Lecture # 07: Laminar and Turbulent Flows

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Sources/ Further reading:
Munson, Young, & Okiishi, “Fundamentals of Fluid Mechanics,” 4th ed, Ch 8
Tritton, “Physical Fluid Dynamics,” 2nd ed, Chs 2, 19–21

Sources/ Further reading:
Schlichting, “Boundary Layer Theory,” any ed
“Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.”

-LF Richardson

Leonardo Da Vinci
Reynolds number: \( \text{Re} = \frac{\rho D U}{\mu} \)

Empirically, when \( \text{Re} < 1000 \), laminar flow, and when \( \text{Re} > 3000 \), turbulent flow.

\( \text{Re}_c \sim \text{critical Reynolds number above which flow exhibits turbulent characteristics} \)

For external flows (e.g., flow around airfoil)
\( (\text{Re}_c)_L \sim 3 \cdot 10^5 \)
For internal flows (e.g., pipe flow)
\( (\text{Re}_c)_\delta \sim 3 \cdot 10^3 \)

From Prandtl: boundary layer (\( \delta \)) vs body size (L) scales like \( L/\delta \sim (\text{Re})_L^{1/2} \).
Thus \( (\text{Re})_L / (\text{Re})_\delta \sim 10^2 \).
Reynolds number: \( Re = \frac{LU}{\nu} \)

At the large scales, the flow is determined by the reference length scale, \( L \), and the reference time scale, \( \tau_0 = \frac{L}{U} \).

The small length scales are governed by the Kolmogorov scales \( \eta = \left( \frac{\nu^3}{\epsilon} \right)^{1/4} \), and \( \tau_\eta = \left( \frac{\nu}{\epsilon} \right)^{1/2} \).

\( \epsilon \) is the turbulent energy dissipation rate.

Scaling:

\[
\frac{L}{\eta} \sim L \left( \frac{U^3 L}{\nu^3} \right)^{1/4} = Re^{3/4}
\]

\[
\frac{\tau_0}{\tau_\eta} \sim \tau_0 \left( \frac{U^3}{L \nu} \right)^{1/2} = Re^{1/2}
\]

For a flow with \( Re \sim 10,000 \):
Flow scales span 3 orders of magnitude in length and 2 orders of magnitude in time.

Turbulent flows contain a vast range of length and time scales that must be resolved!!! Difficult!!
Laminar Flows and Turbulence Flows

• Laminar flow, sometimes known as streamline flow, occurs when a fluid flows in parallel layers, with no disruption between the layers. Viscosity determines momentum diffusion.
  – In nonscientific terms laminar flow is "smooth," while turbulent flow is "rough."

• Turbulent flow is a fluid regime characterized by chaotic, stochastic property changes. Turbulent motion dominates diffusion of momentum and other scalars. The flow is characterized by rapid variation of pressure and velocity in space and time.
  – Flow that is not turbulent is called laminar flow
Turbulent flows in a pipe

\[ \text{Re} = \frac{\rho V D}{\mu} \]
Characterization of Turbulent Flows

\[ u = \bar{u} + u' ; \quad v = \bar{v} + v' \quad w = \bar{w} + w' \]

\[ \bar{u} = \frac{1}{T} \int_{t_0}^{t_0+T} u(x, y, z, t) dt ; \quad \bar{v} = \frac{1}{T} \int_{t_0}^{t_0+T} v(x, y, z, t) dt ; \quad \bar{w} = \frac{1}{T} \int_{t_0}^{t_0+T} w(x, y, z, t) dt \]

**Figure 7.7** Velocity components in a turbulent pipe flow: (a) $x$-component velocity; (b) $r$-component velocity; (c) $\theta$-component velocity.
Turbulence Intensities

\[ \bar{u}' = 0; \quad \bar{v}' = 0 \quad \bar{w}' = 0 \]

\[ (u')^2 = \frac{1}{T} \int_{t_0}^{t_0+T} (u')^2 \, dt > 0; \quad (v')^2 > 0 \quad (w')^2 > 0 \]
Turbulent Shear Stress

