Lecture # 2: Measurement Uncertainties

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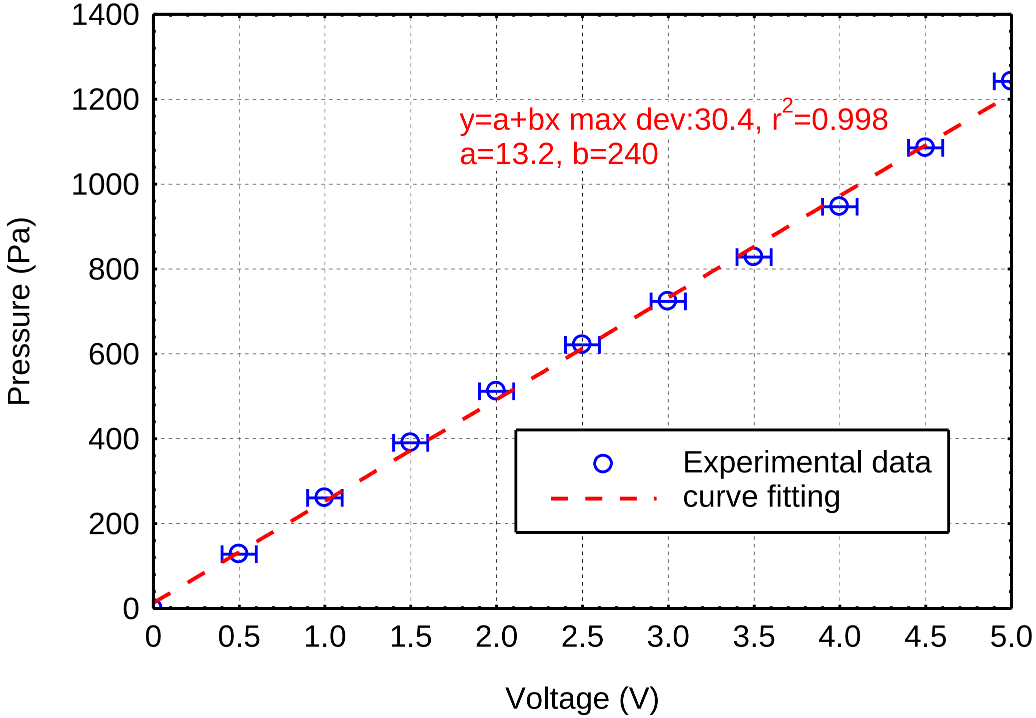
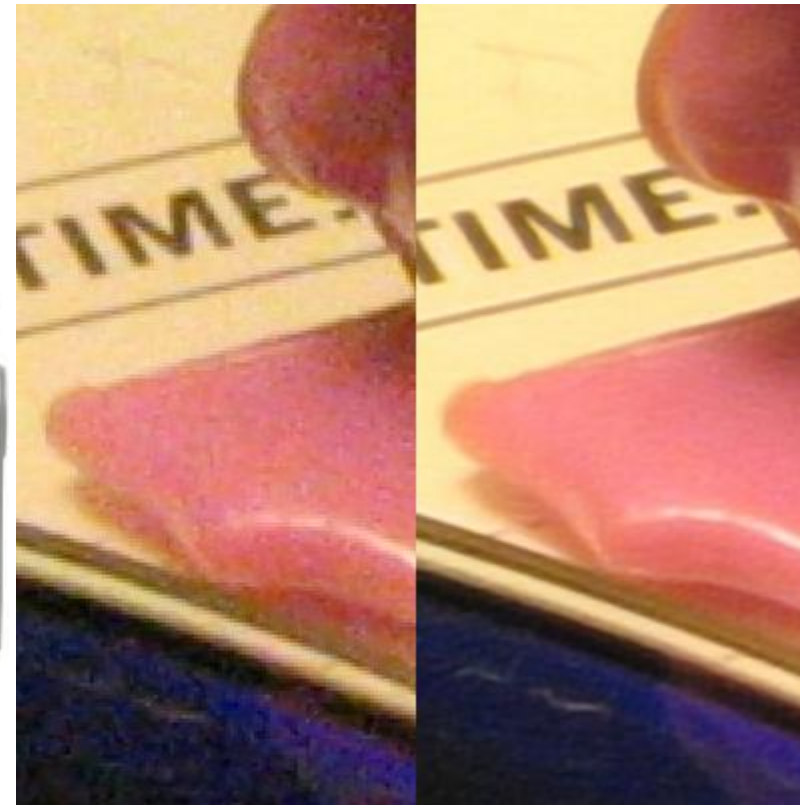
Calibration

- **Calibration: A calibration applies a known input value to a measurement system for the purpose of observing the system output value. It establishes the relationship between the input and output values.**
- **The know value used for the calibration is called standard.**



Instrument Resolution

- ***Instrument Resolution** represents the smallest increment in the measured value that can be discerned by using the instrument. In terms of a measurement system, it is quantified by the smallest scale increscent of least count.*



Measurement Uncertainties

- ***"Accuracy" is generally used to indicate the relative closeness of agreement between an experimentally-determined value of a quantity and its true value.***
- ***"Error" is the difference between the experimentally-determined value and its true value; therefore, as error decreases, accuracy is said to increase.***
- ***Since the true value is not known, it is necessary to estimate error, and that estimate is called an uncertainty, U.***
- ***Uncertainty estimates are made at some confidence level—a 95% confidence estimate, for example, means that the true value of the quantity is expected to be within the ±U interval about the experimentally-determined value 95 times out of 100.***

$$A_{error} = A_{measured} - A_{true} \quad \Rightarrow \quad E = A_m - A_{true}$$

Which case is a more accurate measurement ?

$V_t = 10m/s,$ Measurement error $\Delta V = 1m/s$

$V_t = 100m/s,$ Measurement error $\Delta V = 5m/s$

$$E_{relative} = \frac{A_{error}}{A_{true}}$$

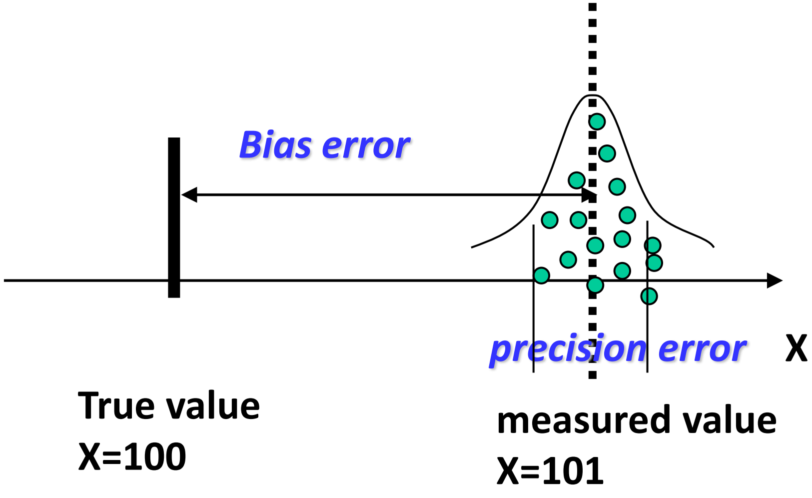
Measurement Uncertainties

- ***Total error, U, can be considered to be composed of two components:***
  - ***a random (precision) component,***
  - ***a systematic (bias) component,***
  - ***We usually don't know these exactly, so we estimate them with P and B, respectively.***

