Lecture # 11: Particle image velocimetry

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Sources/ Further reading:

Raffel, Willert, Wereley, Kompenhans, "Particle image velocimetry: A practical guide" 2nd ed.

Particle-based Flow Diagnostic Techniques

- *Seeded the flow with small particles (~ µm in size)*
- *Assumption: the particle tracers move with the same velocity as local flow velocity!*

Flow velocity V_f = **Particle velocity V_p**

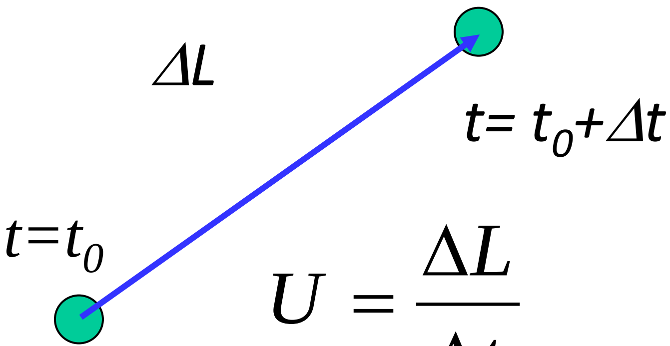
↑

Measurement of particle velocity

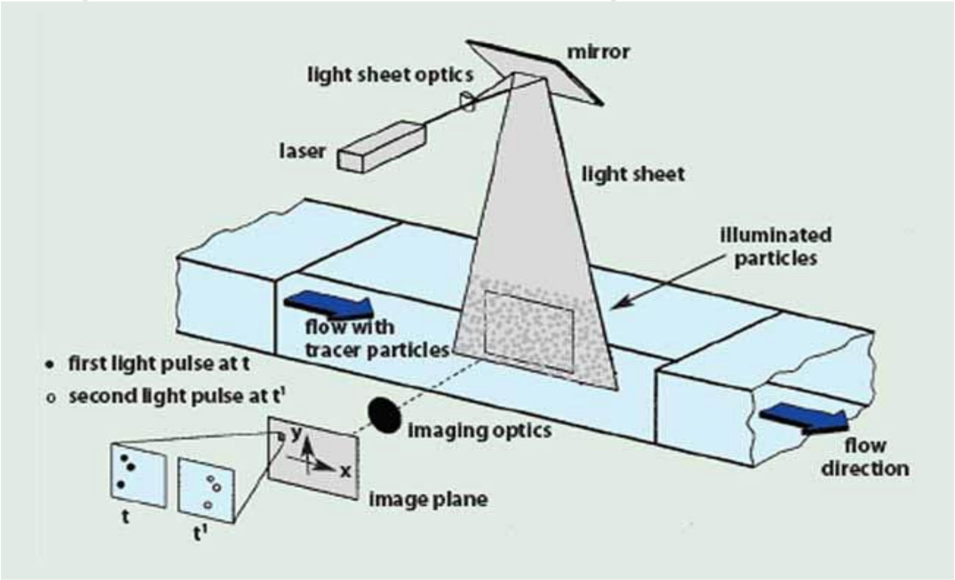
- *Smoke visualization*

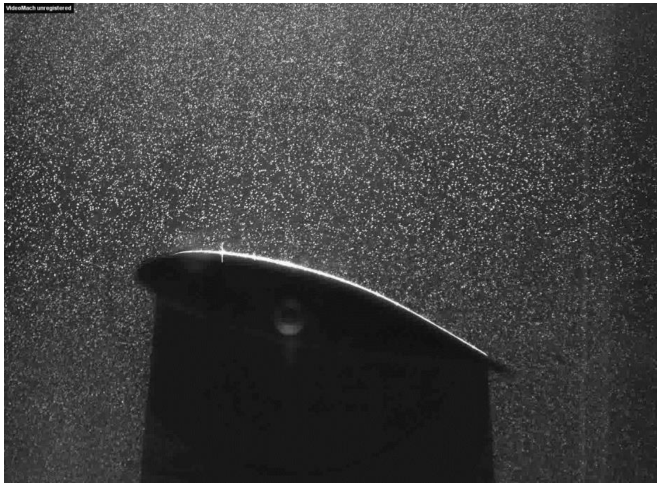
Particle Image Velocimetry (PIV) technique

- ***Time-of-flight method:*** *to measure the displacements of the tracer particles seeded in the flow in a fixed time interval.*

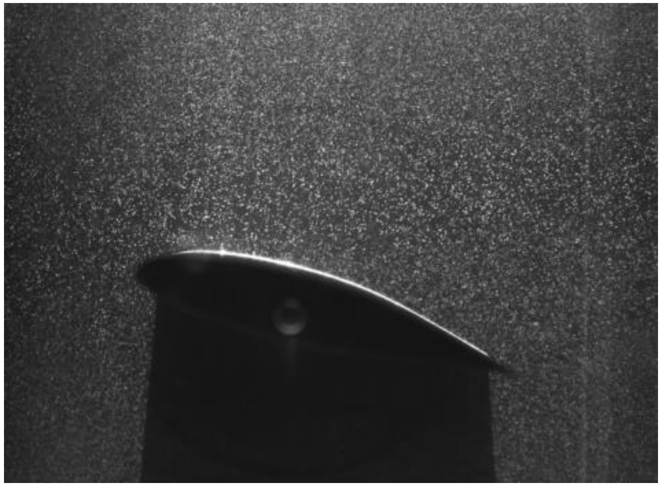


$$U = \frac{\Delta L}{\Delta t}$$

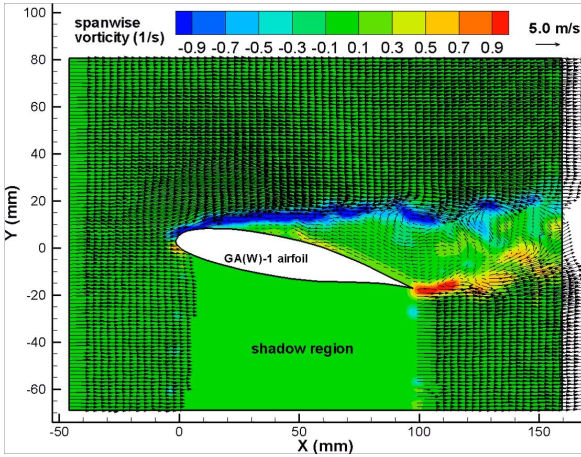




a. T=t0



b. T=t0+10μs



Corresponding Velocity field

PIV System Setup

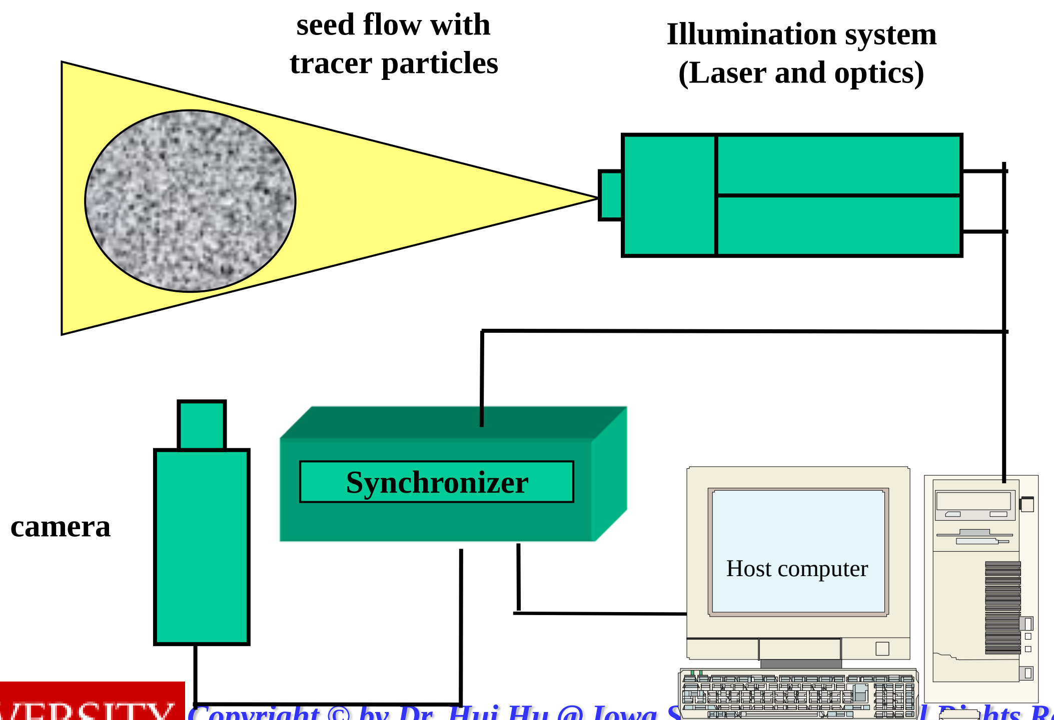
Particle tracers: track the fluid movement.

Illumination system: illuminate the flow field in the interest region.

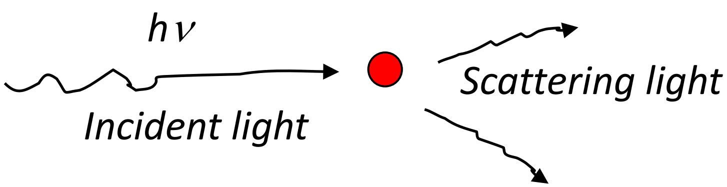
Camera: capture the images of the particle tracers.

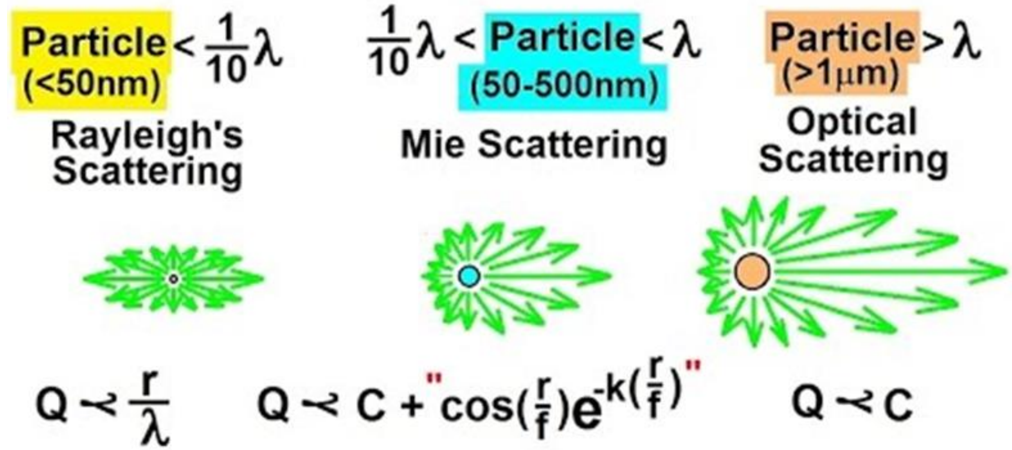
Synchronizer: control the timing of the laser illumination and camera acquisition.

Host computer: to store the particle images and conduct image processing.

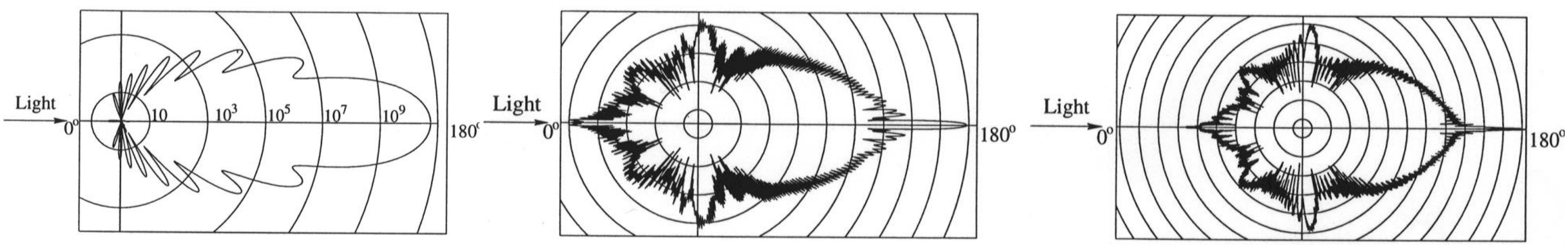


Tracer Particles for PIV





- *Tracer particles should be neutrally buoyant and small enough to follow the flow perfectly.*
- *Tracer particles should be big enough to scatter the illumination lights efficiently .*
- *The scattering efficiency of trace particles also strongly depends on the ratio of the refractive index of the particles to that of the fluid.*
  For example: the refractive index of water is considerably larger than that of air. The scattering of particles in air is at least one order of magnitude more efficient than the same particles size in water.



a). d=1μm *b). d=10μm* *c). d=30μm*

Tracer Particles for PIV

- ***A primary source of measurement error is the influence of gravitational forces when the density of the tracer particles is different to the density of work fluid.***

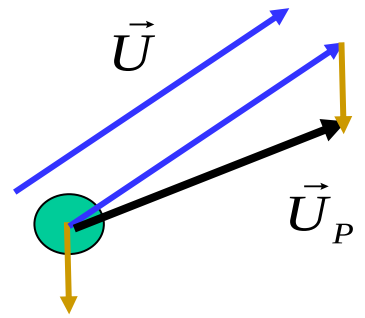
$$U_g = d_p^{\,2}\frac{(\rho_p-\rho)}{18\mu}g$$

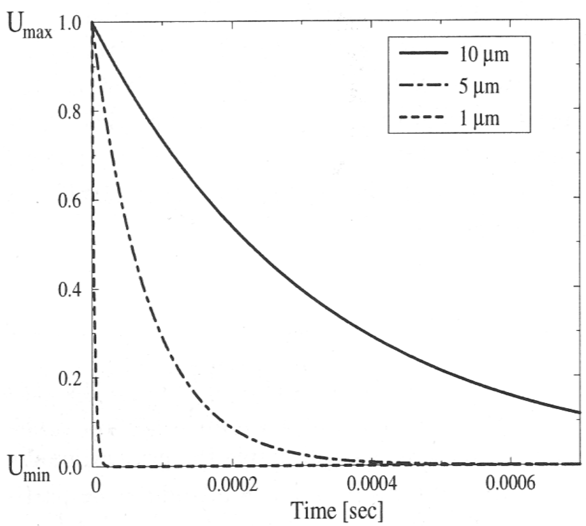
- ***The velocity lag of a particle in a continuously acceleration fluid will be:***

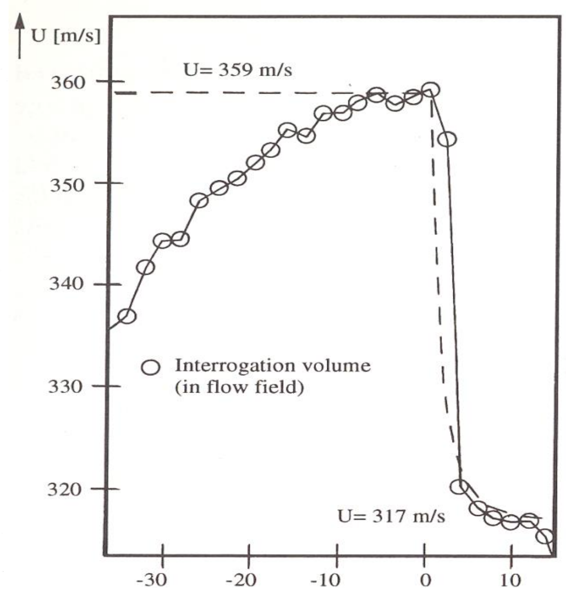
$$\vec{U}_s = \vec{U}_P - \vec{U} = d_p^{\,2}\frac{(\rho_p-\rho)}{18\mu}g$$

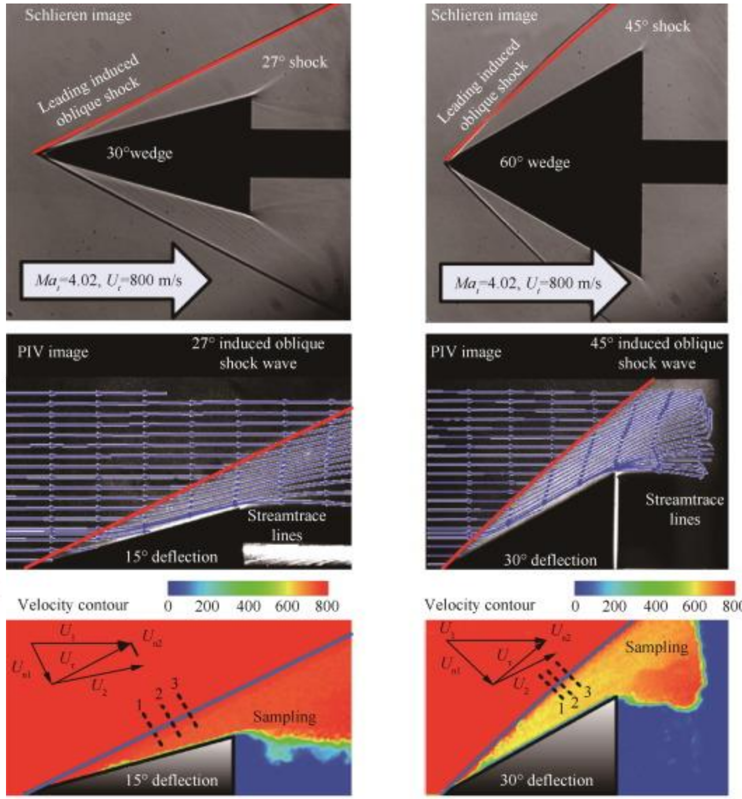
$$\vec{U}_p(t) = \vec{U}(1-\exp(-\frac{t}{\tau_s}));$$

$$\tau_s = d_p^{\,2}\frac{\rho_p}{18\mu}$$









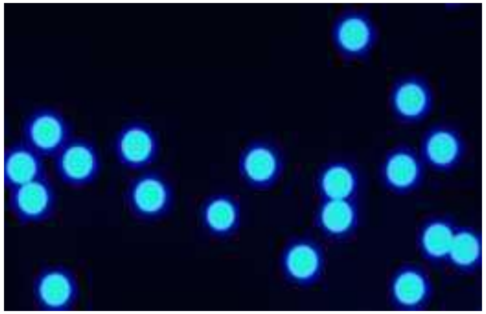
(a) A wedge with deflection angle 15°

(b) A wedge with deflection angle 30°

Tracer Particles for PIV

- *Tracers for PIV measurements in liquids (water):*
  - *Polymer particles (d=10~100 μm, density = 1.03 ~ 1.05 kg/cm^3)*
  - *Silver-covered hollow glass beams (d =1 ~10 μm, density = 1.03 ~ 1.05 kg/cm3)*
  - *Fluorescent particle for micro flow (d=200~1000 nm, density = 1.03 ~ 1.05 kg/cm3).*
  - *Quantum dots (d= 2 ~ 10 nm)*

roduct
Polyamide particles, 55 µm, 1.2g/cm³
Polyamide particles, 100 µm, 1.1 g/cm³
Polyamide particles HQ, 20 µm, 1.03g/cm³
Polyamide particles HQ, 60 µm, 1.03g/cm³



- *Fluorescent particle*

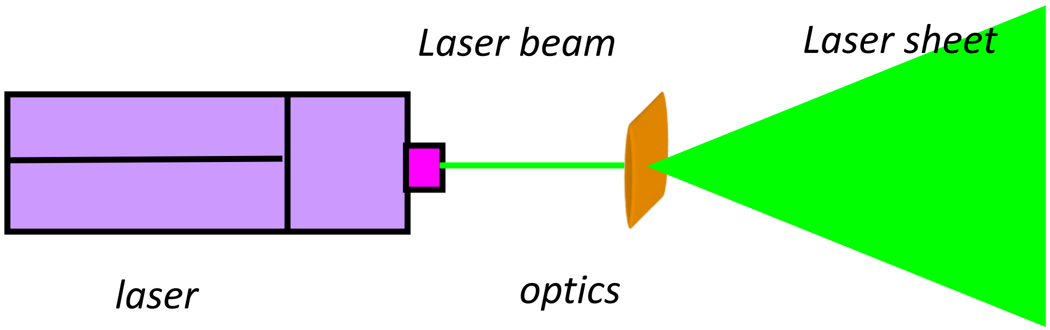
- *Tracers for PIV measurements in gaseous flows:*
  - *Smoke …*
  - *Droplets, mist, vapor…*
  - *Condensations ….*
  - *Hollow silica particles (0.5 ~ 2 µm in diameter and 0.2 g/cm3 in density for PIV measurements in combustion applications.*
  - *Nanoparticles of combustion products*