Laminar flows:

\[ \tau_{lam} = \mu \frac{\partial u}{\partial y} \]

Turbulent flows:

\[ \tau_{turb} = -\rho u'v' \]

\[ \tau = \tau_{lam} + \tau_{turb} = \mu \frac{\partial u}{\partial y} - \rho u'v' \]

(a) laminar flow

(b) turbulent flow
Quantification of Boundary Layer Flow

Displacement thickness:

\[ \delta^* = \int_0^\infty \left( 1 - \frac{u}{U} \right) \, dy \]

Momentum thickness:

\[ \theta = \int_0^\infty \frac{u}{U} \left( 1 - \frac{u}{U} \right) \, dy \]

at \( y = \delta \),

\[ u = 0.99 U_\infty \]
**Blasius solution for laminar boundary layer:**

\[ \frac{\partial p}{\partial y} \approx 0 \]

\[ \Re_x = \frac{\rho U_\infty X}{\mu} \]

\[ C_f = \frac{1.328}{\sqrt{\Re_x}} \]

\[ \delta = \frac{5.0X}{\sqrt{\Re_x}} \]

\[ \delta^* = \frac{1.72X}{\sqrt{\Re_x}} \]

\[ \theta = \frac{0.664X}{\sqrt{\Re_x}} \]
Turbulent boundary layer:

\[ \frac{\partial p}{\partial y} \approx 0 \]

\[ \tau_w = \mu \left. \frac{\partial U}{\partial y} \right|_{wall} \]

\[ \text{Re}_X = \frac{\rho U_\infty X}{\mu} \]

\[ C_f = \frac{0.074}{(\text{Re}_X)^{1/5}} \]

\[ \delta = \frac{0.37X}{(\text{Re}_X)^{1/5}} \]
Boundary Layer Flows

Which one will induce more drag?
Laminar boundary layer?
Turbulent boundary layer?
Which one will induce more drag? Laminar boundary layer? Turbulent boundary layer?
Laminar Flows and Turbulent Flows

Viscous forces important throughout
Re = $U/D
\nu = 0.1$

Viscosity not important
Viscous effects important

Re = 50
Separation location
Separation bubble

Viscosity not important
Boundary layer separation
Viscous effects important

Re = $10^5$

$C_d = \frac{D}{2 \rho U^2 A}$

Re = $\frac{\rho DU}{\mu}$

$C_d = \frac{24}{Re}$

Smooth cylinder
Smooth sphere

$C_d = \frac{D}{2} b^2 \frac{1}{bD}$

Flat plate
Circle
Ellipse
Airfoil
Float plate

Re = $\frac{UD}{\nu}$

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Flow Around A Sphere with laminar and Turbulence Boundary Layer

Top:
Instantaneous flow past a sphere at $Re_D = 15,000$. Dye in water shows a laminar boundary layer separating ahead of the equator and remaining laminar for almost one radius. It then becomes unstable and quickly turns turbulent.

Bottom:
Instantaneous flow past a sphere at $Re_D = 30,000$ with a trip wire. A classical experiment of Prandtl and Wieselsberger is repeated here, using air bubbles in water. A wire hoop ahead of the equator trips the boundary layer. It becomes turbulent, so that it separates farther rearward than if it were laminar (compare with top photograph). The overall drag is thereby dramatically reduced, in a way that occurs naturally on a smooth sphere only at a Reynolds numbers ten times as great.
Aerodynamics of a golfball

$C_D = \frac{1}{2} \rho v^2 \frac{D^2}{4}$

$\frac{\varepsilon}{D}$ = relative roughness

$\frac{\varepsilon}{D} = 1.25 \times 10^{-2}$
$\frac{\varepsilon}{D} = 5 \times 10^{-3}$
$\frac{\varepsilon}{D} = 1.5 \times 10^{-3}$
$\frac{\varepsilon}{D} = 0$ (smooth)

