Lecture #06  Hotwire anemometry: Fundamentals and instrumentation

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• **Thermal anemometers:**
  - Measure the local flow velocity through its relationship to the convective cooling of electrically heated metallic sensors.

• **Hot wire anemometers:**
  - for clean air or other gas flows

• **Hot film anemometers:**
  - for liquid or some gas flows
How a Hot wire Sensor Works

The electric current \( i \) flowing through the wire generates heat \( (i^2R_w) \)

Flow Field

In equilibrium, this must be balanced by heat lost (primarily convective) to the surroundings.

Electric current, \( i \), through wire
**Technical Fundamentals**

- **Heat transfer characteristics:**
  - Convection (nature convection, forced convection or mixed convection depending on Richardson numbers)
  - Conduction to the supporting prong
  - Radiation: <0.1%, is negligible.

\[
Nu = \frac{\dot{q}}{\pi k (T_w - T)}
= Nu (Re, Pr, Gr, M, Kn, a_T, l / d, \theta)
\]

\[
Re = \frac{\rho Ud}{\mu}; \quad Pr = \frac{\nu}{\gamma}
\]

\[
Gr = \frac{g \alpha (T_w - T) d^3}{\nu^2}; \quad M = \frac{V}{c}
\]

\[
Kn = \frac{\lambda}{d} = \sqrt{\frac{1}{2} \frac{\pi c_p}{c_v} \frac{M}{Re}}
\]

\[
a_T = \frac{T_w - T}{T}
\]
Following King’s Law (1915),

\[ Nu = (A + B \text{Re}^n)(1 + \frac{1}{2}a_T)^m \]

\[ Nu = (0.24 + 0.56\text{Re}^{0.45})(1 + \frac{1}{2}a_T)^{0.17}, \quad \text{for} \quad 44 < \text{Re} < 140 \]

\[ Nu = 0.48\text{Re}^{0.51}(1 + \frac{1}{2}a_T)^{0.17}, \quad \text{for} \quad 0.02 < \text{Re} < 44 \]

According to Collis and Williams (1959):

For a given sensor and fixed overheat ratio, The above equation can transfer as the relationship between the voltage output, \( E \), of the hot-wire operation circuit and the flow velocity

\[ \frac{E}{T_w - T} = A + BV^n \]

Wire temperature cannot be measured directly, but can be estimated from its relationship to the wire resistance, \( R_w \), directly measured by the operating bridge. For metallic wires:

\[ R_w = R_r [1 + a_r (T_w - T)] \]

\( a_r \): thermal resistivity coefficient
\( T_r \): reference temperature
The hot wire is electrically heated.

If velocity changes for a unsteady flow, convective heat transfer changes, wire temperature will change and eventually reach a new equilibrium.

The rate of which heat is removed from the sensor is directly related to the velocity of the fluid flowing over the sensor.
For a sensor placed in a unsteady flow, the unsteady energy equation will become:

\[ mc \frac{dT_w}{dt} = i^2 R_w - \dot{q}(V, T_w) \]

- \[ m \]: the mass of the sensor
- \[ c \]: specific heat of the sensor
- \[ \dot{q} \]: convective heat flux \[ \dot{q} = \dot{q}(V, T_w) \]

The above equation has three unknowns: \( i, T_w \) (or \( R_w \)) and \( V \)

To render this equation solvable, one must keep with the electric current, \( i \), or the sensor temperature \( (T_w) \) constant, which can be achieved with the use of suitable electric circuits.

The corresponding methods are known as:

1. Constant-current anemometry
2. Constant-temperature anemometry
The unsteady energy equation is highly-nonlinear. When linearized in the vicinity of an operation point, namely at a particular flow speed, \( V_{op} \) and sensor temperature, \( T_{wop} \), it leads to the following first-order differential equation:

\[
\tau_w \frac{dT_w}{dt} + (T_w - T_{wop}) = K_T (V - V_{op})
\]

\( \tau_w \) : a time constant, which is proportional to the overheat ratio, and a static sensitivity, \( K_T \)

Since voltage, \( E \), is proportional to, \( Rw \), which, in turn, is linearly related to \( Tw \), the linearized \( E \)-\( V \) relationship will be:

\[
\tau_w \frac{dE}{dt} + (E - E_{op}) = K(V - V_{op})
\]

\( \tau_w \) : is usually ~ 1ms for thin hot-wire and ~ 10 ms for slim cylindrical hot-film.

For flow with variable velocity or temperature, overheat ratio will vary as well.
Flow low speed flow, it may result in “burnout”, for high-speed flow, sensitivity is low.
The unsteady energy equation is highly-nonlinear. When linearized in the vicinity of an operation point, namely at a particular flow speed, $V_{op}$, and sensor temperature, $T_{wop}$, it leads to the following first-order differential equation:

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Since voltage, $E$, is proportional to, $R_w$, which, in turn, is linearly related to $T_w$, the linearized $E$-$V$ relationship will be:

$$\tau_w \frac{dE}{dt} + (E - E_{op}) = K(V - V_{op})$$

$\tau_w$: is usually ~ 1ms for thin hot-wire and ~ 10 ms for slim cylindrical hot-film.