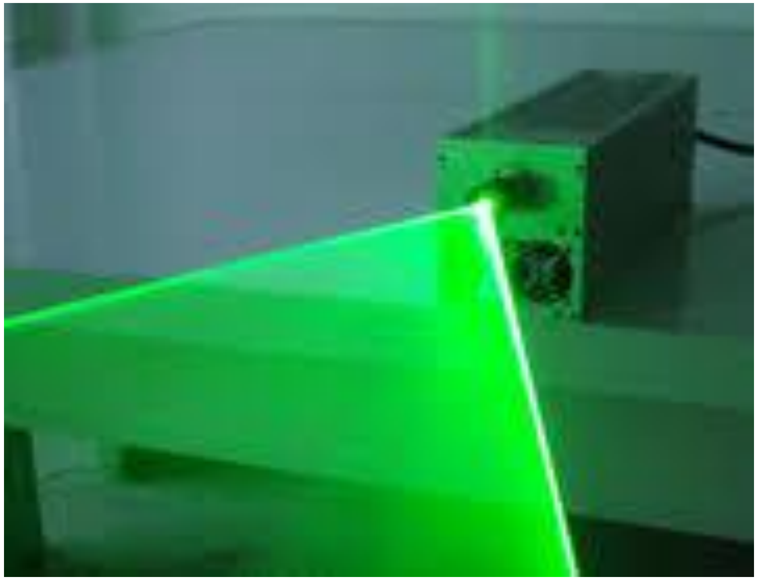
illumination system

• *The illumination system of PIV is always composed of light source and optics.*

• ***Lasers**: such as Argon-ion laser and Nd:YAG Laser, are widely used as light source in PIV systems due to their ability to **emit monochromatic light** with **high energy density** which can easily be bundled into thin light sheet for illuminating and recording the tracer particles without chromatic aberrations.*

• ***Optics**: always consisted by a set of cylindrical lenses and mirrors to shape the light source beam into a planar sheet to illuminate the flow field.*





Double-pulsed Nd:Yag Laser for PIV

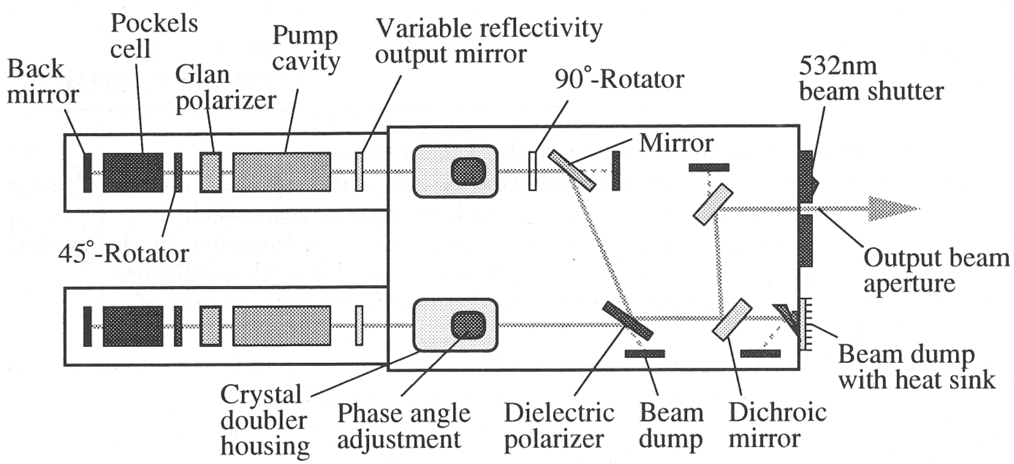
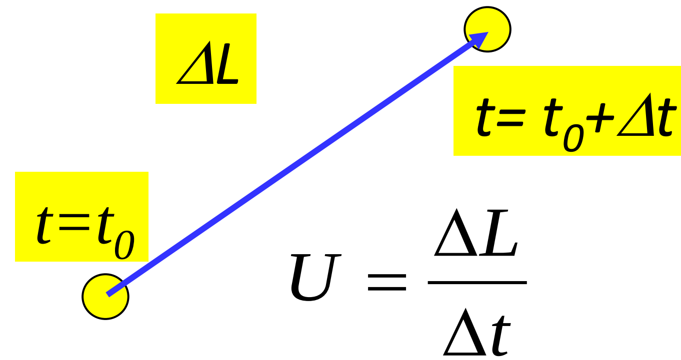
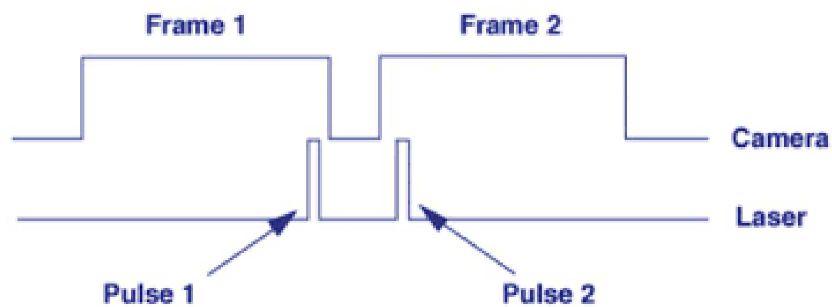


Fig. 2.17. Double oscillator laser system with critical resonators



$$U = \frac{\Delta L}{\Delta t}$$



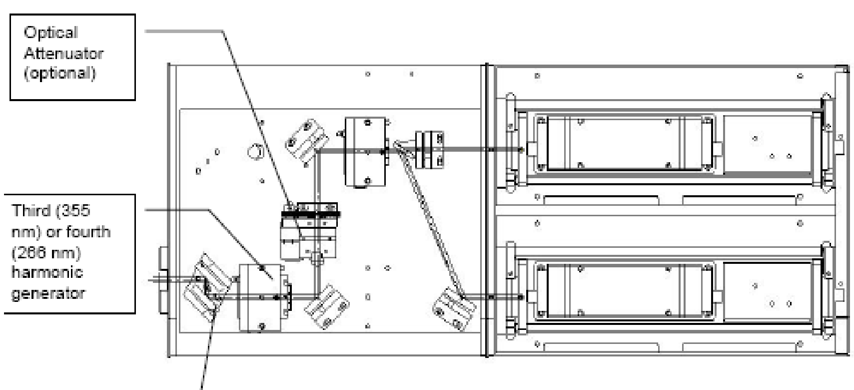
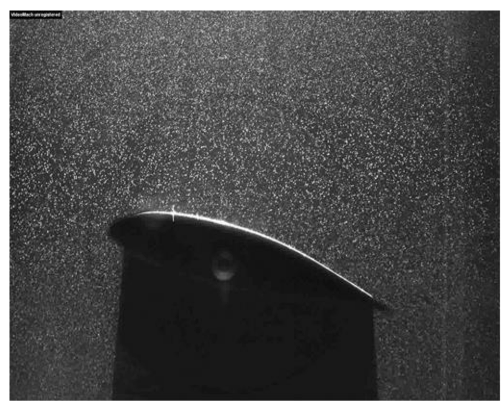
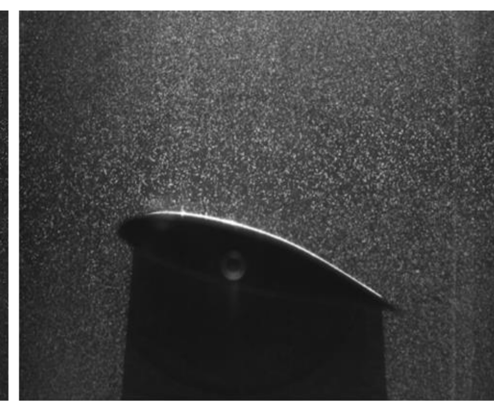


Figure 4-8: Gemini with Extended Base Plate for UV Operation

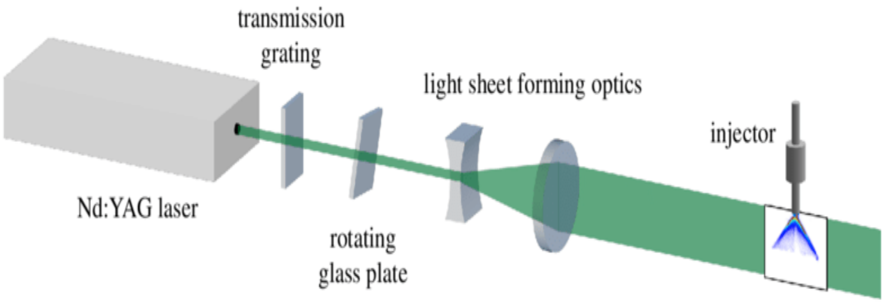


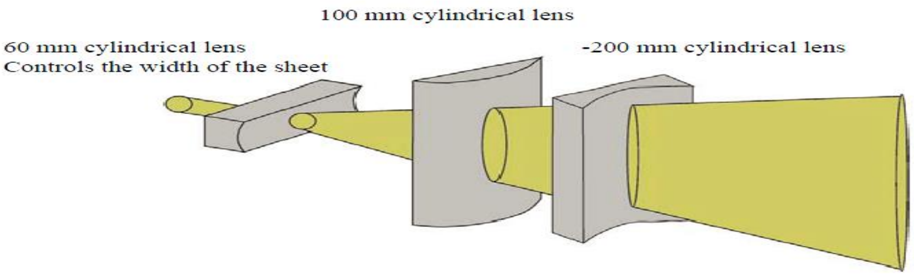
a. T=t0

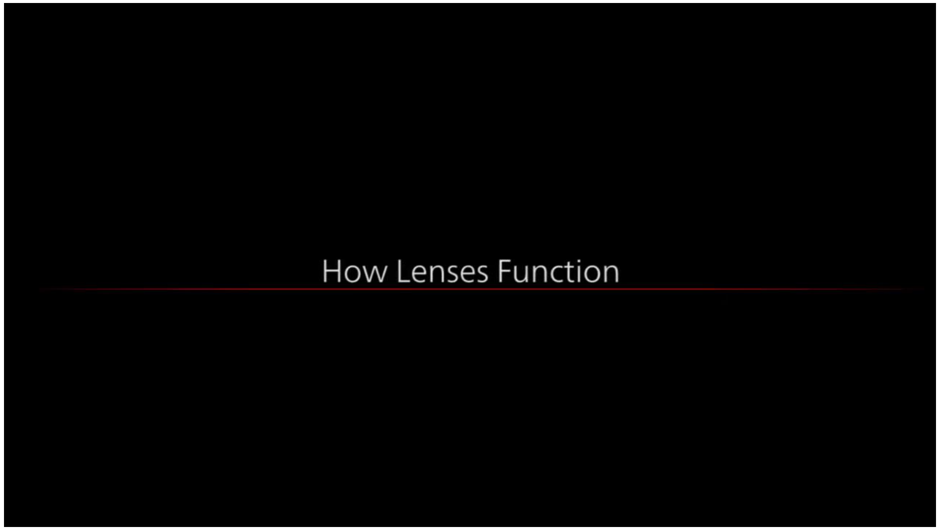


b. T=t0+10μs

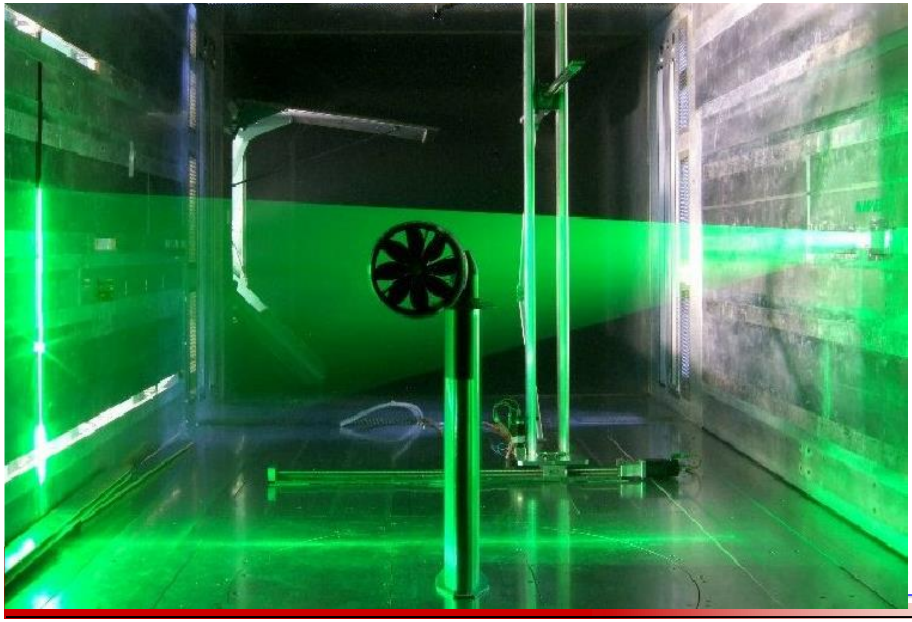
Optics/Lenses to shape Laser Beam to Sheet







- https://www.youtube.com/watch?v=EL9J3Km6wxI



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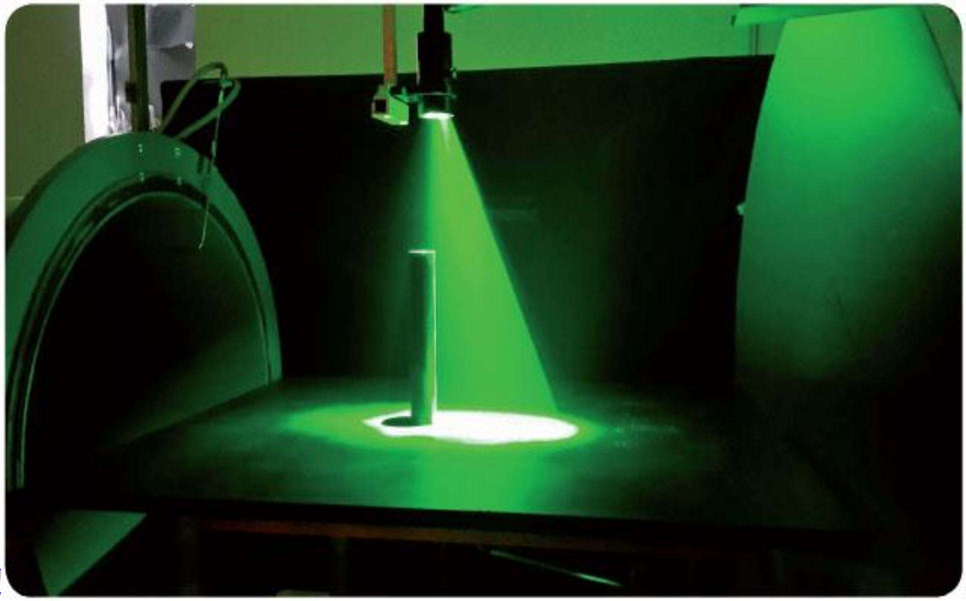
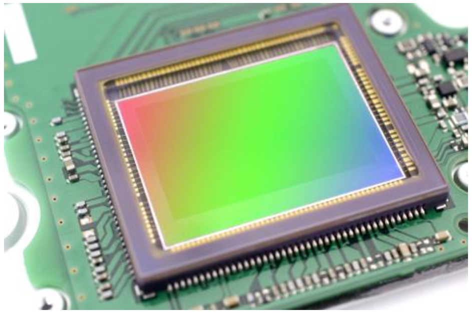


Image Acquisition System: Cameras

- The widely used cameras for PIV:
  - Photographic film-based cameras or digital cameras.

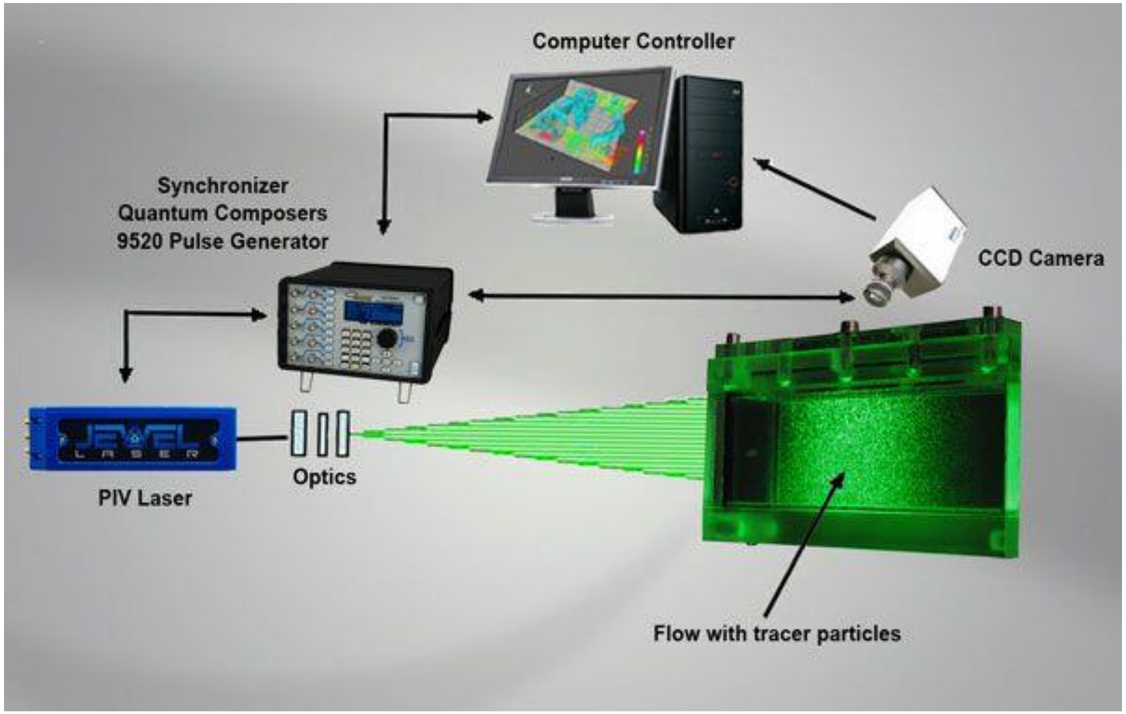
- Advantages of digital cameras:
  - It is fully digitized
  - Various digital techniques can be implemented for PIV image processing.
  - Conventional auto- or cross-correlation techniques combined with special framing techniques can be used to measure higher velocities.