- ***Precision Error: Random error***
  - ***Normal Distribution or Gaussian Distribution***

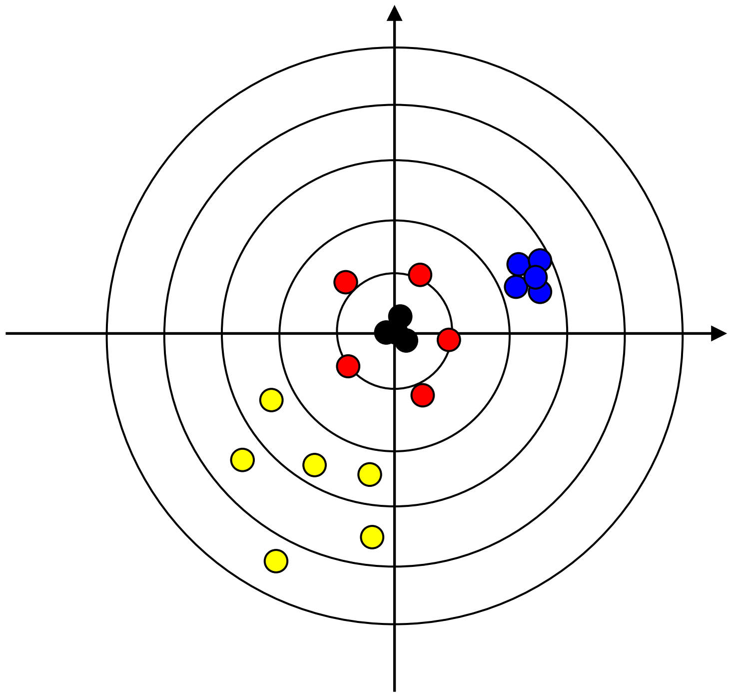
- ***Bias Error: Fixed Error, System Error***
  - ***Constant Throughout the experiment***
  - ***Can be positive or Negative***

$$U^2 = B^2 + P^2$$



Measurement Uncertainties

- ***Precise but biased***
- ***Unbiased but Imprecise***
- ***Biased and Imprecise***
- ***Precise and Unbiased***

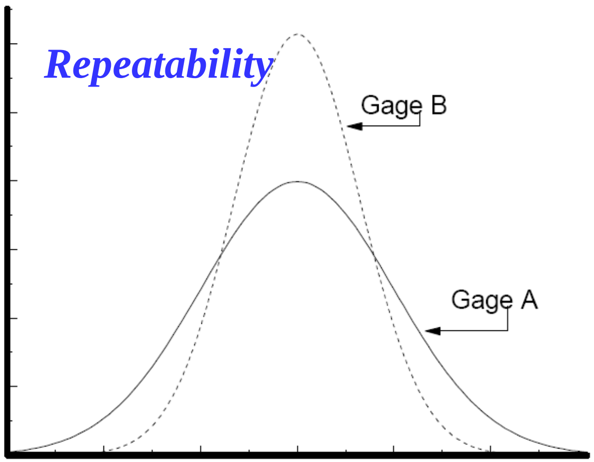


Qualification of measurement error:

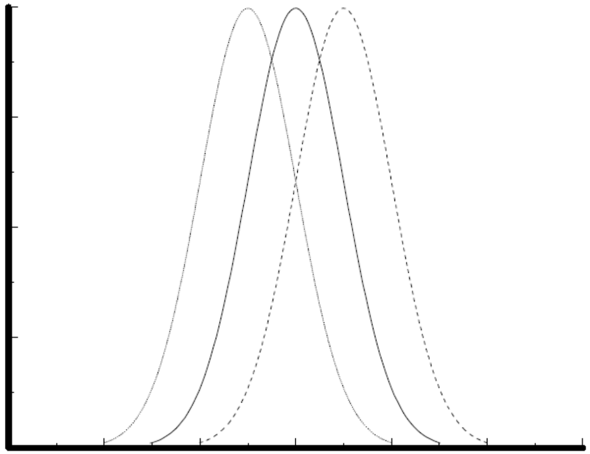
$$E^2 = B^2 + P^2$$

Repeatability and Reproducibility

- *Repeatability is the variability of the measurements obtained by one person while measuring the same item repeatedly. This is also known as the inherent precision of the measurement equipment.*
  - *Consider the probability density functions shown in Figure 1. The density functions were constructed from measurements of the thickness of a piece of metal with Gage A and Gage B. The density functions demonstrate that Gage B is more repeatable than Gage A.*



- *Reproducibility is the variability of the measurement system caused by differences in operator behavior. Mathematically, it is the variability of the average values obtained by several operators while measuring the same item.*
  - *Figure 2 displays the probability density functions of the measurements for three operators. The variability of the individual operators are the same, but because each operator has a different bias, the total variability of the measurement system is higher when three operators are used than when one operator is used.*



Reproducibility

Repeatability	*Precision Error*
Reproducibility	*Both Bias and Precision Errors*

Measurement Uncertainties

- ***We almost always are dealing with a data reduction equation to get to the final results.***
  - ***In this case, we must not only deal with uncertainty in the measured values but uncertainty in the final results.***
- ***A general form looks like this:***

$$R = R\left(X_1, X_2, X_3, \ldots, X_J\right)$$

  - ***R is the result determined from J independent variables.***

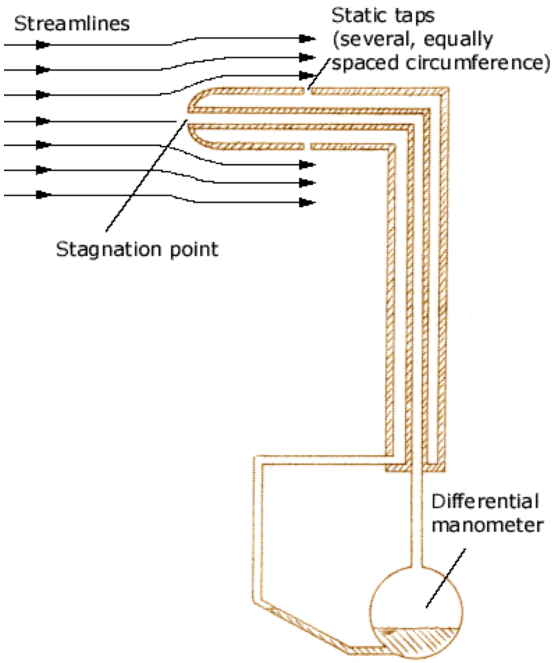
Example

- ***Uncertainty in velocity V:***

$$U_R^2 = B_R^2 + P_R^2$$

$$B_R^2 = \sum_{i=1}^{J}\left[\frac{\partial R}{\partial X_i} B_i\right]^2; \qquad P_R^2 = \sum_{i=1}^{J}\left[\frac{\partial R}{\partial X_i} P_i\right]^2$$

