

AerE 344: Undergraduate Aerodynamics and Propulsion Laboratory

Lab Instructions

Lab #10: Pressure Measurements in a De Laval Nozzle

Instructor: Dr. Hui Hu
Department of Aerospace Engineering
Iowa State University
Office: Room 2251, Howe Hall
Tel: 515-294-0094
Email: huhui@iastate.edu

Many analyses in design problems assume 1D nozzle flow theory in cases of internal supersonic flow. One example of this might be in the prediction of the form drag of a rocket nozzle. This drag term is simply the difference between pressures on the inside and outside of the nozzle integrated over the area of the nozzle. In order to predict the wall pressure on the nozzle interior, 1D nozzle theory may be used. This theory obviously makes many assumptions about the flow and it is desired to know how much confidence can be placed in the resulting solutions.

In this experiment you will measure the wall pressure along a de Laval nozzle. The experiment should be run over a wide range of operating conditions. At the least, these conditions should be included:

- 1) Under-expanded flow
- 2) 3rd critical
- 3) Over-expanded flow with oblique shocks
- 4) 2nd critical
- 5) Normal shock existing inside the nozzle
- 6) 1st critical

The time at which each of the above cases occurs will be determined by observing the shock patterns in the Schlieren image. For each case, record the total pressure (tank pressure) and the wall pressure (see nozzle diagram). The wall pressures will be recorded by using a DSA system.

For all cases, plot the measured wall pressures as a function of distance along the nozzle axis and include in the plot the value of total pressure for that flow condition. For cases 2, 4, 5 and 6 include these same values as predicted by the 1D nozzle theory within the same plot.

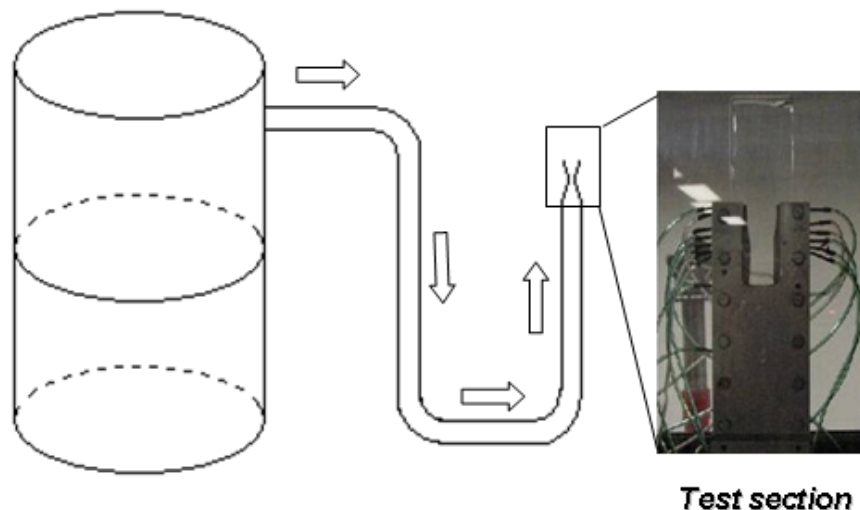
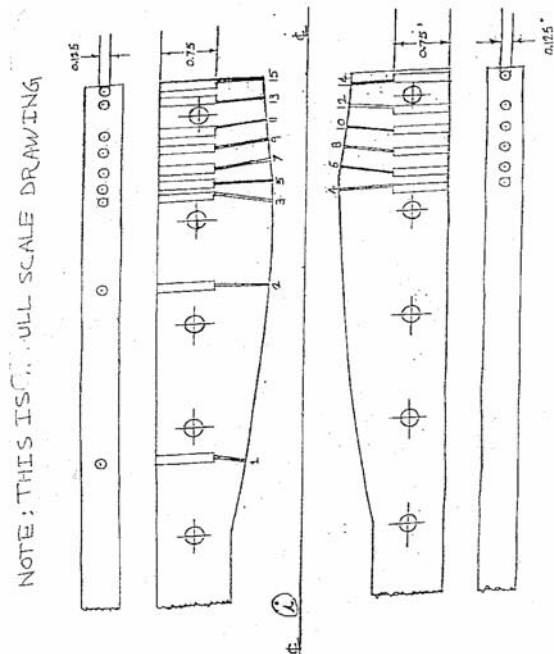
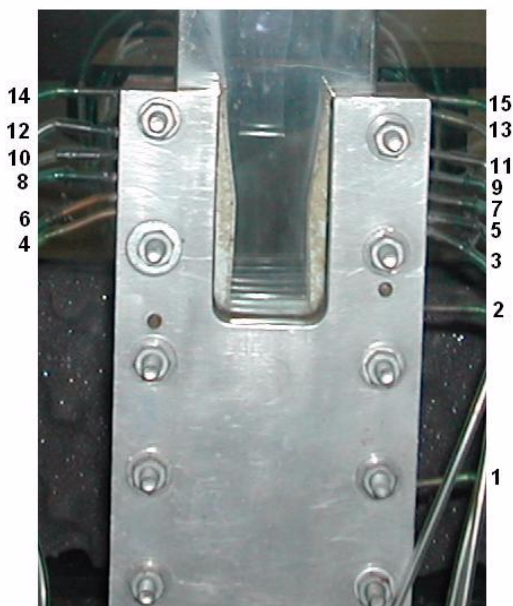


