Lecture # 09: Flow Visualization Techniques: Shadowgraph and Schlieren

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Sources/ Further reading:

Hecht, "Optics" 4th ed.
Raffel, Willert, Wereley, Kompenhans, "Particle image velocimetry: A practical guide" 2nd ed.
Tropea, Yarin, & Foss, "Springer Handbook of Experimental Fluid Mechanics," Part B Ch 6

Schlieren Imaging In Slow Motion

- ***How Well Do Masks Work? (Schlieren Imaging In Slow Motion!)***
  https://www.youtube.com/watch?v=0Tp0zB904Mc

Index of refraction and thermodynamic state

- *Index of refraction is a function of thermodynamic state (density) for homogeneous medium:*

- *Lorenz-Lorentz relationship:* $\frac{1}{\rho}\frac{n^2-1}{n^2+2}=K$

- *When n≈1, for gaseous flow:* $\frac{n-1}{\rho}=K \quad \Rightarrow \quad K\rho=n-1$ ← Gladstone-Dale Eqn

- *At standard condition, with n_o and ρ_o:* $\frac{n_0-1}{\rho_0}=K \quad \Rightarrow \quad n-1=\frac{\rho}{\rho_0}(n_0-1)$

$$\Rightarrow \quad \rho=\rho_0\frac{n-1}{n_0-1}$$

- *First- and second-derivative is determined by schlieren and shadowgraph apparatus:*

$$\frac{\partial\rho}{\partial y}=\frac{1}{const}\frac{\partial n}{\partial y} \quad \Rightarrow \frac{\partial\rho}{\partial y}=\frac{\rho_0}{n_0-1}\frac{\partial n}{\partial y}$$

$$\frac{\partial^2\rho}{\partial y^2}=\frac{1}{const}\frac{\partial^2 n}{\partial y^2} \quad \Rightarrow \frac{\partial^2\rho}{\partial y^2}=\frac{\rho_0}{n_0-1}\frac{\partial^2 n}{\partial y^2}$$

Shadowgraphy and Schlieren Techniques

- *Index of refraction:* $n = c / v = \frac{\lambda_0}{\lambda} > 1$

- *Depends on the variation of the index of refraction in a transparent medium, which affects the light rays passing through.*

- ***Shadowgraphy:*** *used to indicate the variation of the second derivatives (normal to the light beam) of the index of refraction.*

- ***Schlieren systems:*** *used to indicate the variation of the first derivative of the index of refraction*

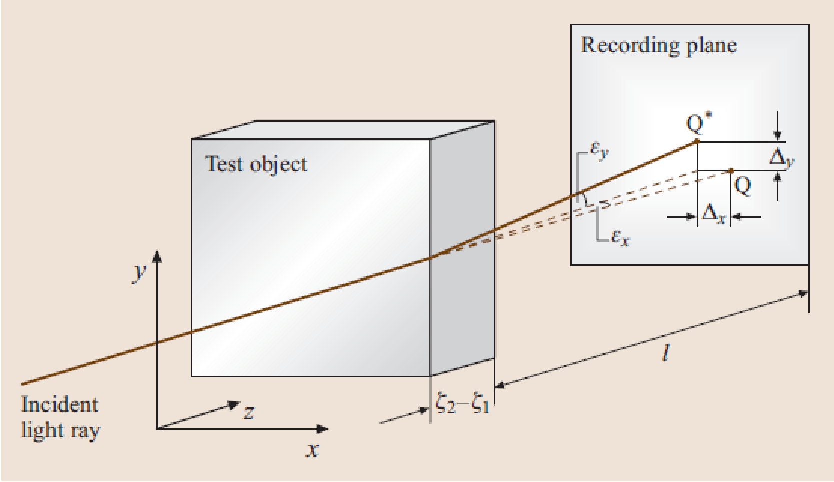


Fig. 6.1 Refractive deflection of a light ray in an object field (flow) with varying refractive index (caused by varying fluid density)

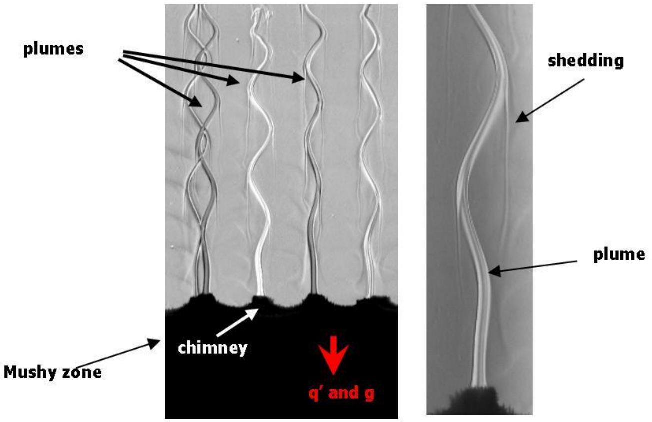
Schlieren of a .30-06 caliber high-powered rifle muzzle blast from (by Gary S. Settles)

Shadowgraphy and Schlieren Techniques

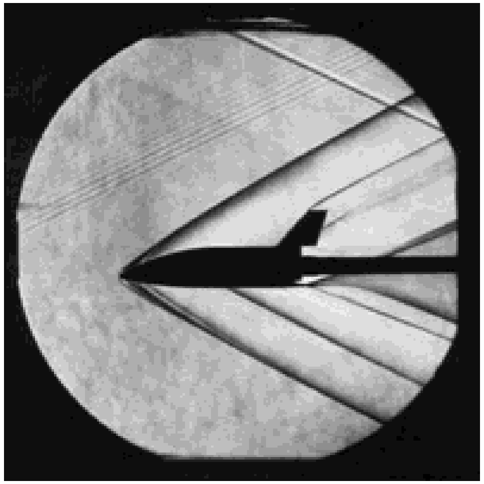
- *Shadowgraphy and Schlieren systems are often used in shock waves and flame phenomena, in which density gradient is quite big.*

- *While these techniques are mostly used for qualitative flow visualization, they can be used to map pressure, density, or temperature measurements theoretically.*

- *These techniques are often used to determine the integrated quantity over the length of light beam.*



shadowgraph image of plumes during solidification process (by Lum Chee)



Schlieren image

Introduction-3

- *Index of refraction is a function of thermodynamic state (density) for homogeneous medium:*

- *Lorenz-Lorentz relationship:*

$$\frac{1}{\rho}\frac{n^2-1}{n^2+2} = const$$

- *When n≈1, for gaseous flow:*

$$\frac{n-1}{\rho} = const \quad \Rightarrow \quad \rho = \frac{n-1}{const}$$

- *at standard condition, with n_o and $\rho_{o,}$:*

$$\frac{n_0-1}{\rho_0} = const \quad \Rightarrow \quad n-1 = \frac{\rho}{\rho_0}(n_0-1)$$

$$\Rightarrow \quad \rho = \rho_0 \frac{n-1}{n_0-1}$$

- *When first and second derivative is determined as in Schlieren and shadowgraph apparatus:*

$$\frac{\partial \rho}{\partial y} = \frac{1}{const}\frac{\partial n}{\partial y} \quad \Rightarrow \frac{\partial \rho}{\partial y} = \frac{\rho_0}{n_0-1}\frac{\partial n}{\partial y}$$

$$\frac{\partial^2 \rho}{\partial y^2} = \frac{1}{const}\frac{\partial^2 n}{\partial y^2} \quad \Rightarrow \frac{\partial^2 \rho}{\partial y^2} = \frac{\rho_0}{n_0-1}\frac{\partial^2 n}{\partial y^2}$$

Introduction-4

- *Application of the Schlieren and shadowgraphy techniques:*
  - *Compressible flow with shock waves* $\Rightarrow$ *density changes*
  - *Natural convective flow* $\Rightarrow$ *density changes*
  - *Flame and combustion system:* $\Rightarrow$ *density changes*

- *Temperature changes inside flows:*
  - *For low speed flow with heat transfer:*
  - *P = constant*

$$\rho = P/RT \Rightarrow \frac{\partial \rho}{\partial y} = \frac{P}{RT^2}\frac{\partial T}{\partial y} = \frac{\rho}{T}\frac{\partial T}{\partial y}$$

$$\Rightarrow \frac{\partial n}{\partial y} = \frac{n_0 - 1}{\rho_0}\frac{\partial \rho}{\partial y} = \frac{n_0 - 1}{T}\frac{\rho}{\rho_0}\frac{\partial T}{\partial y}$$

$$\Rightarrow \frac{\partial T}{\partial y} = \frac{T}{n_0 - 1}\frac{\rho_0}{\rho}\frac{\partial n}{\partial y}$$

$$\Rightarrow \frac{\partial^2 n}{\partial y^2} = \frac{n_0 - 1}{\rho_0}[-\frac{\rho}{T}\frac{\partial^2 T}{\partial y^2} + \frac{2\rho}{T^2}(\frac{\partial T}{\partial y})^2]$$

Deflection of light rays

- According to definition of index of refraction, the light velocity will be $V=C_o/n$.
- The slope of the wave front of the light: $\frac{dy}{dz}$
- If the angle $\Delta\alpha'$ is quite small:

$$\Delta Z = \frac{C_0}{n}\Delta\tau$$

$$\Delta^2 Z = \Delta Z - \Delta Z_{y+\Delta y} = -C_0(\Delta(\frac{1}{n})/\Delta y)\Delta\tau\,\Delta y$$

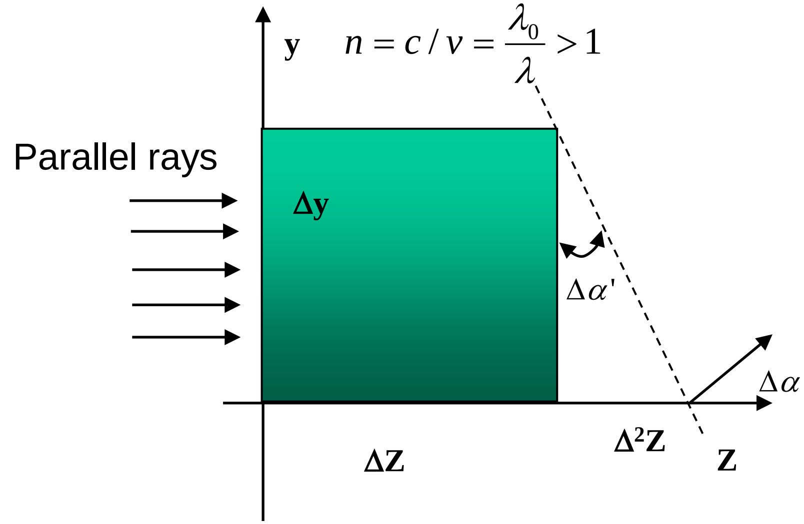
$$\Delta\alpha' = \frac{\Delta^2 Z}{\Delta y} = -n\,(\Delta(\frac{1}{n})/\Delta y)\Delta Z$$

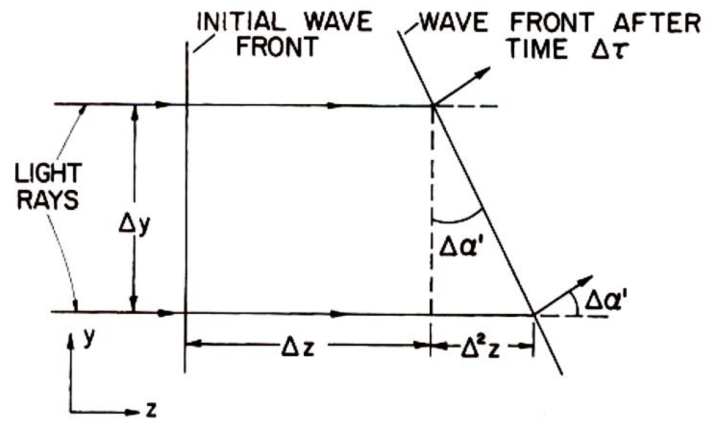
$$\frac{dy}{dz} = d\alpha' = -n[\frac{d(\frac{1}{n})}{dy}]dz = n\frac{1}{n^2}[\frac{dn}{dy}]dz = \frac{1}{n}(\frac{dn}{dy})dz = \frac{d(\ln n)}{dy}dz$$

$$\frac{d^2y}{dz^2} = \frac{d(\ln n)}{dy}$$

$$d\alpha' = -n[\frac{d(\frac{1}{n})}{dy}]dz = n\frac{1}{n^2}[\frac{dn}{dy}]dz = \frac{1}{n}(\frac{dn}{dy})dz = \frac{d(\ln n)}{dy}dz$$

$$\Rightarrow \alpha' = \int\frac{1}{n}(\frac{dn}{dy})dz \quad \overset{n\approx 1}{\Rightarrow} \quad \alpha' = \int\frac{dn}{dy}dz$$



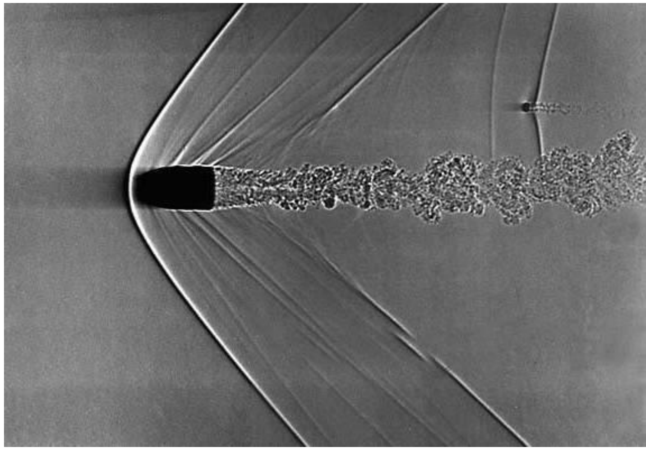


Shadowgraphy

- In shadowgraphy, as light rays pass through the measurement region, the deflection of the light rays as they interact with variations in the optical index lead to an intensity distribution:

$$\frac{\Delta I}{I} = l \int_{\zeta_1}^{\zeta_2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (\ln n) \, dz$$

- For *weak refraction*, and applying the Gladstone-Dale formula reveals a dependence on the second partial derivatives of density.