For flow with variable velocity or temperature, overheat ratio will vary as well. Flow low speed flow, it may result in “burnout”, for high-speed flow, sensitivity is low.
• Electric current through the sensor is adjustable continuously through an electric feedback system, and in response to the changes in convective cooling, to make the temperature of the hot wire keep in constant.

• The unsteady energy equation becomes steady equation

• Dynamic response of the anemometer is the same as its static response with a wide frequency range.

\[ mc \frac{dT_w}{dt} = i^2 R_w - \dot{q}(V, T_w) \Rightarrow i^2 R_w - \dot{q}(V) = 0 \]
Sensor, Rw, comprises one leg of the Wheatstone bridge.

An adjustable decade resistor array, Rd, compress opposite leg of the bridge.

The bridge ratio $R_2/R_1$ is fixed, and $R_2/R_1 \approx 10$ to 20 to make sure to supply most of the available power to the sensor.

The two midpoints of the bridge are connected the input of a high-gain, low noise differential amplifier, whose out put is fed back to the top of the bridge.

If $R_2/R_d = R_1/R_w$, then $E_B - E_w = 0$, the amplifier output will be zero.

If $R_d$ is increased to a value $R_{d}'$, the resulting bridge imbalance will generate an input imbalance to the amplifier.

The amplifier will create some current through both legs of the bridge. The additional current through the hot wire will create additional joule heating, which tend to increase its temperature and thus its resistance, until the resistance increasing sufficiently to balance the bridge once more.
Various effects and error source

- **Velocity orientation effects:**
  - Effective cooling velocity \( V_{\text{eff}} = V \cos \theta \).
  - In reality, flow velocity tangential to the sensor would result in cooling.
  - \( V_{\text{eff}} = V (\cos^2 \theta + k^2 \sin^2 \theta)^{1/2} \)
  - Typical values of \( k^2 \) are 0.05 and 0.20.
Various effects and error source

**Prong interference effects:**

- **Interference of the prongs and the probe body may produce additional complications of the heat transfer characteristics.**
- **For example a stream in binormal direction will produce higher cooling than a stream with the same velocity magnitude but in the normal direction.**
- **In reality,** \( V_{\text{eff}} = (V_N^2 + K^2 V_T^2 + h^2 V_B^2)^{1/2} \)
- **\( V_N, V_T \) and \( V_B \) are the normal tangential and binormal velocity components.**
- **Typically,** \( h^2 = 1.1 \sim 1.2 \)
- **To minimize the effect, it usually use long and thin prongs. Tapered prongs are also recommended.**
Various effects and error source

- **Heat conduction effects:**
  - Previous analysis is based on 2-D assumption with $l/d = \infty$.
  - In reality, the effect of end conduct may affect the accuracy of the measurement results.
  - Cold length, $l_c = 0.5d \left( \frac{K_w^2}{K} \left( 1 + a_R \right) / \text{Nu} \right)^{1/2}$
  - $K_w$ is thermal conductivity of the sensor
  - $K$ is thermal conductivity of the fluid
  - $a_R$ is overheat ratio
  - Effect of the sensor length $l/l_c$
  - A recent study has demonstrated that end conduction effects are expected to decrease significantly as the Reynolds number increasing.
Various effects and error source

- **Compressibility effects:**
  - The velocity and temperature fields around the sensor become quite complicated when $M > 0.6$.

  \[
  V \Rightarrow S_V \\
  \rho \Rightarrow S_\rho \\
  T_0 \Rightarrow S_{T_0}
  \]

  For $M \geq 1.2$ \( S_V = S_\rho \)

**Modified King's law for compressible flow:**

\[
E^2 = A + B(\rho V)^n
\]

\( n \approx 0.55 \)
Various effects and error source

- **Temperature variation effects:**
  - Calibration at Temperature $T_1$.
  - Correlation is needed if real measurements will be conducted at Temperature $T_2$.
  - When the flow temperature varies from position to position or contain turbulent fluctuations, corrections is much more complicated.
  - It requires simultaneous flow temperature measurements.
  - $S_v$ is increasing with overheat ratio $a_T$.
  - At extremely low $a_T$, a thermal anemometer is totally insensitive to velocity variations, and becomes a resistance thermometer. The sensor is called cold wire.
Various effects and error source

- **Composition effects:**
  - Composition of flow may affect the convective heat transfer from a thermal anemometer in as much as it affect the heat conductivity of surrounding fluid.
  - It requires simultaneous measurements of fluid species concentration.
Various effects and error source