Fig. 1: Experiment rig

We will use the nozzle that is generally used in the laboratory at ISU which has the following properties:

Tap No.	Distance downstream of throat (inches)	Area (Sq. inches)
1	-4.00	0.800
2	-1.50	0.529
3	-0.30	0.480
4	-0.18	0.478
5	0.00	0.476
6	0.15	0.497
7	0.30	0.518
8	0.45	0.539
9	0.60	0.560
10	0.75	0.581
11	0.90	0.599
12	1.05	0.616
13	1.20	0.627
14	1.35	0.632
15	1.45	0.634

Nozzle Pressure Tap Numbering Diagram



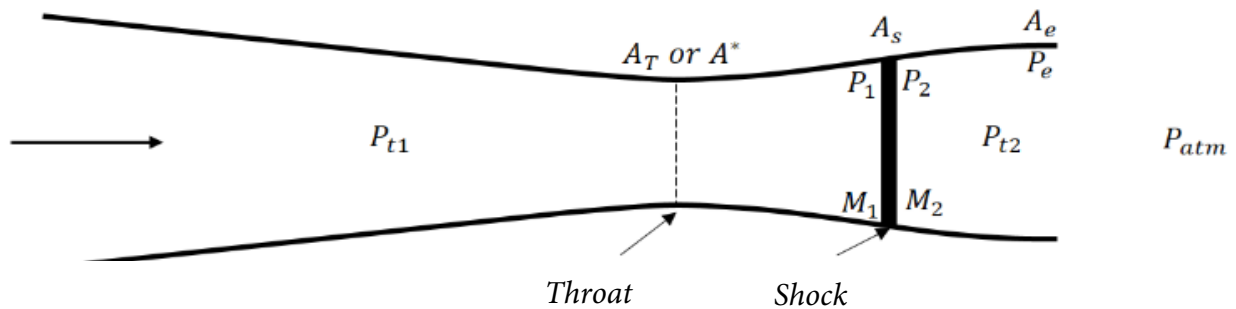
Questions need to be answered:

- What can you say about the predicted pressures in comparison to the measured pressures?
- That is, does the theory under- or over-predict the wall pressure?
- Give some possible reasons for the differences.
- What might this mean for the prediction of other flow quantities such as Mach number, temperature, etc.?
- Point out any interesting anomalies you might see in the measured data.
- Present your results in formal report format.

Pressure distribution in a Supersonic Nozzle

This experiment is concerned with the measurement of pressure in a converging/diverging (de laval) nozzle. The goal is to compare the measurements with the values predicted by the theory of one-dimensional inviscid compressible flow. As the name implies, the two major assumptions in the theory is that the flow is inviscid and uniform across the cross-sectional area at any given nozzle location. It will be seen by comparing with experiments that these assumptions can have significant influence on the pressures being measured along the wall which in reality is covered by a boundary layer with its own unique shock system.

There are two possible ways to formulate this problem as far as the theory is concerned. One way is to specify a total pressure of the flow upstream of the throat and then proceed to calculate the resulting shock location (if any) within the nozzle's diverging section. The second approach is to specify a shock location within the nozzle and then calculate the corresponding total pressure upstream of the throat. In both cases, it is assumed that the exit (usually atmospheric) pressure is known. This document describes the latter approach. In either case, the goal is to find the pressure distribution throughout the nozzle. A typical de Laval nozzle is shown in the following figure along with the definition of several key variables.