Shadowgraph of a bullet (by Andrew Davidhazy)

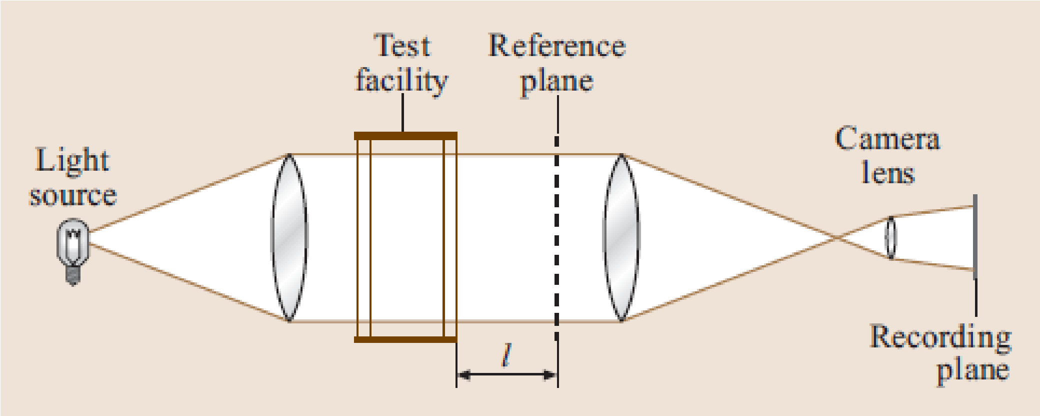


Fig. 6.2 Shadowgraph setup with parallel beams through the test object

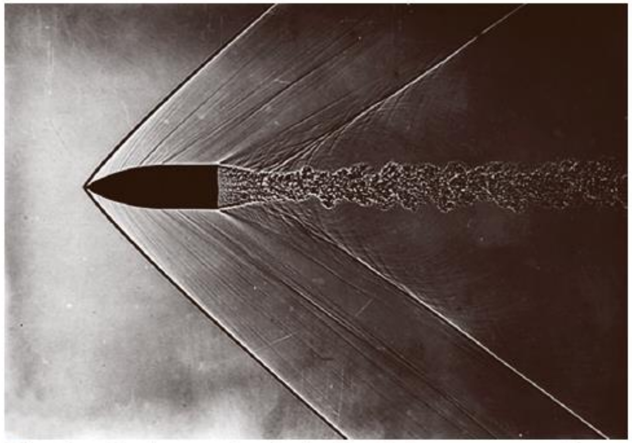
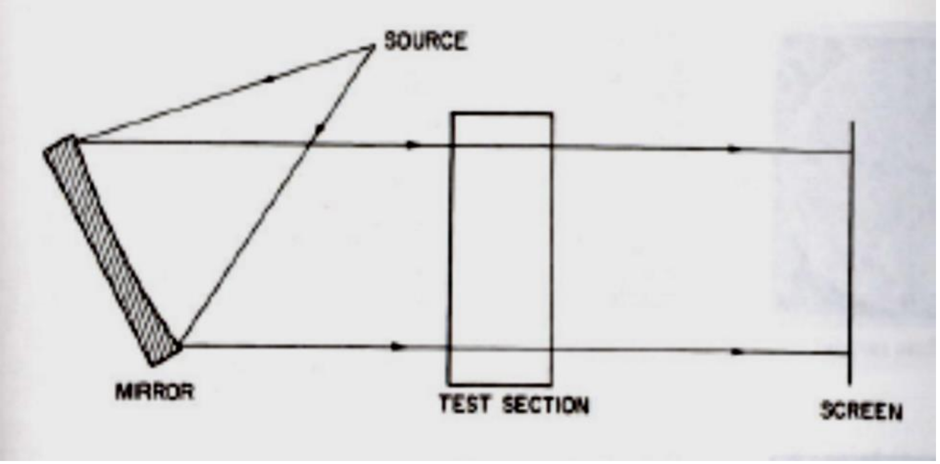
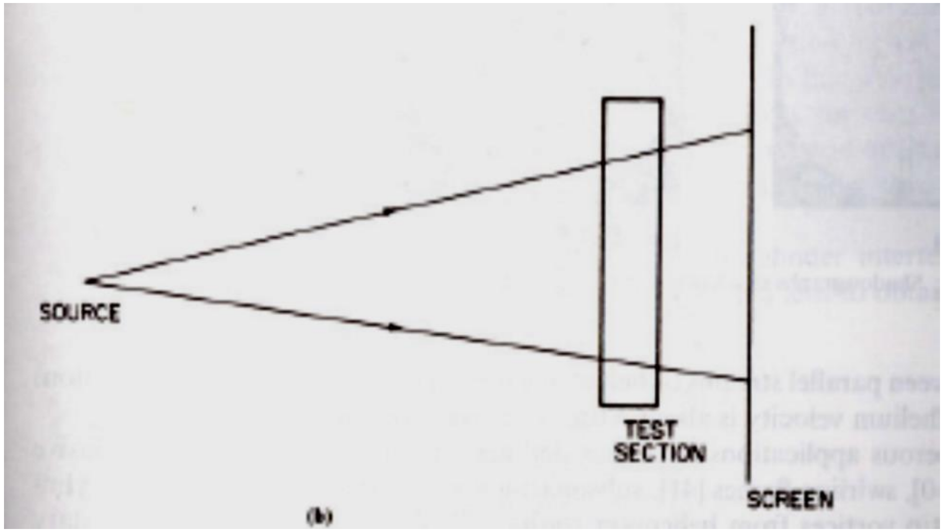


Fig. 6.3 Shadowgraph of a bullet flying at supersonic velocity (courtesy Deutsch-Französisches Forschungsinstitut, ISL, St. Louis, France)

Setup of a Shadowgraph imaging system

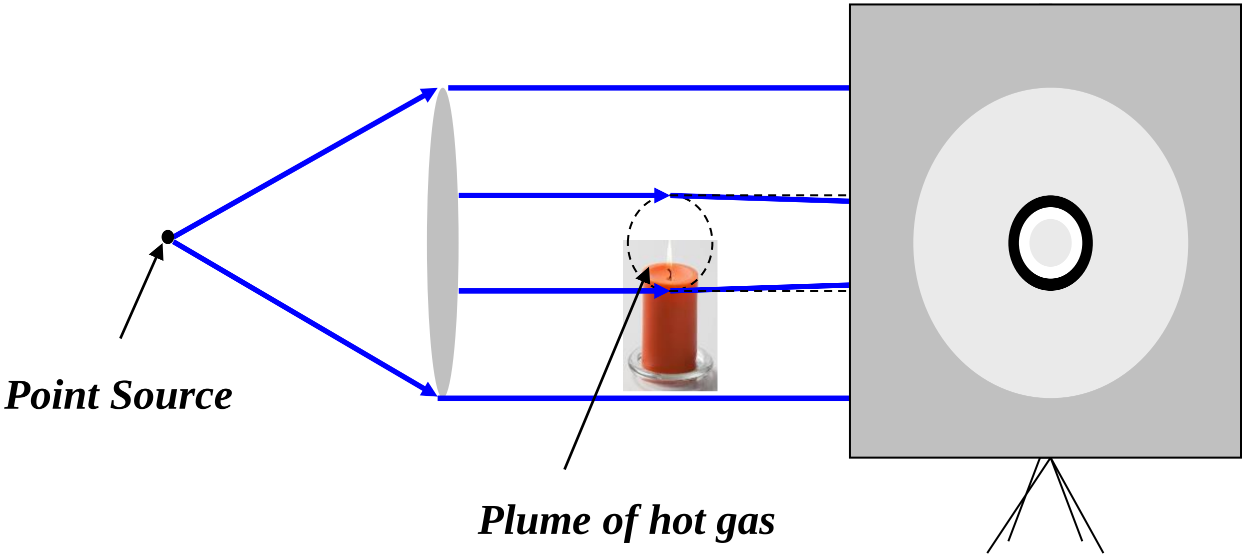


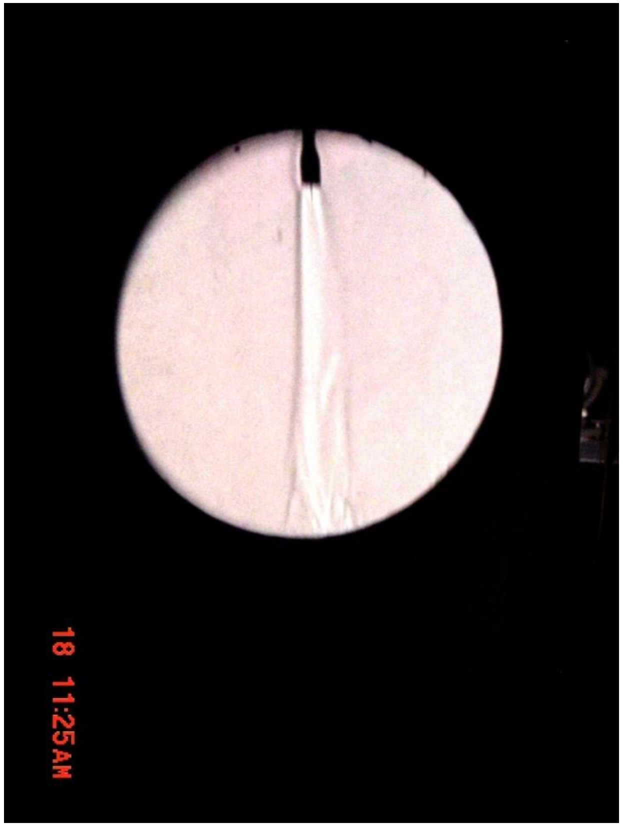
Experimental setup with one converging mirror



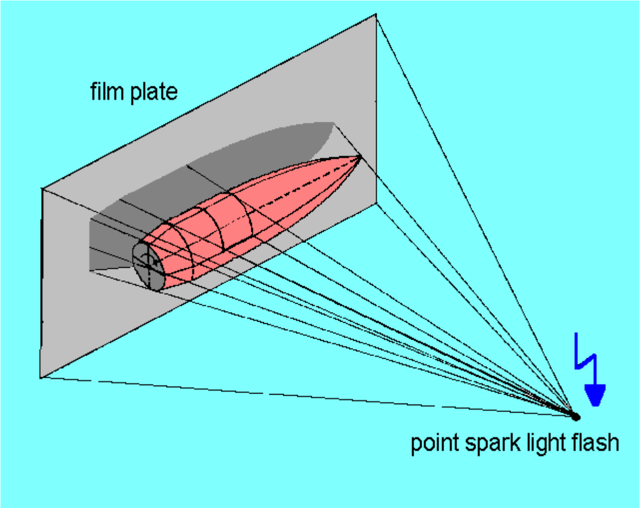
Experimental setup without lens or mirror