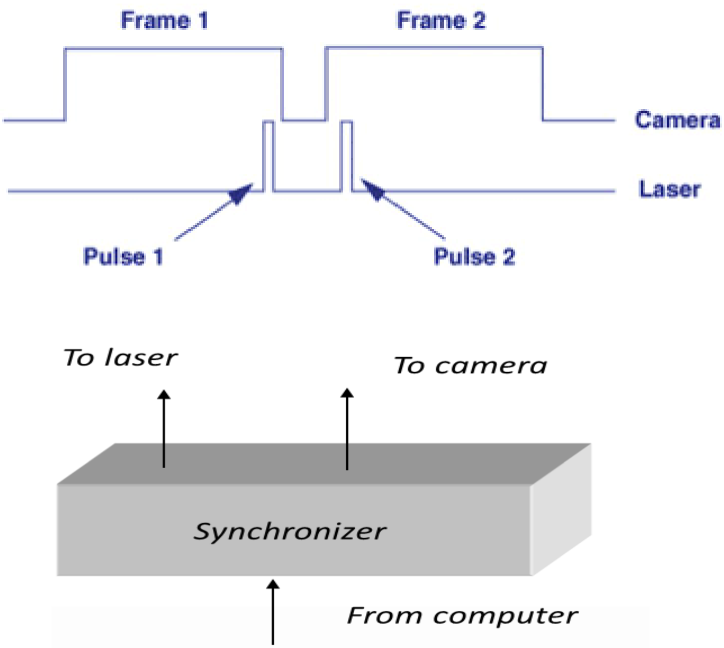


- Disadvantages of digital cameras:
  - Low temporal resolution (defined by the video framing rate):
  - Low spatial resolution:

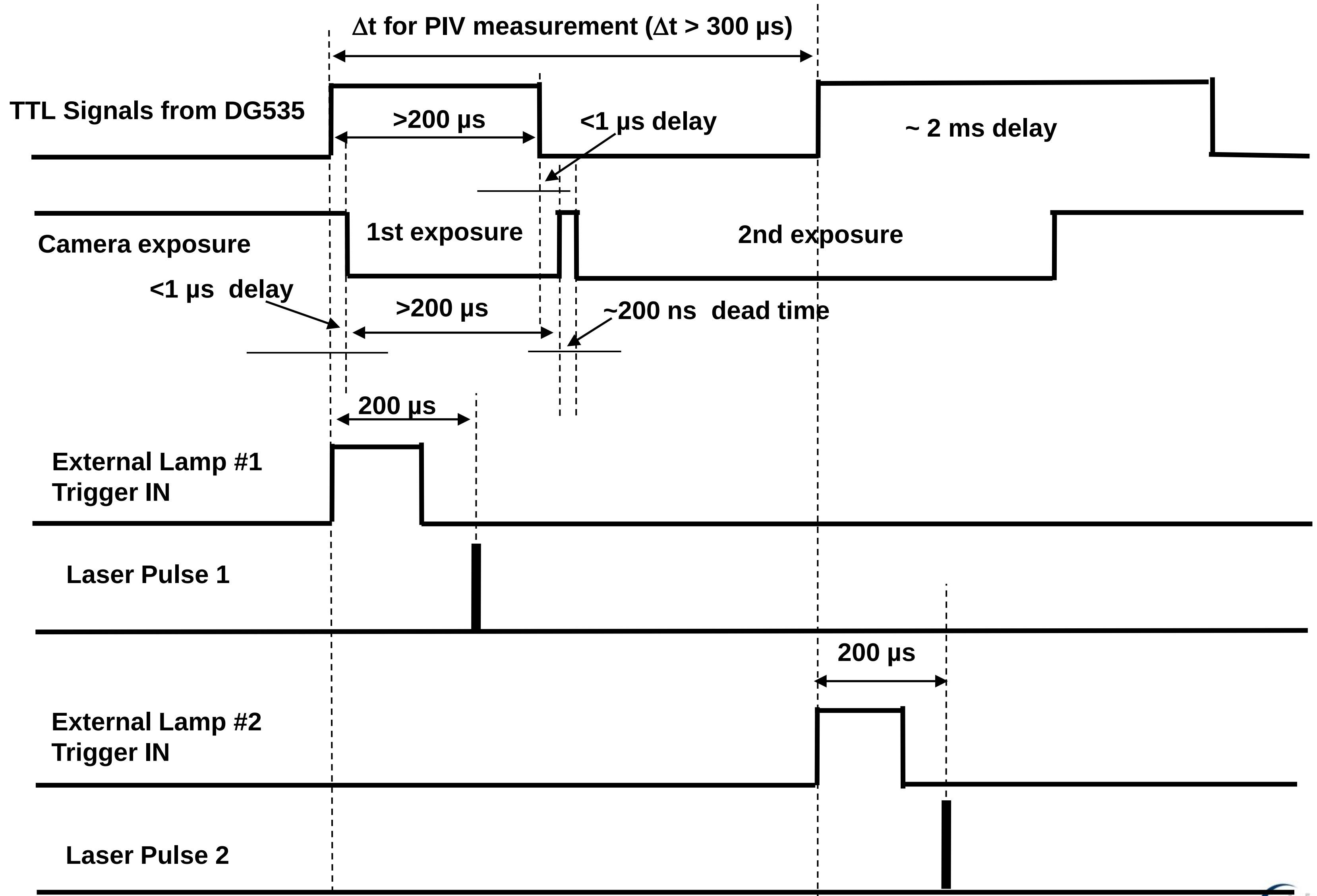
Synchronizer

- *Function of Synchronizer:*
  - *To control the timing of the laser illumination and camera acquisition*



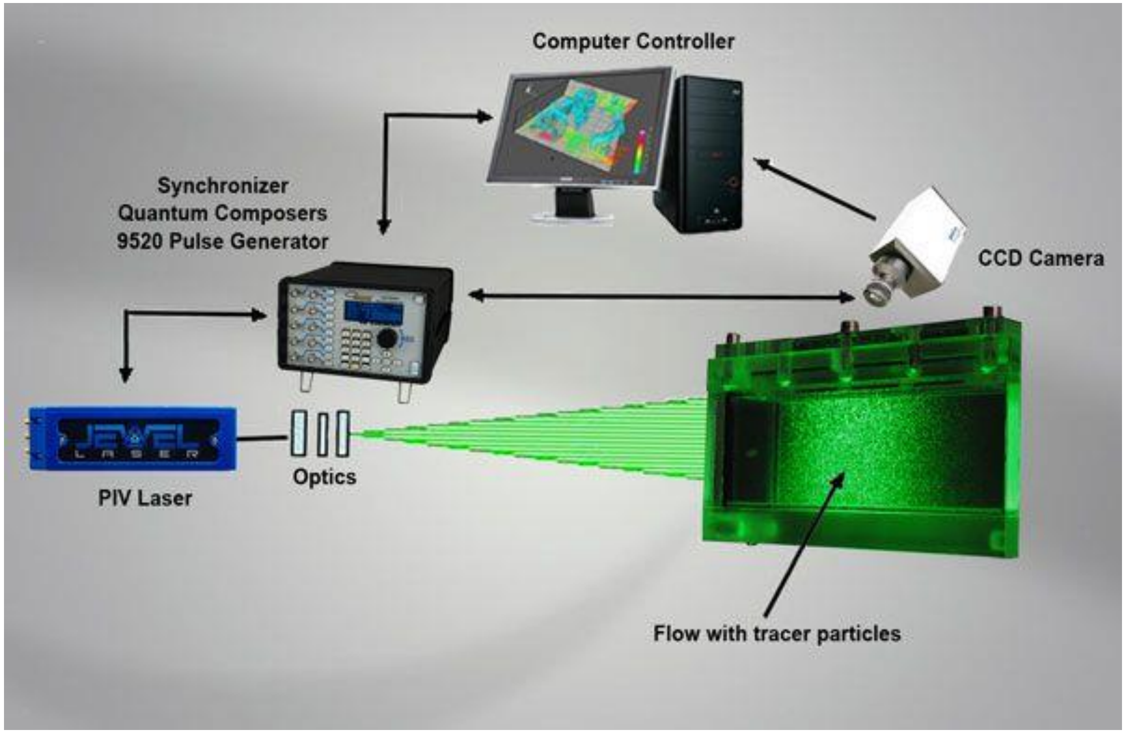


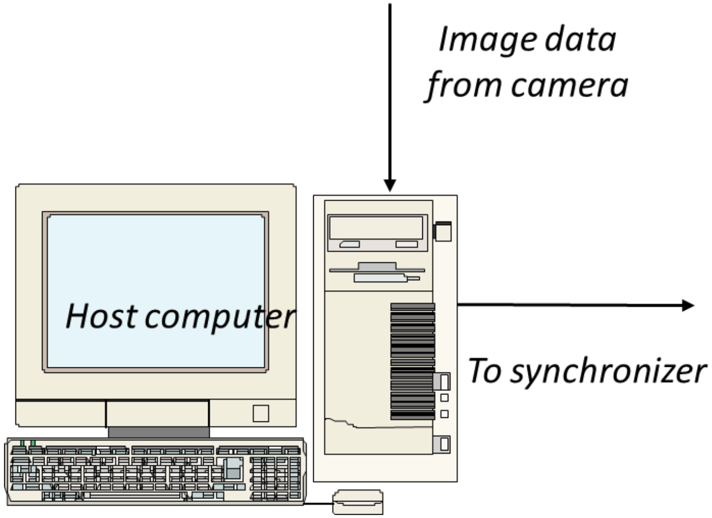
Time Chart of the PIV Measurements (Δt > 300 μs)



Host computer

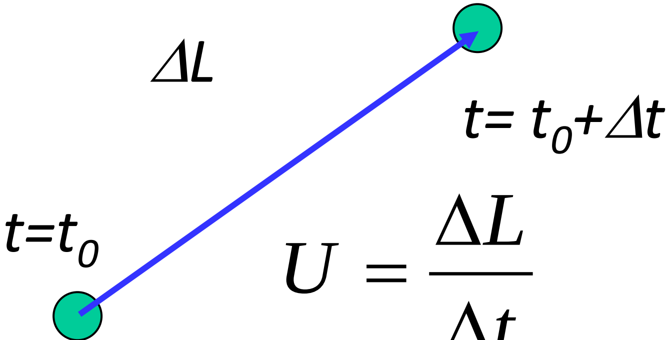
- *To send timing control parameter to synchronizer.*
- *To store the particle images and conduct image processing.*



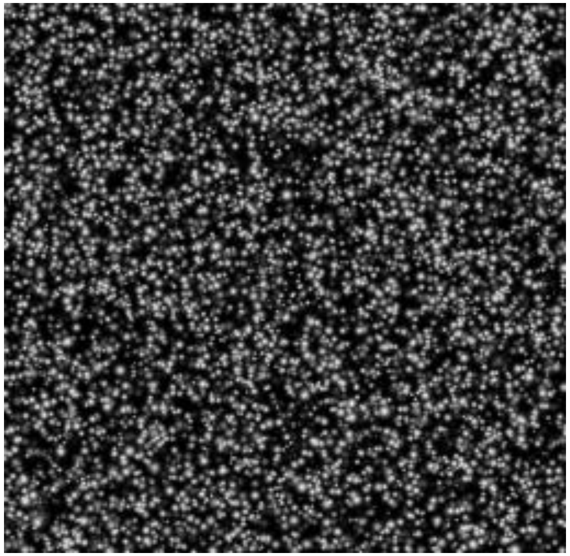


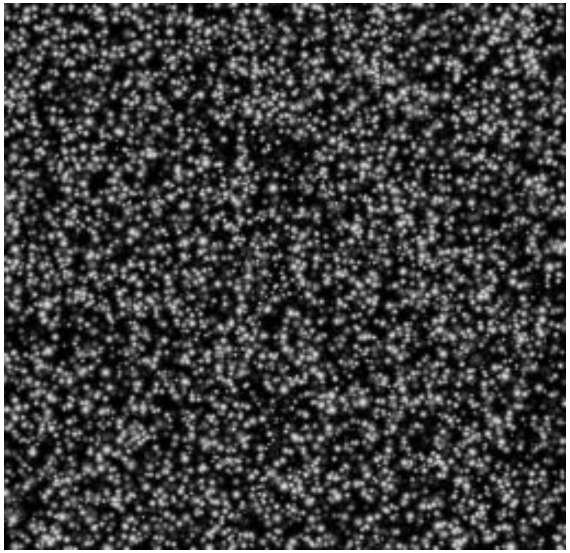
PIV image Processing

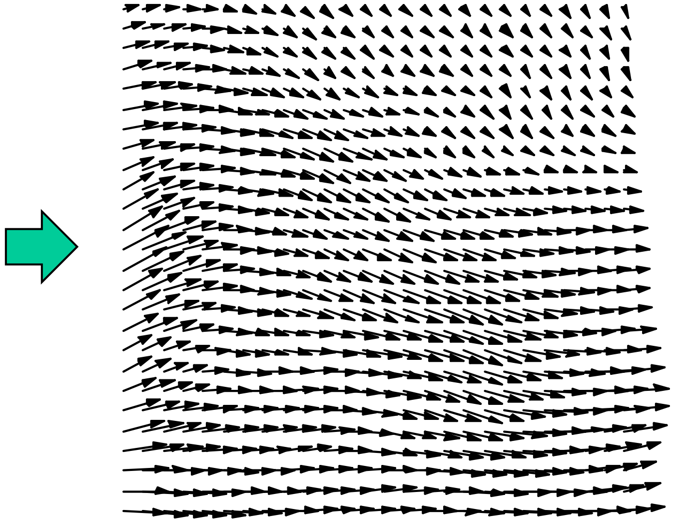
- *Time-of-flight method: to measure the displacements of the tracer particles seeded in the flow in a fixed time interval.*



$$U = \frac{\Delta L}{\Delta t}$$





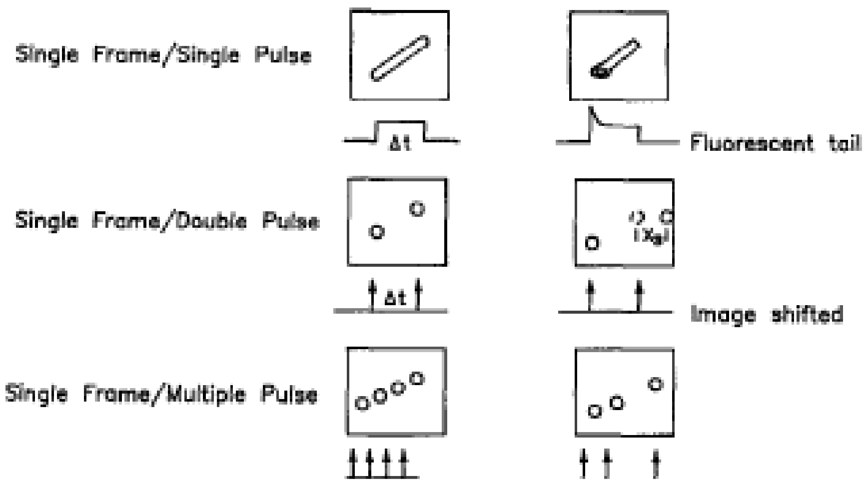


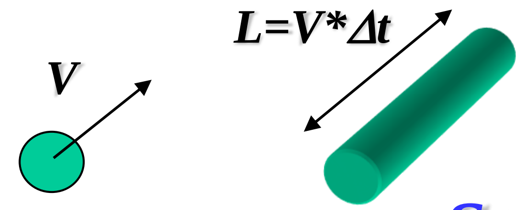
a. T=t0

b. T=t0+4ms

Corresponding Velocity field

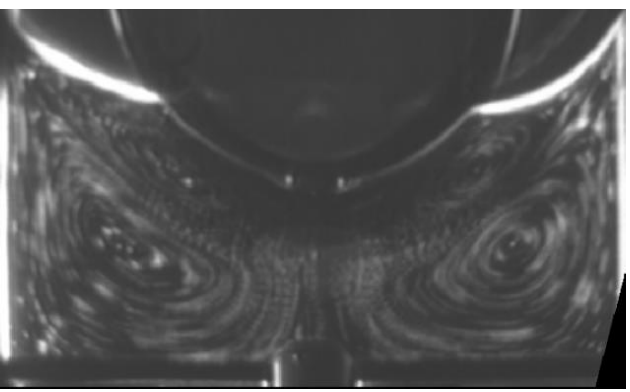
Single-frame technique

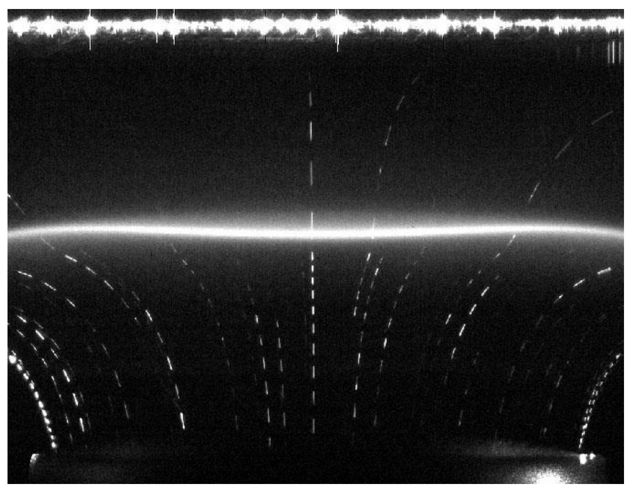


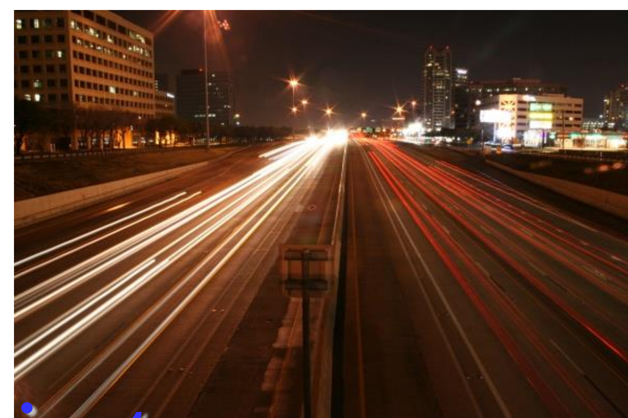


particle

Streak line







single-pulse

Multiple-pulse

Particle streak velocimetry

Multi-frame technique

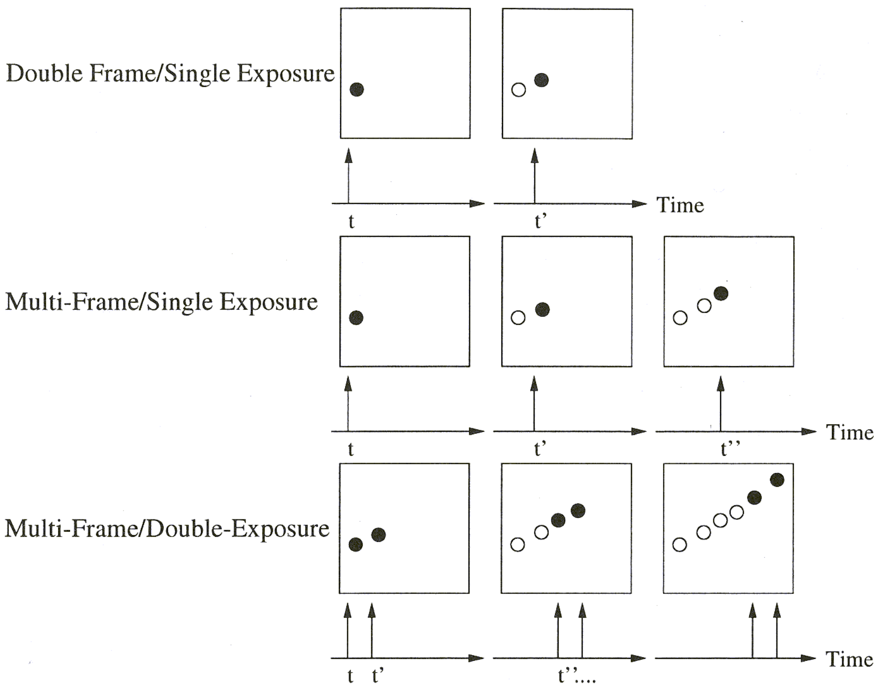
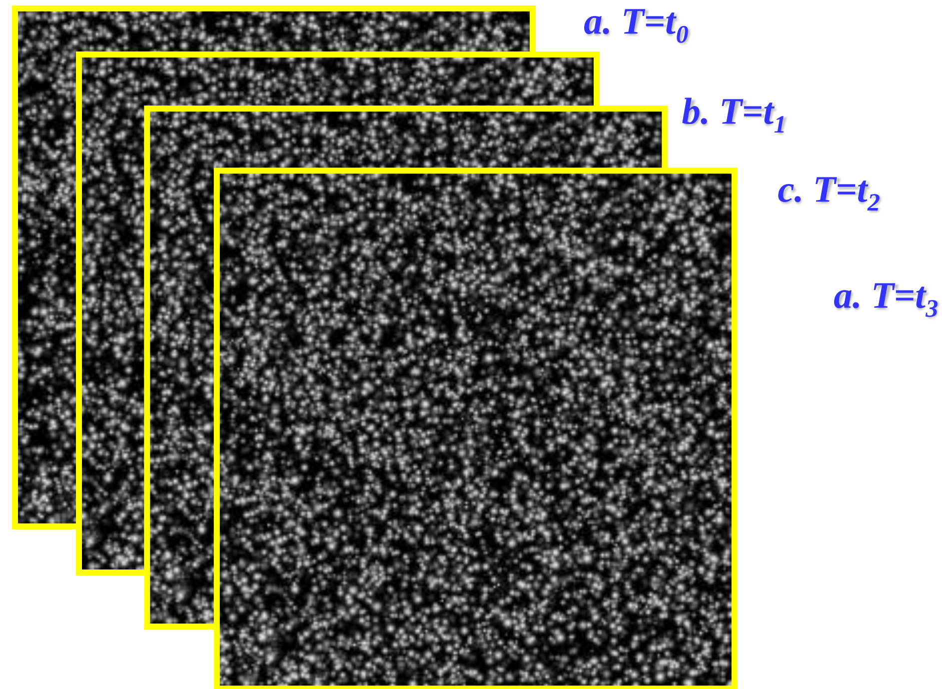
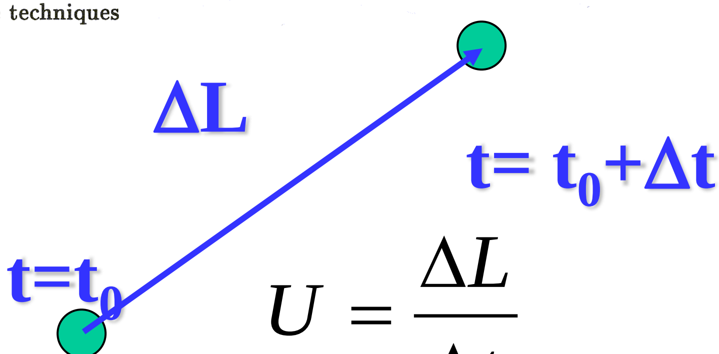