$$B_i = \sqrt{\sum_{j=1}^{M} B_{i_j}^2}$$



- *For a large number of samples (N>10)*

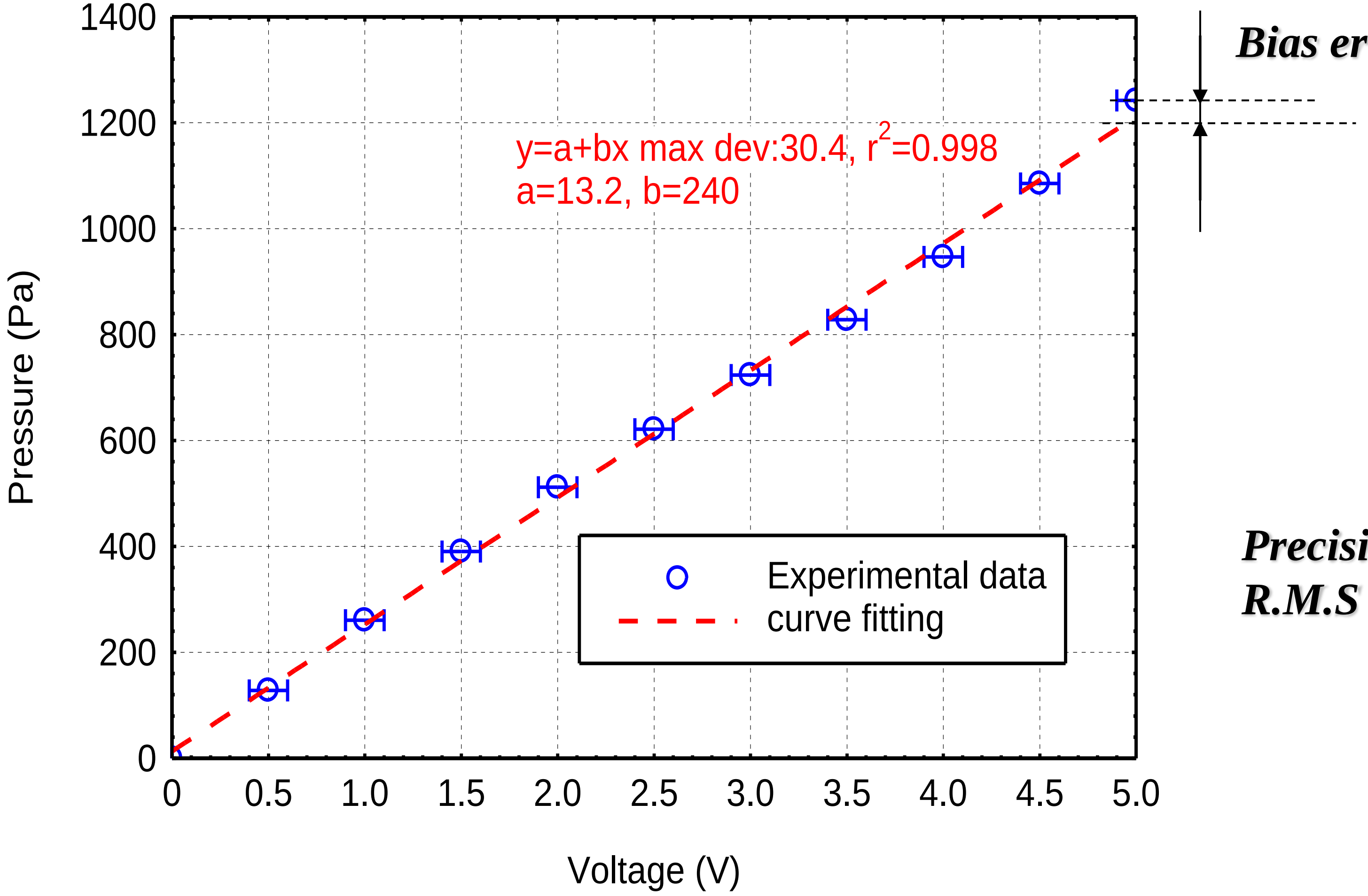
$$P_i = 2S_i$$

$$S_i = \left[\frac{1}{N-1}\sum_{k=1}^{N}\left[(X_i)_k - \bar{X}_i\right]^2\right]^{1/2}; \qquad \bar{X}_i = \frac{1}{N}\left[\sum_{k=1}^{N}(X_i)_k\right]$$

$$p_{total} = p_{static} + \frac{1}{2}\rho V^2, (Bernoulli)$$

$$V = \sqrt{\frac{2(p_{total} - p_{static})}{\rho}} = \sqrt{\frac{2\Delta p}{\rho}}$$