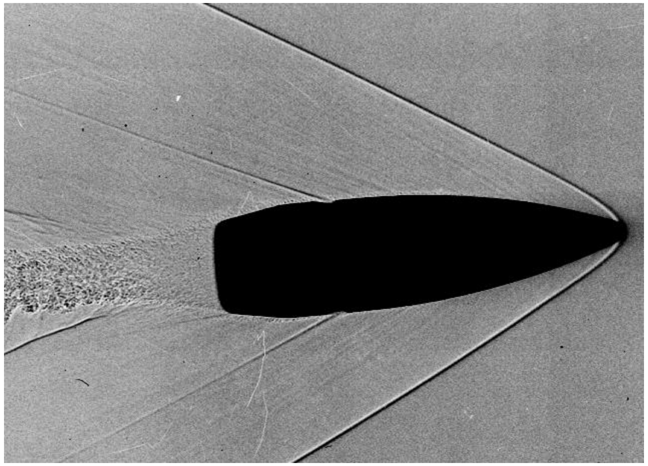
Direct Shadowgraph

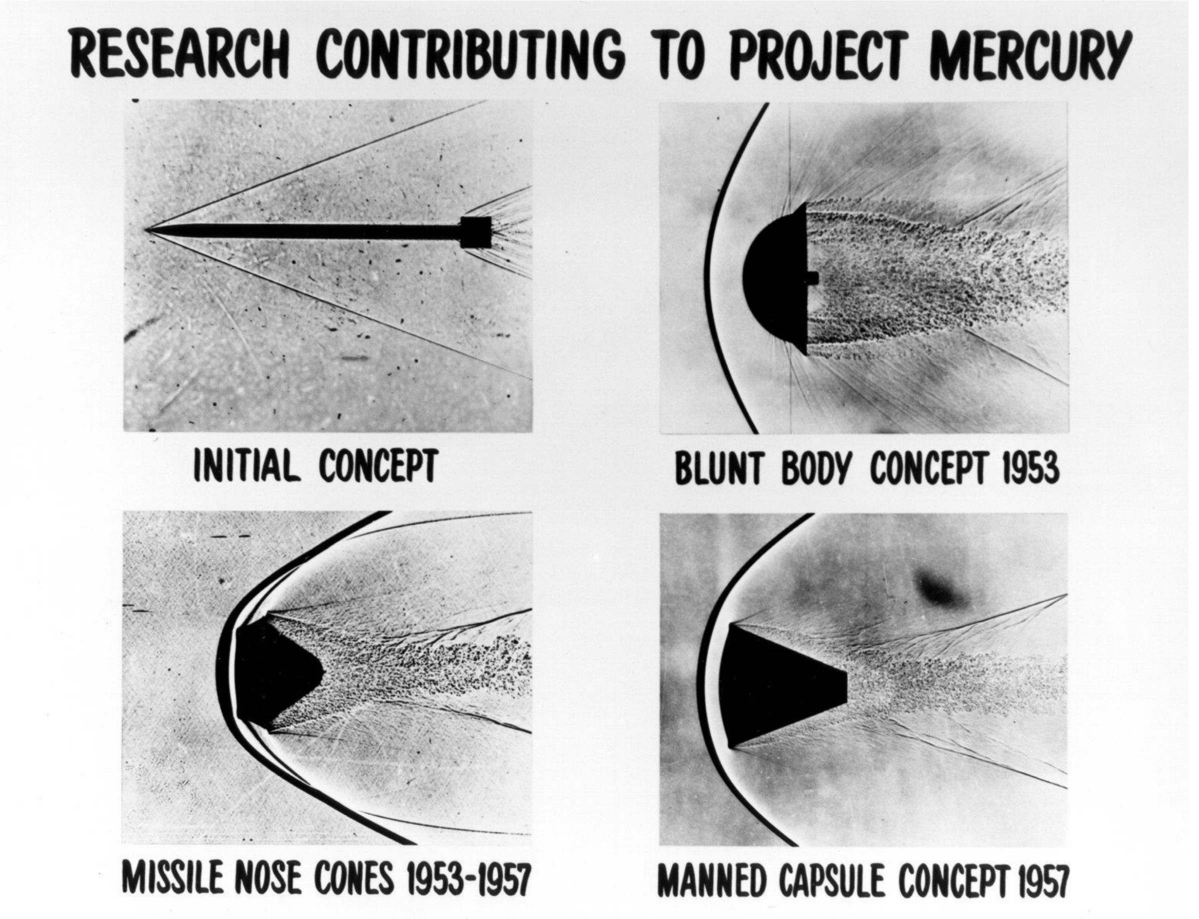




Examples: Shadowgraph images



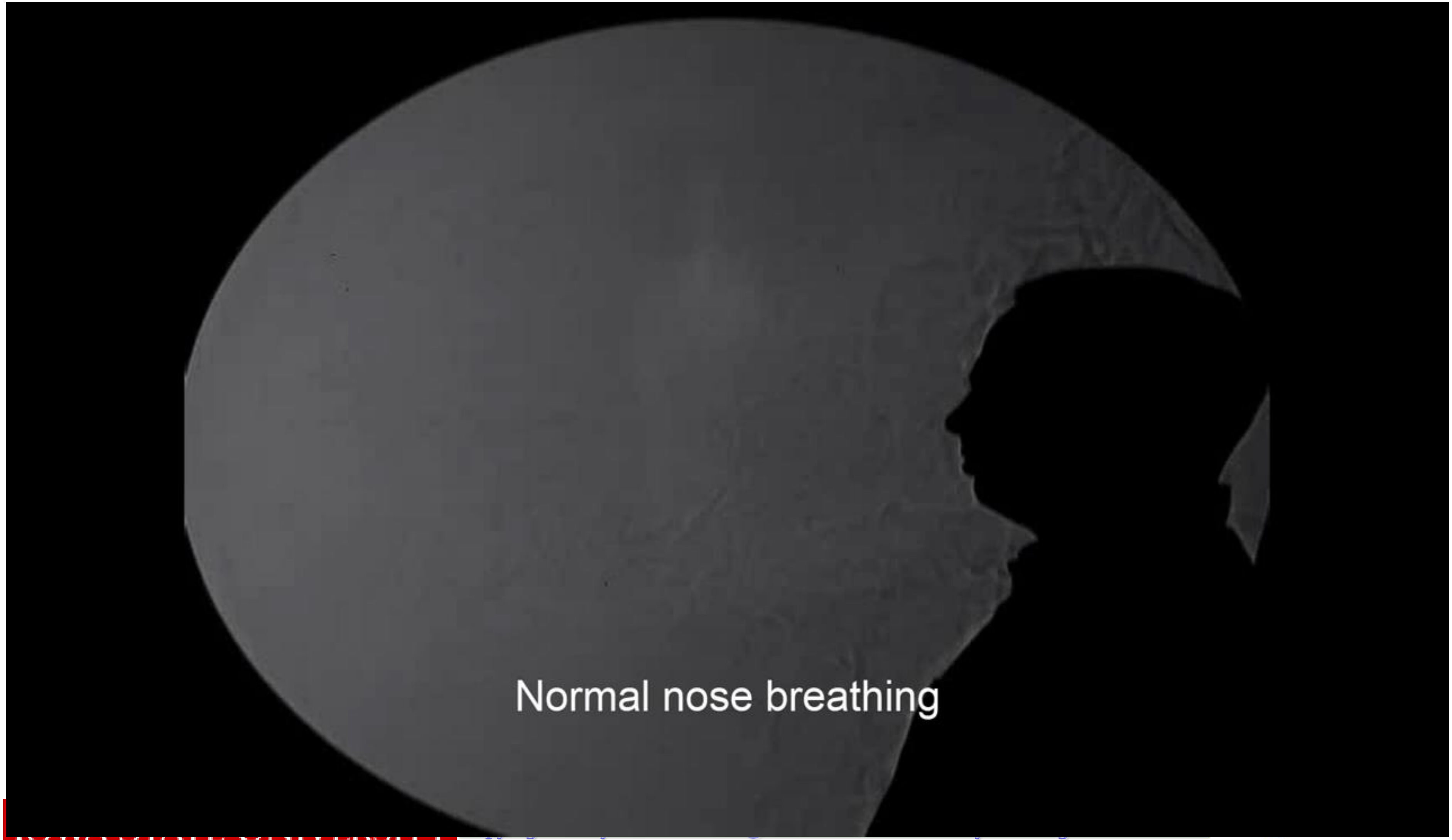




Shadowgraph Images of Re-entry Vehicles

Shadowgraph Imaging Example

- *Shadowgraph Imaging of Human Exhaled Airflows: An Aid to Aerosol Infection Control*
- *https://www.youtube.com/watch?v=gElHX1AIlOY*



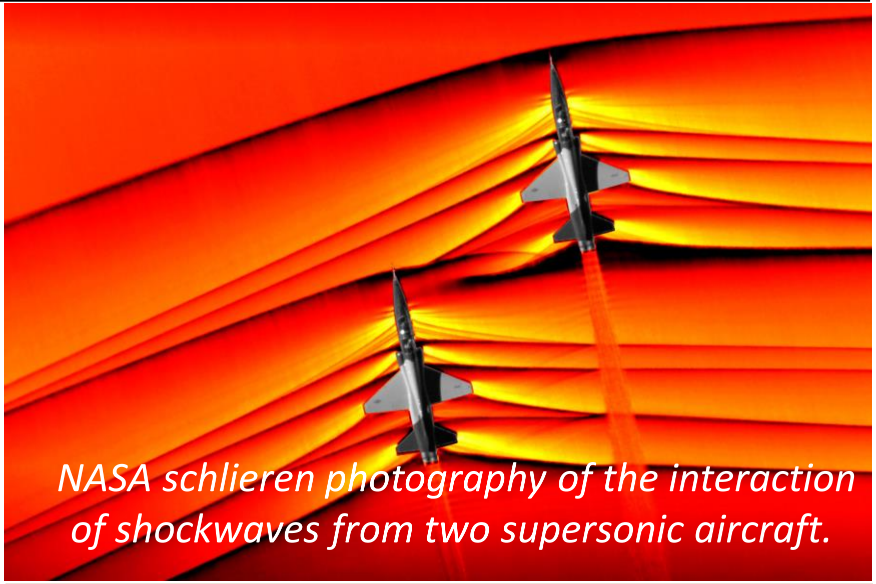
Schlieren

- *In Schlieren, as light rays pass through index variations in the measurement region, the deflection of the light rays cause them to be either blocked or pass a knife edge:*

$$\frac{\Delta I}{I} = \frac{f_2}{a} \int_{\zeta_1}^{\zeta_2} \frac{1}{n} \frac{\partial n}{\partial y} \mathrm{d}z \, .$$

- *For small angles of deflection, and applying the Gladstone-Dale formula reveals a dependence on the partial derivatives of density.*

$$\frac{\Delta I}{I} = \frac{K f_2}{a} \int_{\zeta_1}^{\zeta_2} \frac{\partial \rho}{\partial y} \mathrm{d}z$$


NASA schlieren photography of the interaction of shockwaves from two supersonic aircraft.

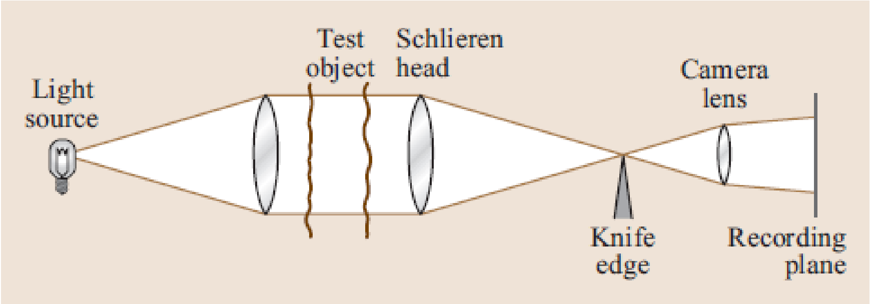


Fig. 6.5 Schlieren setup with parallel light through the test field

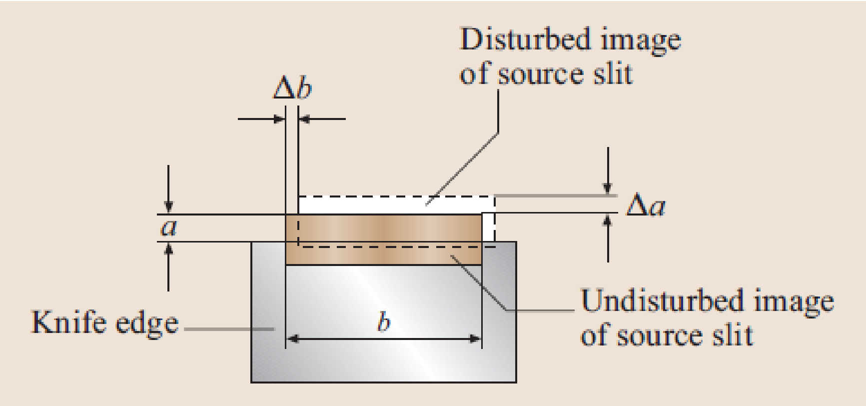


Fig. 6.6 Image of a light source of size $a \times b$ in the focal plane of the schlieren head, as seen in the direction of the optical axis; shift of the light source by Δa and Δb, respectively, caused by light deflection in the refractive index object

FUNDAMENTALS OF SCHLIEREN TECHNIQUE

- *According to definition of index of refraction, the light velocity will be $V=C_o/n$.*
- *The slope of the wave front of the light:* $\dfrac{dy}{dz}$
- *If the angle $\Delta\alpha'$ is quite small.*

$$\Delta Z = \frac{C_0}{n}\Delta\tau$$

$$\Delta^2 Z = \Delta Z - \Delta Z_{y+\Delta y} = -C_0(\Delta(\frac{1}{n})/\Delta y)\Delta\tau\,\Delta y$$

