Lecture # 07: Laminar and Turbulent Flows

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Sources/ Further reading:
Munson, Young, & Okiishi, “Fundamentals of Fluid Mechanics,” 4th ed, Ch 8
Tritton, “Physical Fluid Dynamics,” 2nd ed, Chs 2, 19–21
“Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.”

-LF Richardson
Reynolds number: \( \text{Re} = \frac{\rho D U}{\mu} \)

Empirically, when \( \text{Re} < 1000 \), laminar flow, and when \( \text{Re} > 3000 \), turbulent flow.

\( \text{Re}_C \sim \text{critical Reynolds number above which flow exhibits turbulent characteristics} \)

For external flows (e.g., flow around airfoil)
\( (\text{Re}_C)_L \sim 3 \cdot 10^5 \)

For internal flows (e.g., pipe flow)
\( (\text{Re}_C)_\delta \sim 3 \cdot 10^3 \)

From Pradtl: boundary layer (\( \delta \)) vs body size (\( L \)) scales like \( L/\delta \sim (\text{Re})_L^{1/2} \).
Thus \( (\text{Re})_L / (\text{Re})_\delta \sim 10^2 \).
Reynolds number: \( \text{Re} = \frac{LU}{\nu} \)

At the large scales, the flow is determined by the reference length scale, \( L \), and the reference time scale, \( \tau_0 = L/U \).

The small length scales are governed by the Kolmogorov scales \( \eta = (\nu^3 / \varepsilon)^{1/4} \), and \( \tau_\eta = (\nu / \varepsilon)^{1/2} \).

\( \varepsilon \) is the turbulent energy dissipation rate.

Scaling:

\[
\frac{L}{\eta} \sim L \left( \frac{U^3 L}{\nu^3} \right)^{1/4} = \text{Re}^{3/4}
\]

\[
\frac{\tau_0}{\tau_\eta} \sim \tau_0 \left( \frac{U^3}{L \nu} \right)^{1/2} = \text{Re}^{1/2}
\]

For a flow with \( \text{Re} \sim 10,000 \):
Flow scales span 3 orders of magnitude in length and 2 orders of magnitude in time.

Turbulent flows contain a vast range of length and time scales that must be resolved!!! Difficult!!
Laminar Flows and Turbulence Flows

- Laminar flow, sometimes known as streamline flow, occurs when a fluid flows in parallel layers, with no disruption between the layers. Viscosity determines momentum diffusion.
  - In nonscientific terms laminar flow is "smooth," while turbulent flow is "rough."

- Turbulent flow is a fluid regime characterized by chaotic, stochastic property changes. Turbulent motion dominates diffusion of momentum and other scalars. The flow is characterized by rapid variation of pressure and velocity in space and time.
  - Flow that is not turbulent is called laminar flow
Turbulent flows in a pipe

\[ \text{Re} = \frac{\rho V D}{\mu} \]
Characterization of Turbulent Flows

\[ u = \bar{u} + u'; \quad v = \bar{v} + v'; \quad w = \bar{w} + w' \]

\[ \bar{u} = \frac{1}{T} \int_{t_0}^{t_0+T} u(x, y, z, t) dt; \quad \bar{v} = \frac{1}{T} \int_{t_0}^{t_0+T} v(x, y, z, t) dt; \quad \bar{w} = \frac{1}{T} \int_{t_0}^{t_0+T} w(x, y, z, t) dt \]

**FIGURE 7.7** Velocity components in a turbulent pipe flow: (a) \(x\)-component velocity; (b) \(r\)-component velocity; (c) \(\theta\)-component velocity.
Turbulence Intensities

\[ u' = 0; \quad v' = 0 \quad w' = 0 \]

\[ (u')^2 = \frac{1}{T} \int_{t_0}^{t_0+T} (u')^2 \, dt > 0; \quad (v')^2 > 0 \quad (w')^2 > 0 \]
Turbulent Shear Stress

Laminar flows:
\[ \tau_{\text{lam}} = \mu \frac{\partial u}{\partial y} \]

Turbulent flows:
\[ \tau_{\text{turb}} = -\rho u'v' \]
\[ \tau = \tau_{\text{lam}} + \tau_{\text{turb}} = \mu \frac{\partial u}{\partial y} - \rho u'v' \]

(a) laminar flow
(b) turbulent flow
Laminar Flows and Turbulence Flows

\[ \frac{C_d}{2 \rho U^2} = \begin{cases} \frac{24}{Re} & \text{for smooth cylinder} \\ \frac{C_{D_{\infty}}}{Re} & \text{for smooth sphere} \end{cases} \]

\[ Re = \frac{\rho U D}{\mu} \]