- Reverse flow and high-turbulence effects:
  - thermal anemometer could not resolve velocity orientation.
  - Forward flow can not be identified from reversing flow
  - In highly turbulent flow (turbulent intensity >25%), reverse flow will occur statistically some time, therefore, using thermal anemometer for the flow velocity measurement may result quite large measurement uncertainty.
  - Pulsed Hot–wire concept
• **Cross-wire (X-wire) design:**

\[
V_{\text{eff-}A} = \frac{\sqrt{2}}{2} (V_1 + V_2)
\]

\[
V_{\text{eff-}B} = \frac{\sqrt{2}}{2} (V_1 - V_2)
\]

\[
V_1 = \frac{\sqrt{2}}{2} (V_{\text{eff-}A} + V_{\text{eff-}B})
\]

\[
V_2 = \frac{\sqrt{2}}{2} (V_{\text{eff-}A} - V_{\text{eff-}B})
\]
Multi-sensor probes

- Three sensor design
- Four sensor design:

Figure 11.7. Sketches of multi-sensor hot-wire probes for three-dimensional velocity measurement: (a) a three-sensor probe and (b) and (c) two four-sensor probes; the probe shown in (c) may be also used for streamwise vorticity measurement.
Diameter of hot wires

- $L = 0.8 \sim 1.5 \text{ mm}$
- $D = \sim 5 \mu m$ for conventional applications
- $D = \sim 10 \mu m$ for high-speed applications
- $D = \sim 2 \mu m$ for low speed applications
- Prongs: usually tapered to be $d \leq 1 \text{ mm}$
Lecture #05  Determination of the Aerodynamic Performance of a Low Speed Airfoil based on Pressure Distribution Measurements

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Aerodynamic Performance of An Airfoil

\[ C_l = \frac{L}{\frac{1}{2} \rho V_a^2 c} \]

\[ C_d = \frac{D}{\frac{1}{2} \rho V_a^2 c} \]

Before stall

After stall

Airfoil stall

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Determination of the Aerodynamic Performance of a Low Speed Airfoil based on Pressure Distribution Measurements

low pressure

high pressure

E 64

total pressure force

lift

direction of onset flow

drag

UPPER SURFACE LIFT

CENTER OF PRESSURE

LOWER SURFACE FORCE

SYMMETRICAL AIRFOIL AT ZERO LIFT

UPPER SURFACE LIFT

CENTER OF PRESSURE

LOWER SURFACE FORCE

SYMMETRICAL AIRFOIL AT POSITIVE LIFT

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Determination of the Aerodynamic Performance of a Low Speed Airfoil based on Pressure Distribution Measurements
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\[ C_p = \frac{P - P_\infty}{\frac{1}{2} \rho V_\infty^2} \]

NACA0012 airfoil with 49 pressure tabs

Before stall

After stall
Determination of the Aerodynamic Performance of a Low Speed Airfoil based on Pressure Distribution Measurements

What you will have available to you for this portion of the lab:

• A Pitot probe already mounted to the floor of the wind tunnel for acquiring dynamic pressure throughout your tests.

• A Setra manometer to be used with the Pitot tube to measure the incoming flow velocity.

• A thermometer and barometer for observing ambient lab conditions (for calculating atmospheric density).

• A computer with a data acquisition system capable of measuring the voltage from your manometer.

• The pressure sensor you calibrated last week

• A NACA 0012 airfoil that can be mounted at any angle of attack up to 15.0 degrees.

• Two 16-channel Scanivalve DSA electronic pressure scanners.
Determining the Aerodynamic Performance of a Low Speed Airfoil based on Pressure Distribution Measurements

- Calculating airfoil lift coefficient and drag coefficient by numerically integrating the surface pressure distribution around the airfoil:

\[
\begin{align*}
 p_{i+1/2} &= \frac{1}{2} (p_i + p_{i+1}) \\
 p_{N+1/2} &= \frac{1}{2} (p_N + p_1)
\end{align*}
\]

\[
\begin{align*}
 \Delta x_i &= x_{i+1} - x_i, \quad \Delta y_i = y_{i+1} - y_i \\
 \Delta x_N &= x_1 - x_N, \quad \Delta y_N = y_1 - y_N
\end{align*}
\]

\[
\begin{align*}
 \partial A'_i &= -p_{i+1/2} \Delta y_i \\
 \partial N'_i &= p_{i+1/2} \Delta x_i \\
 N' &= \sum_{i=1}^{N} \partial N'_i = \sum_{i=1}^{N} p_{i+1/2} \Delta x_i \\
 A' &= \sum_{i=1}^{N} \partial A'_i = -\sum_{i=1}^{N} p_{i+1/2} \Delta y_i \tag{6}
\end{align*}
\]

\[
\begin{align*}
 L' &= N' \cos \alpha - A' \sin \alpha \\
 D' &= N' \sin \alpha + A' \cos \alpha \tag{7}
\end{align*}
\]
Required Plots for the Lab Report

- You must generate plots of $C_p$ for the upper and lower surfaces of the airfoil for the angles of attack that you tested.
- Make comments on the characteristics of the $C_p$ distributions.
- Calculate $C_L$ and $C_D$ by numerical integration $C_p$ for the angles of attack assigned to your group.
- You must report the velocity of the test section and the Reynolds number (based on airfoil chord length) for your tests.
- You must provide sample calculations for all the steps leading up to your final answer.
- You should include the first page of the spreadsheet used to make your calculations.