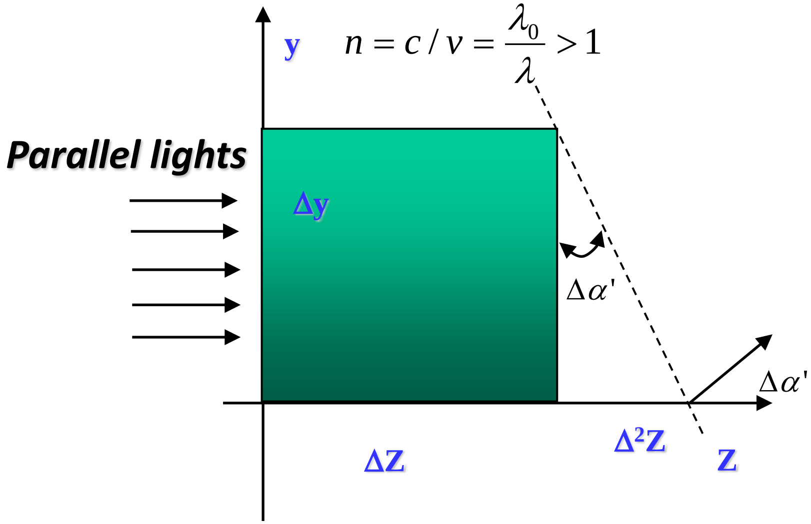
$$\Delta\alpha' = \frac{\Delta^2 Z}{\Delta y} = -n\,(\Delta(\frac{1}{n})/\Delta y)\Delta Z$$

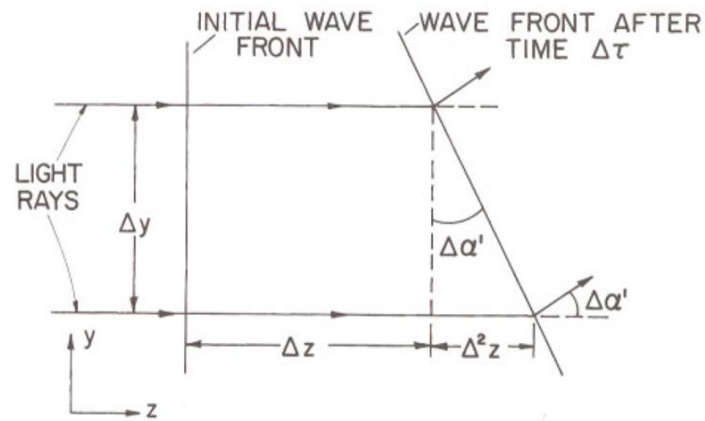
$$\frac{dy}{dz} = d\alpha' = -n[\frac{d(\frac{1}{n})}{dy}]dz = n\frac{1}{n^2}[\frac{dn}{dy}]dz = \frac{1}{n}(\frac{dn}{dy})dz = \frac{d(\ln n)}{dy}dz$$

$$\frac{d^2 y}{dz^2} = \frac{d(\ln n)}{dy}$$

$$d\alpha' = -n[\frac{d(\frac{1}{n})}{dy}]dz = n\frac{1}{n^2}[\frac{dn}{dy}]dz = \frac{1}{n}(\frac{dn}{dy})dz = \frac{d(\ln n)}{dy}dz$$

$$\Rightarrow \alpha' = \int\frac{1}{n}(\frac{dn}{dy})dz \quad \overset{n\approx 1}{\Rightarrow} \quad \alpha' = \int\frac{dn}{dy}dz$$





Shadowgraph technique

$$I_{sc} = \frac{\Delta y}{\Delta y_{sc}} I_0$$

$$\Delta y_{sc} = \Delta y + Z_{sc} \cdot d\alpha$$

$$\frac{\Delta I}{I_0} = \frac{I_{sc} - I_0}{I_0} = \frac{\Delta y}{\Delta y_{sc}} - 1$$

$$= -Z_{sc} \cdot \frac{d\alpha}{\Delta y_{sc}} \approx -Z_{sc} \cdot \frac{d\alpha}{dy}$$

$$\Rightarrow \frac{\Delta I}{I_0} = \approx -Z_{sc} \cdot \frac{d\alpha}{dy}$$

since $\alpha = \frac{1}{n_a} \int \frac{dn}{dy} dz$

$$\Rightarrow \frac{\Delta I}{I_0} = \frac{-Z_{sc}}{n_a} \cdot \int \frac{d^2 n}{dy^2} dz$$

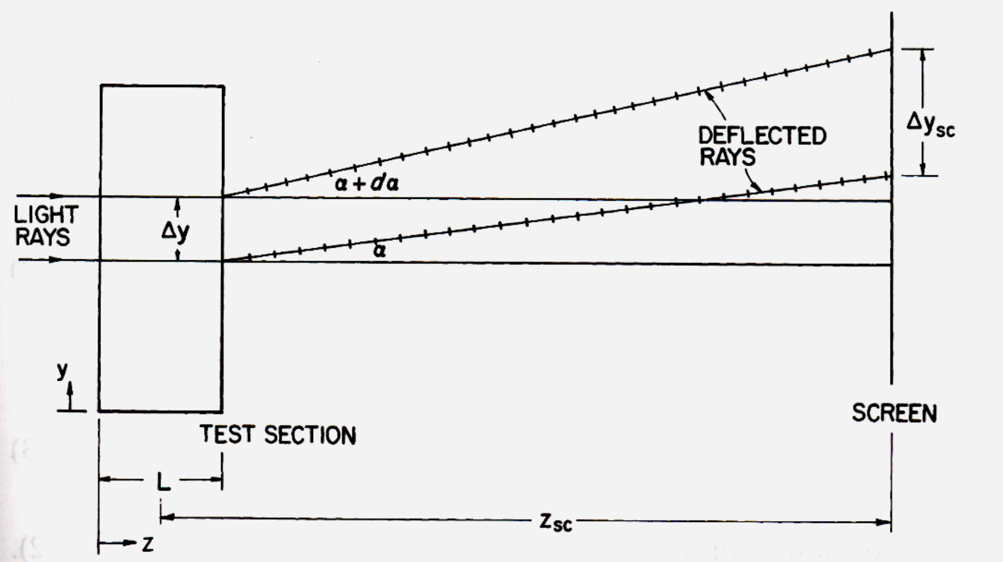


Figure 7.12 Displacement of light beam for shadowgraph evaluation

- Sensitivity is proportional to index of refraction 1/n, and screen distance Z_{sc}

Schlieren concept

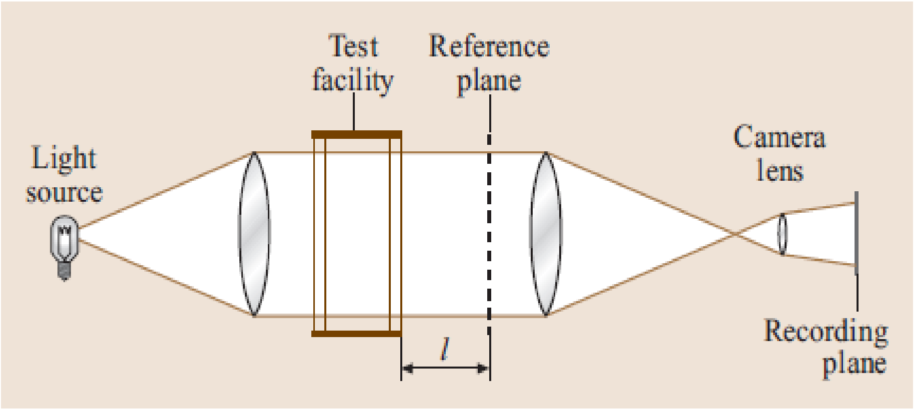


Fig. 6.2 Shadowgraph setup with parallel beams through the test object

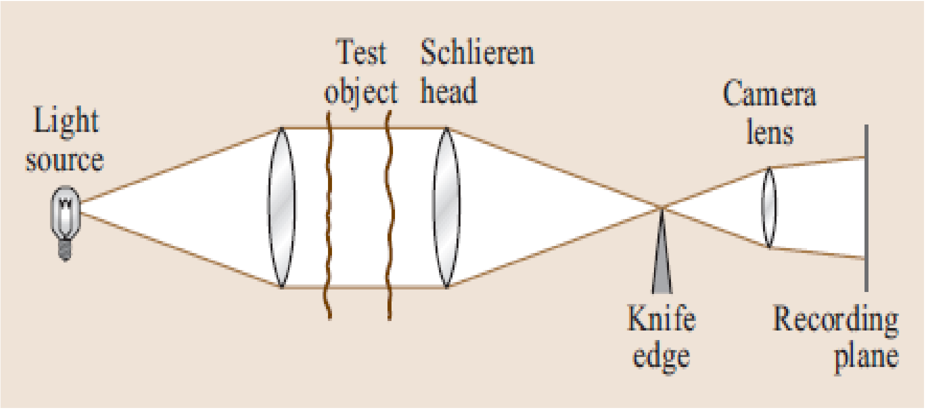
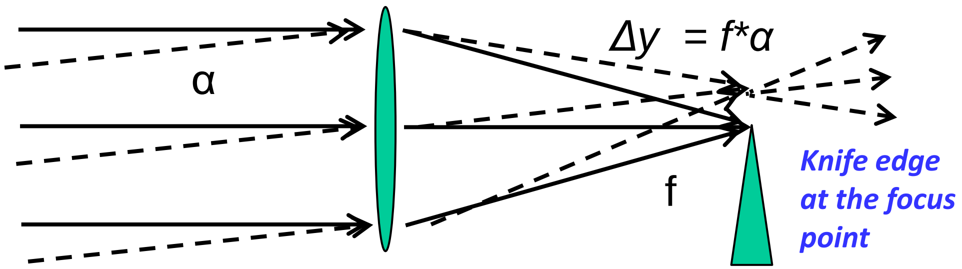
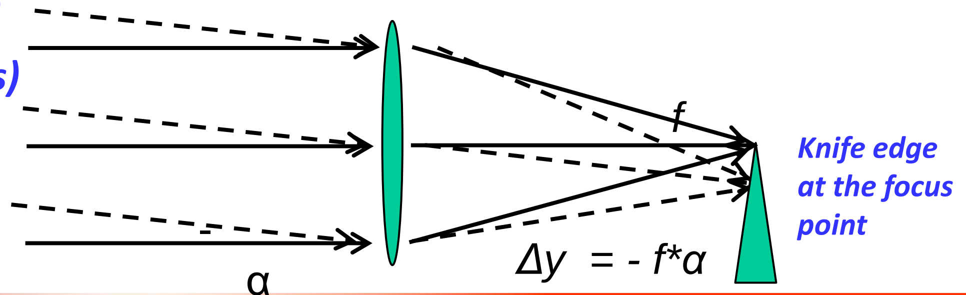


Fig. 6.5 Schlieren setup with parallel light through the test field

- *Parallel rays are focused at len's focal distance*
- *Deflected rays are focused off-axis*
- *Parallel rays at angle α to optical axis are displaced Δy = f*α*
- *Suppose a knife edge is added*
- *Rays deflected away are passed (bright regions)*
- *Rays deflected toward are blocked (dark regions)*



- *Schlieren technique*



FUNDAMENTALS OF SCHLIEREN TECHNIQUE

- *The intensity after the shape razor blade (knife edge) before the experiment*

$$I_k = \frac{a_K}{a_0} I_0$$

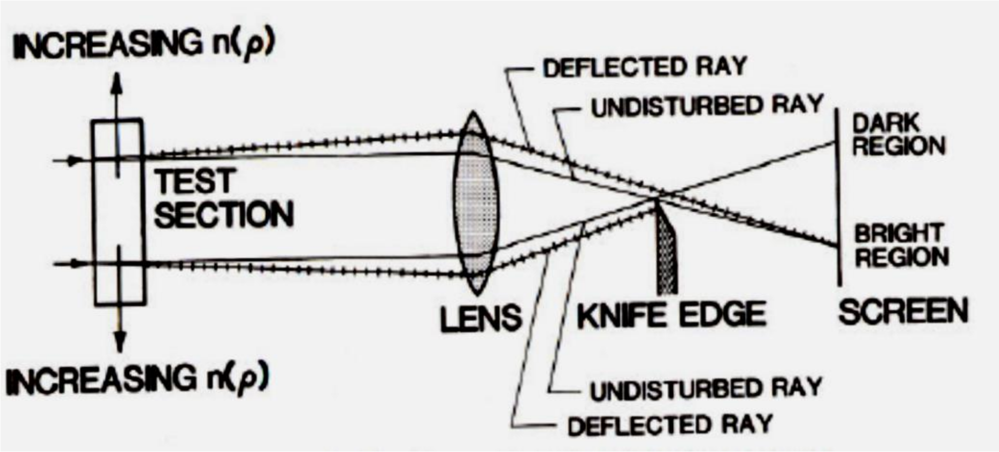
- *The intensity after the deformation due to the variation of the index of refraction*