We will outline two methods which can be used to calculate the pressure distribution throughout the nozzle. The first method will be a numerical method by which a continuous pressure distribution can be obtained. The second method will be used to calculate the pressure at a discrete number of points along the nozzle. This method will use the NACA 1135 Tables in order to calculate the pressure.

A). Numerical Approach

One can also calculate the pressure in the nozzle using the derived equations. This would allow you to calculate several shock locations using the same program or spreadsheet without consulting the shock tables for every value.

1. Find Mach number along the de Laval nozzle

a. Using the area ratio, can calculate the Mach number at any point up to the shock using:

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}} \quad (1)$$

Where A is the cross section area up to the shock, A^* is the cross section area of the throat of the nozzle, M is the Mach number up to the shock, and γ is heat capacity ratio which is 1.4 in our experiments.

b. After finding Mach number at the front of shock (M_1), calculate Mach number after shock (M_2) using:

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \quad (2)$$

Where M_1 is the Mach number at the front of shock wave, and M_2 is the Mach number at the point behind shock wave.

Then calculate A_2^* using:

$$(A_2^*)^2 = M_2^2 A_s^2 \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M_2^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}} \quad (3)$$

Where A_2^* is a virtual throat cross section area, A_s is the cross section area at the shock wave point.

Then, the Mach number at any point down to the shock can be calculated by using formula (1).

2. Find pressure distribution along the de Laval nozzle

a). Pressure at exit is same as atmospheric pressure for the condition when the shock is inside nozzle ($P_e = P_{atm}$). For the condition when shock is after lip of nozzle (exit of the nozzle), total pressure is constant throughout the interior of the nozzle ($P_{t2} = P_{t1}$).

b). Find total pressure behind the shock:

$$P_{t2} = \frac{P_{t2}}{P_e} P_e, \text{ and } \frac{P_{t2}}{P_e} = \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (4)$$

Where P_{t2} is the total pressure after shock wave, P_e is the static pressure at the exit of the nozzle, which is equal to the atmospheric pressure $P_{atm} = 1010 \text{ millibar}$, and M_e is the Mach number at the exit of the nozzle.

c). Any pressure behind the shock wave is therefore:

$$P = P_{t2} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{-\gamma}{\gamma-1}} \quad (5)$$

d). Calculate total pressure ahead the shock (P_{t1}) wave:

$$P = P_{t2} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{-\gamma}{\gamma-1}} \quad (5)$$

d). Calculate total pressure ahead of the shock (P_{t1}) wave:

$$P_{t1} = \frac{P_{t1}}{P_1} \frac{P_1}{P_2} \frac{P_2}{P_{t2}} P_{t2} \quad (6)$$

Where we can use Total-Static relation for the first and the third ratios, and for the middle ratio:

$$\frac{P_1}{P_2} = \frac{1 + \gamma M_2^2}{1 + \gamma M_1^2} \quad \text{or} \quad \frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \quad (7)$$

Where P_1 is the static pressure at front of shock, P_2 is the static pressure at behind of shock.

The Total-Static relation is:

$$\frac{P_{total}}{P_{static}} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (8)$$

Where M is the Mach number.

B). Experimental Approach (Experimental part for the measured Mach number and measured pressure)

1. Measured pressure

The measured pressure can be read directly from the DSA file.

Note that the pressure read from the DAS file is gauge pressure, and the absolute pressure is calculated by adding gauge pressure and reference pressure, and the reference pressure is atmospheric pressure, which is 1010 Millibar.

2. Measured Mach number

a). Calculate the measured total pressure ahead of the shock wave

The Mach number at the throat of the nozzle is 1 during whole process from condition 1 to condition 6, so that the total pressure ahead the shock wave can be calculated by using the Total-Static relation:

$$\frac{P_{mt1}}{P_{mst}} = \left(1 + \frac{\gamma-1}{2} M_{st}^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (9)$$

Where P_{mt1} is the measured total pressure ahead the shock wave, P_{mst} is the measured static pressure at the throat of the nozzle, M_{st} is the Mach number at the throat of the nozzle, which is 1.

b). Calculate the measured Mach number ahead of the shock wave

Since the total pressure ahead of the shock wave is same, then the Mach number ahead of the shock wave can be calculated by using Total-Static relation too:

$$\frac{P_{mt1}}{P_{msa}} = \left(1 + \frac{\gamma - 1}{2} M_{ma}^2\right)^{\frac{\gamma}{\gamma - 1}} \quad (10)$$