Viscous forces important throughout \( Re = UD/\nu = 0.1 \)

Viscosity not important

Re = 50

Viscosity not important

Boundary layer separation

Re = 10^5

Separated region

\[ C_d = \frac{D}{2 \rho U^2 A} \]

Flat plate

Circle

Ellipse

Airfoil

Float plate

\[ Re = \frac{UD}{\nu} \]

\[ b = \text{length} \]
Flow Around A Sphere with laminar and turbulent boundary layers

Top:

Instantaneous flow past a sphere at Re_D = 15,000. Dye in water shows a laminar boundary layer separating ahead of the equator and remaining laminar for almost one radius. It then becomes unstable and quickly turns turbulent.

Bottom:

Instantaneous flow past a sphere at Re_D = 30,000 with a trip wire. A classical experiment of Prandtl and Wieselsberger is repeated here, using air bubbles in water. A wire hoop ahead of the equator trips the boundary layer. It becomes turbulent, so that it separates farther rearward than if it were laminar (compare with top photograph). The overall drag is thereby dramatically reduced, in a way that occurs naturally on a smooth sphere only at a Reynolds numbers ten times as great.
Aerodynamics of a golfball

\[ C_D = \frac{1}{\frac{1}{2} \rho u^2 \pi D^2} \]

\[ \frac{\varepsilon}{D} = \text{relative roughness} \]

\[ \frac{\varepsilon}{D} = 1.25 \times 10^{-2} \]

\[ \frac{\varepsilon}{D} = 5 \times 10^{-3} \]

\[ \frac{\varepsilon}{D} = 1.5 \times 10^{-3} \]

\[ \frac{\varepsilon}{D} = 0 \text{ (smooth)} \]

\[ \text{Re} = \frac{UD}{v} \]

Separation

Smooth Sphere

Thick wake

Laminar boundary layer

Turbulent boundary layer

Transition

Thin wake
Laminar and turbulent flows

Re = 100,000

Smooth ball

Rough ball

Golf ball

Distance (X/D)

Centerline Velocity (U/U_∞)

Laminar and turbulent flows

Re = 100,000
Automobile aerodynamics

- Chart showing the change in drag coefficient ($C_D$) from 1920 to 2000.
- Image of a modern Chevrolet Volt,
- Diagram illustrating flow patterns and pressure distributions:
  - Gray indicates $p > p_0$,
  - Blue indicates $p < p_0$.

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Automobile aerodynamics

Mercedes Boxfish

Vortex generator above a Mitsubishi rear window
Aerodynamic performance of an airfoil

\[ C_l = \frac{L}{\frac{1}{2} \rho V^2 c} \]

\[ C_d = \frac{D}{\frac{1}{2} \rho V^2 c} \]

\[ \frac{X}{C*100} \]
\[ \frac{Y}{C*100} \]

Airfoil stall

Before stall

After stall

Experimental data

Drag Coefficient, \( C_d \)

Angle of Attack (degrees)

Lift Coefficient, \( C_l \)
Flow Separation and Transition on Low-Reynolds-number Airfoils

- Low-Reynolds-number airfoil (with Re<500,000) aerodynamics is important for both military and civilian applications, such as propellers, sailplanes, ultra-light man-carrying/man-powered aircraft, high-altitude vehicles, wind turbines, unmanned aerial vehicles (UAVs) and Micro-Air-Vehicles (MAVs).

- Since laminar boundary layers are unable to withstand any significant adverse pressure gradient, laminar flow separation is usually found on low-Reynolds-number airfoils. Post-separation behavior of the laminar boundary layers would affect the aerodynamic performances of the low-Reynolds-number airfoils significantly.

- Separation bubbles are usually found on the upper surfaces of low-Reynolds-number airfoils. Separation bubble bursting can cause airfoil stall at high AOA when the adverse pressure gradients become too big.
Surface Pressure Coefficient distributions (Re=68,000)

Typical surface pressure distribution when a laminar separation bubble is formed (Russell, 1979)

GA (W)-1 airfoil
(also labeled as NASA LS(1)-0417)
Laminar Separation Bubble on a Low-Reynolds-number Airfoil

PIV measurement results at AOA = 10 deg, Re=68,000

(Hu et al., ASME Journal of Fluid Engineering, 2008)
Stall Hysteresis Phenomena

- **Stall hysteresis**, a phenomenon where stall inception and stall recovery do not occur at the same angle of attack, has been found to be relatively common in low-Reynolds-number airfoils.
- When stall hysteresis occurs, the coefficients of lift, drag, and moment of the airfoil are found to be multiple-valued rather than single-valued functions of the angle of attack.
- **Stall hysteresis** is of practical importance because it produces widely different values of lift coefficient and lift-to-drag ratio for a given airfoil at a given angle of attack. It could also affect the recovery from stall and/or spin flight conditions.