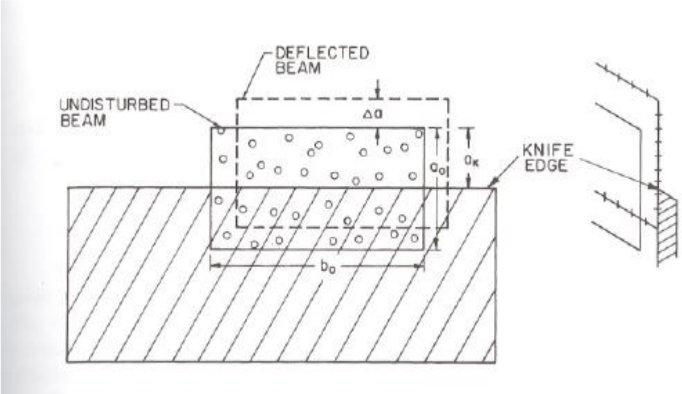
$$I_d = I_k + \frac{\Delta a}{a_K} I_k = (1 + \frac{\Delta a}{a_K}) I_k$$

$$contrast = \frac{\Delta I}{I_k} = \frac{I_d - I_k}{I_k} = \frac{\Delta a}{a_K} = \pm \frac{\alpha f_2}{a_K}$$

$$sensitivity: \quad \frac{d(contrast)}{d\alpha} = \frac{f_2}{a_K}$$

- *Sensitivity is proportional to f_2 and inversely to a_k.*





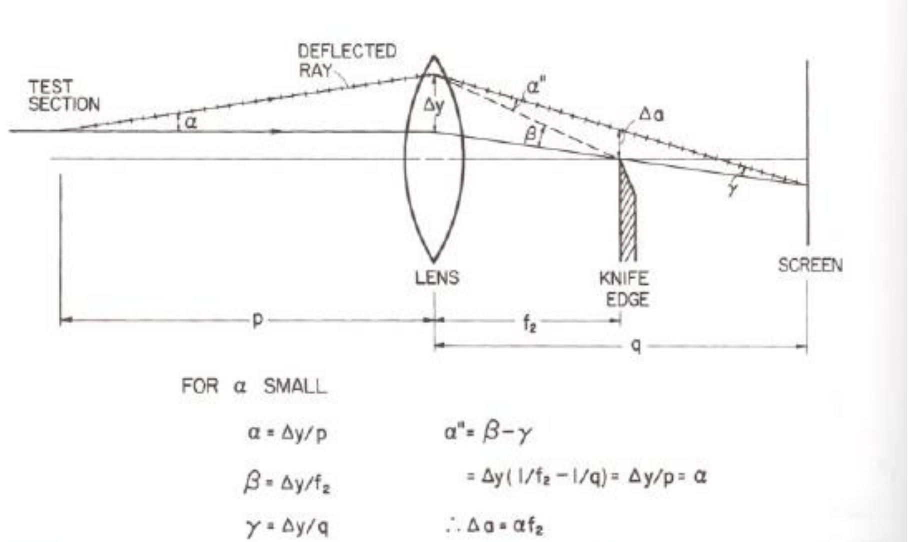
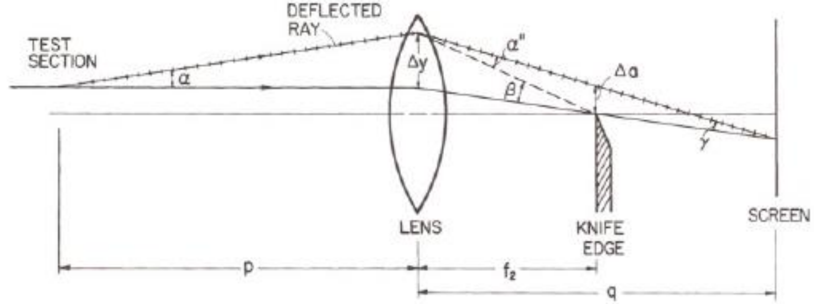


Figure 7.4 Ray displacement at knife-edge for a given angular deflection

Fundamentals of Schlieren Technique

- *For a gas flow with density change:*



FOR α SMALL

$\alpha = \Delta y/p$ $\qquad \alpha'' = \beta - \gamma$

$\beta = \Delta y/f_2$ $\qquad = \Delta y(1/f_2 - 1/q) = \Delta y/p = \alpha$

$\gamma = \Delta y/q$ $\qquad \therefore \Delta a = \alpha f_2$

Figure 7.4 Ray displacement at knife-edge for a given angular deflection

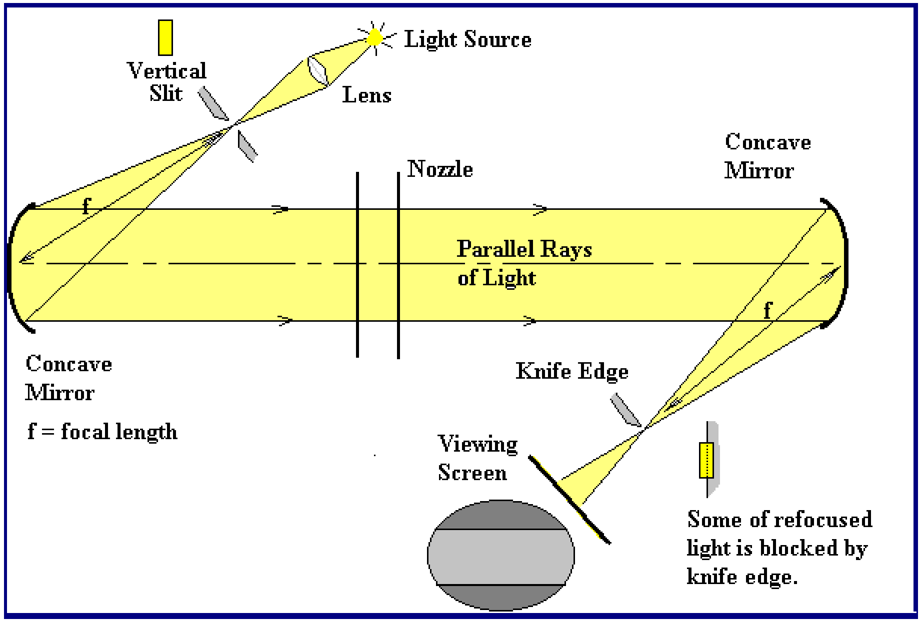
$$\frac{\Delta I}{I_k} = \pm \frac{\alpha\, f_2}{a_K}$$

$$\alpha' = \int \frac{dn}{dy} dz \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \int \frac{dn}{dy} dz$$

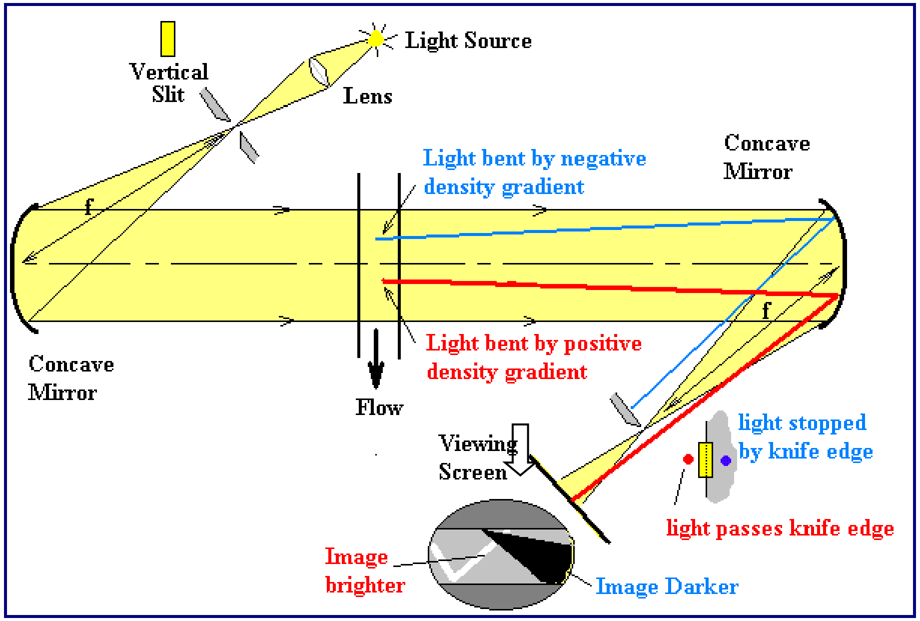
$$\frac{\partial \rho}{\partial y} = \frac{\rho_0}{n_0 - 1} \frac{\partial n}{\partial y} \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \int \frac{d\rho}{dy} dz$$

$$n \approx 1 \quad \Rightarrow \quad \frac{\Delta I}{I_k} = \pm \frac{f_2}{a_K} \frac{n_0 - 1}{\rho_0} \frac{d\rho}{dy} L$$

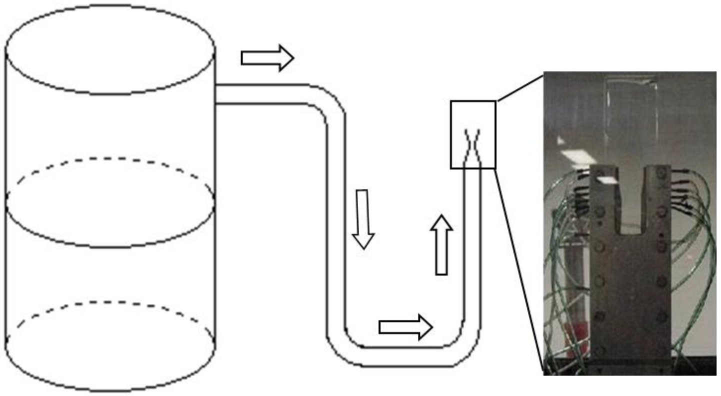
Visualization of shock waves in a transonic/supersonic nozzle using Schlieren technique

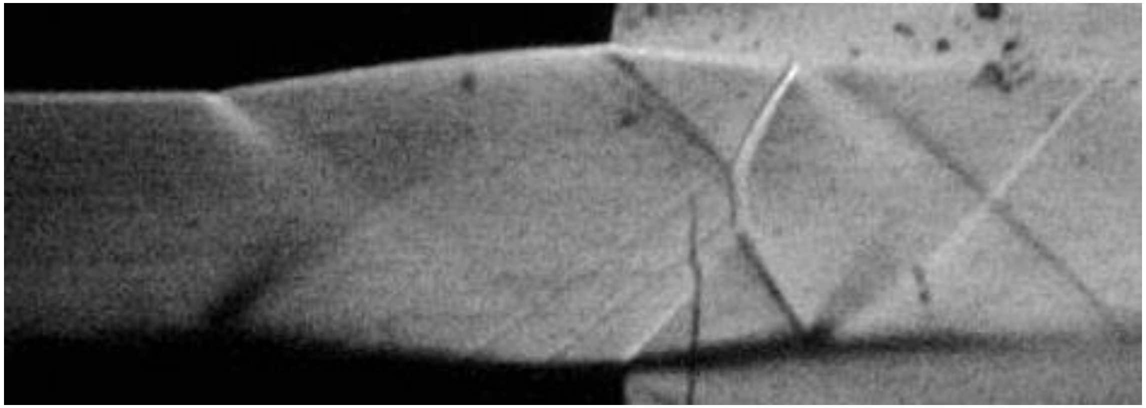


Before turning on the Supersonic jet



After turning on the Supersonic jet





- *Over-expanded flow*

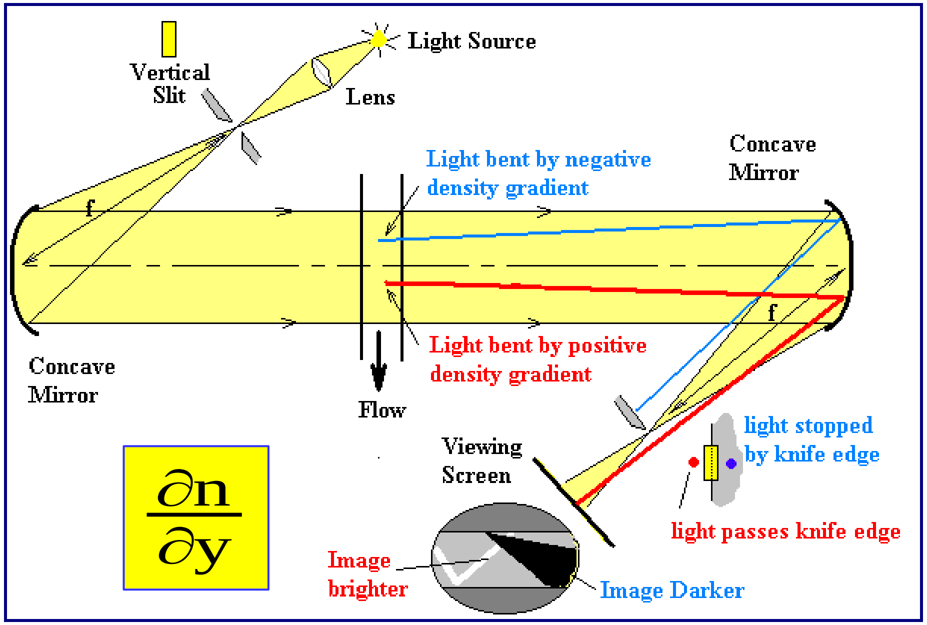
Comparison of Schlieren vs. Shadowgraph

Schlieren:

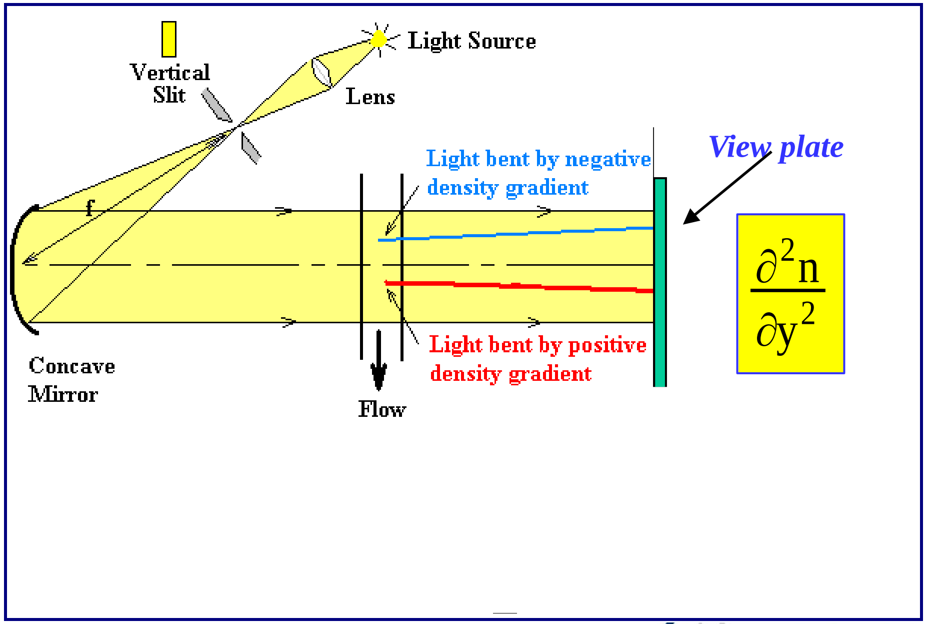
- *Displays a focused image*
- *Shows ray refraction angle, ε*
- *Contrast level responds to the 1st derivative of refractive index changes.*
- *Knife edge used for cutoff*

Shadowgraph:

- *Displays a mere shadow*
- *Shows light ray displacement*
- *Contrast level responds to the 2nd derivative of refractive index changes.*
- *No knife edge used*



$$\frac{\partial n}{\partial y}$$



$$\frac{\partial^2 n}{\partial y^2}$$

Schlieren & Shadowgraph for Quantitative Measurements

- *Application of the Schlieren and shadowgraph techniques:*
  - *Compressible flow with shock waves* $\Rightarrow$ *density changes*
  - *Natural convective flow* $\Rightarrow$ *density changes*
  - *Flame and combustion system:* $\Rightarrow$ *density changes*

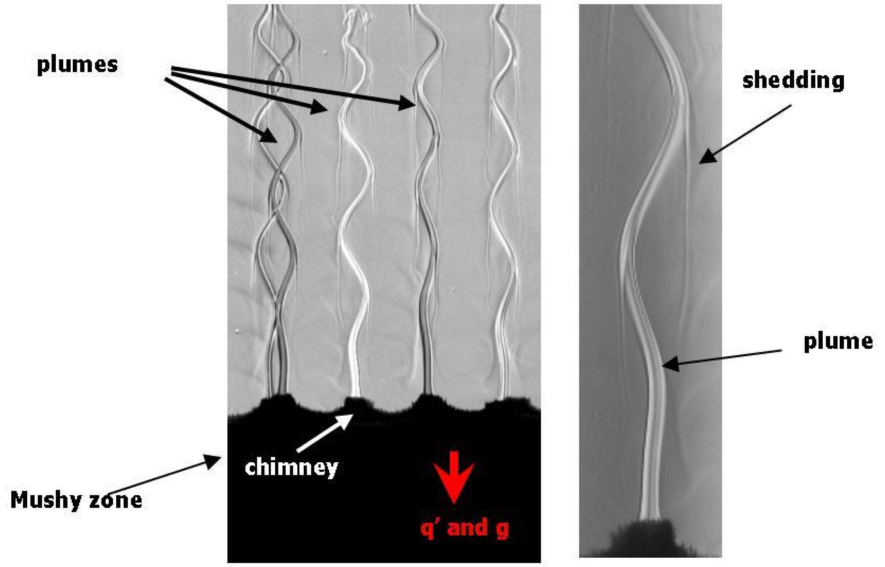
- *Temperature changes inside flows:*
  - *For low speed flow with heat transfer:*
  - *P = constant*

$$\rho = P/RT \Rightarrow \frac{\partial \rho}{\partial y} = \frac{P}{RT^2}\frac{\partial T}{\partial y} = \frac{\rho}{T}\frac{\partial T}{\partial y}$$

$$\Rightarrow \frac{\partial n}{\partial y} = \frac{n_0 - 1}{\rho_0}\frac{\partial \rho}{\partial y} = \frac{n_0 - 1}{T}\frac{\rho}{\rho_0}\frac{\partial T}{\partial y}$$

$$\Rightarrow \frac{\partial T}{\partial y} = \frac{T}{n_0 - 1}\frac{\rho_0}{\rho}\frac{\partial n}{\partial y}$$

$$\Rightarrow \frac{\partial^2 n}{\partial y^2} = \frac{n_0 - 1}{\rho_0}\left[-\frac{\rho}{T}\frac{\partial^2 T}{\partial y^2} + \frac{2\rho}{T^2}\left(\frac{\partial T}{\partial y}\right)^2\right]$$



Schlieren & Shadowgraph for Quantitative Density Measurement

- *Index of refraction is a function of thermodynamic state (density) for homogeneous medium:*

- *Lorenz-Lorentz relationship:*

$$\frac{1}{\rho}\frac{n^2-1}{n^2+2}=const$$

- *When n≈1, for gaseous flow:*

$$\frac{n-1}{\rho}=const \quad \Rightarrow \quad \rho=\frac{n-1}{const}$$

- *at standard condition, with n_o and ρ_o, :*

$$\frac{n_0-1}{\rho_0}=const \quad \Rightarrow \quad n-1=\frac{\rho}{\rho_0}(n_0-1)$$

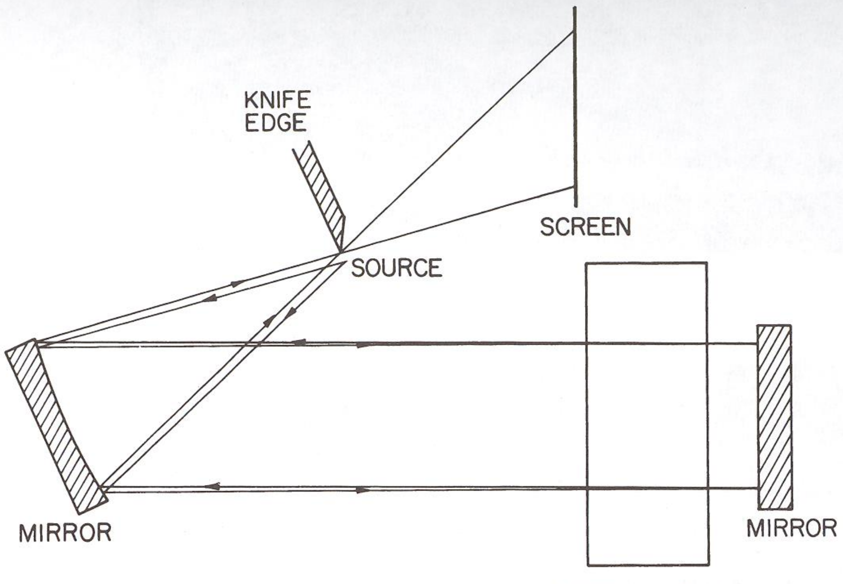
$$\Rightarrow \quad \rho=\rho_0\frac{n-1}{n_0-1}$$

- *When first and second derivative is determined as in Schlieren and shadowgraph apparatus:*

$$\frac{\partial\rho}{\partial y}=\frac{1}{const}\frac{\partial n}{\partial y} \quad \Rightarrow \frac{\partial\rho}{\partial y}=\frac{\rho_0}{n_0-1}\frac{\partial n}{\partial y}$$

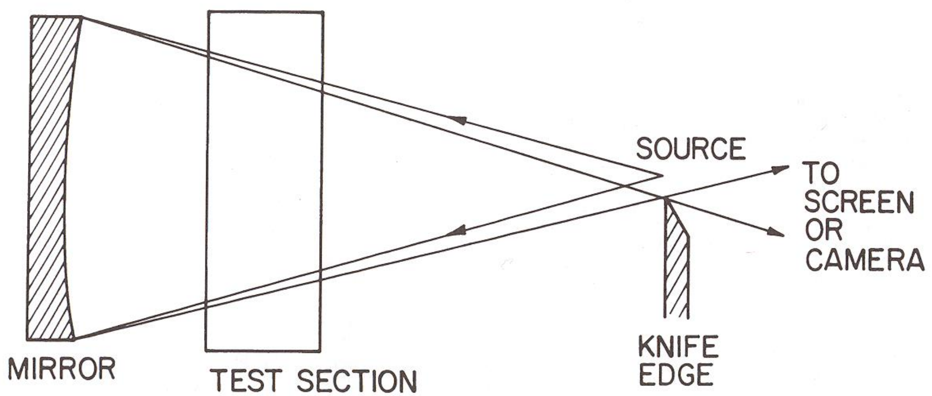
$$\frac{\partial^2\rho}{\partial y^2}=\frac{1}{const}\frac{\partial^2 n}{\partial y^2} \quad \Rightarrow \frac{\partial^2\rho}{\partial y^2}=\frac{\rho_0}{n_0-1}\frac{\partial^2 n}{\partial y^2}$$

Alternative Schlieren system



(a)

A. Setup with one converging and one plane mirror



(b)