$Re = \frac{U D}{v}$

Separation
Smooth Sphere
Thick wake
Laminar boundary layer
Transition
Turbulent boundary layer
Golf Ball
Thin wake
Laminar and turbulent flows

Smooth ball

Rough ball

Golf ball

Re = 100,000

Distance (X/D)

Centerline Velocity (U/U∞)

smooth-ball
rough-ball
golf-ball

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Automobile aerodynamics

![Graph showing the change in aerodynamic drag coefficient over time.](image1)

![Image of a modern car in a wind tunnel.](image2)

![Diagram illustrating air flow and pressure distributions.](image3)
Automobile aerodynamics

Mercedes Boxfish

Vortex generator above a Mitsubishi rear window
Flow Separation on an Airfoil

- Separation points
- Turbulent wake
- Separation point moves slightly forward
- Maximum lift
- Separation point jumps forward
- Separated flow region expands and reduces lift
- Large turbulent wake (Reduced lift and large pressure drag)

- Shoulder of airfoil - maximum speed outside of the boundary layer
- Note: Flow outside boundary layer is inviscid flow
- Turbulent boundary layer

- Stagnation point pressure = Total pressure $p_t$
- Transition (laminar becomes turbulent)
- Separation point (Stalled flow)

- Free-stream air flow
- Airfoil surface
- Separation point
- Increasing distance downstream

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Conventional vs Laminar Airfoils

- Laminar flow airfoils are usually thinner than the conventional airfoil.
- The leading edge is more pointed and its upper and lower surfaces are nearly symmetrical.
- The major and most important difference between the two types of airfoil is this, the thickest part of a laminar wing occurs at 50% chord while in the conventional design the thickest part is at 25% chord.
- Drag is considerably reduced since the laminar airfoil takes less energy to slide through the air.
- Extensive laminar flow is usually only experienced over a very small range of angles-of-attack, on the order of 4 to 6 degrees.
- Once you break out of that optimal angle range, the drag increases by as much as 40% depending on the airfoil.

FIGURE 2: Extent of laminar flow on some famous airfoils.
Aerodynamic performance of an airfoil

\[ C_l = \frac{L}{\frac{1}{2} \rho V_w^2 c} \]

Airfoil stall

Before stall

\[ C_{d} = \frac{D}{\frac{1}{2} \rho V_w^2 c} \]

After stall

\[ C_l = 2\alpha \]

Experimental data

\[ C_{d} \]

Experimental data
Flow Separation and Transition on Low-Reynolds-number Airfoils

- Low-Reynolds-number airfoil (with Re<500,000) aerodynamics is important for both military and civilian applications, such as propellers, sailplanes, ultra-light man-carrying/man-powered aircraft, high-altitude vehicles, wind turbines, unmanned aerial vehicles (UAVs) and Micro-Air-Vehicles (MAVs).

- Since laminar boundary layers are unable to withstand any significant adverse pressure gradient, laminar flow separation is usually found on low-Reynolds-number airfoils. Post-separation behavior of the laminar boundary layers would affect the aerodynamic performances of the low-Reynolds-number airfoils significantly.

- Separation bubbles are usually found on the upper surfaces of low-Reynolds-number airfoils. Separation bubble bursting can cause airfoil stall at high AOA when the adverse pressure gradients become too big.
Surface Pressure Coefficient distributions (Re=68,000)

Typical surface pressure distribution when a laminar separation bubble is formed (Russell, 1979)

GA (W)-1 airfoil
(also labeled as NASA LS(1)-0417)
Laminar Separation Bubble on a Low-Reynolds-number Airfoil

PIV measurement results at AOA = 10 deg, Re=68,000

(Hu et al., ASME Journal of Fluid Engineering, 2008)
Active flow control: Plasma actuators

Use methods to actively control boundary layer separation:

- Suction
- Blowing
- **Plasma actuators**: Applies a body force acting on weakly ionized air to couple momentum into the flow
- Synthetic Jets

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**Figure 1**
Schematic illustration of a single-dielectric barrier discharge plasma actuator (a) and photograph of ionized air at 1-atm pressure that forms over an electrode covered by a dielectric layer (b).