Fig. 4.2. Multiple frame techniques





$$U = \frac{\Delta L}{\Delta t}$$

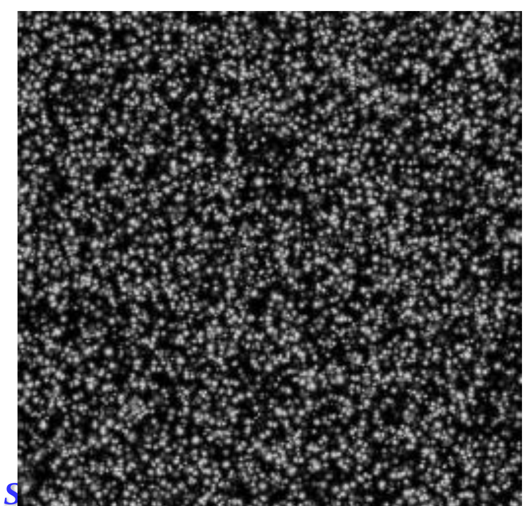
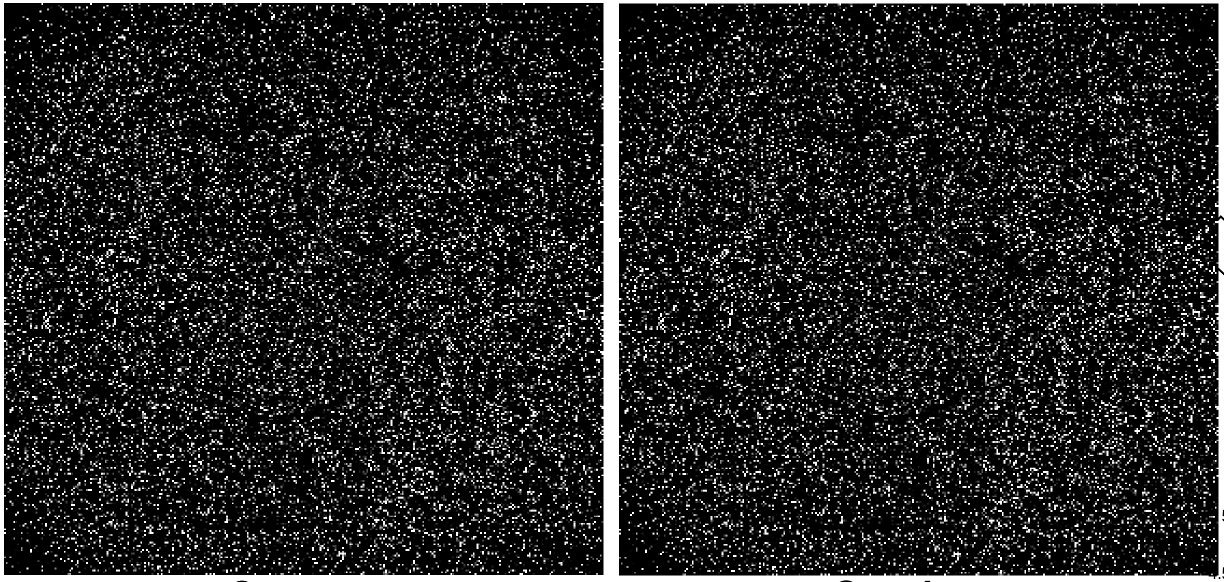


Image Processing for PIV

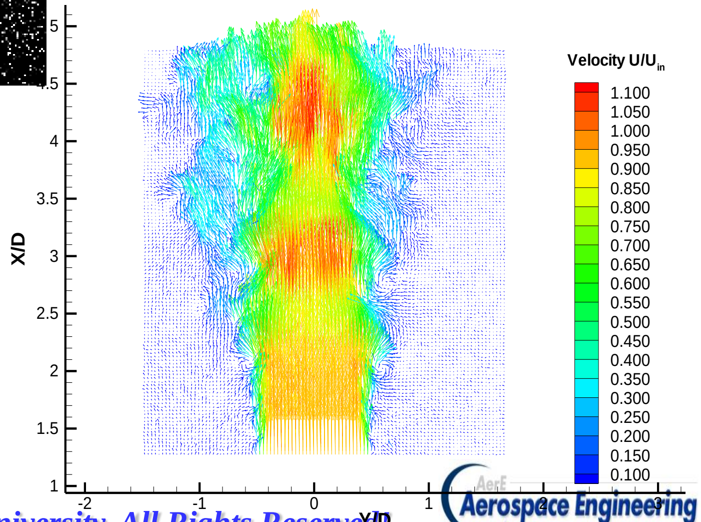
- ***To extract velocity information from particle images.***



t=t0 *t=t0+4ms*

A typical PIV raw image pair

Image processing



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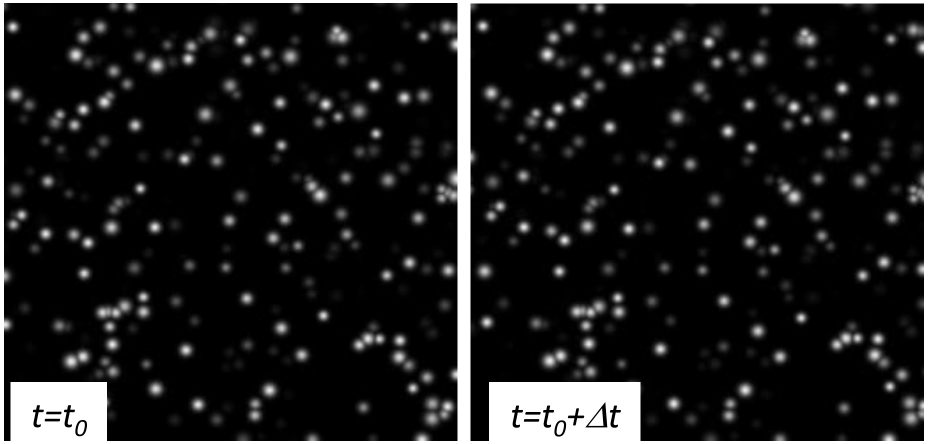
Particle Tracking Velocimetry (PTV)

1. *Find position of the particles at each images*
2. *Find corresponding particle image pair in the different image frame*
3. *Find the displacements between the particle pairs.*
4. *Velocity of particle equates the displacement divided by the time interval between the frames.*



Low particle-image density case

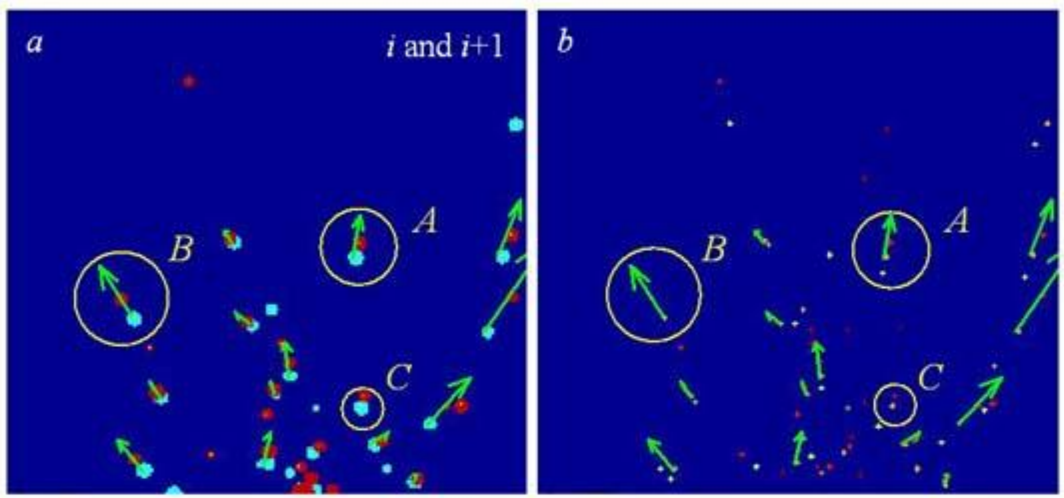
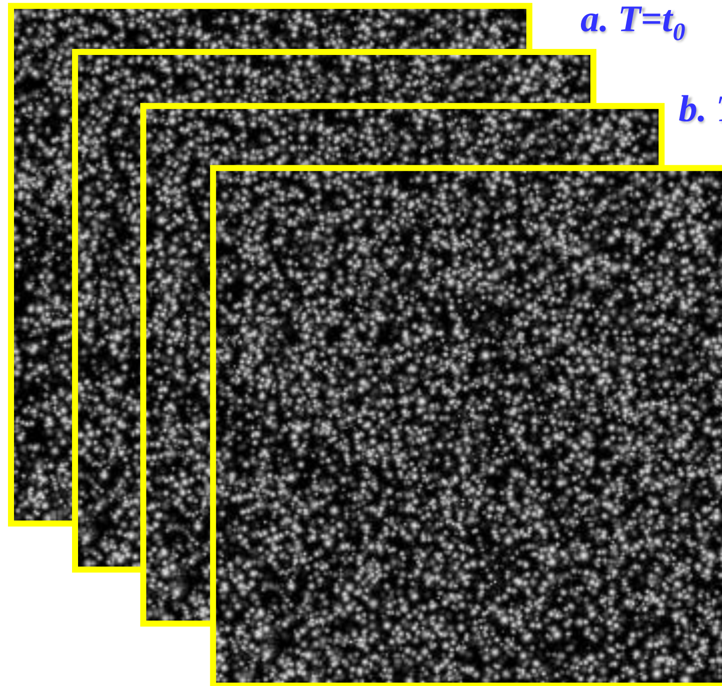


Figure 2. The particle-tracking algorithm applied to the sequence of images (i-1) to (i+2). (a) Detected particles in frames (i) (light blue) and (i+1) (dark red) with overlapped centroids and velocity vectors. (b) Detected centroids of particles in all four frames with overlaid velocity vectors. Consecutive frames are colored from light to dark.

Particle Tracking Velocimetry (PTV)-2

1. *Find position of the particles at each images*
2. *Find corresponding particle image pair in the different image frame*
3. *Find the displacements between the particle pairs.*
4. *Velocity of particle equates the displacement divided by the time interval between the frames.*

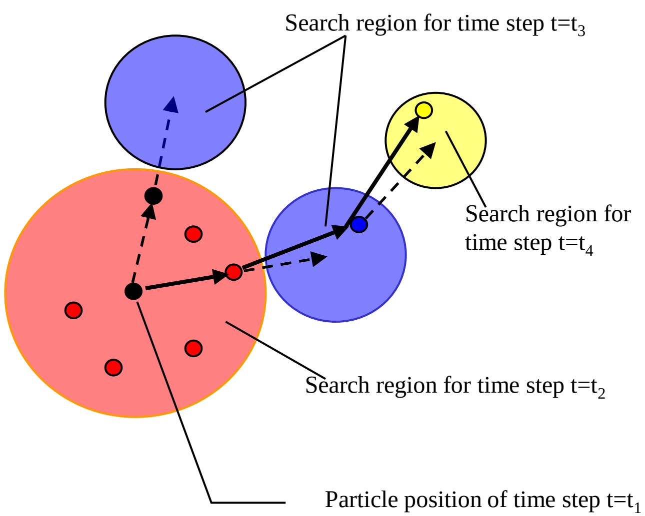


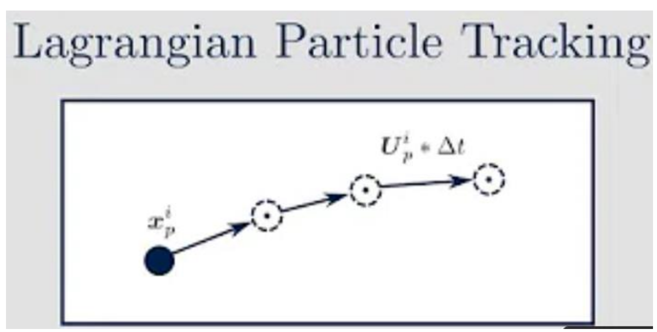
a. $T=t_0$

b. $T=t_1$

c. $T=t_2$

a. $T=t_3$



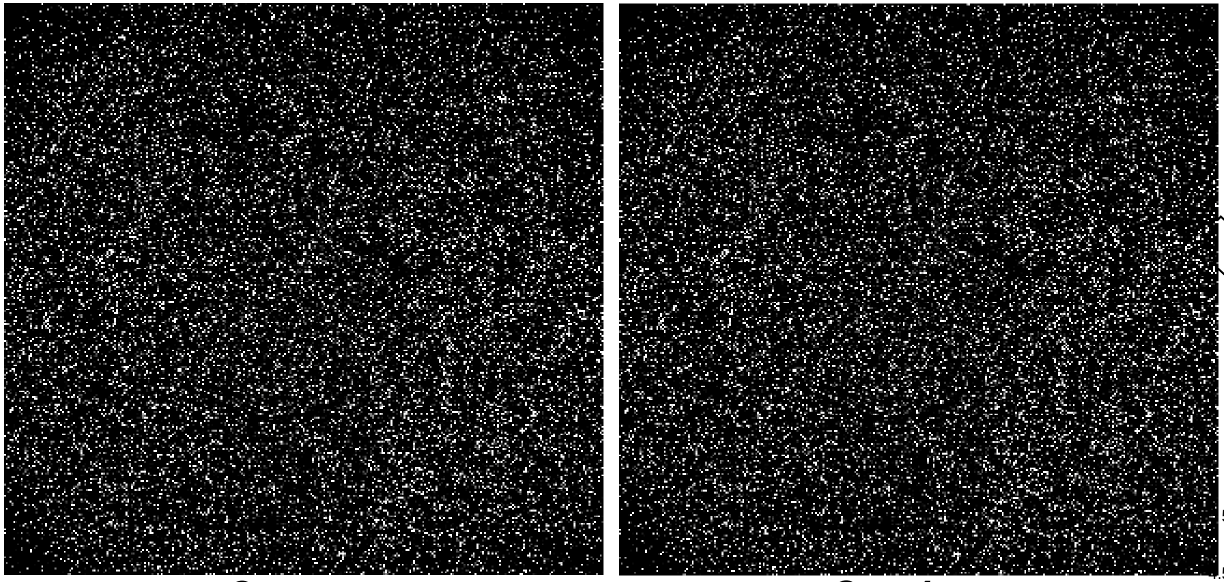


results

- *Four-frame-particle tracking algorithm*

Image Processing for PIV

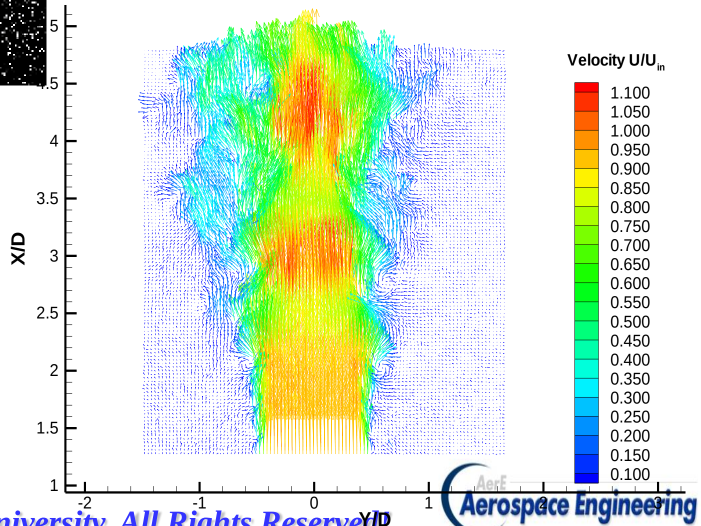
- *To extract velocity information from particle images.*



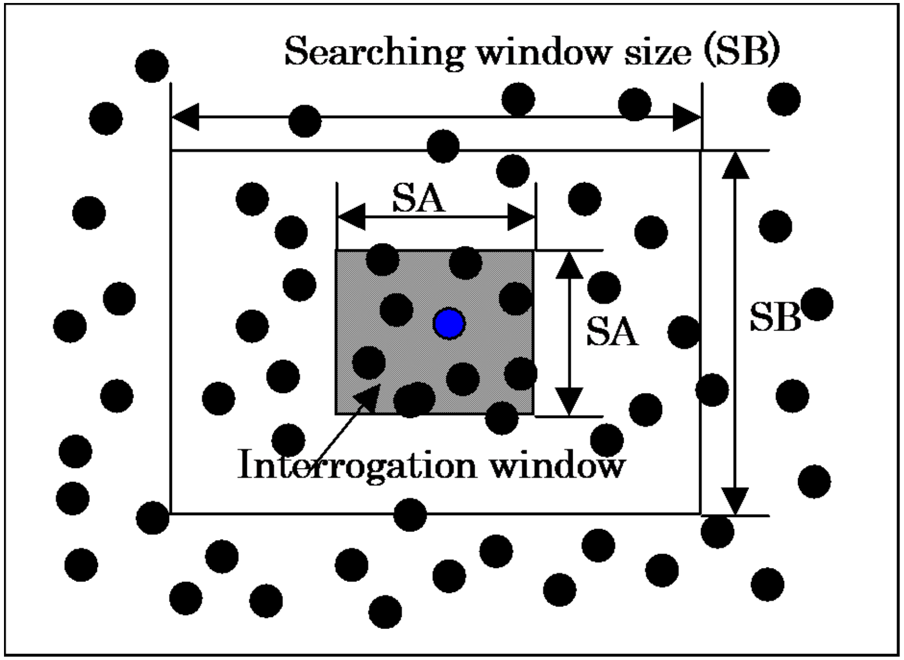
t=t0 *t=t0+4ms*

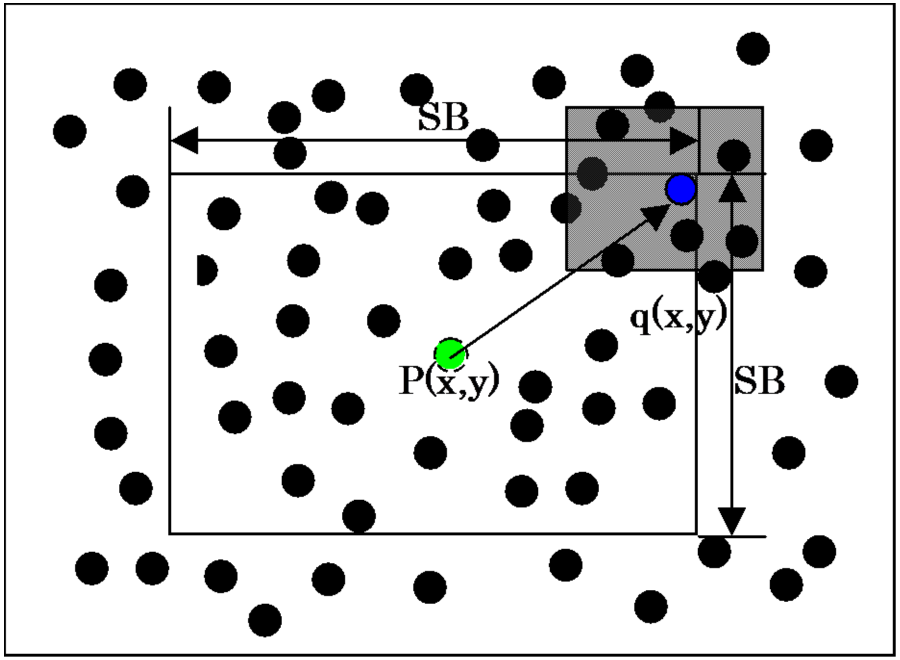
A typical PIV raw image pair

Image processing



Correlation-based PIV methods





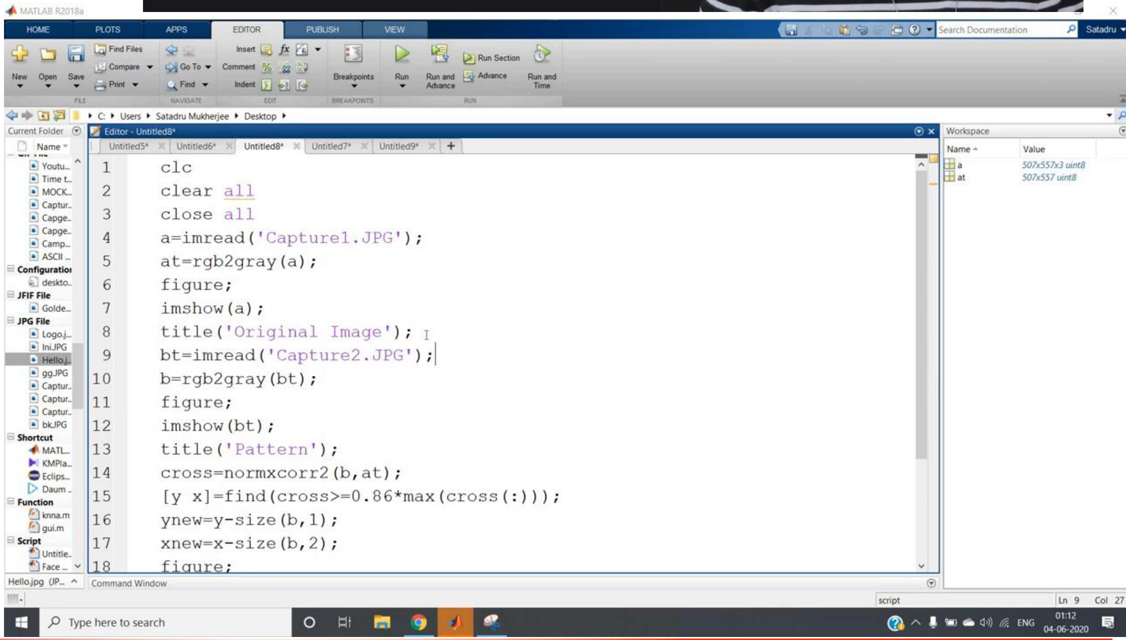
t=t$_0$ *t=t*$_0$*+Δt*

Correlation coefficient function

$$R(p,q)=\frac{\int (f(x,y)-\bar{f})(g(x,y)-\bar{g})dv}{\sqrt{\int (f(x,y)-\bar{f})^2 dv \int (g(x,y)-\bar{g})^2 dv}}$$