A. Setup with one converging mirror

Holographic Schlieren system

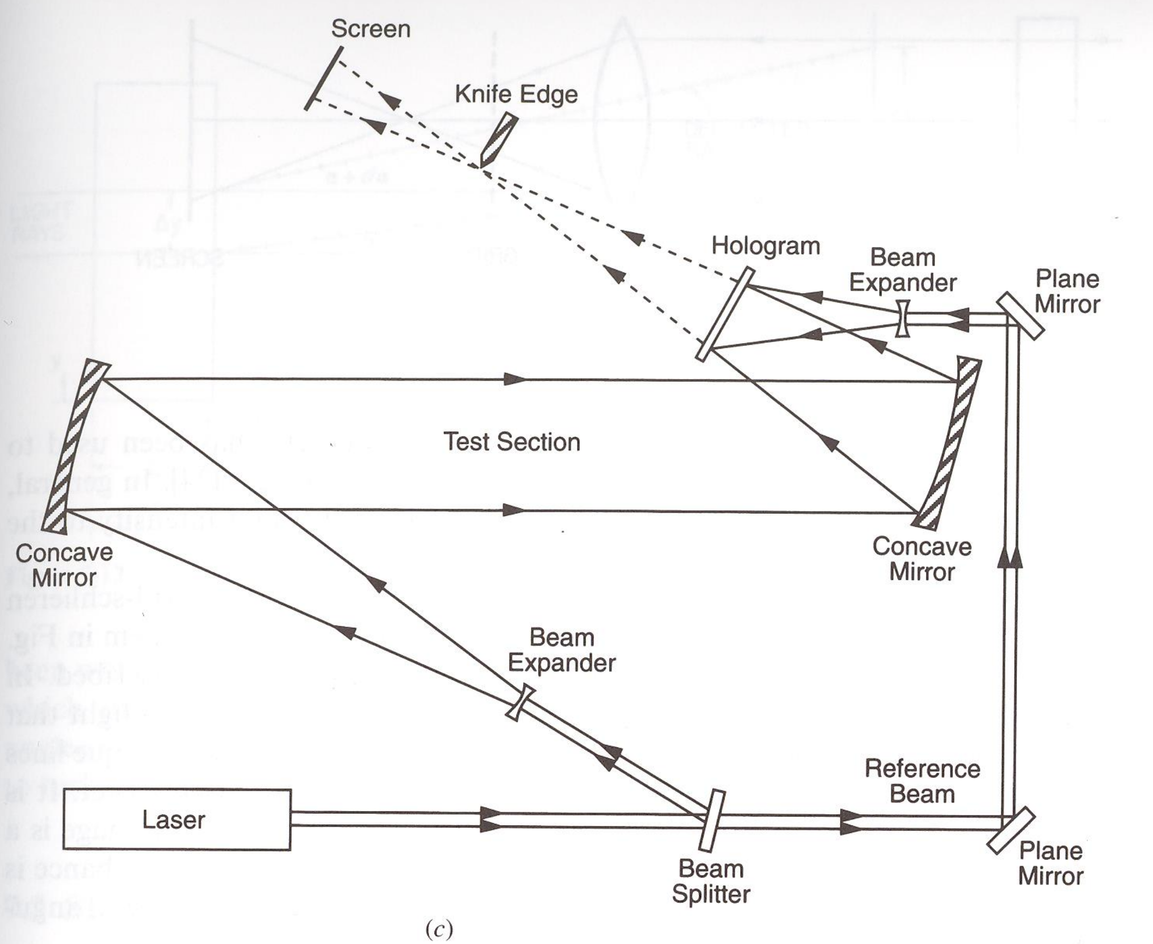


Figure 7.10 Continued

Fundamentals of Schlieren System

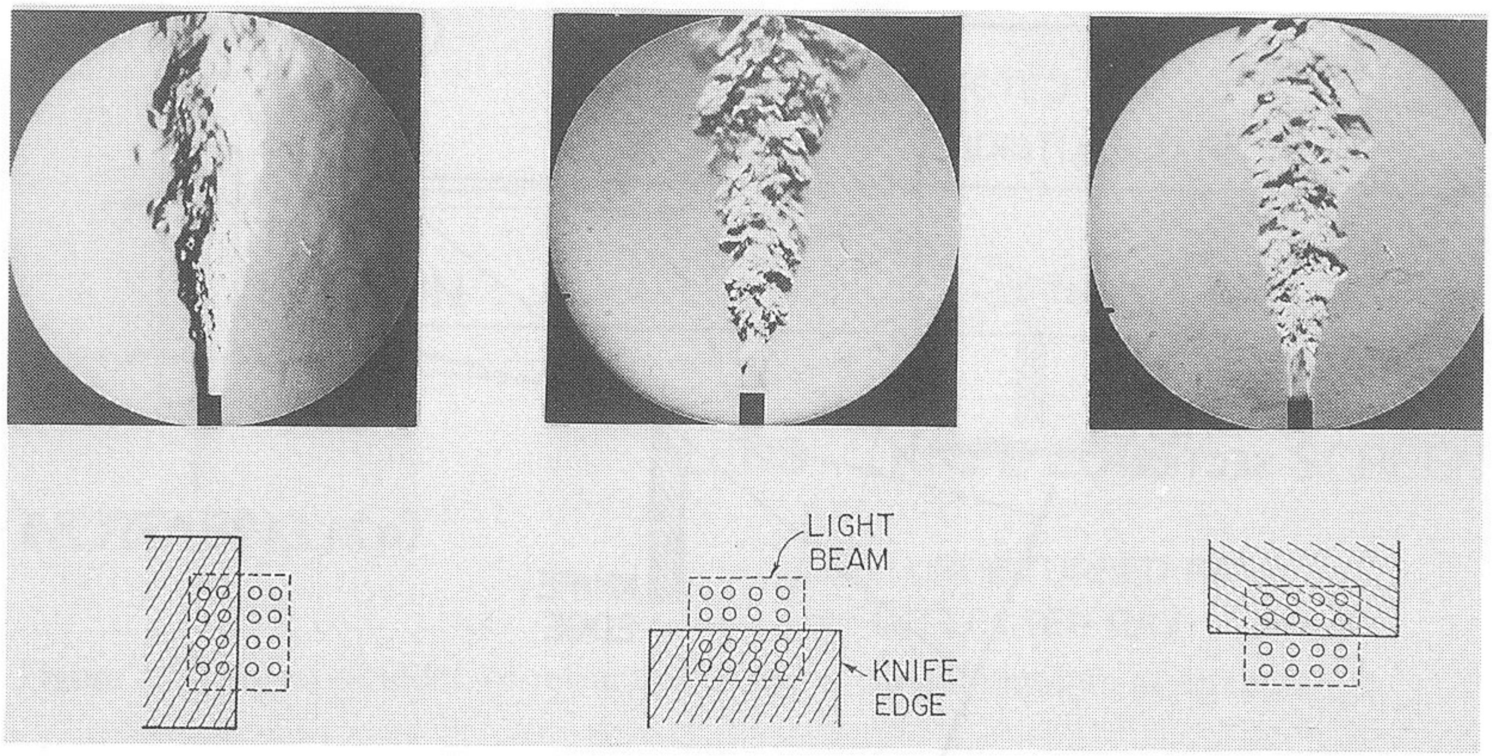


Figure 7.7 Schlieren images of a helium jet entering an atmosphere of air: The effect of knife-edge orientation (Re = 630)

Fundamentals of Schlieren System

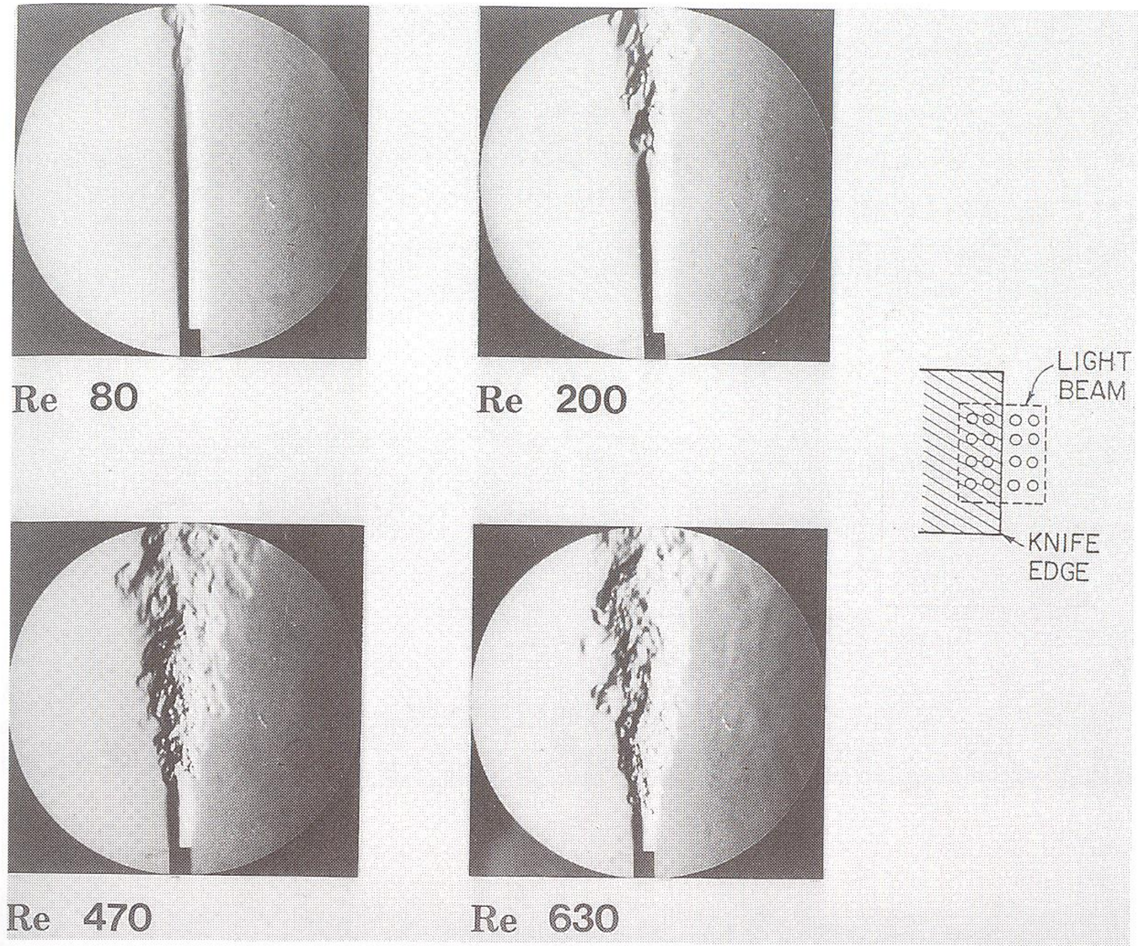
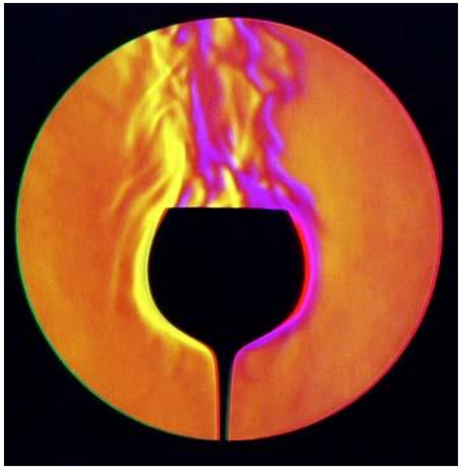


Figure 7.8 Schlieren images of the flow structure of a helium jet entering air at differ numbers

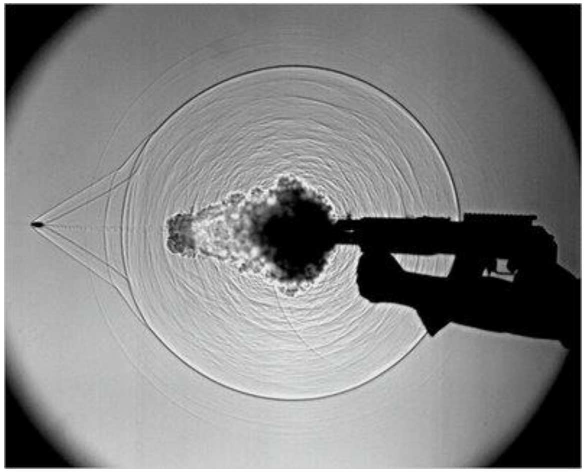
Examples: Shlieren Photography



Warm water

A cough

A gas leak



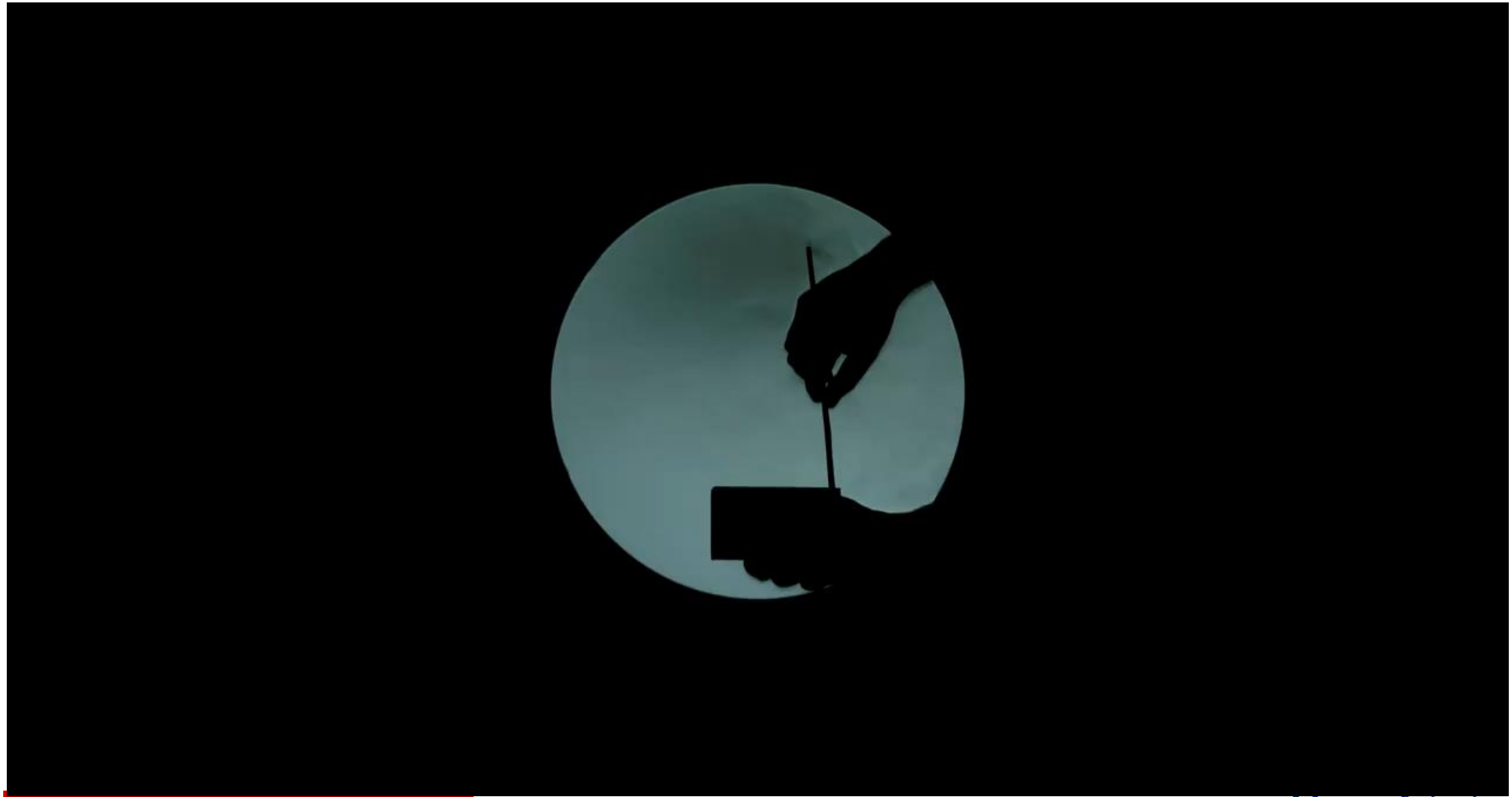
The firing of an AK-47.

