  \[
  \Delta \alpha' = \frac{\Delta^2 Z}{\Delta y} = -n \left( \frac{\Delta}{\Delta y} \right) \Delta Z
  \]
  \[
  \frac{dy}{dz} = d \alpha' = -n \frac{d(\frac{1}{n})}{dy} \frac{dy}{dz} = n \frac{1}{n^2} \frac{dn}{dy} \frac{dy}{dz} = \frac{1}{n} \left( \frac{dn}{dy} \right) \frac{dy}{dz} = \frac{d(ln n)}{dy} \frac{dy}{dz}
  \]
  \[
  \frac{d^2 y}{dz^2} = \frac{d(ln n)}{dy}
  \]
  \[
  d \alpha' = d \left( \frac{\frac{1}{n}}{\frac{dy}{dy}} \right) \frac{dz}{dz} = n \frac{1}{n^2} \left[ \frac{dn}{dy} \frac{dy}{dz} \right] \frac{dz}{dz} = \frac{1}{n} \left( \frac{dn}{dy} \right) \frac{dz}{dz} = \frac{d(ln n)}{dy} \frac{dz}{dz}
  \]
  \[
  \Rightarrow \alpha' = \int \frac{1}{n} \left( \frac{dn}{dy} \right) dz
  \]
  \[
  \Rightarrow \alpha' = \int \frac{1}{n} \frac{dz}{dy}
  \]
Fundamentals of Schlieren System

- The intensity after the shape razor blade (knife edge) before the experiment
  \[ I_k = \frac{a_K}{a_0} I_0 \]
- The intensity after the deformation due to the variation of the index of refraction
  \[ I_d = I_k + \frac{\Delta a}{a_K} I_k = (1 + \frac{\Delta a}{a_K}) I_k \]
  \[ \text{contrast} = \frac{\Delta I}{I_k} = \frac{I_d - I_k}{I_k} = \frac{\Delta a}{a_K} = + \frac{\alpha f_2}{a_K} \]
  \[ \text{sensitivity} : \frac{d(\text{contrast})}{d\alpha} = \frac{f_2}{a_K} \]
- Sensitivity is proportional to \( f_2 \) and inversely to \( a_k \).
Fundamentals of Schlieren System

- The intensity after the shape razor blade (knife edge) before the experiment
  \[ I_k = \frac{a_K}{a_0} I_0 \]

- The intensity after the deformation due to the variation of the index of refraction
  \[ I_d = I_k + \frac{\Delta a}{a_K} I_k = (1 + \frac{\Delta a}{a_K}) I_k \]

- Contrast
  \[ \text{contrast} = \frac{\Delta I}{I_k} = \frac{I_d - I_k}{I_k} = \frac{\Delta a}{a_K} = \pm \frac{\alpha}{a_K} f_2 \]

- Sensitivity
  \[ \text{sensitivity} : \frac{d(\text{contrast})}{d\alpha} = \frac{f_2}{a_K} \]

- Sensitivity is proportional to \( f_2 \) and inversely to \( a_K \).
Fundamentals of Schlieren System

FOR $\alpha$ SMALL

\[ \alpha = \Delta y/p \]
\[ \beta = \Delta y/f_2 \]
\[ \gamma = \Delta y/q \]

\[ \alpha'' = \beta - \gamma \]
\[ \Delta \alpha = \alpha'' \cdot f_2 \]

Figure 7.4 Ray displacement at knife-edge for a given angular deflection

INCREASING $n(\rho)$

Increasing $n(\rho)$
Figure 7.7  Schlieren images of a helium jet entering an atmosphere of air: The effect of knife-edge orientation (Re = 630)
Fundamentals of Schlieren System

Figure 7.8 Schlieren images of the flow structure of a helium jet entering air at different Reynolds numbers.

Re 80  Re 200
Re 470  Re 630

LIGHT BEAM
KNIFE EDGE
Fundamentals of Schlieren System

- For a gas flow with density change:

\[
\frac{\Delta I}{I_k} = \pm \alpha \frac{f_2}{a_K} \\
\alpha' = \int \frac{dn}{dy} \, dz \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \int \frac{dn}{dy} \, dz \\
\frac{\partial \rho}{\partial y} = \frac{\rho_0}{n_0 - 1} \frac{\partial n}{\partial y} \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \int \frac{d\rho}{dy} \, dz \\
n \approx 1 \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \int \frac{d\rho}{dy} \, L
\]
Fundamentals of Schlieren System