Measurement Results

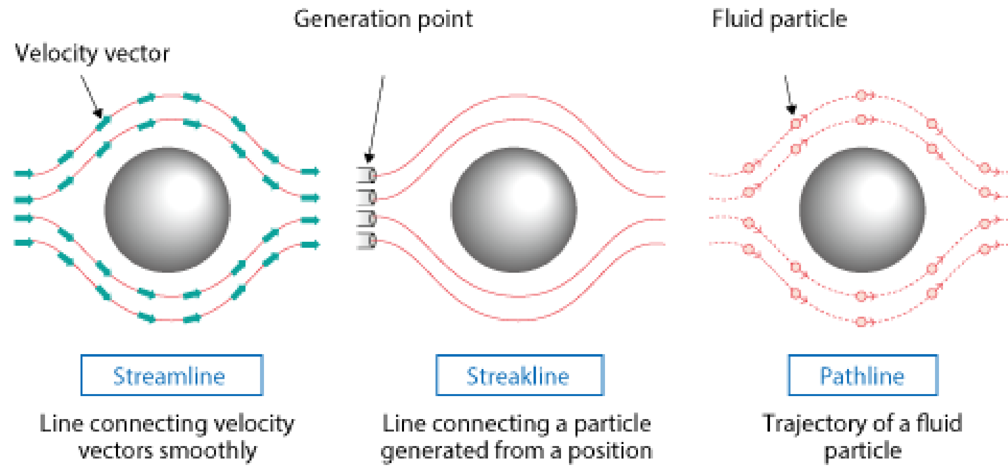


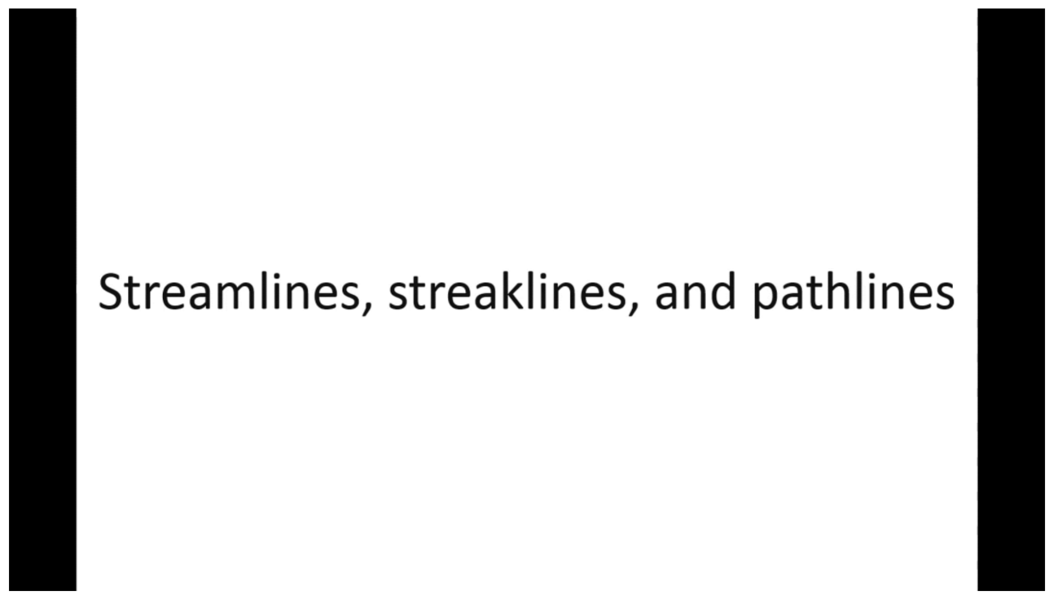
Bias error

Precision error:
R.M.S of your data

• Streamlines; Streaklines, and Pathline

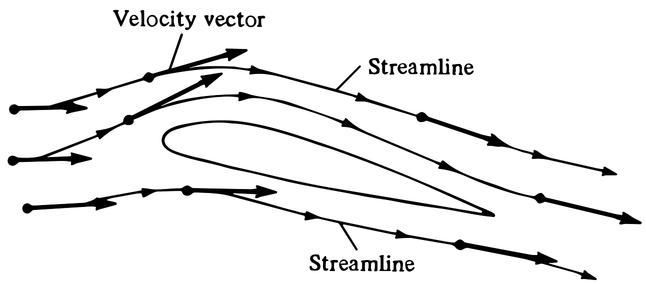
- ***Streamlines*** *are a family of curves that are instantaneously tangent to the velocity vector of the flow. These show the direction in which a massless fluid element will travel at any point in time (Eularian approach).*
- ***Streaklines*** *are the loci of points of all the fluid particles that have passed continuously through a particular spatial point in the past. Dye steadily injected into the fluid at a fixed point extends along a streakline (Langragian approach).*
- ***Pathlines*** *are the trajectories that individual fluid particles follow. These can be thought of as "recording" the path of a fluid element in the flow over a certain period. The direction the path takes will be determined by the streamlines of the fluid at each moment in time (Langragian approach).*

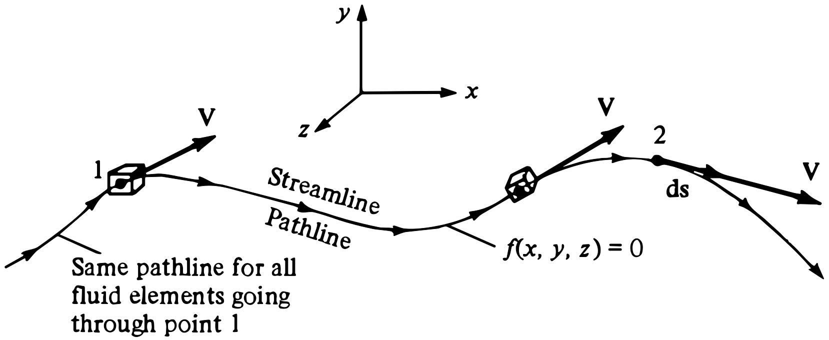




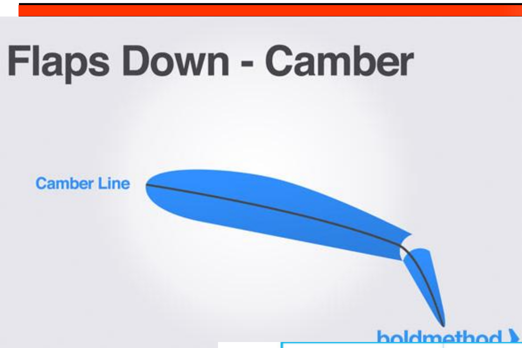
Lab #1: Flow visualization by using smoke wind tunnel

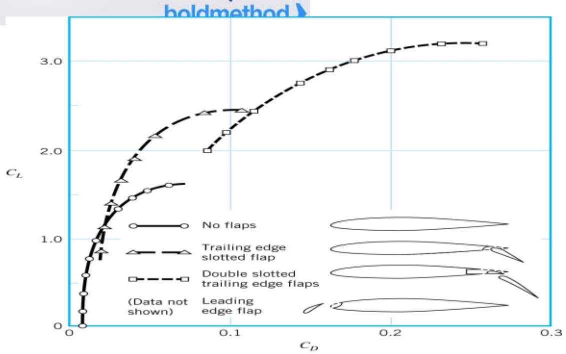
- *Path line*
- *Streak lines*
- *Streamline*

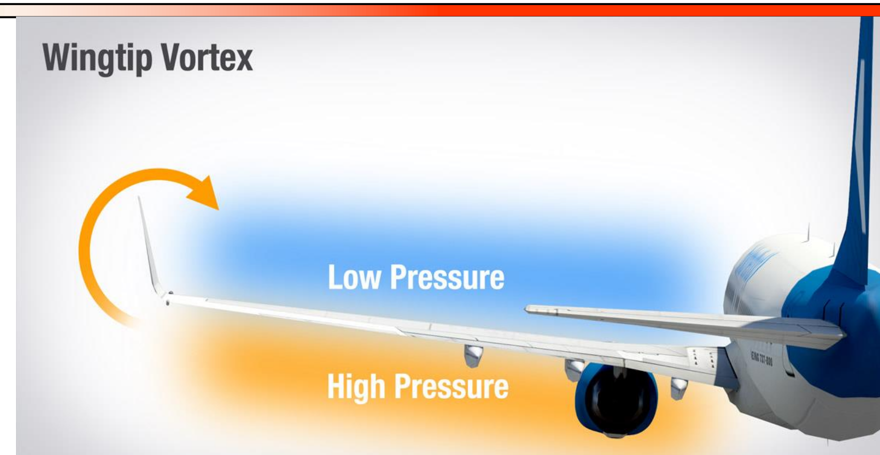


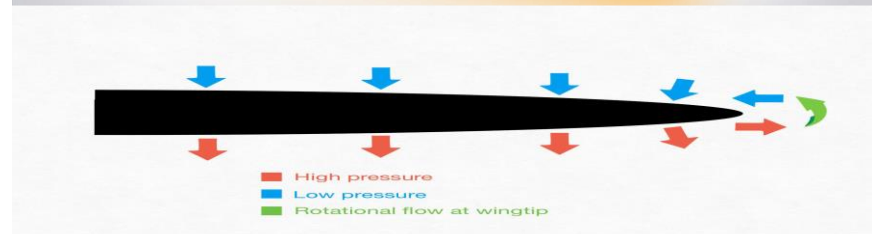


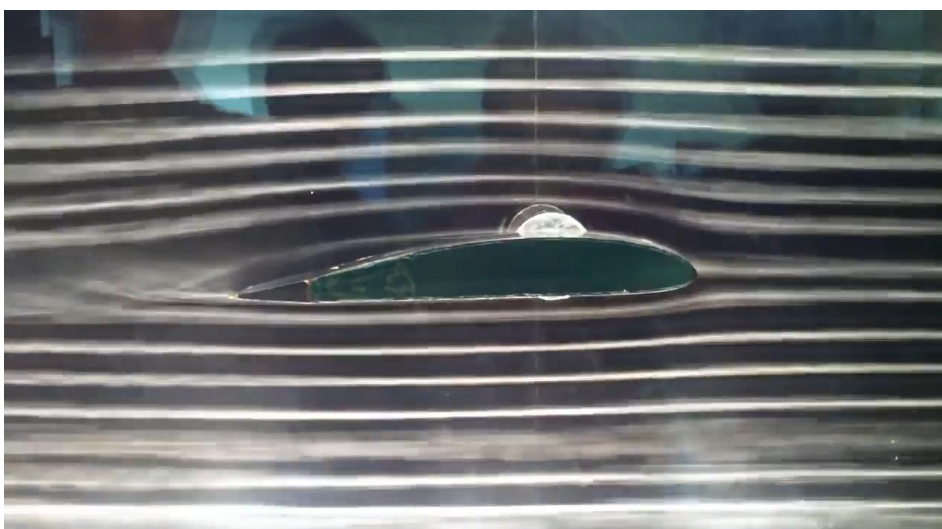
Lab #1: Flow visualization by using smoke wind tunnel