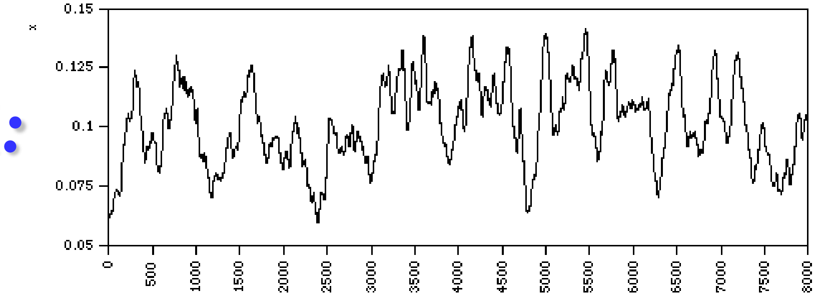
Cross Correlation Operation

$$R(p,q)=\frac{\int (f(x,y)-\overline{f})(g(x,y)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2 dv \int (g(x,y)-\overline{g})^2 dv}}$$

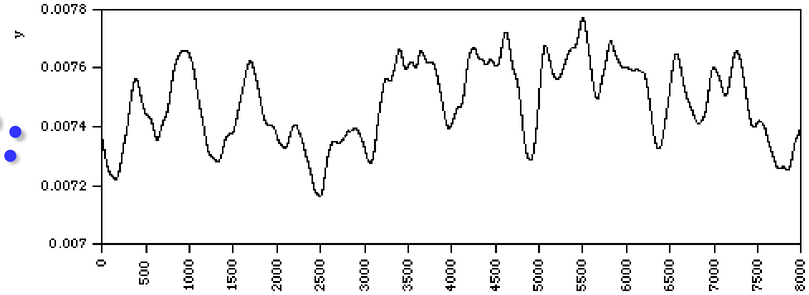


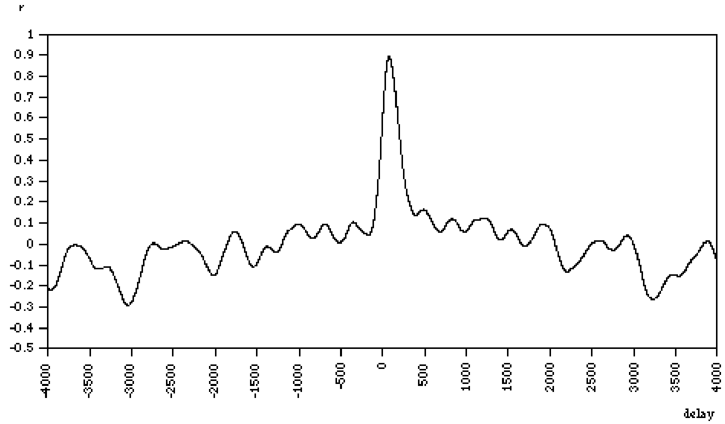
Cross Correlation Operation

Signal A:

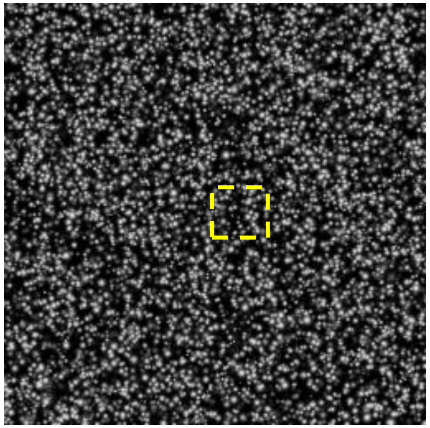


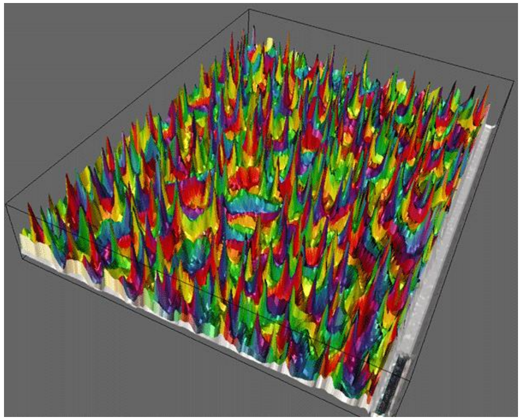
Signal B:

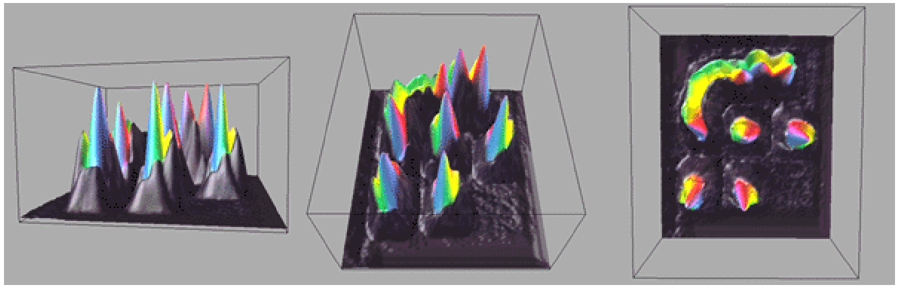




$$R(u) = \frac{\int [f(x) * g(x+u)]dx}{\sqrt{\int [f(x)^2]dx * \int [g(x+u)^2]dx}}$$

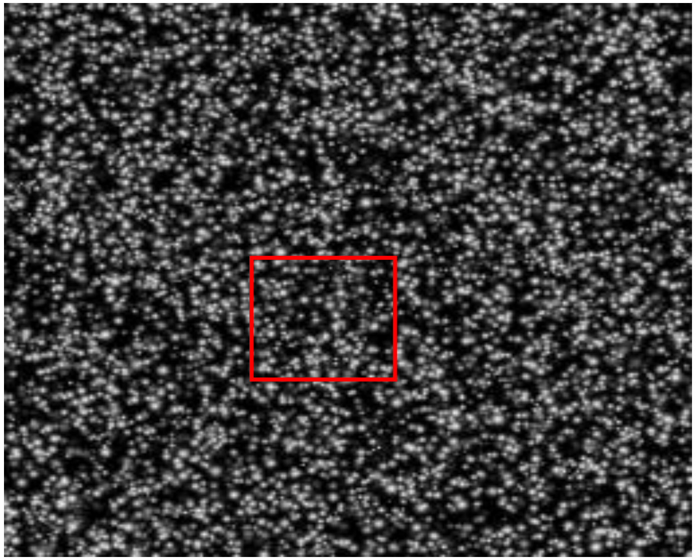




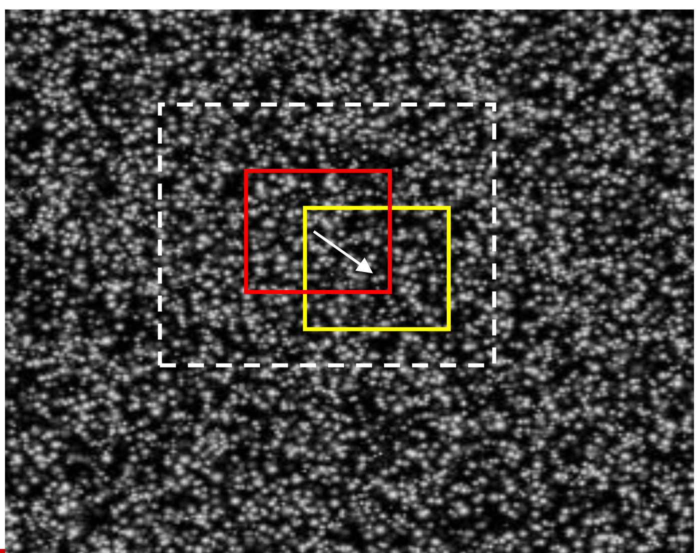


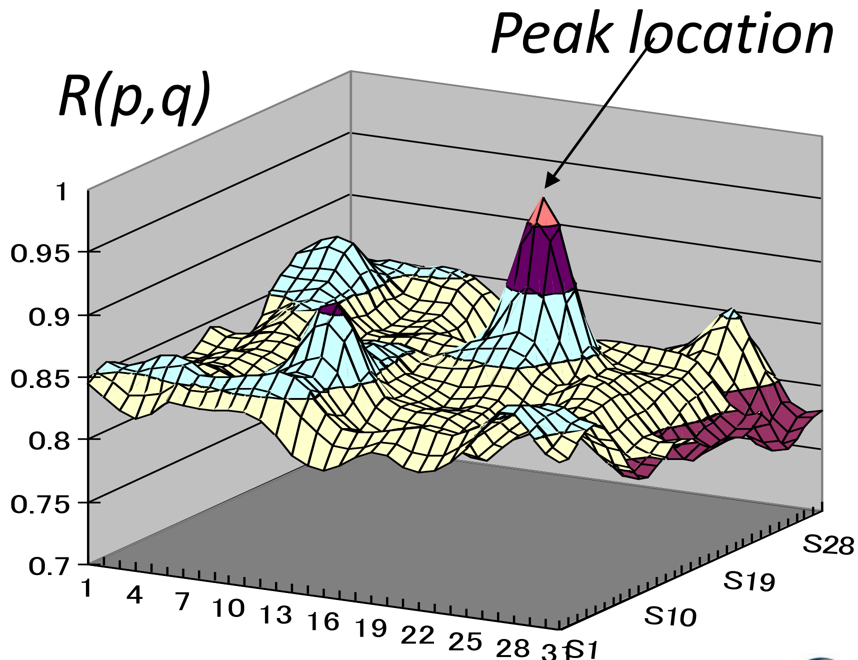
$$c(u,v) = \sum_{x,y} f(x,y)t(x-u, y-v)$$

Correlation coefficient distribution

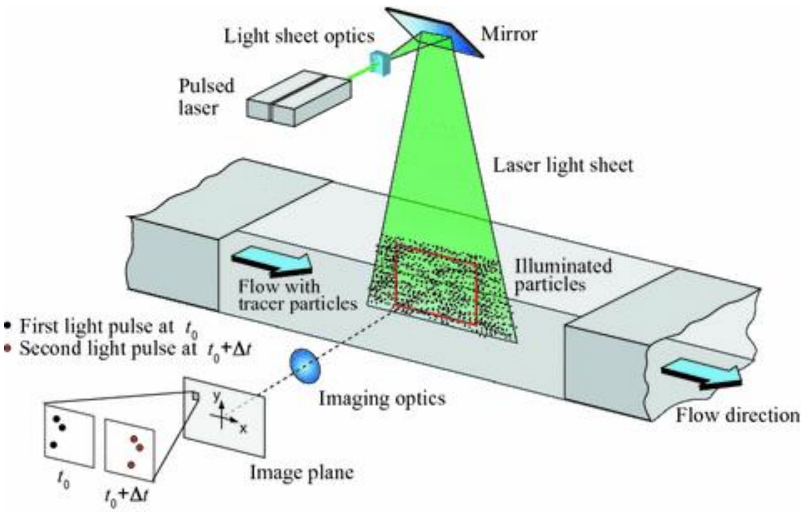


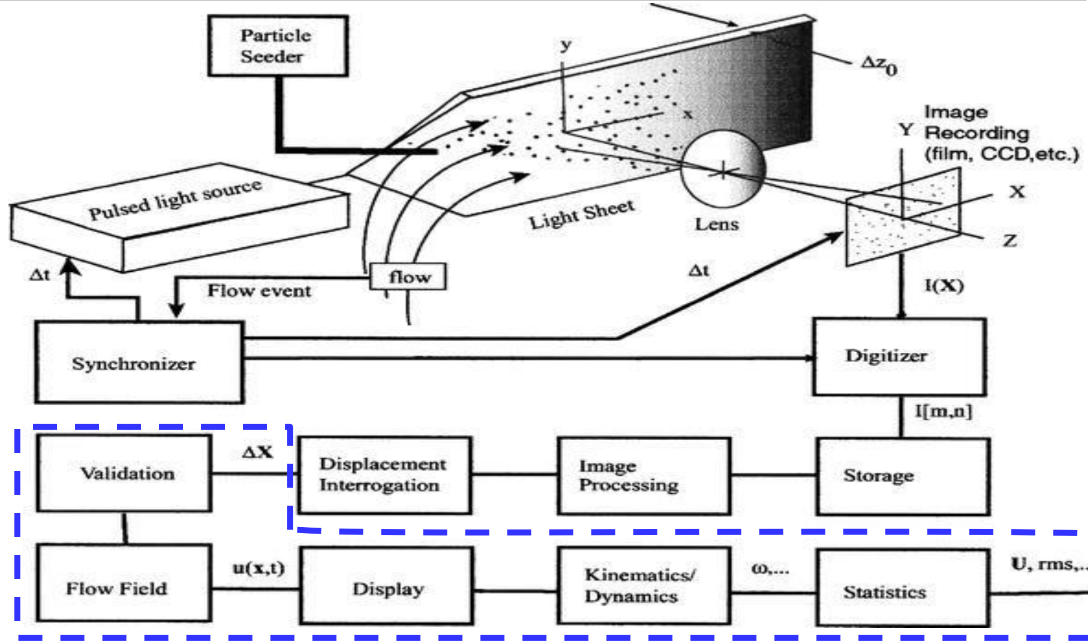
$$R(p,q)=\frac{\int (f(x,y)-\bar{f})(g(x,y)-\bar{g})dv}{\sqrt{\int (f(x,y)-\bar{f})^2 dv \int (g(x,y)-\bar{g})^2 dv}}$$

