Lecture #2: Measurement Uncertainties

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Labs begin!

- Labs begin this week
- Please do the pre-lab assignment prior to your lab (available on the website)
- Please be at your lab on time
- Lab reports are due the Friday the week after the lab

- Office hours
  - Tuesday 3–4
  - Thursday 3–4

Sources/ Further reading:
Dally & Riley, “Experimental Stress Analysis,” Part V Ch 21
Calibration

- **Calibration**: A calibration applies a known input value to a measurement system for the purpose of observing the system output value. It establishes the relationship between the input and output values.
- The known value used for the calibration is called **standard**.

![Graph showing experimental data and curve fitting](graph.png)

\[ y = a + bx \]

- Maximum deviation: 30.4
- \( r^2 = 0.998 \)
- \( a = 13.2, b = 240 \)
Example

Wind tunnel test section

Pitot tube measured freestream, $U$ [m/s]

Hotwire voltage, $V$ [V]

Least-squares fit,

$U \approx 1.0 + 7.5V + 16.7V^2 + 22.4V^3$ [m/s]
• **Instrument Resolution** represents the smallest increment in the measured value that can be discerned by using the instrument. In terms of a measurement system, it is quantified by the smallest scale increase of least count.

• **Dynamic Range** is the ratio of the extent of values that can be measured with the instrument: $\text{DR} = \frac{\text{Full scale}}{\text{Resolution}}$

  Often given as number of “counts” “bits” or “digits”
Measurement Uncertainties

• “Accuracy” is generally used to indicate the relative closeness of agreement between an experimentally-determined value of a quantity and its true value.
• “Error” is the difference between the experimentally-determined value and its true value; therefore, as error decreases, accuracy is said to increase.
• Since the true value is not known, it is necessary to estimate error, and that estimate is called an uncertainty, U.
• Uncertainty estimates are made at some confidence level—e.g., a 95% confidence estimate, means that the true value of the quantity is expected to be within the ±U interval about the experimentally-determined value 95 times out of 100.

\[
A_{\text{error}} = A_{\text{measured}} - A_{\text{true}} \quad \Rightarrow \quad E = A_m - A_{\text{true}}
\]

Which case is the more accurate measurement?

\[
V_t = 10 \text{m/s}, \quad \text{Measurement error } \Delta V = 1 \text{m/s}
\]
\[
V_t = 100 \text{m/s}, \quad \text{Measurement error } \Delta V = 5 \text{m/s}
\]

\[
E_{\text{relative}} = \frac{A_{\text{error}}}{A_{\text{true}}}
\]
Measurement Uncertainties

- Total error, $U$, can be considered to be composed of two components:
  - a random (precision) component,
  - a systematic (bias) component,
  - We usually don’t know these exactly, so we estimate them with $P$ and $B$, respectively.

- Precision Error: Random error
  - e.g., Normal Distribution or Gaussian Distribution

- Bias Error: Fixed error, system error
  - Constant throughout the experiment
  - Can be positive or negative

\[ U^2 = B^2 + P^2 \]
Measurement Uncertainties

- Precise but biased
- Unbiased but imprecise
- Biased and imprecise
- Precise and unbiased

Qualification of measurement error:

\[ E^2 = B^2 + P^2 \]
Measurement Results

Experimental data curve fitting

$y = a + bx$ max dev: 30.4, $r^2 = 0.998$

$a = 13.2$, $b = 240$

Precision error: R.M.S of your data

Bias error
• **Repeatability** is the variability of the measurements obtained by one person while measuring the same item repeatedly. This is also known as the inherent precision of the measurement equipment.

Consider the probability density functions shown in Figure 1. The density functions were constructed from measurements of the thickness of a piece of metal with Gage A and Gage B. The density functions demonstrate that Gage B is more repeatable than Gage A.
Repeatability and Reproducibility

- **Reproducibility** is the variability of the measurement system caused by differences in operator behavior. Mathematically, it is the variability of the average values obtained by several operators while measuring the same item.

Figure 2 displays the probability density functions of the measurements for three operators. The variability of the individual operators are the same, but because each operator has a different bias, the total variability of the measurement system is higher when three operators are used than when one operator is used.

<table>
<thead>
<tr>
<th>Repeatability</th>
<th>Precision Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reproducibility</td>
<td>Both Bias and Precision Errors</td>
</tr>
</tbody>
</table>

![Figure 1. Repeatability](image1)

![Figure 2. Reproducibility](image2)
Measurement Uncertainties

- We almost always are dealing with a data reduction equation to get to our final results.
  - In this case, we must not only deal with uncertainty in the measured values but uncertainty in the final results.

- A general form looks like this:

\[ R = R(X_1, X_2, X_3, \ldots, X_J) \]

- \( R \) is the result determined from \( J \) independent variables.

\[ dR = \frac{\partial R}{\partial X_1} dX_1 + \frac{\partial R}{\partial X_2} dX_2 + \ldots + \frac{\partial R}{\partial X_J} dX_J \]

“Propagation of errors”
Uncertainty in velocity $V$:

$$U^2_R = B^2_R + P^2_R$$

$$B^2_R = \sum_{i=1}^{J} \left[ \frac{\partial R}{\partial X_i} B_i \right]^2; \quad P^2_R = \sum_{i=1}^{J} \left[ \frac{\partial R}{\partial X_i} P_i \right]^2$$

$$B_i = \sqrt{\sum_{j=1}^{M} B_{i,j}^2}$$

For a large number of samples ($N>10$) $P_i = 2S_i$

$$S_i = \left[ \frac{1}{N-1} \sum_{k=1}^{N} [(X_i)_k - \overline{X}_i]^2 \right]^{1/2}; \quad \overline{X}_i = \frac{1}{N} \left[ \sum_{k=1}^{N} (X_i)_k \right]$$

$$p_{total} = p_{static} + \frac{1}{2} \rho V^2, \text{ (Bernoulli)}$$

$$V = \sqrt{\frac{2(p_{total} - p_{static})}{\rho}} = \sqrt{\frac{2\Delta p}{\rho}}$$
Example

For a large number of samples ($N > 10$)

$$S_i = \left[ \frac{1}{N-1} \sum_{k=1}^{N} \left( (X_i)_k - \bar{X}_i \right)^2 \right]^{\frac{1}{2}}; \quad \bar{X}_i = \frac{1}{N} \sum_{k=1}^{N} (X_i)_k$$

$$p_{total} = p_{static} + \frac{1}{2} \rho V^2, \text{(Bernoulli)}$$

$$V = \sqrt{\frac{2(p_{total} - p_{static})}{\rho}} = \sqrt{\frac{2\Delta p}{\rho}}$$
Lab #1: Flow visualization by using smoke wind tunnel

- Path line
- Streak lines
- Streamline
Lab #1: Flow visualization by using smoke wind tunnel

Streamlines (experiment)