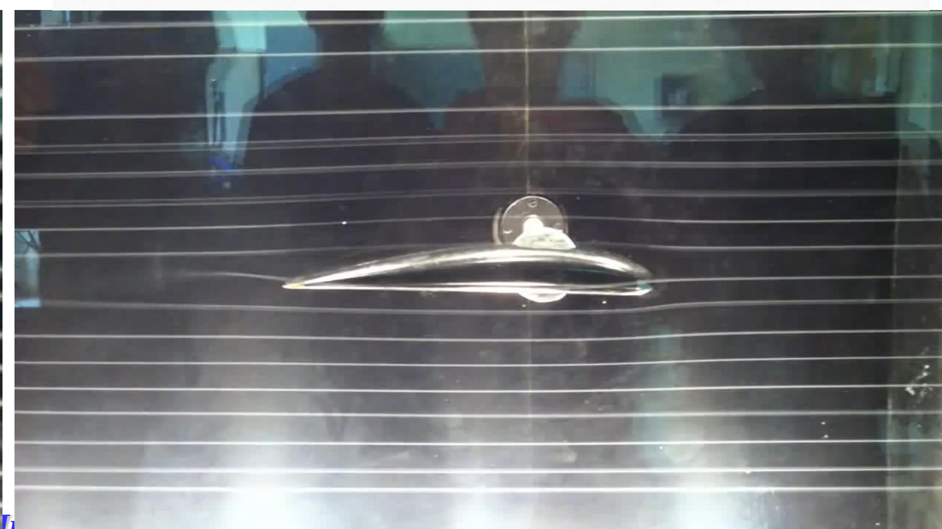








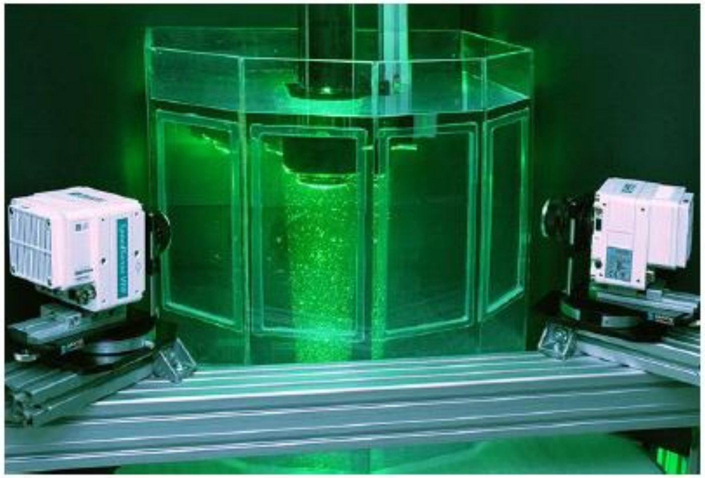




❑ Measurement Signal

Signal:
Signal is a function that conveys information about a phenomenon.

- Voltage, current
- Sound strength, light intensity or other electromagnetic wave that carries information

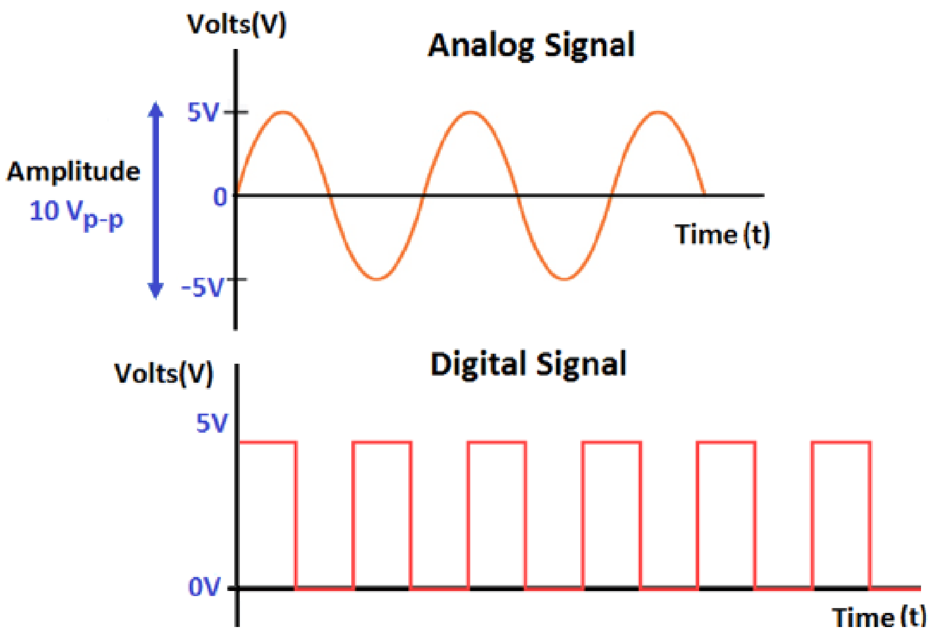


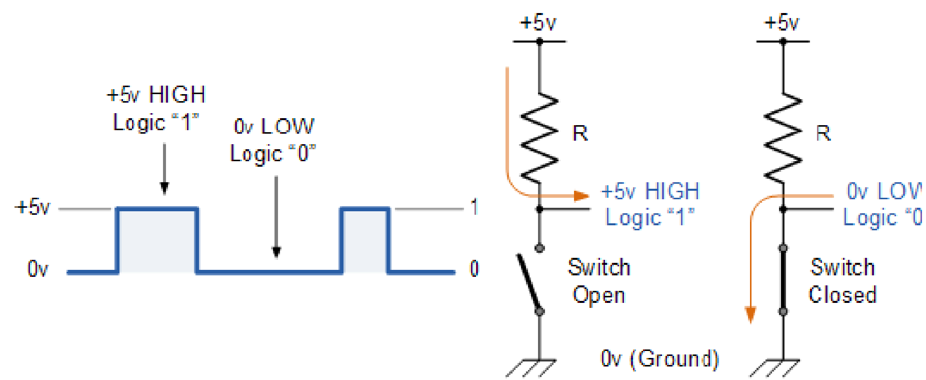
❑ MEASUREMENT SIGNAL

Signal classifications:

- Analog signal: Continuous signal for which its variation represents some other varying quantity.

- Digital signal: a signal is constructed from a discrete set of values to represent a physical quantity.