A simulated explosion in an airplane cabin.

Hair dryer

Schlieren Application Examples

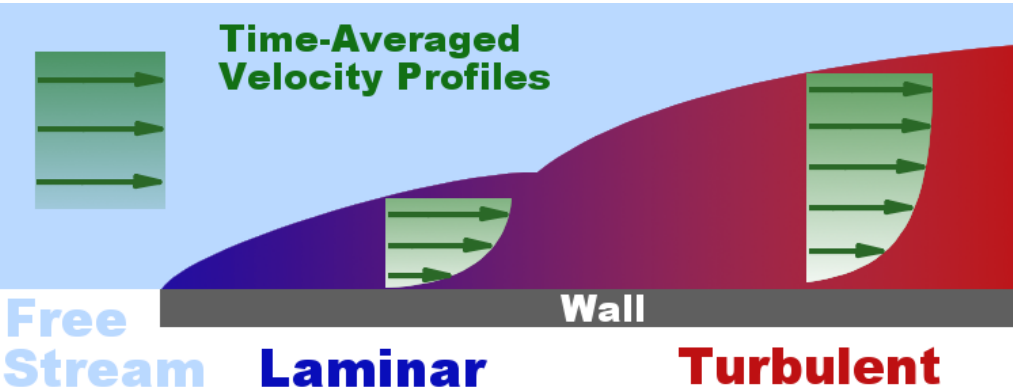
- ***Seeing the Invisible: SLOW MOTION Schlieren Imaging results*** ***https://www.youtube.com/watch?v=4tgOyU34D44***

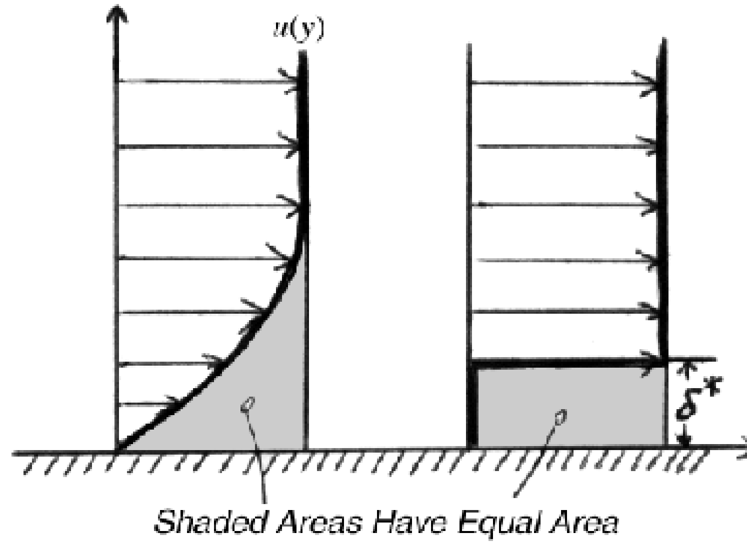


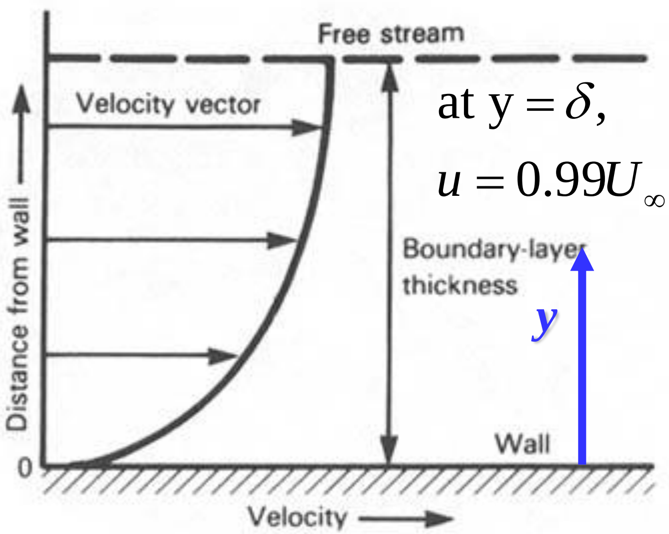
Summary of the 1st Survey of AerE344 Course

- ***Group size:*** *Most of the students prefer smaller group*
- ***Lecture:*** *Most of students think lectures are helpful*
  - *Sound of the embedded videos is not good…*
  - *Better linkage to the labs*
  - *More "Pre-Lab Videos“ and more on the data analysis*
  - *On what the week's lab is about first, then tie it to the material next.*
- ***Lab:*** *Most of students feel AerE344 labs are helpful for you to better understanding the concepts and principles taught in AerE310 and AerE311.*
- ***TAs:*** *Most of students think the TAs are doing nice jobs.*
  - *Some TAs is restricted from offering as much help as they could, especially when it comes to Matlab.*
- ***Other comments:***
  - *It would be nice to have more guidelines around lab writing and the coding.*
  - *What does the final exam look like*

AerE344 Lab#8: Measurements of Boundary Layer over a Flat Plate







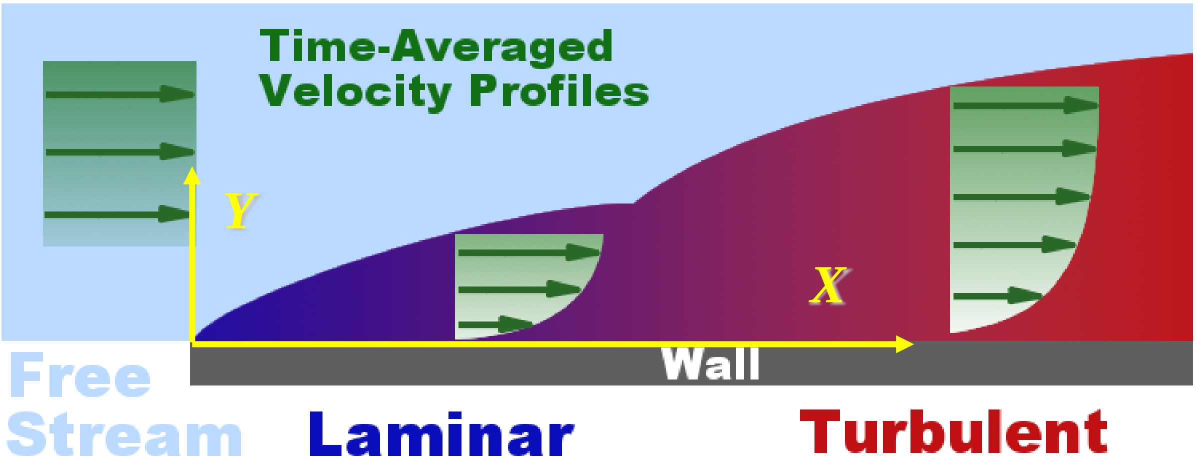
at $y = \delta$,

$u = 0.99U_\infty$

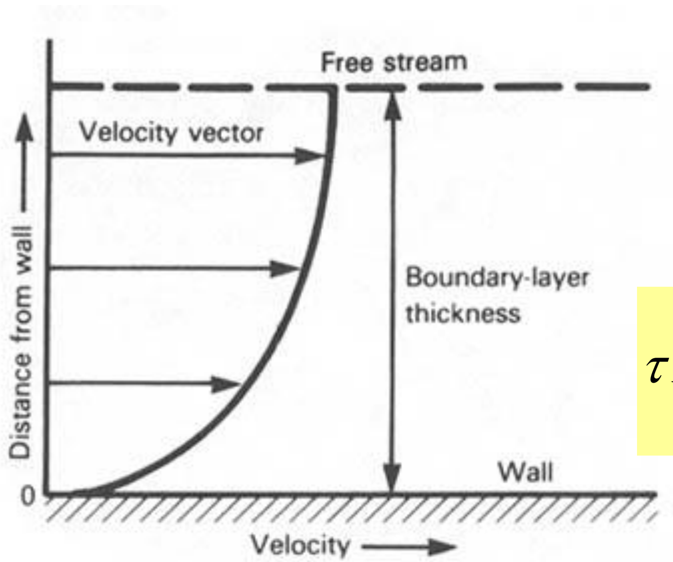
Displacement thickness: $\delta^* \equiv \int_0^\infty \left(1 - \frac{u}{U}\right) dy$

Momentum thickness: $\theta \equiv \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

AerE344 Lab#8: Measurements of Boundary Layer over a Flat Plate



$$\frac{\partial p}{\partial y} \approx 0$$



$$\tau_w = \mu \frac{\partial U}{\partial y}\bigg|_{wall}$$

Laminar boundary layer:

$$Re_X = \frac{\rho U_\infty X}{\mu}$$

$$C_f = \frac{1.328}{\sqrt{Re_X}}$$

$$\delta = \frac{5.0X}{\sqrt{Re_X}}$$

$$\delta^* = \frac{1.72X}{\sqrt{Re_X}}$$

$$\theta = \frac{0.664X}{\sqrt{Re_X}}$$

Turbulent boundary layer:

$$Re_X = \frac{\rho U_\infty X}{\mu}$$

$$C_f = \frac{0.074}{(Re_X)^{1/5}}$$

$$\delta = \frac{0.37X}{(Re_X)^{1/5}}$$

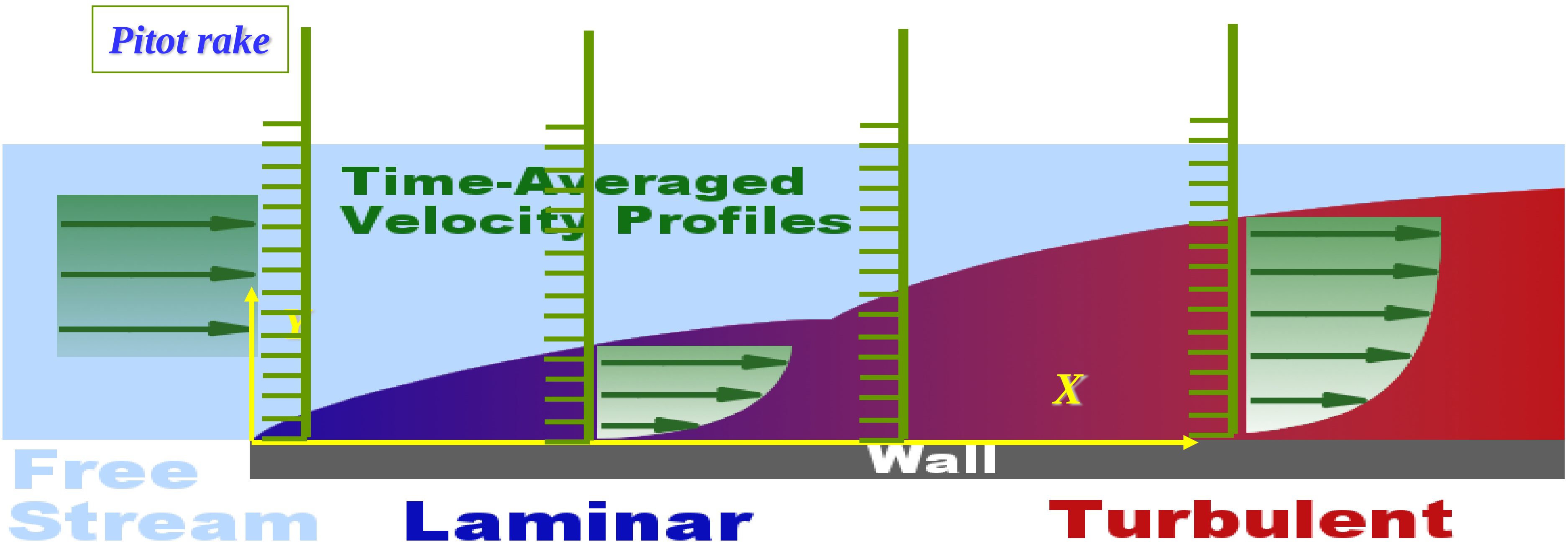
AerE344L Lab#8: Measurements of Boundary Layer over a Flat Plate

- *To conduct velocity profile measurements at 10 downstream locations.*
- *To determine boundary layer thickness and drag coefficient based on the velocity measurement results.*

Displacement thickness: $\delta^* \equiv \int_0^\infty \left(1 - \frac{u}{U}\right) dy$

Momentum thickness: $\theta \equiv \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

Drag coefficient: $C_d = 2\frac{\theta}{L}$



AerE344 Lab#8: Measurements of Boundary Layer over a Flat Plate

Plots needed for lab report:

- *Mean velocity profiles based on your measurements:* U/U_∞ vs y/δ
- *Plot the experimental values of $\delta(x)$ and θ x). The plot should also include comparison to the analytical expressions .*
- *Using the momentum thickness and the integral momentum equations to estimate:*
  - *Local shear stress coefficient C_f, , as a function of x.*
  - *Find the drag coefficient Cd*