Lecture # 2: Measurement Uncertainties

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**Calibration**

- **Calibration**: A calibration applies a known input value to a measurement system for the purpose of observing the system output value. It establishes the relationship between the input and output values.
- The known value used for the calibration is called **standard**.

![Experimental data curve fitting](image)

\[ y = a + bx \]

- Maximum deviation: 30.4
- \( r^2 = 0.998 \)
- \( a = 13.2 \), \( b = 240 \)
Instrument Resolution

- **Instrument Resolution** represents the smallest increment in the measured value that can be discerned by using the instrument. In terms of a measurement system, it is quantified by the smallest scale increment of least count.
Measurement Uncertainties

- “Accuracy” is generally used to indicate the relative closeness of agreement between an experimentally-determined value of a quantity and its true value.
- “Error” is the difference between the experimentally-determined value and its true value; therefore, as error decreases, accuracy is said to increase.
- Since the true value is not known, it is necessary to estimate error, and that estimate is called an uncertainty, $U$.
- Uncertainty estimates are made at some confidence level—a 95% confidence estimate, for example, means that the true value of the quantity is expected to be within the $\pm U$ interval about the experimentally-determined value 95 times out of 100.

\[
A_{\text{error}} = A_{\text{measured}} - A_{\text{true}} \quad \Rightarrow \quad E = A_m - A_{\text{true}}
\]

Which Case is more accurate measurement?

- $V_t = 10 \, \text{m/s}$, Measurement error $\Delta V = 1 \, \text{m/s}$
- $V_t = 100 \, \text{m/s}$, Measurement error $\Delta V = 5 \, \text{m/s}$
Measurement Uncertainties

- Total error, $U$, can be considered to be composed of two components:
  - a random (precision) component,
  - a systematic (bias) component,
  - We usually don’t know these exactly, so we estimate them with $P$ and $B$, respectively.

- Precision Error: Random error
  - Normal Distribution or Gaussian Distribution

- Bias Error: Fixed Error, System Error
  - Constant Throughout the experiment
  - Can be positive or Negative

$$U^2 = B^2 + P^2$$
Measurement Uncertainties

- Precise but biased
- Unbiased but Imprecise
- Biased and Imprecise
- Precise and Unbiased

Qualification of measurement error:

\[ E^2 = B^2 + P^2 \]
**Repeatability and Reproducibility**

- **Repeatability** is the variability of the measurements obtained by one person while measuring the same item repeatedly. This is also known as the inherent precision of the measurement equipment.
  - Consider the probability density functions shown in Figure 1. The density functions were constructed from measurements of the thickness of a piece of metal with Gage A and Gage B. The density functions demonstrate that Gage B is more repeatable than Gage A.

- **Reproducibility** is the variability of the measurement system caused by differences in operator behavior. Mathematically, it is the variability of the average values obtained by several operators while measuring the same item.
  - Figure 2 displays the probability density functions of the measurements for three operators. The variability of the individual operators are the same, but because each operator has a different bias, the total variability of the measurement system is higher when three operators are used than when one operator is used.

<table>
<thead>
<tr>
<th>Repeatability</th>
<th>Precision Error</th>
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<tbody>
<tr>
<td>Reproducibility</td>
<td>Both Bias and Precision Errors</td>
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• **We almost always are dealing with a data reduction equation to get to our final results.**
  – In this case, we must not only deal with uncertainty in the measured values but uncertainty in the final results.

• **A general form looks like this:**

\[ R = R(X_1, X_2, X_3, \ldots, X_J) \]

  – *R* is the result determined from *J* independent variables.
**Uncertainty in velocity V:**

\[ U_R^2 = B_R^2 + P_R^2 \]

\[ B_R^2 = \sum_{i=1}^{J} \left[ \frac{\partial R}{\partial X_i} B_i \right]^2; \quad P_R^2 = \sum_{i=1}^{J} \left[ \frac{\partial R}{\partial X_i} P_i \right]^2 \]

\[ B_i = \sqrt{\sum_{j=1}^{M} B_{i,j}^2} \]

For a large number of samples (\( N > 10 \))

\[ P_i = 2S_i \]

\[ S_i = \left[ \frac{1}{N-1} \sum_{k=1}^{N} [(X_i)_k - \bar{X}_i]^2 \right]^{1/2} \]

\[ \bar{X}_i = \frac{1}{N} \sum_{k=1}^{N} (X_i)_k \]

**Bernoulli's Equation**

\[ p_{total} = p_{static} + \frac{1}{2} \rho V^2, \text{(Bernoulli)} \]

\[ V = \sqrt{\frac{2(p_{total} - p_{static})}{\rho}} = \sqrt{\frac{2\Delta p}{\rho}} \]
Measurement Results

Experimental data curve fitting

\[ y = a + bx \]

Max dev: 30.4, \( r^2 = 0.998 \)

\[ a = 13.2, \ b = 240 \]

Bias error

Precision error: R.M.S of your data
Lab #1: Flow visualization by using smoke wind tunnel

- **Path line**
- **Streak lines**
- **Streamline**

![Flow visualization diagram with path line, streak lines, and streamline examples](image-url)
Lab #1: Flow visualization by using smoke wind tunnel

Streamlines (experiment)