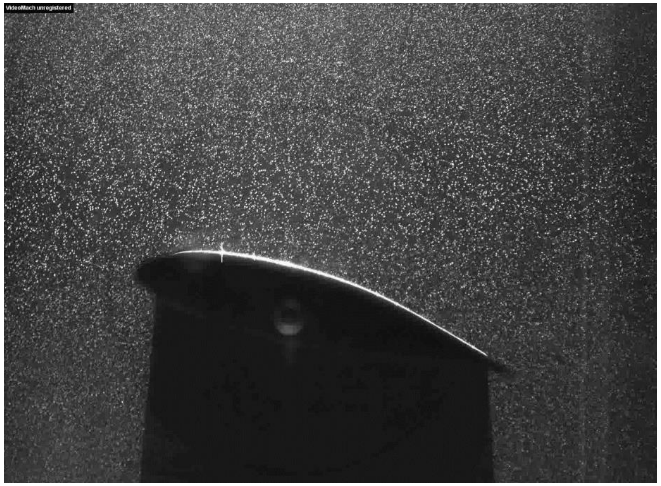


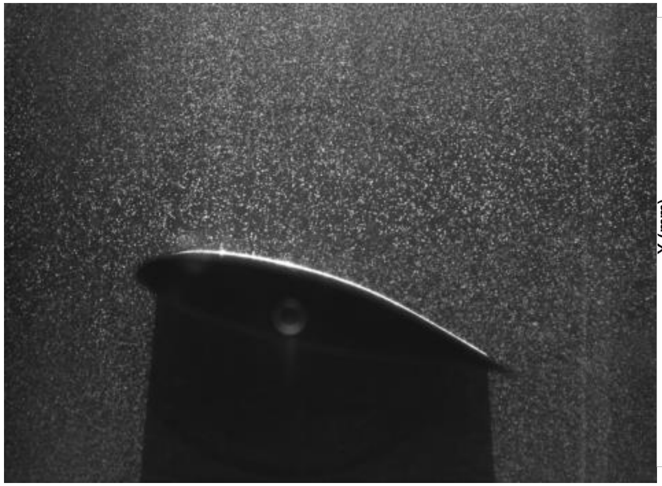


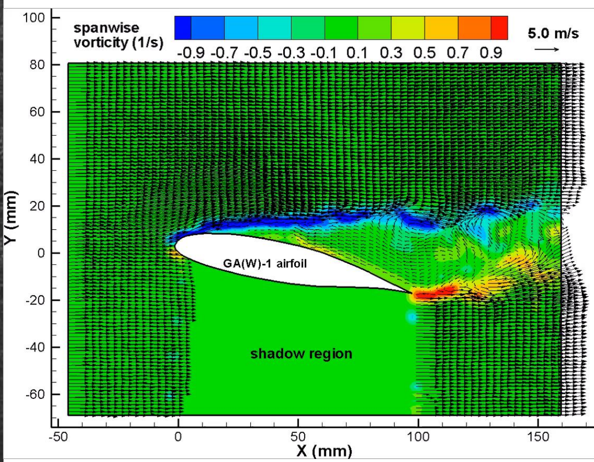
Post processing of PIV measurements











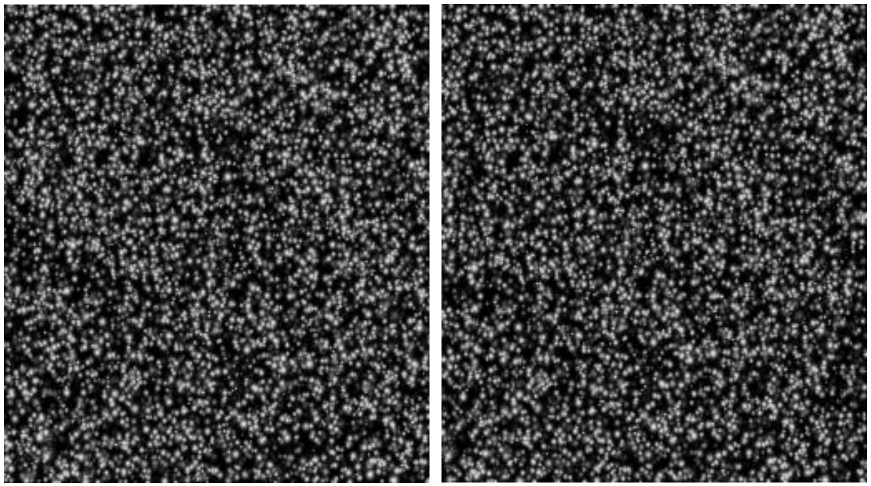
a. T=t0

b. T=t0+10μs

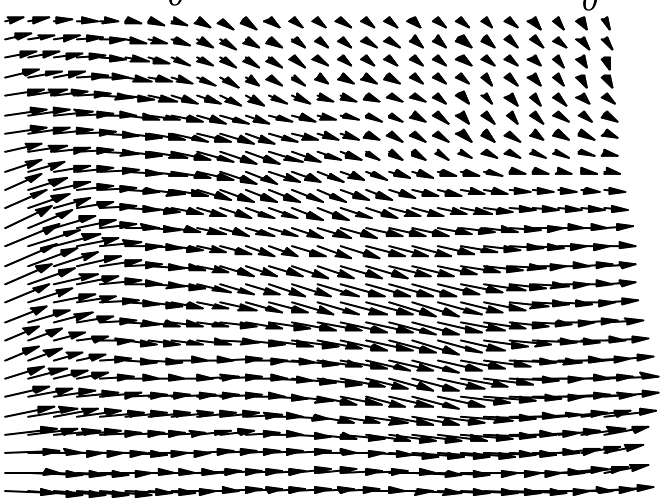
Corresponding Velocity field

Post processing: Detection of spurious vectors

- *Ideal case*

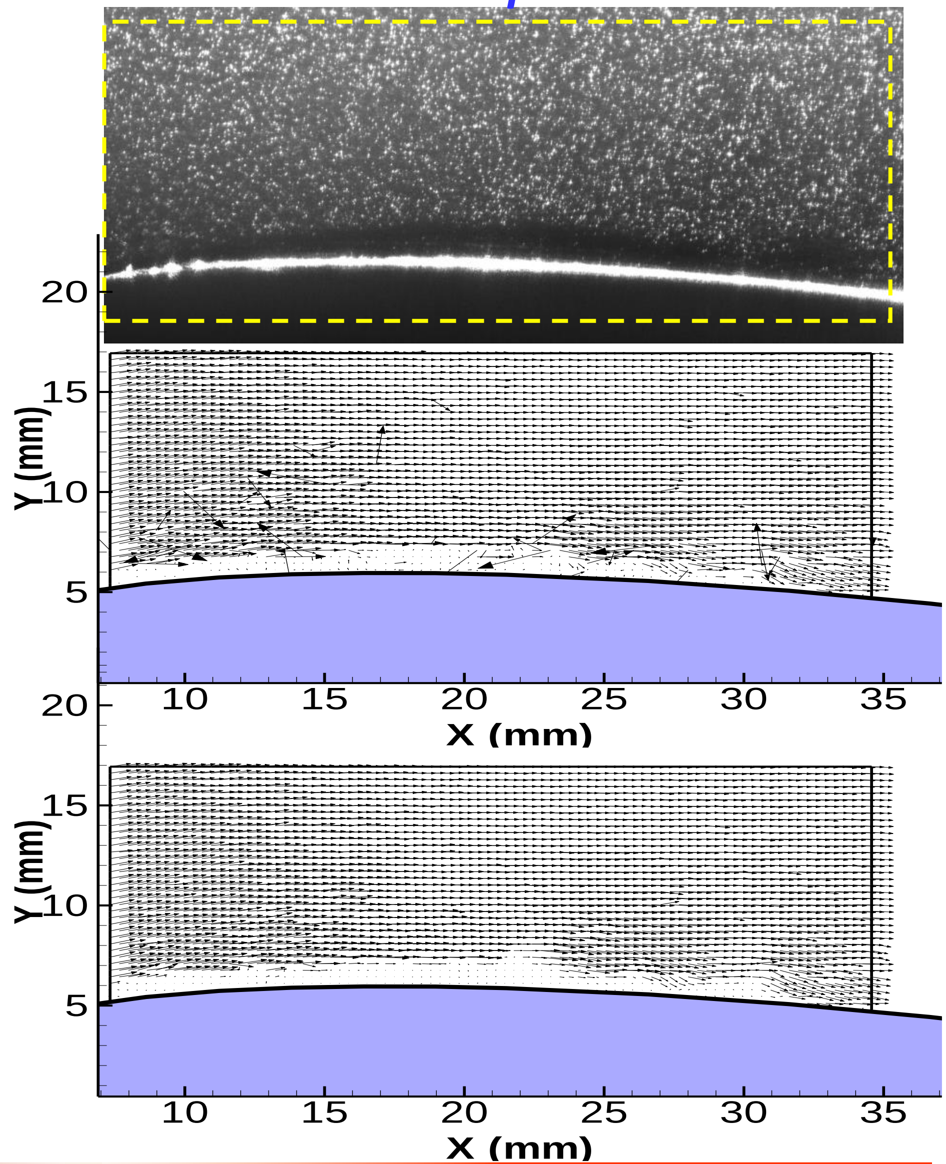


a. $t=t_0$ *b. $t=t_0+\Delta t$*

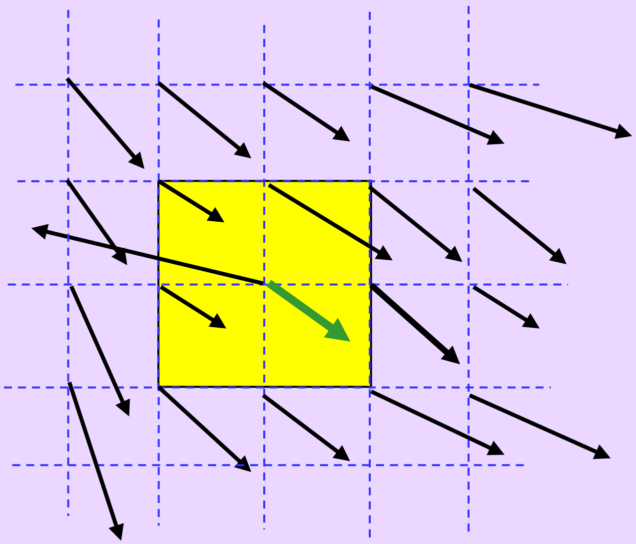


Corresponding flow velocity field

- *Real experiment*



Detection of spurious vectors



$$Um = \frac{U_{i+1,j-1} + U_{i+1,j} + U_{i+1,j+1} + U_{i,j-1} + U_{i,j+1} + U_{i-1,j-1} + U_{i-1,j} + U_{i-1,j+1}}{8}$$

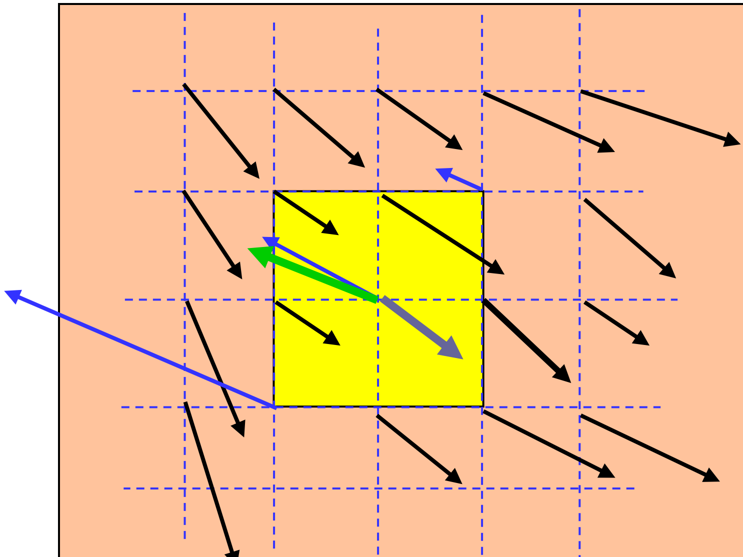
$$Vm = \frac{V_{i+1,j-1} + V_{i+1,j} + V_{i+1,j+1} + V_{i,j-1} + V_{i,j+1} + V_{i-1,j-1} + V_{i-1,j} + V_{i-1,j+1}}{8}$$

$$R\_value = \frac{\sqrt{(U_{i,j} - Um)^2 + (V_{i,j} - Vm)^2}}{\sqrt{Um^2 + Vm^2}} \approx \frac{\sqrt{(U_{i,j} - Um)^2 + (V_{i,j} - Vm)^2}}{\sqrt{Um^2 + Vm^2} + \varepsilon}$$

ε: is a very small number, such as 0.000000001.

$R\_value > R_{thresh\_hold}$ for a spurious vector

- ***Regular method***



$$Um = \frac{U_{i+1,j-1} + U_{i+1,j} + U_{i+1,j+1} + U_{i,j-1} + U_{i,j+1} + U_{i-1,j-1} + U_{i-1,j} + U_{i-1,j+1} - U\max - U\min}{6}$$

$$Vm = \frac{V_{i+1,j-1} + V_{i+1,j} + V_{i+1,j+1} + V_{i,j-1} + V_{i,j+1} + V_{i-1,j-1} + V_{i-1,j} + V_{i-1,j+1} - V\max - V\min}{6}$$

$$R\_value = \frac{\sqrt{(U_{i,j} - Um)^2 + (V_{i,j} - Vm)^2}}{\sqrt{Um^2 + Vm^2}} \approx \frac{\sqrt{(U_{i,j} - Um)^2 + (V_{i,j} - Vm)^2}}{\sqrt{Um^2 + Vm^2} + \varepsilon}$$

ε: is a very small number, such as 0.000000001.

$R\_value > R_{thresh\_hold}$ for a spurious vector

- ***Median test method***

Estimation of differential quantities

$$\frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}\boldsymbol{X}} = \begin{bmatrix} \frac{\partial U}{\partial X} & \frac{\partial V}{\partial X} & \frac{\partial W}{\partial X} \\ \frac{\partial U}{\partial Y} & \frac{\partial V}{\partial Y} & \frac{\partial W}{\partial Y} \\ \frac{\partial U}{\partial Z} & \frac{\partial V}{\partial Z} & \frac{\partial W}{\partial Z} \end{bmatrix} \tag{6.7}$$

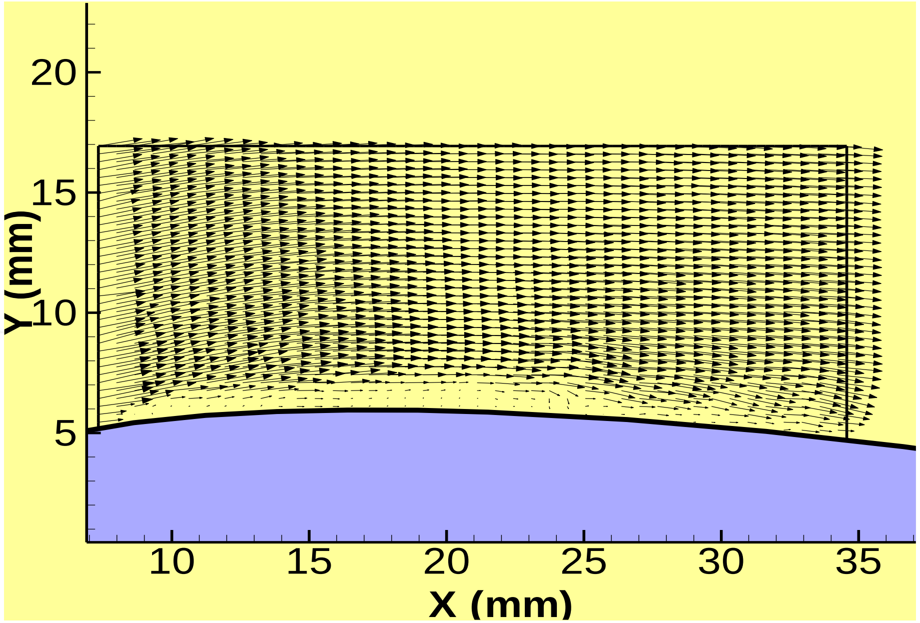
This deformation tensor can be decomposed into a symmetric part and an antisymmetric part:

$$\frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}\boldsymbol{X}} = \begin{bmatrix} \frac{\partial U}{\partial X} & \frac{1}{2}\left(\frac{\partial V}{\partial X}+\frac{\partial U}{\partial Y}\right) & \frac{1}{2}\left(\frac{\partial W}{\partial X}+\frac{\partial U}{\partial Z}\right) \\ \frac{1}{2}\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right) & \frac{\partial V}{\partial Y} & \frac{1}{2}\left(\frac{\partial W}{\partial Y}+\frac{\partial V}{\partial Z}\right) \\ \frac{1}{2}\left(\frac{\partial U}{\partial Z}+\frac{\partial W}{\partial X}\right) & \frac{1}{2}\left(\frac{\partial V}{\partial Z}+\frac{\partial W}{\partial Y}\right) & \frac{\partial W}{\partial Z} \end{bmatrix} \tag{6.8}$$

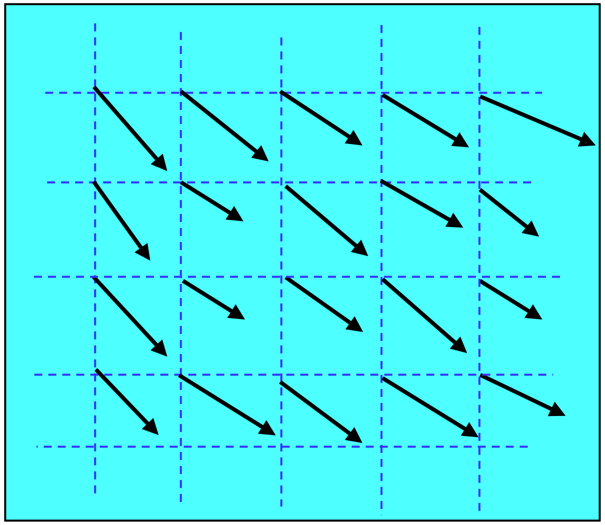
$$+ \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}\right) & \frac{1}{2}\left(\frac{\partial W}{\partial X}-\frac{\partial U}{\partial Z}\right) \\ \frac{1}{2}\left(\frac{\partial U}{\partial Y}-\frac{\partial V}{\partial X}\right) & 0 & \frac{1}{2}\left(\frac{\partial W}{\partial Y}-\frac{\partial V}{\partial Z}\right) \\ \frac{1}{2}\left(\frac{\partial U}{\partial Z}-\frac{\partial W}{\partial X}\right) & \frac{1}{2}\left(\frac{\partial V}{\partial Z}-\frac{\partial W}{\partial Y}\right) & 0 \end{bmatrix} \tag{6.9}$$

A substitution of the strain and vorticity components yields:

$$\frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}\boldsymbol{X}} = \begin{bmatrix} \epsilon_{XX} & \frac{1}{2}\epsilon_{XY} & \frac{1}{2}\epsilon_{XZ} \\ \frac{1}{2}\epsilon_{YX} & \epsilon_{YY} & \frac{1}{2}\epsilon_{YZ} \\ \frac{1}{2}\epsilon_{ZX} & \frac{1}{2}\epsilon_{ZY} & \epsilon_{ZZ} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2}\omega_Z & -\frac{1}{2}\omega_X \\ -\frac{1}{2}\omega_Z & 0 & \frac{1}{2}\omega_Y \\ -\frac{1}{2}\omega_X & \frac{1}{2}\omega_Y & 0 \end{bmatrix} \tag{6.10}$$

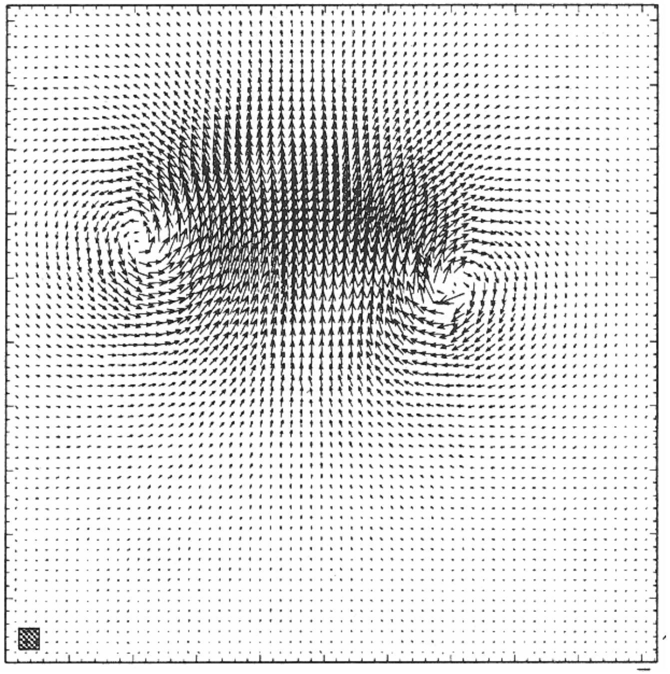


Operator	Implementation	Accuracy	Uncertainty
Forward difference	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{i+1/2} \approx \frac{f_{i+1}-f_i}{\Delta X}$	$O(\Delta X)$	$\approx 1.41\frac{\epsilon_U}{\Delta X}$
Backward difference	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{i-1/2} \approx \frac{f_i-f_{i-1}}{\Delta X}$	$O(\Delta X)$	$\approx 1.41\frac{\epsilon_U}{\Delta X}$
Center difference	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{i} \approx \frac{f_{i+1}-f_{i-1}}{2\Delta X}$	$O(\Delta X^2)$	$\approx 0.7\frac{\epsilon_U}{\Delta X}$
Richardson extrapol.	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{i} \approx \frac{f_{i-2}-8f_{i-1}+8f_{i+1}-f_{i+2}}{12\Delta X}$	$O(\Delta X^3)$	$\approx 0.95\frac{\epsilon_U}{\Delta X}$
Least squares	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{i} \approx \frac{2f_{i+2}+f_{i+1}-f_{i-1}-2f_{i-2}}{10\Delta X}$	$O(\Delta X^2)$	$\approx 1.0\frac{\epsilon_U}{\Delta X}$



Estimation of Vorticity distribution

$$\varpi_z = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}$$



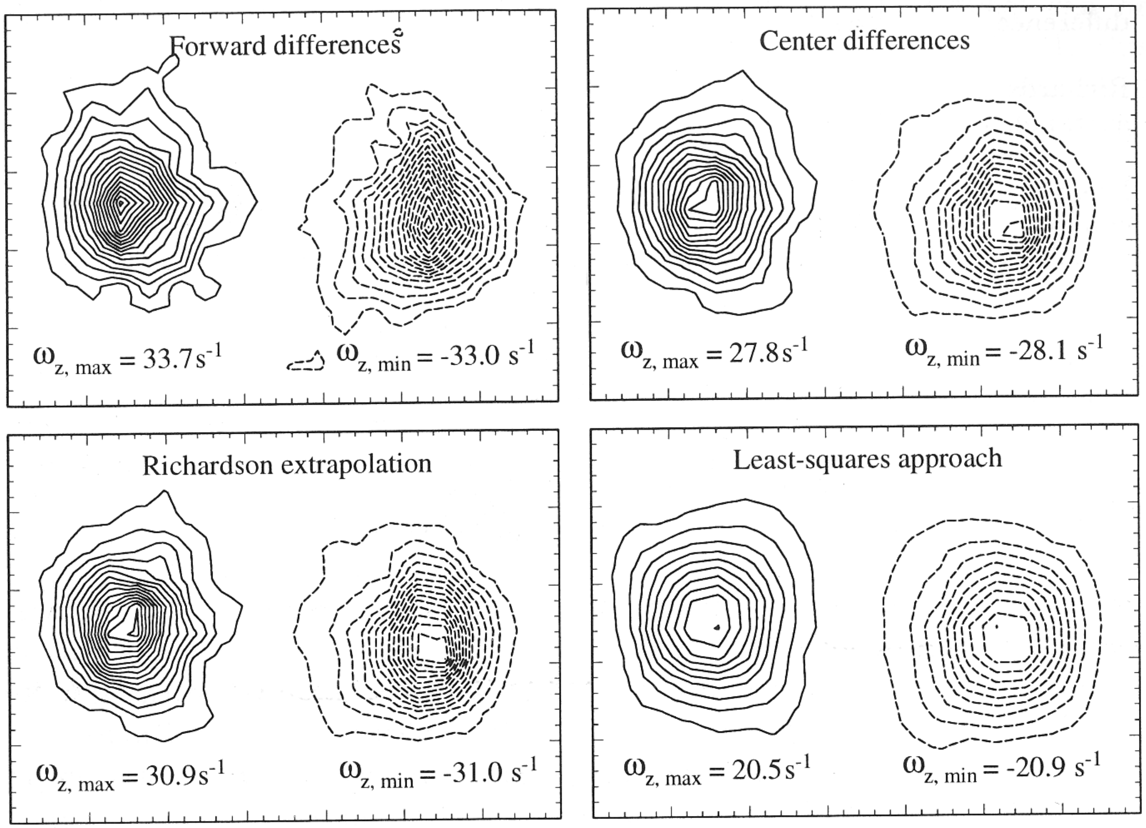


Fig. 6.7. Vorticity field estimates obtained from twice oversampled PIV data, e.g. the interrogation window overlap is 50%. The vortex pair is known to be laminar and thus should have smooth vorticity contours

Estimation of Vorticity distribution

- ***Stokes Theorem:***

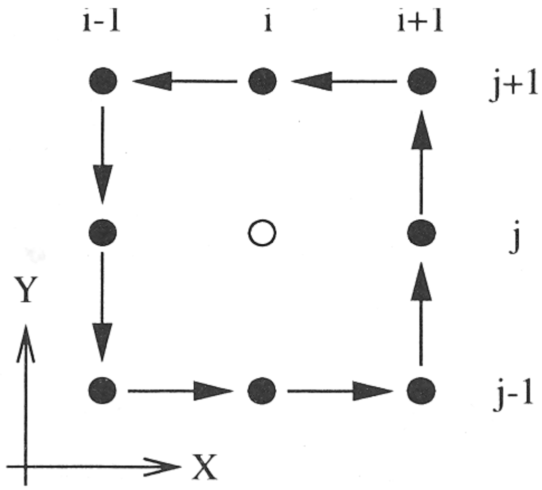
$$\Gamma = -\oint_C \vec{V}\, d\vec{l} = -\iint_S \vec{\varpi} \bullet d\vec{A}$$

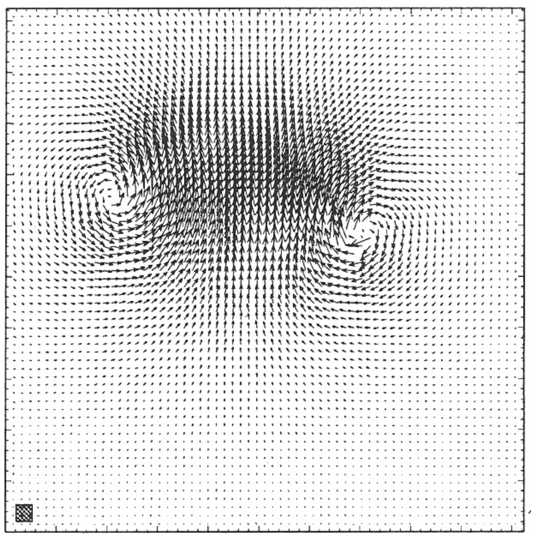
$$\Rightarrow \varpi_z = \frac{\Gamma_{x-y}}{dA}$$

$$(\omega_Z)_{i,j} \;\widehat{=}\; \frac{\Gamma_{i,j}}{4\Delta X \Delta Y}$$

with

$$\begin{aligned}\Gamma_{i,j} =\ & \frac{1}{2}\Delta X (U_{i-1,j-1} + 2U_{i,j-1} + U_{i+1,j-1}) \\ +\ & \frac{1}{2}\Delta Y (V_{i+1,j-1} + 2V_{i+1,j} + V_{i+1,j+1}) \\ -\ & \frac{1}{2}\Delta X (U_{i+1,j+1} + 2U_{i,j+1} + U_{i-1,j+1}) \\ -\ & \frac{1}{2}\Delta Y (V_{i-1,j+1} + 2V_{i-1,j} + V_{i-1,j-1})\end{aligned}$$





