AerE 344: Undergraduate Aerodynamics and Propulsion Laboratory

Lab Instructions

Lab #08: Visualization of the Shock Waves in a Supersonic Jet by using Schlieren technique

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AerE344 Lab09: Visualization of Shock Waves in a Supersonic Jet by using Schlieren technique

Why look at flow through a nozzle?

The nozzle is one of the most important physical systems on a rocket. Whatever technology drives the propellant, the nozzle is where that combustion energy is turned into thrust, and the rocket is ultimately dependent on the nozzle for its performance.

A given nozzle will produce the most thrust at a specific altitude: only when the exit pressure of the nozzle and the ambient pressure are matched does the nozzle reach its peak performance. At this point, called third critical, flow exits smoothly from the nozzle with no shock waves. At altitudes above or below the design altitude, shock waves will develop outside, or inside the nozzle, respectively.

This would present no difficulty if rockets were mainly used at one altitude, but nozzle efficiency will drop off severely above or below the altitude for which it was designed. This problem of matching the exit pressure to the ambient pressure is why the area ratios of the Shuttle Main Engine and the Solid Rocket Boosters are so different. The SRBs, which run through the lower portion of the ascent have an area ratio of about 7:1, while the main engines, which fire the entire ascent, have a much higher design altitude and thus a higher area ratio of ~77:1. This also the reason for the translatable “skirt” some rockets employ. These allow the nozzle to have more than one design altitude, and thus, travel further and faster with less propellant.

Photo courtesy of NASA GSFC
Quasi-1D Nozzle Review

Example: Want to find P, T, M, etc. given \( P_o, P_e, \) and nozzle shape.

Quasi – Area is allowed to vary along x coordinate, but flow variables are functions of x only.

\[
\text{Quasi 1D} \quad \begin{array}{c}
1 \\
\text{2}
\end{array} \\
\text{1D} \quad \begin{array}{c}
1 \\
\text{2}
\end{array}
\]

Start out with governing conservation equations:

Mass:
\[
\iiint_V \frac{\partial \rho}{\partial t} dV + \int_S \rho \vec{U} \cdot \vec{n} dS = 0 \tag{1.1}
\]

Momentum:
\[
\frac{\partial}{\partial t} \iiint_V \rho \vec{U} dV + \int_S \rho \left( \vec{U} \cdot \vec{n} \right) \vec{U} dS = -\int_S p dS + \iiint_V \rho \vec{f} dV + F_{\text{viscous}} \tag{1.2}
\]

Energy:
\[
\frac{\partial}{\partial t} \iiint_V \rho \left( e + \frac{U^2}{2} \right) dV + \int_S \rho \left( e + \frac{U^2}{2} \right) \vec{U} \cdot \vec{n} dS = -\int_S p \vec{U} \cdot \vec{n} dS + \iiint_V \rho \frac{\partial q}{\partial t} dV + \iiint_V \rho \left( \vec{f} \cdot \vec{U} \right) dV \tag{1.3}
\]

Assuming:
1. steady
2. inviscid
3. no body forces
4. 2D flow

Quasi-1D: Area is allowed to vary but flow variables are a function of x only

\[
A, u, \rho \quad \text{A+}dA, u+du, \rho +d\rho
\]

Mass:
\[-\rho uA + (u + du)(\rho + d\rho)(A + dA) = 0\]  
(1.4)

\[-\rho u A + \rho u A + \rho u dA + \rho du A + d\rho u A + \text{higher order terms} = 0\]  
(1.5)

Divided by through \(\rho u A\)

\[\frac{dA}{A} + \frac{du}{u} + \frac{d\rho}{\rho} = 0\]

or

\[d(\rho u A) = 0\]  
(1.6)

Momentum:

\[-\rho u^2 A + (\rho + d\rho)(u + du)(A + dA) = PA - (P + dP)(A + dA) + 2 \left(\frac{PdA}{2}\right)\]  
(1.7)

\[-\rho u^2 A + \rho u^2 A + \rho u dA + u^2 Ad \rho + \rho u Ad u + \rho u Ad u = PA - PA - PdA - AdP + PdA\]  
(1.8)

\[u\left(\rho u dA + u Ad \rho + \rho Ad u\right) + \rho u Ad u = -AdP\]  
(1.9)

\[dP = -\rho ud\]  
(1.10)

\[\frac{dP}{\rho} = \frac{dP}{d\rho} \frac{d\rho}{\rho} = -ud\]  
(1.11)

\[\frac{d\rho}{\rho} = -\frac{u}{a^2} du\]  
(1.12)

Substituting equation (1.12) into (1.6) to get:

\[\frac{dA}{A} + \frac{du}{u} - \frac{u}{a^2} du = 0\]  
(1.13)

which can be rearranged to get:

\[\frac{dA}{A} + \frac{du}{u} \left(1 - \frac{u^2}{a^2}\right) = \frac{dA}{A} + \frac{du}{u} (1 - M^2) = 0\]  
(1.14)

\[\frac{dA}{A} = \frac{du}{u} (M^2 - 1)\]  
(1.15)

which can be used to determine general flow behavior in a converging-diverging nozzle, as below:
Energy:

We will not go through the derivation for the energy equation but, applying analysis as before will give:

\[ dh + u du = 0 \quad \text{or} \quad c_p T = -u du \]  \hspace{1cm} (1.16)

**Total-Static-Mach Relations**

Isentropic Relations:

\[ \frac{P_2}{P_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma = \left( \frac{T_2}{T_1} \right)^{\gamma} \]  \hspace{1cm} (1.17)

**Static-Total**

From Energy Equation with \( u_o = 0 \) (total)

\[ c_p T_o = c_p T_1 + \frac{u_1^2}{2} \]  \hspace{1cm} (1.18)

Rearranging

\[ \frac{T_o}{T} = 1 + \frac{u^2}{2c_p T} \]  \hspace{1cm} (1.19)

using \( c_p = \gamma R / \gamma - 1 \),
\[
\frac{T_\infty}{T} = 1 + \frac{\gamma - 1}{2} \frac{u^2}{\gamma RT}
\]  
(1.20)

Inserting \( a = \sqrt{\gamma RT} \) and \( M = u/a \) gives

\[
\frac{T_\infty}{T} = 1 + \frac{\gamma - 1}{2} M^2
\]  
(1.21)

\[
\frac{P_\infty}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}
\]  
(1.22)

\[
\frac{\rho_\infty}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}
\]  
(1.23)

Given Eq.’s (1.21),(1.22), and (1.23) we can now:

- Find any static property in an isentropic flow given Mach #, \( P_\infty, T_\infty, \) and \( \rho_\infty. \)
- Use/control known total conditions to find mach # through nozzle

**Area-Mach Relations**

From mass

\[
\rho^* u^* A^* = \rho u A
\]  
(1.24)

at \( A^* M = \frac{u^*}{a^*} = 1 \rightarrow u^* = a^* \), giving

\[
\left(\frac{A}{A^*}\right)^2 = \left(\frac{\rho^*}{\rho}\right)^2 \left(\frac{a^*}{u}\right)^2
\]  
(1.25)

or

\[
\left(\frac{A}{A^*}\right)^2 = \left(\frac{\rho^*}{\rho}\right)^2 \left(\frac{P^*}{P}\right)^2 \left(\frac{a^*}{u}\right)^2
\]  
(1.26)

using isentropic relations for the density terms

\[
\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{\frac{\gamma + 1}{\gamma - 1}}
\]  
(1.27)

Mach # is a function of this area ratio only. Must find \( A^*. \)
**Completely Subsonic Flow**

\[ P_e = P_{\text{atm}} \]  \hspace{1cm} (1.28)

From isentropic relation

\[ M_e = \sqrt{\frac{2}{\gamma-1} \left( \left( \frac{P_o}{P_e} \right)^{\gamma-1} - 1 \right)} \]  \hspace{1cm} (1.29)

Can now determine \( A^* \) and entire Mach # distribution

If: \( \frac{P_o}{P_e} = 1 \) Then: \( M_e = 0, \frac{A_e}{A^*} \to \infty, A^* = 0 \) NO FLOW

As \( \frac{P_o}{P_e} \) increases, \( M_e \) increases, \( \frac{A_e}{A^*} \) decreases, \( A^* \) increases

Note that \( A/A^* < 1 \) is not physically possible. That is, after 1\(^{\text{st}}\) critical is reached, must have A\(_{\text{min}}\) = A*

**Supersonic Flow**

Subsonic Flow ahead of throat. Follow supersonic \( A/A^* \) branch after throat.

\[ A^* = A_i \]  \hspace{1cm} (1.30)

\[ M_e = f\left( \frac{A_e}{A^*} \right) \]  \hspace{1cm} (1.31)

\[ P_e = P_o \left( 1 + \frac{\gamma-1}{2} M_e^2 \right)^{\frac{1}{\gamma-1}} \]  \hspace{1cm} (1.32)
Suppose we pick Po so that Pe = Patm

- If we decrease Po, then Pe < Patm because Me is unchanged. Need weak oblique shocks to get a small pressure jump.
- As Po decreases, need stronger oblique shocks until normal shock at exit, 2nd critical.
- As Po decreases, shock moves up the nozzle. Eventually get to 1st critical.
- Increasing Po from 3rd critical, Pe > Patm. Get Prandtl-Meyer expansion fan to get pressure decrease

Summary:

For the nozzle used in the lab:

\[
\text{\(P_{o,3rd} \approx 60\text{ psig}\)}
\]
\[
\text{\(P_{o,2nd} \approx 20\text{ psig}\)}
\]
\[
\text{\(P_{o,1st} \approx 8\text{ psig}\)}
\]
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(a). Before turning on the Supersonic jet

(b). After turning on the Supersonic jet

Schematic of the Z-type Schlieren system used in the present experiment

Under-expanded flow

Flow close to 3rd critical

Over-expanded flow
Schlieren images of the shock waves in the supersonic jet flow

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Writeup Guidelines

This is a demonstration experiment. There is not lab report required for this lab!