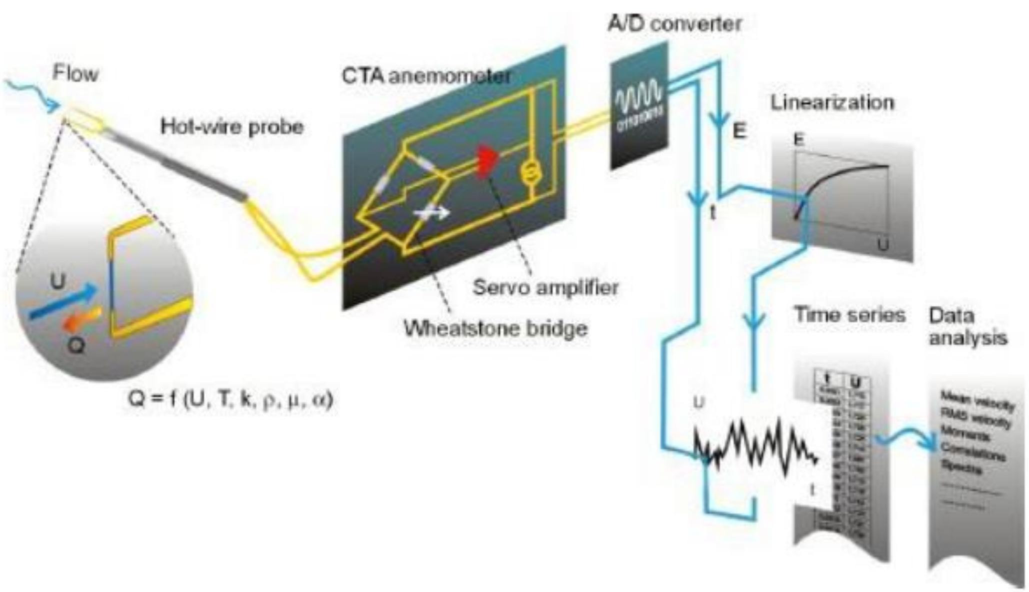




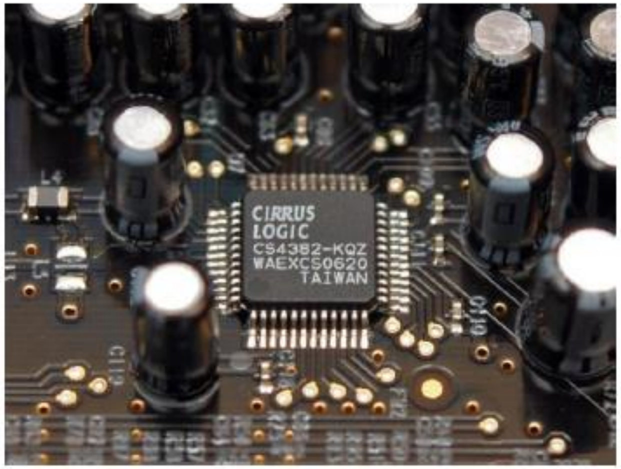
❑ Measurement Signal

Analog-to-digital converter (ADC):
ADC is a system that converts an analog signal into a digital signal.

Reverse ADC:
Digital-to-Analog converter (DAC)