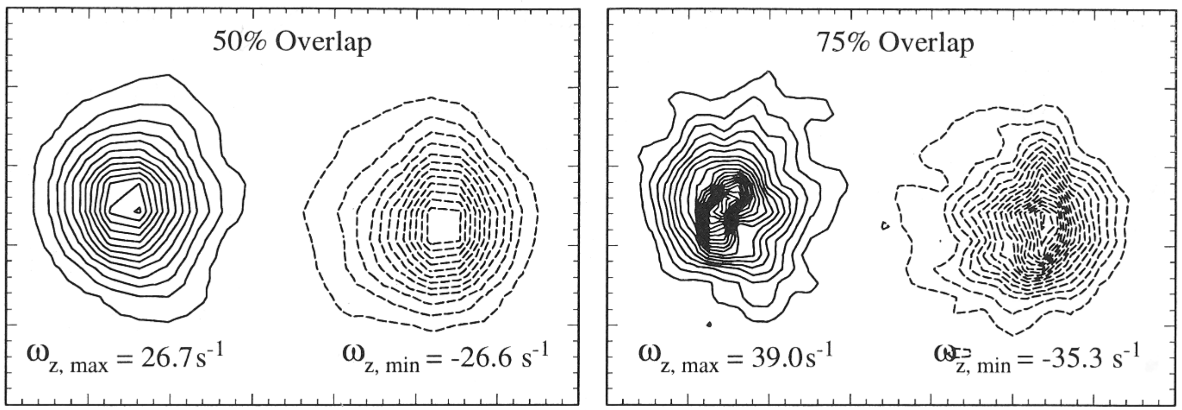
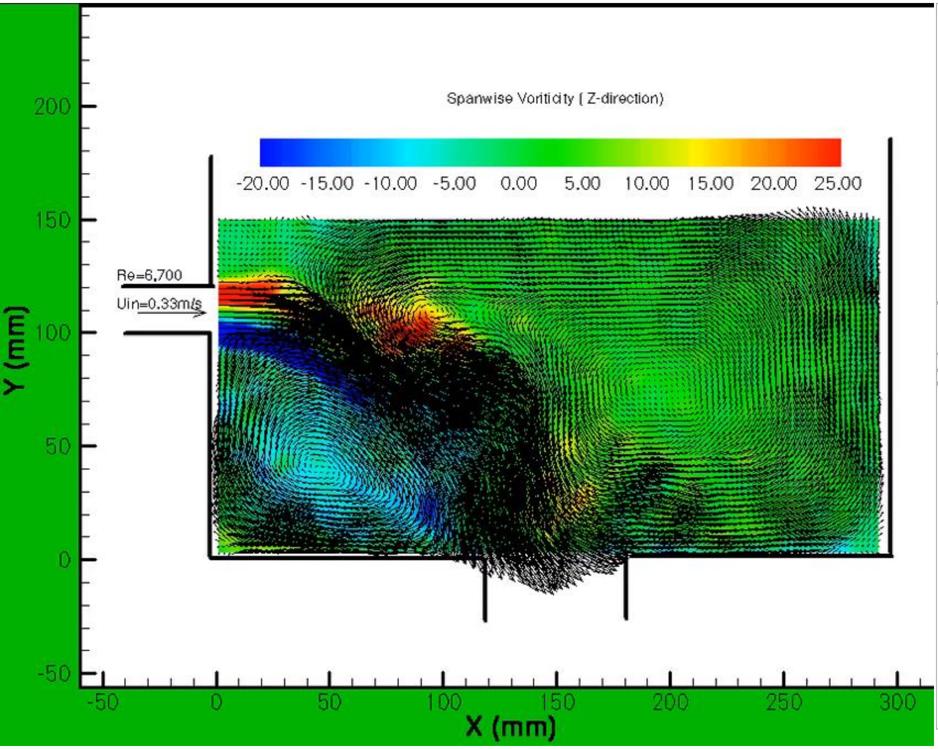
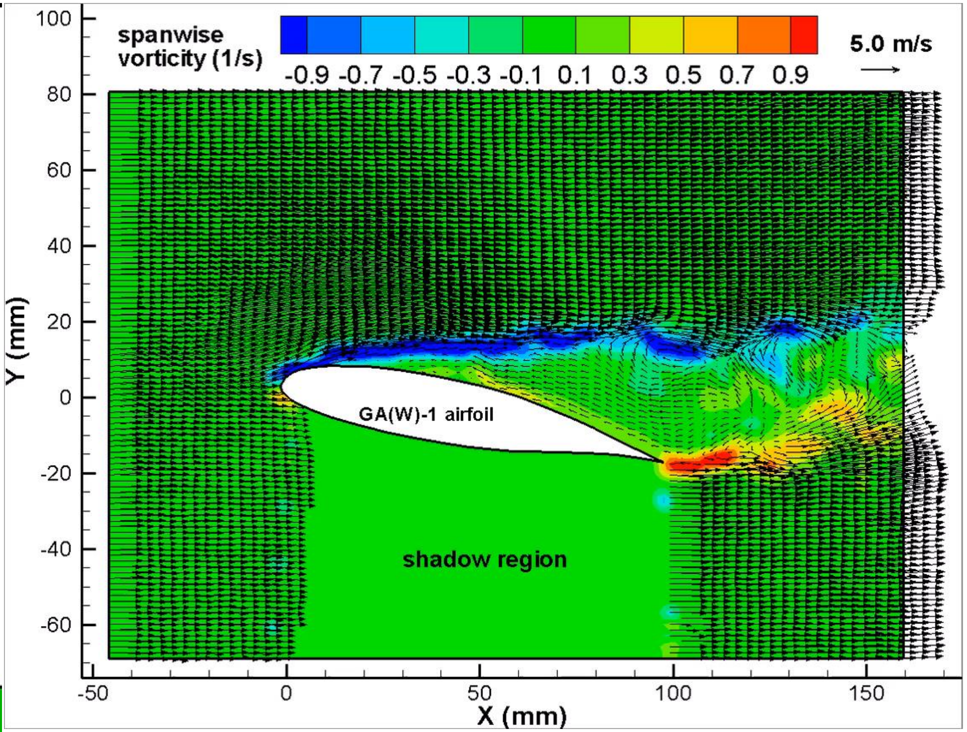


Fig. 6.10. Vorticity field estimates obtained from PIV velocity fields by the circulation method: (left) the velocity field is twice oversampled, (right) four times oversampled. The contours of this laminar vortex pair are known to be smooth such that the nonuniformities are due to measurement noise

Vorticity distribution Examples





IOWA STATE UNIVERSITY

AerE Aerospace Engineering

Ensemble-averaged quantities

- ***Mean velocity components in x, y directions:*** $U = \sum_{i=1}^{N} u_i / N \quad V = \sum_{i=1}^{N} v_i / N$

- ***Turbulent velocity fluctuations:*** $\overline{u'} = \sqrt{\sum_{i=1}^{N} (u_i - U)^2 / N} \quad \overline{v'} = \sqrt{\sum_{i=1}^{N} (v_i - V)^2}$

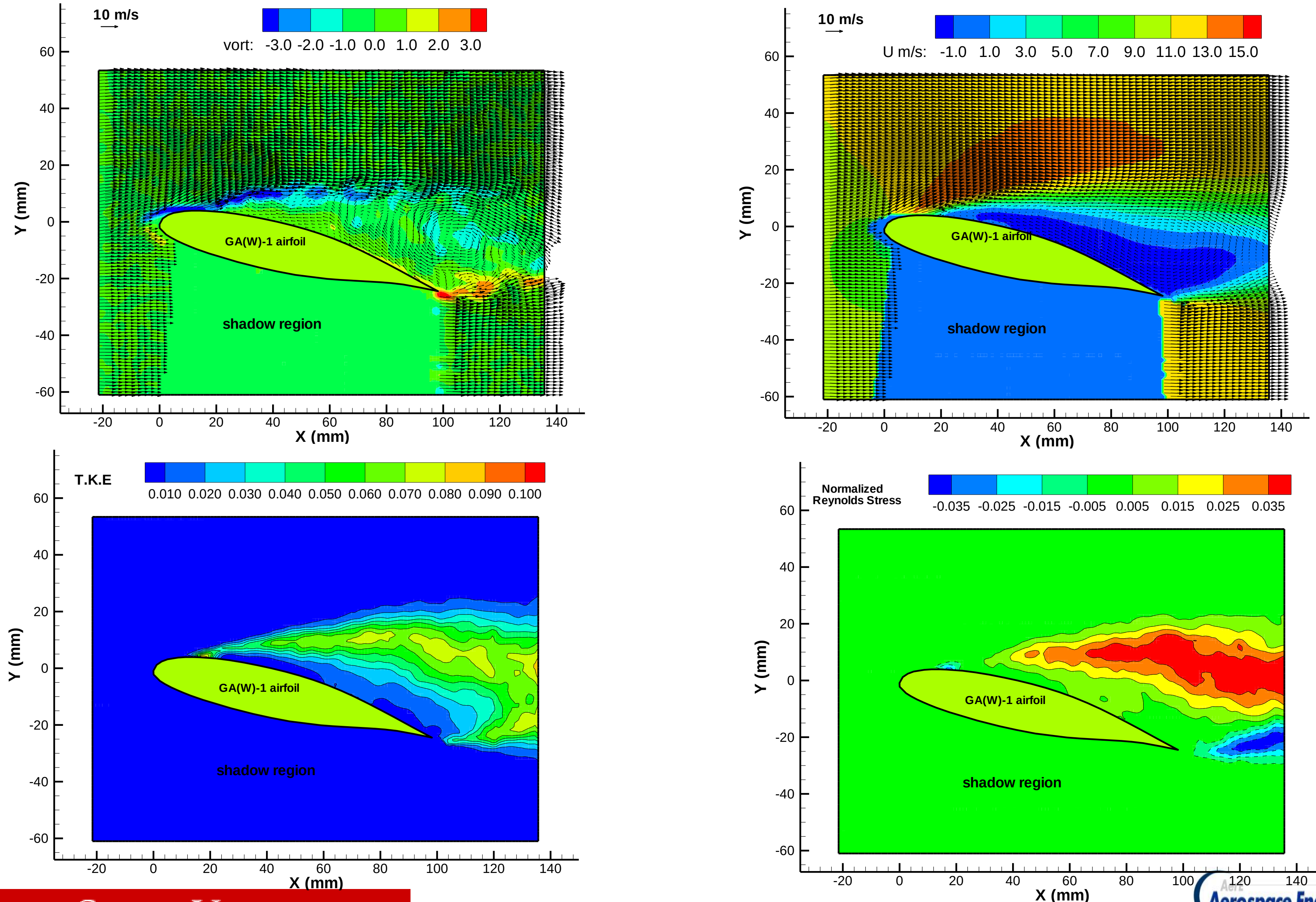
- ***Turbulent Kinetic energy distribution:***

$$TKE = \frac{1}{2}\rho(\overline{u'}^2 + \overline{v'}^2)$$

- ***Reynolds stress distribution:***

$$\tau = -\rho\overline{u'v'} = -\rho\sum_{i=1}^{N}\frac{(u_i - U)(v_i - U)}{N}$$

Ensemble-averaged quantities



Pressure field estimation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

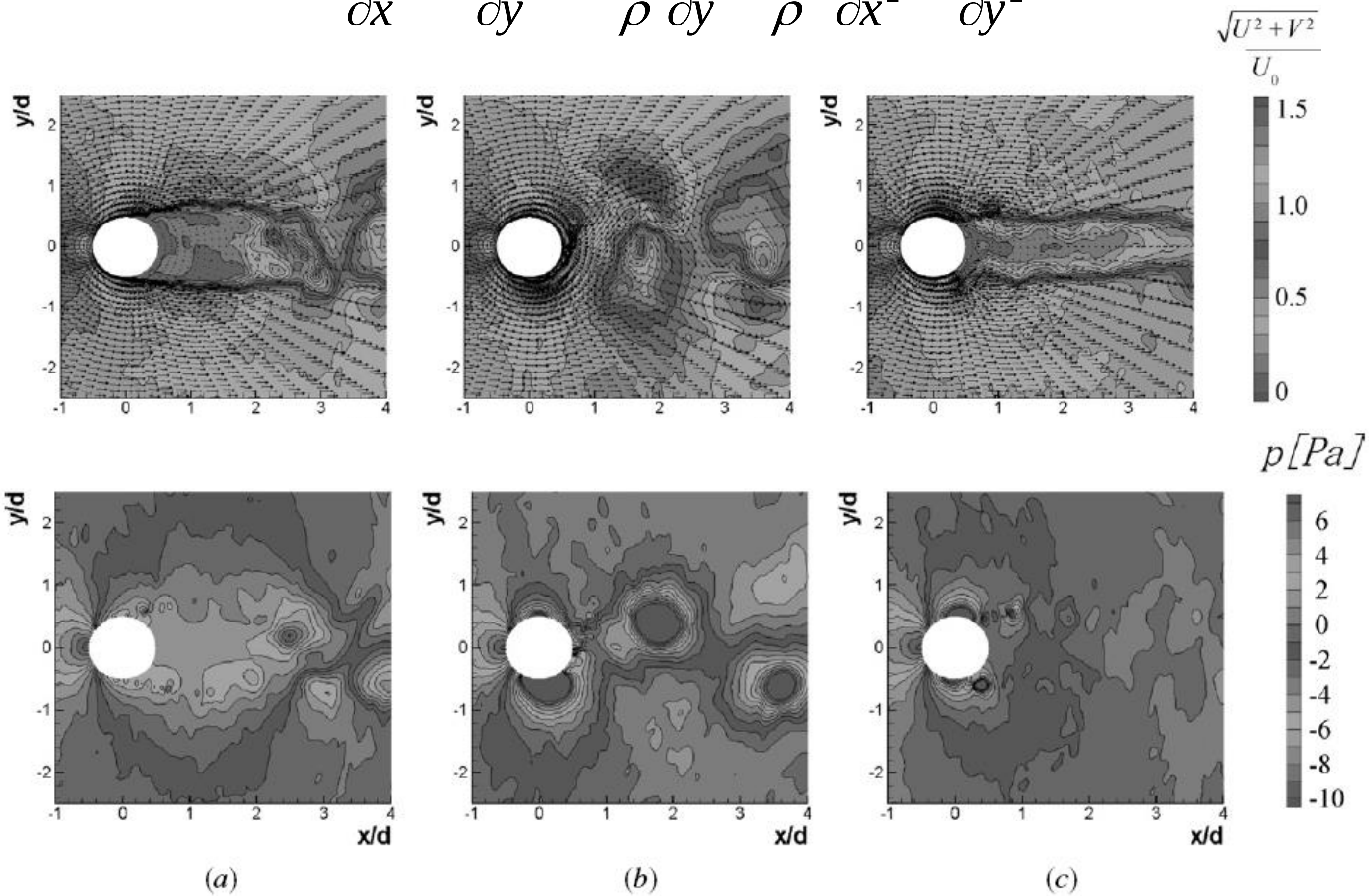
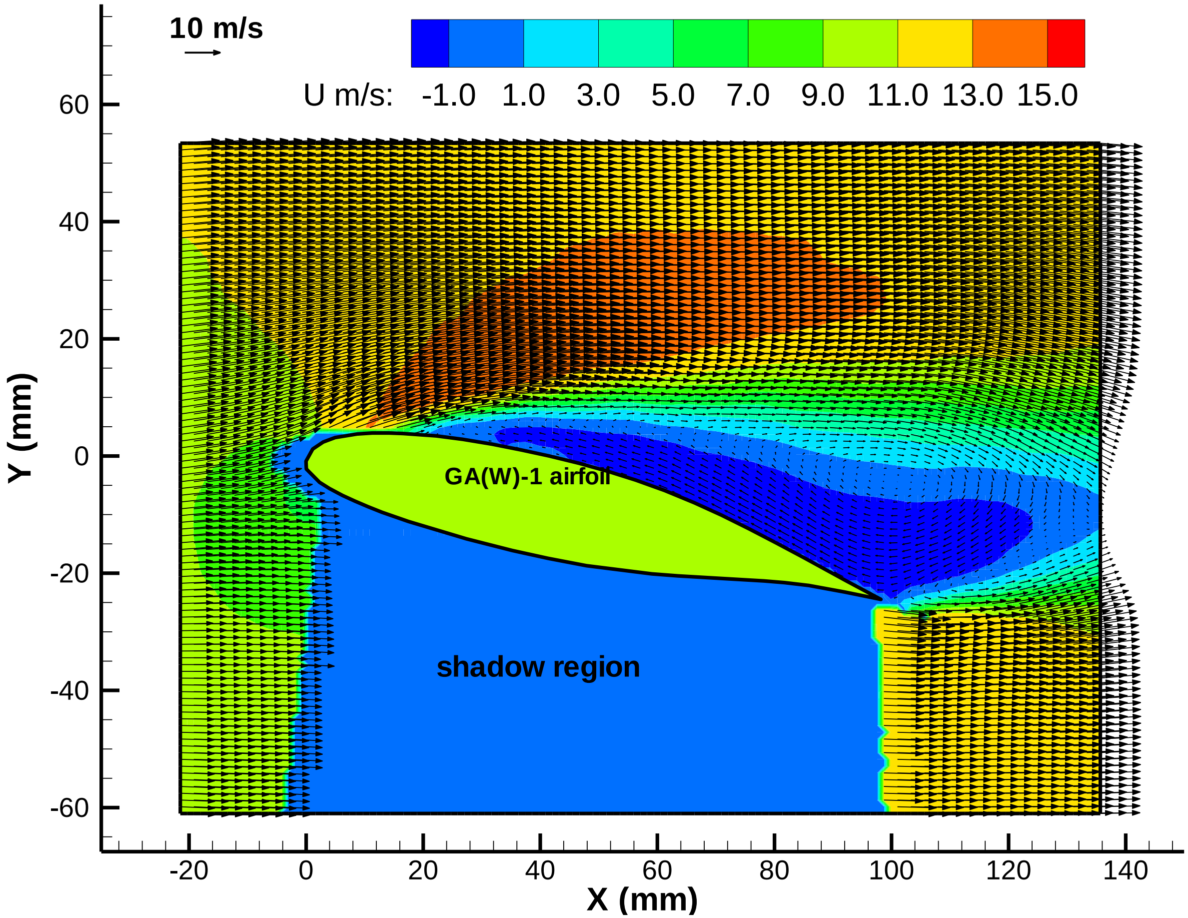


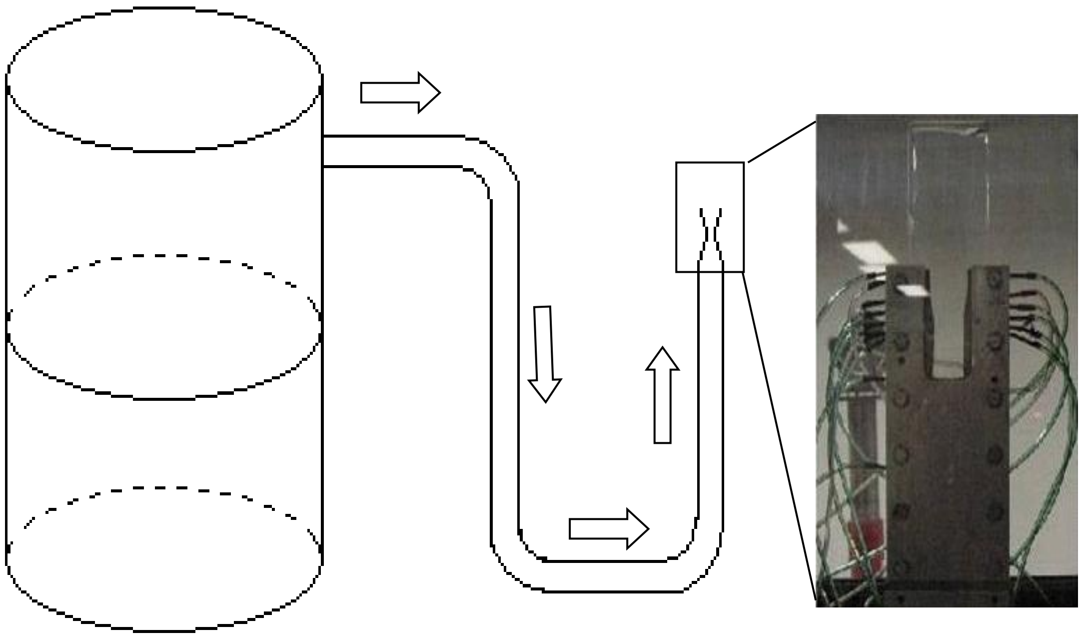
Figure 6. Instantaneous pressure field around a circular cylinder. (*a*) Stationary cylinder, (*b*) low-frequency oscillation ($S_f = 0.2$, $V_r = 2$) and (*c*) high-frequency oscillation ($S_f = 1$, $V_r = 2$).