**Figure 12**
Particle-image-velocimetry images of the flow behind a circular cylinder at $Re_D = 33,000$ with plasma actuators on the lee side of the cylinder off (a) and on (b). Figure taken from Thomas et al. 2008.
Active flow control: Synthetic Jets

Synthetic jets: zero mass flux actuators that inject momentum into the flow


Figure 1  (a) Schematic diagram of a synthetic jet actuator and (b) Schlieren image of a rectangular synthetic jet. $Re_{D} = 18, 124$ ($Re_{D_{0}} = 383$), $b = 0.5$ mm, and $f = 1140$ Hz.

Figure 14  Normalized vorticity, $Re_{D} = 21,500$, $y = 63^\circ$, $C_{J} = 5.1 \times 10^{-3}$, $f = 0.035$.
(a) baseline; actuated: (b) phase locked and (c) time averaged.

Figure 7  Smoke of the flow around a circular cylinder visualization: (a) baseline; and (b) actuated: $\phi = 0$, $y = 60^\circ$ and (c) $180^\circ$, and (d) $\phi = 120^\circ$, $y = 180^\circ$. 
Passive flow control: Shark Skin

Dean & Bhushan, Phil. Trans. R. Soc. A (2010) 368, 4775–4806
Vortex Structure over Riblets
Shark Skin
Shark Skin Swimming Suits

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Forces on CV = Fluid momentum change

Forces on CV:
\[ \sum F_x = -D + \int_{CS} (p\hat{n}dA)_x = -D + \int p_{up} dA - \int p(y) dA \]

Since \( p_{up} = p_\infty \), \( p(y) \approx p_\infty \)
\[ \Rightarrow \sum F_x = -D \]

Momentum change:
\[ \int_2 \rho U(y)(U(y) - U_\infty) dA_2 = \sum F_x = -D \]
\[ \Rightarrow D = \rho U_\infty^2 \int_2 \left[ \frac{U(y)}{U_\infty} \left(1 - \frac{U(y)}{U_\infty}\right) \right] dA_2 \]

\[ C_D = \frac{D}{\frac{1}{2} \rho U_\infty^2 C} = \frac{\rho U_\infty^2}{\frac{1}{2} \rho U_\infty^2 C} \]
\[ \Rightarrow C_D = \frac{2}{C} \int_2 \left[ \frac{U(y)}{U_\infty} \left(1 - \frac{U(y)}{U_\infty}\right) \right] dy \]

Compare with the drag coefficients obtained based on airfoil surface pressure measurements at the same angles of attack!
Pressure rake with 41 total pressure probes (the distance between the probes d=2mm)
Flow Field

\[ \text{Current flow through wire} \]

\[ mc \frac{dT_w}{dt} = i^2 R_w - \dot{q}(V, T_w) \]

- Constant-temperature anemometry

CTA hotwire probe
Hotwire Anemometer Calibration

- Quantify the relationship between the flow velocity and voltage output from the CTA probe

\[ y = a + bx + cx^2 + d x^3 + e x^4 \]

max dev: 0.166, \( r^2 = 1.00 \)

\( a = 10.8, \ b = 3.77, \ c = -26.6, \ d = 13.2 \)
Required Measurement Results

NOTE: We will be using the **GA(W)-1** airfoil from the previous lab for the wake pressure measurements.

Required Plots:

- $C_p$ distribution in the wake (for each angle of attack) for the airfoil wake measurements
- $C_d$ vs angle of attack (do your values look reasonable?) based on the airfoil wake measurements
- Your hot wire anemometer calibration curve: Velocity versus voltage output of hotwire anemometer (including a 4th order polynomial fit)

Please briefly describe the following details:

- How you calculated your drag—you should show your drag calculations
- How these drag calculations compared with the drag calculations you made in the previous experiment
- Reynolds number of tests and the incoming flow velocity