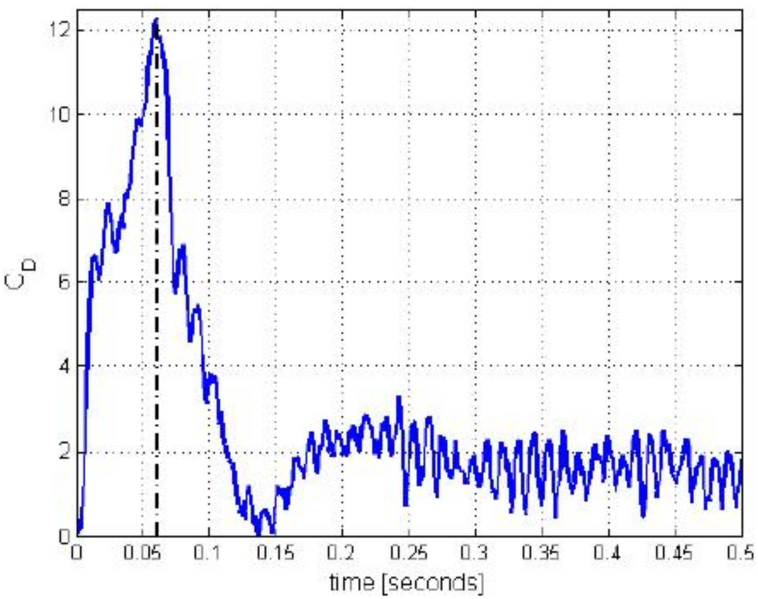


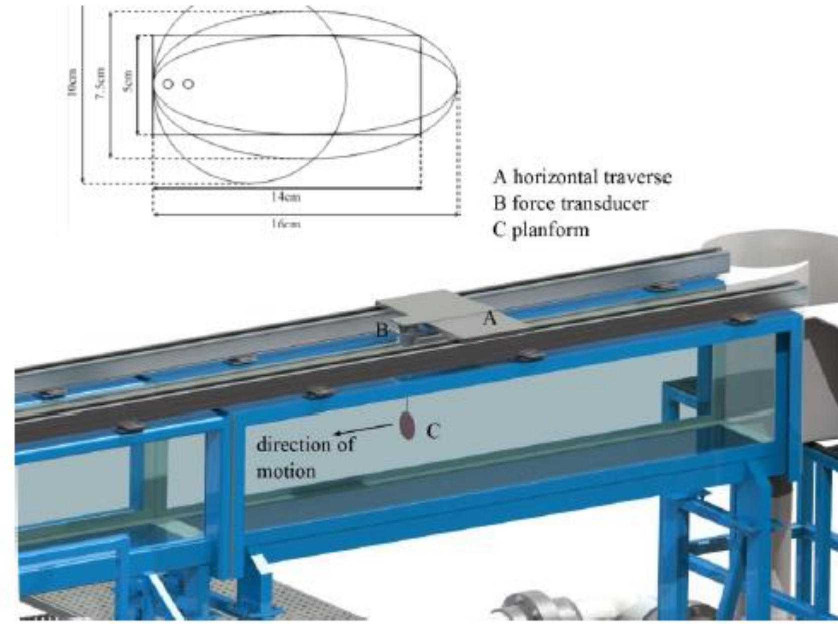
Dantec CTA

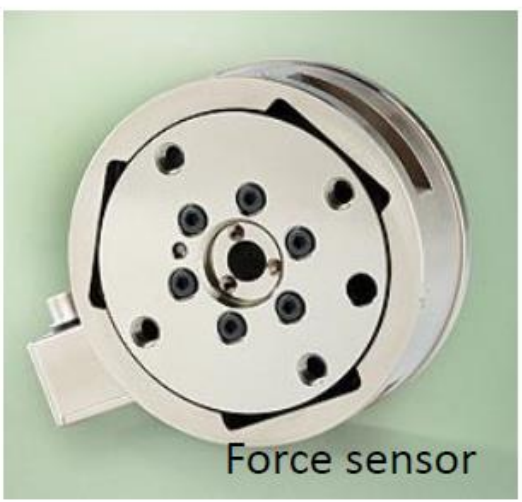


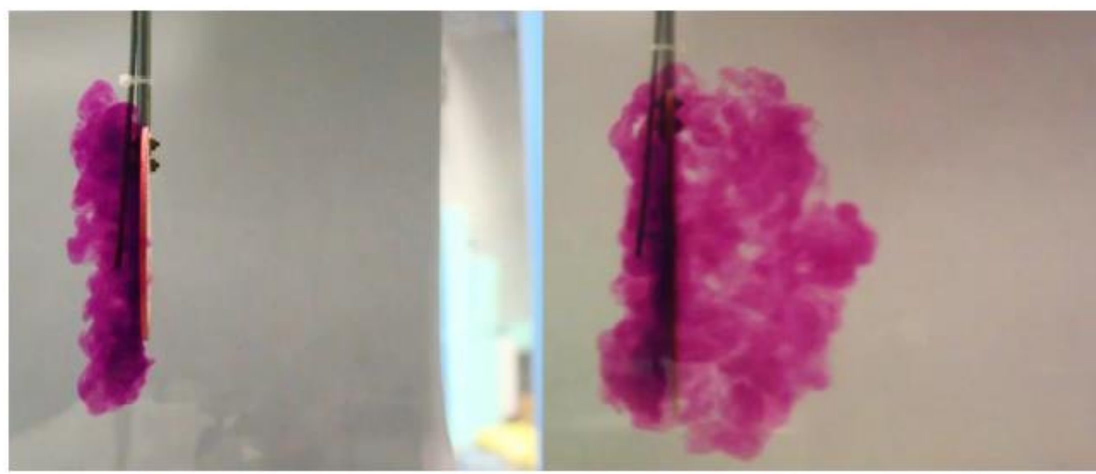
ADC clip in a sound card

Measurement Signal



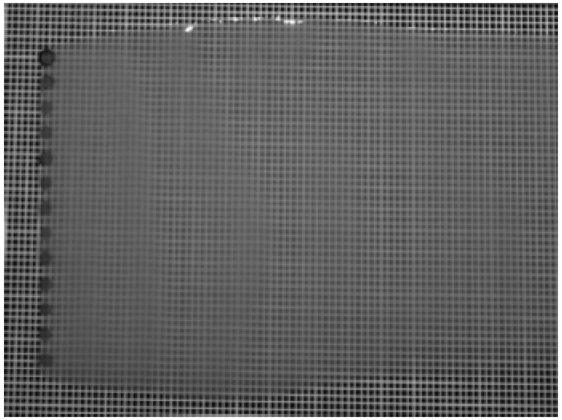


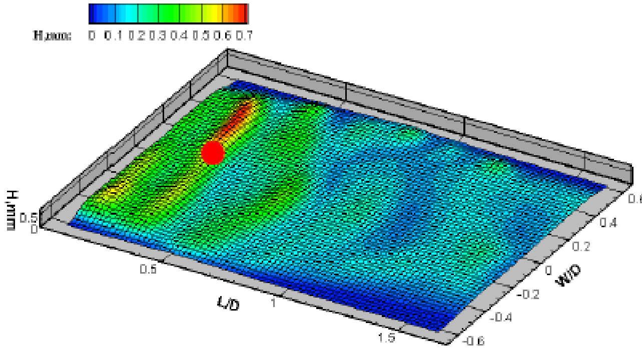


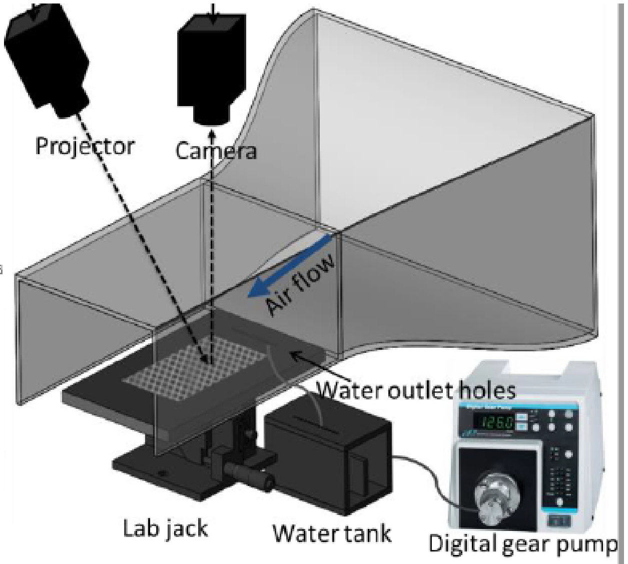


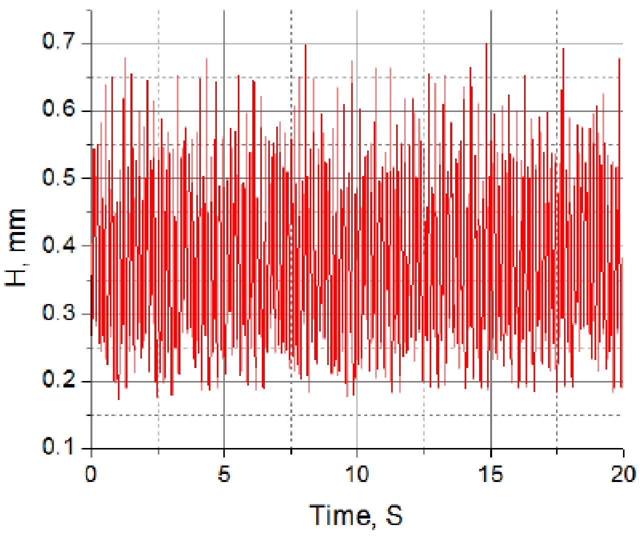
In fluence of shape on vortex development (drag)

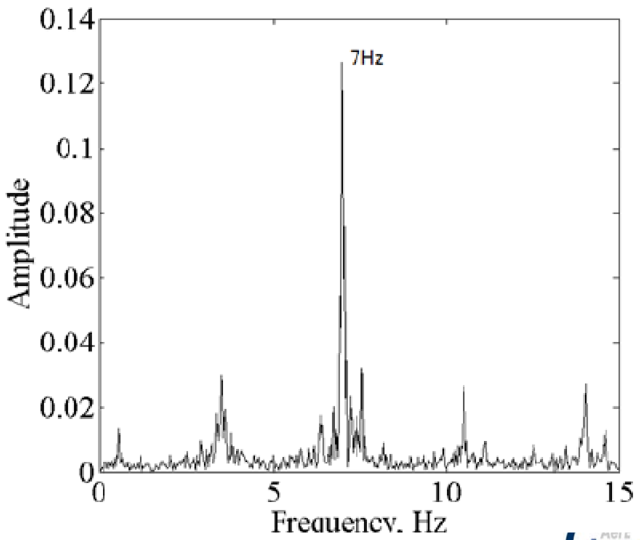
Measurement Signal







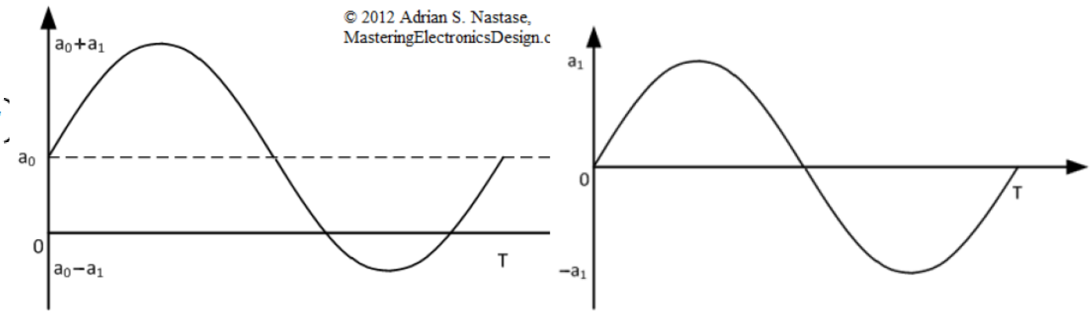




FOURIER TRANSFORM AND SPECTRAL ANALYSIS

- ***Any complex signal can be broken into set of sine and cosine waves of different periods and amplitudes.***