Integral Force estimation

$$\frac{\partial}{\partial t}\int_{C.V.}\rho\vec{V}\,d\forall+\int_{C.S.}(\rho\vec{V}\vec{V})\bullet d\vec{A}=\int_{C.S.}\tilde{P}\bullet d\vec{A}+\int_{C.V.}\rho\,\vec{f}\,d\forall+\vec{F}$$

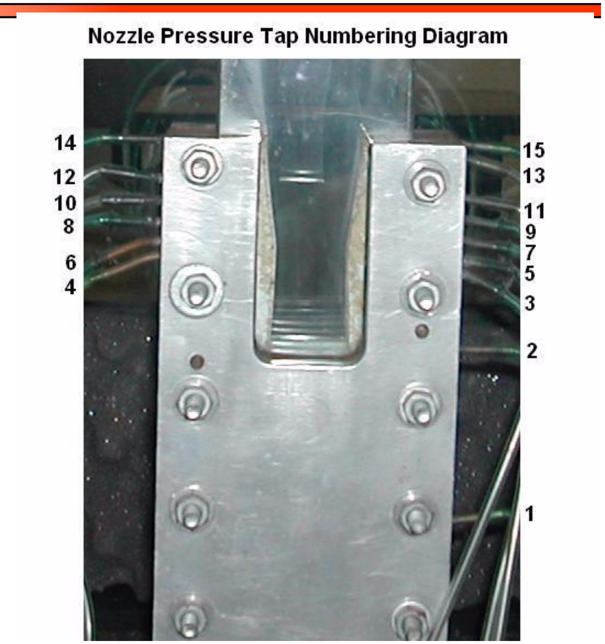


AerE344 Lab # 10: Pressure Measurements in a de Laval Nozzle



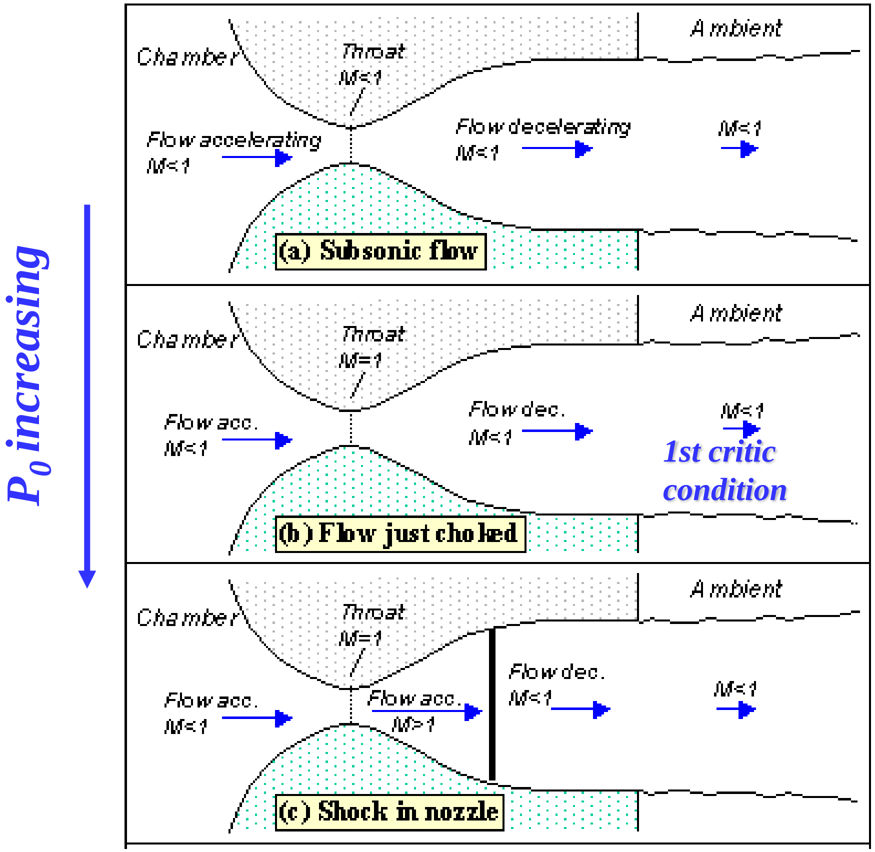
Tank with compressed air

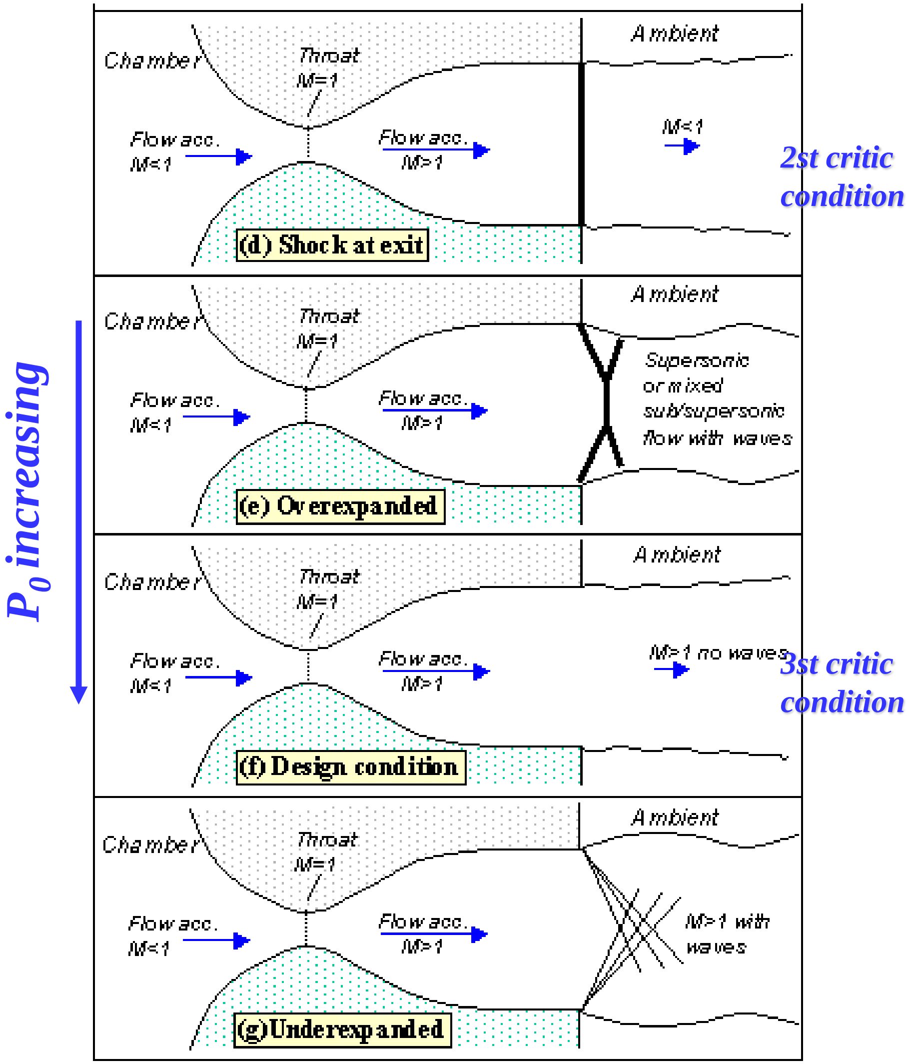
Test section



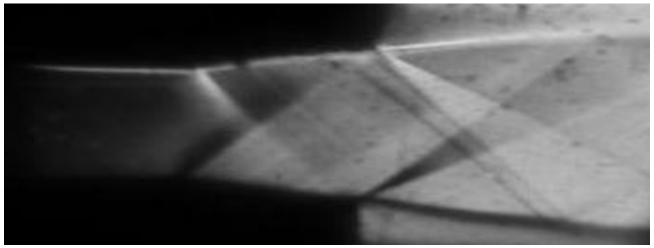
Tap No.	Distance downstream of throat (inches)	Area (Sq. inches)
1	-4.00	0.800
2	-1.50	0.529
3	-0.30	0.480
4	-0.18	0.478
5	0.00	0.476
6	0.15	0.497
7	0.30	0.518
8	0.45	0.539
9	0.60	0.560
10	0.75	0.581
11	0.90	0.599
12	1.05	0.616
13	1.20	0.627
14	1.35	0.632
15	1.45	0.634

1st, 2nd and 3rd critic conditions

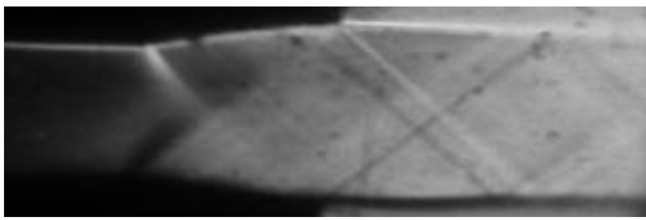




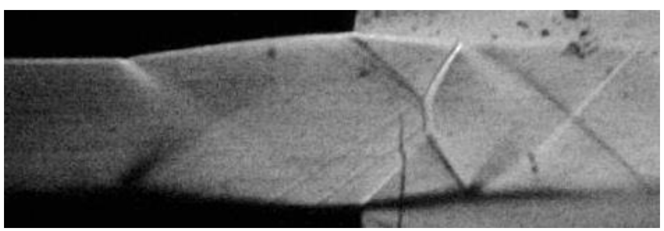
1st, 2nd, and 3rd critical conditions



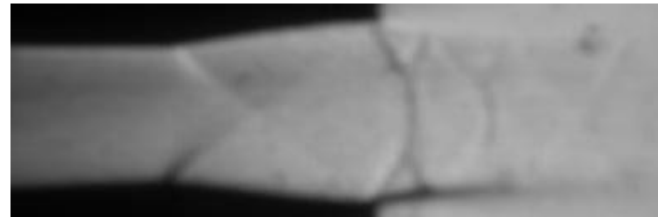
Under-expanded flow



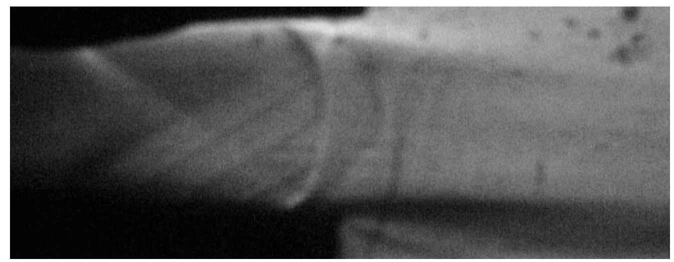
Flow close to 3rd critical



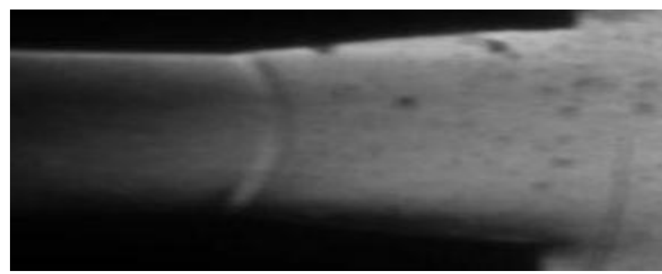
Over-expanded flow



2nd critical – shock is at nozzle exit

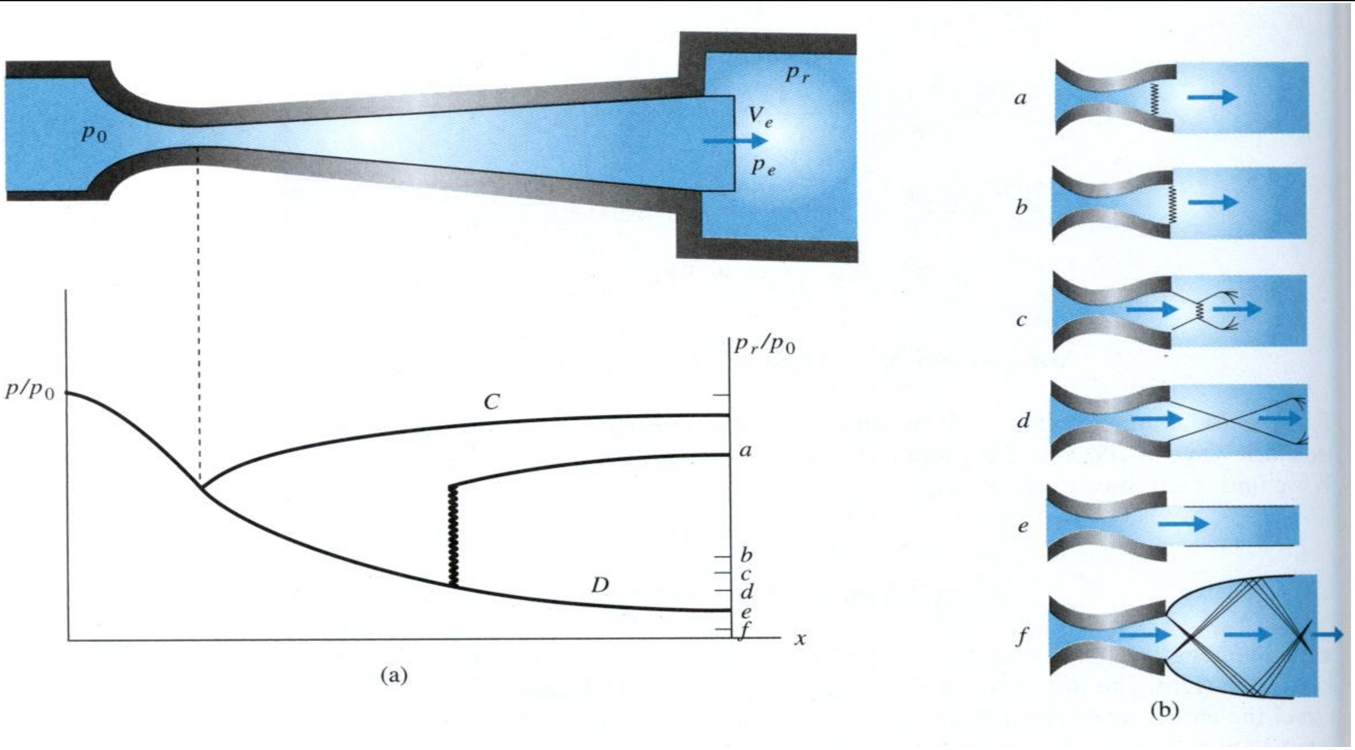


Over-expanded flow with shock between nozzle exit and throat



1st critical – shock is almost at the nozzle throat.

AerE344 Lab#10: Pressure Measurements in a de Laval Nozzle

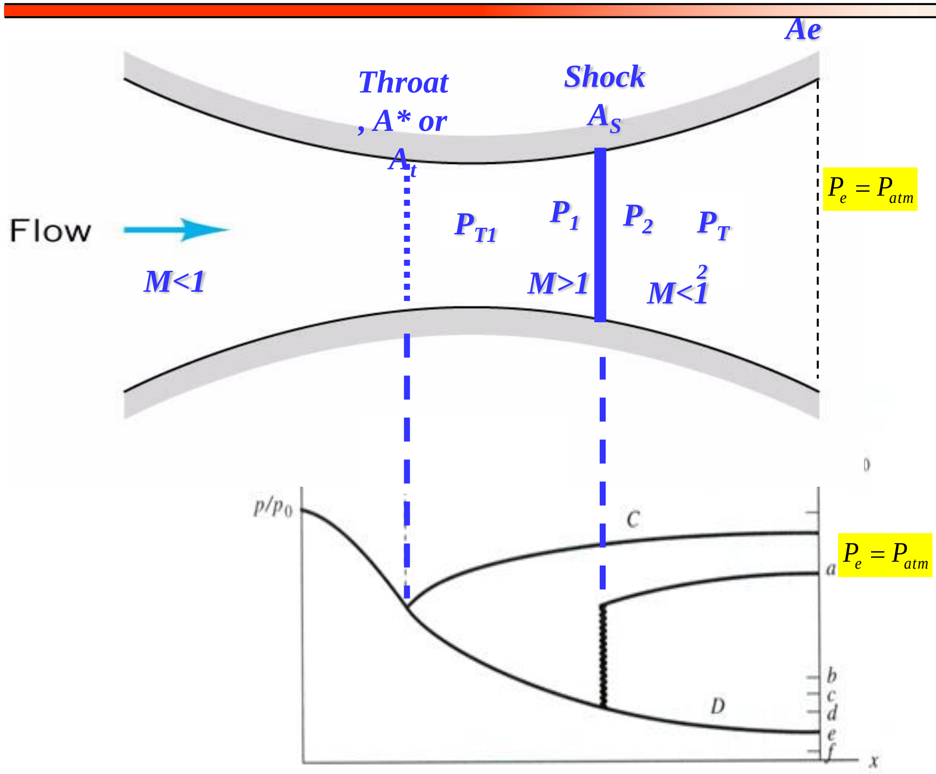


1. Under-expanded flow
2. 3rd critical
3. Over-expanded flow with oblique shocks
4. 2nd critical
5. Normal shock existing inside the nozzle
6. 1st critical

Required Plots:

- *Plots of the measured pressure (static and total pressure) as a function of distance along the nozzle axis for the cases 2, 4, 5 and 6.*
- *Plots of the theoretically predicted pressure (static and total pressure) as a function of distance along the nozzle axis for the cases 2, 4, 5 and 6.*
- *Plots with the measured and predicted values of the wall pressure distribution on the same graphs for the cases 2, 4, 5 and 6 for comparison.*
- *Plots of the theoretically predicted and measured Mach number as a function of distance along the nozzle axis for the cases 2, 4, 5 and 6.*

❑ Prediction of the Pressure Distribution within a De Laval Nozzle by using Numerical Approach



- ***Using the area ratio, the Mach number at any point up to the shock can be determined:***

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

- ***After finding Mach number at front of shock, calculate Mach number after shock using:***

$$M_2^{\,2} = \frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{\gamma M_1^{\,2} - \frac{\gamma-1}{2}}$$

- ***Then, calculate the A_2****

$$\left(A_2^*\right)^2 = M_2^{\,2} A_s^{\,2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\right]^{-\frac{\gamma+1}{\gamma-1}}$$

which allows us calculate the remaining Mach number distribution

$$\left(\frac{A}{A_2^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

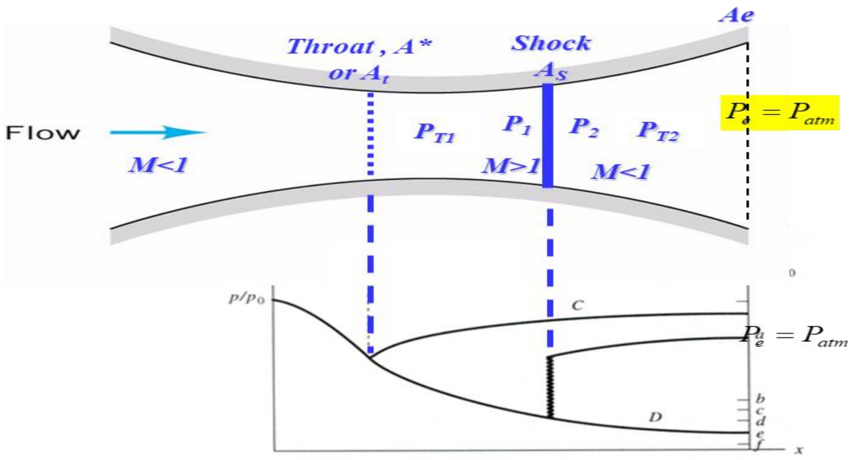
d. To calculate Mach number given the Mach-Area relation, can use Newton iteration to find M

$$F = \left(\frac{A}{A^*}\right)^2 = M^2\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} \quad (2.8)$$

$$F' = \frac{dF}{dM} = \frac{2}{M^3}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} - \frac{2}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{2}{\gamma-1}} \quad (2.9)$$

$$M^{n+1} = M^n - \frac{F}{F'} \quad (2.10)$$

Prediction of the Pressure Distribution within a De Laval Nozzle by using Shock Table Method



Tap No.	Distance downstream of throat (inches)	Area (Sq. inches)
1	-4.00	0.800
2	-1.50	0.529
3	-0.30	0.480
4	-0.18	0.478
5	0.00	0.476
6	0.15	0.497
7	0.30	0.518
8	0.45	0.539
9	0.60	0.560
10	0.75	0.581
11	0.90	0.599
12	1.05	0.616
13	1.20	0.627
14	1.35	0.632
15	1.45	0.634

- ***Method #1: To solve the equation numerically***

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

- ***Method #2: To use Isentropic Flow properties table (Appendix-A of Anderson's textbook)***
  - *If the shockwave is located at position of tab#12:*

Tap No.	A/A*	Mach #	P/P_t	P	P_g
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post-shock					
13					
15					

Examples of the previous lab reports