• For a gas flow with constant pressure distribution:

\[
\frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \int \frac{dn}{dy} \, dz
\]

\[
\frac{\partial T}{\partial y} = \frac{T}{n_0 - 1} \frac{\rho}{\rho_0} \frac{\partial n}{\partial y}
\]

\[
\Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \int \frac{\rho}{T} \frac{dT}{dy} \, dz
\]

\[
\Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \int \frac{P}{RT^2} \frac{dT}{dy} \, dz
\]

\[
\Rightarrow \quad \frac{\Delta I}{I_k} \approx \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \frac{P}{RT^2} \frac{dT}{dy} L
\]
For a liquid flow:

- $n$ is a function of temperature $T$.

$$\frac{dn}{dy} = \frac{\partial n}{\partial T} \frac{\partial T}{\partial y}$$

$$\frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \int \frac{dn}{dy} \, dz = \frac{f_2}{a_K} \int \frac{\partial n}{\partial T} \, dz$$

If $n \to 1 \Rightarrow \frac{\partial n}{\partial T} \frac{\partial T}{\partial y} \approx \text{const}$

$$\Rightarrow \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{\partial n}{\partial T} \frac{\partial T}{\partial y} L$$
Lab #3 - Visualization of shock wave in a transonic/supersonic nozzle using Schlieren technique.

Diagram showing the setup of the Schlieren technique: Light Source, Vertical Slit, Lens, Nozzle, Concave Mirror, Knife Edge, Viewing Screen, and Flow direction. The diagram explains how light is bent by negative and positive density gradients and how the images appear brighter or darker depending on whether light passes through the knife edge.
1st, 2nd and 3rd critic conditions

- **Under-expanded flow**
- **Flow close to 3rd critical**
- **Over-expanded flow**
- **Over-expanded flow with shock between nozzle exit and throat**
- **1st critical – shock is almost at the nozzle throat.**
- **2nd critical – shock is at nozzle exit**
Alternative Schlieren system

A. Setup with one converging and one plane mirror

A. Setup with one converging mirror
Holographic Schlieren system

Figure 7.10  Continued
Fundamentals of Schlieren System

- For a gas flow with density change:

\[
\frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \int \frac{dn}{dy} \, dz
\]

\[
\Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} n_0 \frac{\rho_0 - 1}{\rho_0} \int \frac{d\rho}{dy} \, dz
\]

\[
\Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{\partial n}{\partial T} \frac{\partial T}{\partial y} \, L
\]
Advanced Schlieren technique- Background Oriented Schlieren (BOS)

Figure 1. Schematic of BOS technique.

Figure 6. Schlieren image using the S-BOS technique.

Colored Grid Background Oriented Schlieren (CGBOS) Technique

Figure 4. CGBOS image taken through Mach 2.0 flow.

Figure 5. Green channel of CGBOS image.

Figure 6. Red channel of CGBOS image.

Figure 7. Asymmetric body.

Figure 8. Schlieren image taken with vertical knife edge.

Figure 9. Gray-scale image of horizontal displacement of red stripe.

Figure 10. Schlieren image taken with vertical knife edge.

Figure 11. Reconstructed density distribution on central plane.

Figure 12. Isopycnic surface ($\rho/\rho_0 = 0.74$) and 3-D density distribution.


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Shadowgraph Technique

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Shadowgraph technique

\[ I_{sc} = \frac{\Delta y}{\Delta y_{sc}} I_0 \]

\[ \Delta y_{sc} = \Delta y + Z_{sc} \cdot d\alpha \]

\[ \frac{\Delta I}{I_0} = \frac{I_{sc} - I_0}{I_0} = \frac{\Delta y_{sc}}{I_0} - 1 \approx \frac{\Delta y_{sc}}{I_0} \]

\[ \Rightarrow \frac{\Delta I}{I_0} \approx -Z_{sc} \cdot \frac{d\alpha}{dy} \]

since \[ \alpha = \frac{1}{n_a} \int \frac{dn}{dy} dz \]

\[ \Rightarrow \frac{\Delta I}{I_0} = - \frac{Z_{sc}}{n_a} \cdot \int \frac{d^2 n}{dy^2} dz \]

- Sensitive is proposal to \( Z_{sc} \)
**Shadowgraph technique**

Experimental setup with one converging mirror

Experimental setup without lens or mirror
Lab #3 - Visualization of shock wave in a transonic/supersonic nozzle using Schlieren technique
Schlieren vs. Shadowgraph

**Shadowgraph**
- Displays a mere shadow
- Shows light ray displacement
- Contrast level responds to $\frac{\partial^2 n}{\partial y^2}$
- No knife edge used