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$



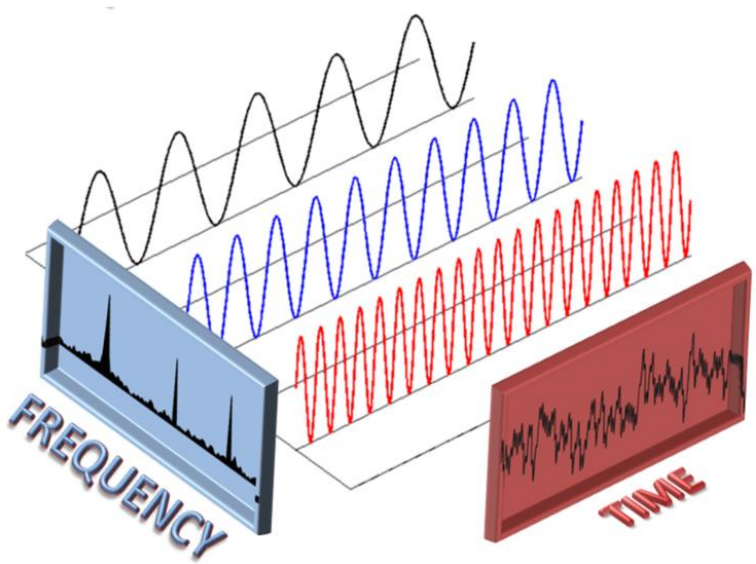
Fourier series for a periodic function with period of 2τ

$$f(x) = \sum_{n=0}^{\infty} \left(a_n \cos\frac{n\pi x}{\tau} + b_n \sin\frac{n\pi x}{\tau}\right)$$

$$a_0 = \frac{1}{2\tau}\int_{-\tau}^{\tau} f(x)dx\,, a_n = \frac{1}{\tau}\int_{-\tau}^{\tau} f(x)\cos\frac{n\pi x}{\tau}dx, b_n = \frac{1}{\tau}\int_{-\tau}^{\tau} f(x)\sin\frac{n\pi x}{\tau}dx$$

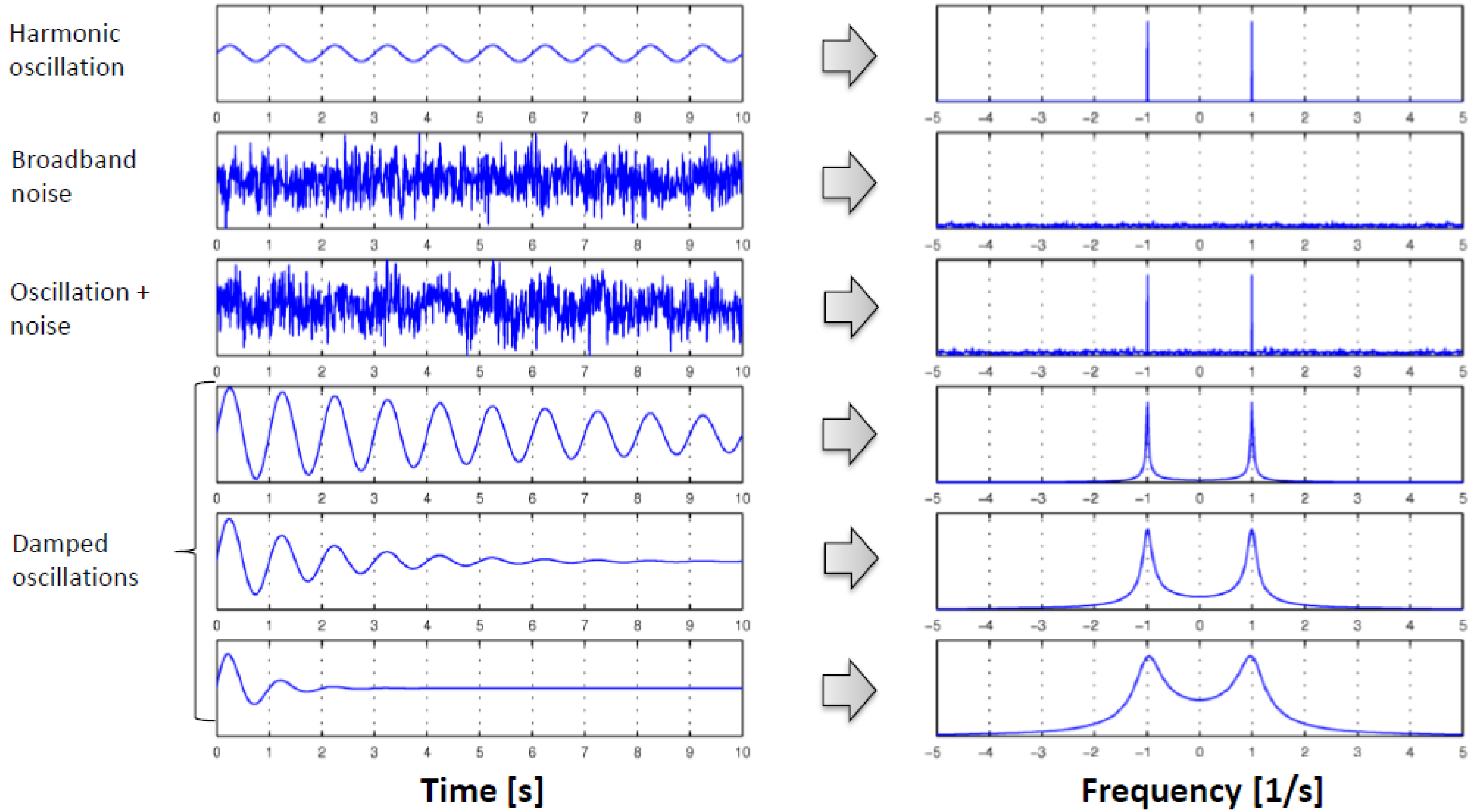
Let $\tau \to \infty$, then a nonperiodic function can be considered as periodic but with an infinite period $2\tau \to \infty$.

$\frac{n\pi}{\tau}$'s are the set of frequencies (spatial or temporal), called **frequency spectrum** $0, \frac{\pi}{\tau}, \frac{2\pi}{\tau}, \frac{3\pi}{\tau}, \ldots$

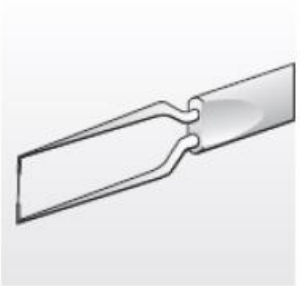


Fourier transform and Spectral Analysis

Examples of Fourier transform:



❑ Fourier transform and Spectral Analysis



Hotwire