![Lift coefficient curve of a “typical” airfoil](image1)

![Lift coefficient curve with stall hysteresis](image2)
**Measured airfoil lift and drag coefficient profiles**

- The hysteresis loop is **clockwise in the lift coefficient profiles**, and **counter-clockwise in the drag coefficient profiles**.
- The aerodynamic hysteresis resulted in **significant variations of lift coefficient, C_l**, and lift-to-drag ratio, l/d, for the airfoil at a given angle of attack.
- The lift coefficient and lift-to-drag ratio at AOA = 14.0 deg were $C_l = 1.33$ and $l/d = 23.5$ when the angle was in the **increasing angle branch of the hysteresis loop**.
- The values were $C_l = 0.8$ and $l/d = 3.66$ for the same AOA=14.0 degrees when the angle was at the **decreasing angle branch of the hysteresis loop**.

**GA(W)-1 airfoil, $Re_C = 160,000$**
PIV Measurement results

(Hu, Yang, Igarashi, Journal of Aircraft, Vol. 44. No. 6, 2007)
Refined PIV Measurement Results

(Hu, Yang, Igarashi, Journal of Aircraft, Vol. 44. No. 6, 2007)
Lab 6: Airfoil Wake Measurements and Hotwire Anemometer Calibration

\[
\sum F_x = -D + \int_{cs} (p \hat{n} dA)_x \\
= -D + \int_1 p_{up} dA - \int_2 p(y) dA
\]

Since \( p_{up} = p_\infty \), \( p(y) \approx p_\infty \)

\[
\Rightarrow \sum F_x = -D \\
\Rightarrow \sum F_x = -D = \rho U_\infty^2 \int_2 \left[ \frac{U(y)}{U_\infty} \left( \frac{U(y)}{U_\infty} - 1 \right) \right] dA_2
\]

\[
\Rightarrow D = \rho U_\infty^2 \int_2 \left[ \frac{U(y)}{U_\infty} \left( 1 - \frac{U(y)}{U_\infty} \right) \right] dA_2
\]

\[
C_D = \frac{D}{\frac{1}{2} \rho U_\infty^2 C} = \frac{\rho U_\infty^2 \int_2 \left[ \frac{U(y)}{U_\infty} \left( 1 - \frac{U(y)}{U_\infty} \right) \right] dA_2}{\frac{1}{2} \rho U_\infty^2 C}
\]

\[
\Rightarrow C_D = \frac{2}{C} \int_2 \left[ \frac{U(y)}{U_\infty} \left( 1 - \frac{U(y)}{U_\infty} \right) \right] dy
\]

Compare with the drag coefficients obtained based on airfoil surface pressure measurements at the same angles of attack!
Lab 6: Airfoil Wake Measurements and Hotwire Anemometer Calibration

Pressure rake with 41 total pressure probes (the distance between the probes d=2mm)
Lab 6: Airfoil Wake Measurements and Hotwire Anemometer Calibration

Flow Field

\[
mc \frac{dT_w}{dt} = i^2 R_w - q(V, T_w)
\]

• Constant-temperature anemometry

CTA hotwire probe
Hotwire Anemometer Calibration

- Quantify the relationship between the flow velocity and voltage output from the CTA probe

\[ y = a + bx + cx^2 + d x^3 + e x^4 \]

Max dev: 0.166, \( r^2 = 1.00 \)

\[ a = 10.8, b = 3.77, c = -26.6, d = 13.2 \]
Required Measurement Results

Required Plots:
• \( C_p \) distribution in the wake (for each angle of attack) for the airfoil wake measurements
• \( C_d \) vs angle of attack (do your values look reasonable?) based on the airfoil wake measurements
• Your hot wire anemometer calibration curve: Velocity versus voltage output of hotwire anemometer (including a 4\(^{th}\) order polynomial fit)

Please briefly describe the following details:
• How you calculated your drag—you should show your drag calculations
• How these drag calculations compared with the drag calculations you made in the previous experiment
• Reynolds number of tests and the incoming flow velocity