**Schlieren**
- Displays a focused image
- Shows ray refraction angle, $\varepsilon$
- Contrast level responds to $\frac{\partial n}{\partial y}$
- Knife edge used for cutoff
Examples

Figure 7.14  Shadowgraphs of a helium jet entering an atmosphere of air

Figure 7.7  Schlieren images of a helium jet entering an atmosphere of air: The effect of knife-edge orientation (Re = 630)

Figure 7.15  Shadowgraph of mixing of parallel-flowing streams of helium (above) and nitrogen.
Interferometers

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Interferometers

- Unlike the Schlieren and shadowgraph systems, an interferometer does not depend upon the deflection of a light beam to determine density or index of refraction variation.
- Interferometers are often used for quantitative measurements
Interference of Waves from Two Sources

Constructive interference

In some places the water wavefronts are in phase (bright spots).

In other places the fronts overlap with peak and valley and interfere destructively (darker spots).
Coherent light Source

- Coherent sources...
- Two sources of light are said to be coherent if the waves emitted from them have the same frequency and are 'phase-linked'; that is, they have a zero or constant phase difference.
Interference of two coherence light waves

Amplitude of a plane light wave in a homogeneous medium can be expressed as:

\[ A = A_0 \sin \frac{2\pi}{\lambda} (ct - z) \]

Therefore:

wave 1: \( A_1 = A_{01} \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 \right) \)

wave 2: \( A_2 = A_{02} \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \Delta \right) \)

if \( A_0 = A_{01} = A_{02} \)

then: \( A_r = A_1 + A_2 \)

\[ = A_0 \left[ \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 \right) + \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \Delta \right) \right] \]

\[ = 2A_0 \cos \frac{\Delta}{2} \sin \left( \frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \frac{\Delta}{2} \right) \]

Therefore, the intensity of the combined wave (which is proportional to the square of the peak amplitude) will be:

\[ I \sim 4A_0^2 \cos^2 \frac{\Delta}{2} \]
Interference of light waves

Thomas Young (1801)
The particle path length along a light beam is defined as:

$$PL = \int ndz$$

or

$$PL = \int \frac{C_0}{C} dz = \frac{1}{\lambda_0} \int \frac{dz}{\lambda}$$

Therefore, the difference between path 1 and path 2:

$$\Delta PL = PL_1 - PL_2 = \int_{path-1} ndz - \int_{path-2} ndz$$

$$= \frac{1}{\lambda_0} (\int_{path-1} \frac{dz}{\lambda} - \int_{path-1} \frac{dz}{\lambda})$$

The phase difference between the two waves will be:

$$\Delta = 2\pi (\int_{path-1} \frac{dz}{\lambda} - \int_{path-1} \frac{dz}{\lambda})$$

or

$$\frac{\Delta}{2\pi} = \frac{\Delta PL}{\lambda_0}$$
Interferometers

\[ \varepsilon = \frac{1}{\lambda_0} \int (n - n_{\text{ref}}) \, dz \]

According to the Glasstone-Dale equation: \[ \rho = \frac{n - 1}{\text{Const}} \]

\[ \Rightarrow \varepsilon = \frac{\text{Const}}{\lambda_0} \int (\rho - \rho_{\text{ref}}) \, dz \]

if only varies over a length \( L \), then, the fringe shift will be:
\[ \varepsilon = \frac{n - n_{\text{ref}}}{\lambda_0} L \]

for gaseous flows
\[ \varepsilon = \frac{\text{Const}}{\lambda_0} (\rho - \rho_{\text{ref}}) L \]

or
\[ \rho - \rho_{\text{ref}} = \frac{\lambda_0 \varepsilon}{\text{Const} \cdot L} = \frac{\lambda_0 \varepsilon}{n_o - 1} \frac{\rho_o}{L} \]

for temperature measurements in gaseous flows
\[ T - T_{\text{ref}} = \frac{\lambda_0 \varepsilon}{L} \frac{1}{dn/DT} \]
Examples

Figure 7.20  Interferograms of a low-Reynolds-number helium jet entering a
Examples

Figure 7.24  Flow over sharp-tipped spike with conical flare; pressure 100 psia; $Ma = 2$.
(a) Infinite fringe interferogram. (b) Wedge-fringe interferogram. (From [46]).

Figure 7.26  Wedge-fringe interferograms used for visualizing flow over a heated gas-turbine blade held in a cascade; oncoming flow direction is parallel to the visible wire carrying the heating current. (From [53]).
Thank you Very Much for Your Time!

Questions?