Where P_{msa} is the measured static pressure ahead of the shock wave, M_{ma} is the Mach number ahead of the shock wave.

c). Calculate the measured Mach number at the behind of the shock wave

From b), we can calculate the Measured Mach number at the front of the shock wave, so that the measured Mach number at behind the shock wave can be calculated:

$$M_{m2}^2 = \frac{1 + \frac{\gamma - 1}{2} M_{m1}^2}{\gamma M_{m1}^2 - \frac{\gamma - 1}{2}} \quad (11)$$

Where M_{m1} is the measured Mach number at front of shock wave, and M_{m2} is the measured Mach number at the behind of shock wave.

d). Calculate the measured total pressure down to the shock wave

From c), we calculate the measured Mach number at behind shock wave, so that the measured total pressure down to the shock wave can be calculated:

$$\frac{P_{mt2}}{P_{m2}} = \left(1 + \frac{\gamma - 1}{2} M_{m2}^2\right)^{\frac{\gamma}{\gamma - 1}} \quad (12)$$

Where P_{mt2} is the measured total pressure down to the shock wave, P_{m2} is the measured static pressure at behind the shock wave, M_{m2} is the Mach number at the behind of the shock wave.

e). Calculate the measured Mach number down to the shock wave

Since we can calculate the measured total pressure down to the shock wave, the measured Mach number down to the shock wave can be calculated by using Total-Static relation too:

$$\frac{P_{mt2}}{P_{msd}} = \left(1 + \frac{\gamma - 1}{2} M_{md}^2\right)^{\frac{\gamma}{\gamma - 1}} \quad (13)$$

Where P_{msd} is the measured static pressure down to the shock wave, M_{md} is the Mach number down to the shock wave.

* Notes

a. For 3rd Critical

1. $P_1 = P_2 = P_e$
2. $M_1 = M_2 = M_e$ (supersonic)

b. For 1st Critical

1. Same as 3rd critical, but M_e is subsonic

c. For 2nd Critical

1. $M_2 = M_e$
2. $P_2 = P_e$

d. To calculate Mach number given the Mach-Area relation, can use Newton iteration to find M

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}} \quad (14)$$

$$F' = \frac{dF}{dM} = \frac{2}{M^3} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}} - \frac{2}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{2}{\gamma-1}} \quad (15)$$

$$M^{n+1} = M^n - \frac{F}{F'} \quad (16)$$

C). Shock Tables Method and Example

In this approach, the geometry of the nozzle is known at a discrete number of points along its length. The method then consists of using the tables to systematically arrive at a Mach number distribution and ultimately a pressure distribution. This method is best to illustrated by an example. We will use the nozzle that is generally used in the laboratory at ISU which has the following properties:

Tap No.	Distance downstream of throat (inches)	Area (Sq. inches)
1	-4.00	0.800
2	-1.50	0.529
3	-0.30	0.480
4	-0.18	0.478
5	0.00	0.476
6	0.15	0.497
7	0.30	0.518
8	0.45	0.539
9	0.60	0.560
10	0.75	0.581
11	0.90	0.599
12	1.05	0.616
13	1.20	0.627
14	1.35	0.632
15	1.45	0.634

We will start by setting up a table with columns that we will need to fill in as we go. We will evaluate flow properties at a limited number of tap points. Specifically, we will use taps

1,2,3,5,7,9,11,13 and 15. Also, we will prescribe the shock to be located at tap 12. It is useful to assume the shock is located at one of the taps because there is no need to interpolate the nozzle area. The table we want to fill out is

Tap No.	A/A*	Mach #	P/P _t	P	P _g
1	1.681				
2	1.111				
3	1.008				
5	1.000				
7	1.088				
9	1.176				
11	1.258				
pre-shock	1.294				
post-shock					
13					
15					

Note that we have two rows at the shock, one labeled pre-shock and the other post-shock. This is because properties, such as A*, change across the shock and must be evaluated on either side of it. We have already carried out the first step in the above table. That is, we have found A/A* from the tables using the value of the nozzle area at each tap location and the fact that A* is equal to the nozzle throat area in front of the shock. We next use the isentropic flow part of the tables to find the Mach number corresponding to each A/A*. We can also use the isentropic flow part of the tables to find the corresponding P/P_t.

Tap No.	A/A*	Mach #	P/P _t	P	P _g
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post-shock					
13					
15					

We don't yet know the total pressure in front of the shock, P_{t1}. We will now have to find some of the values behind the shock. We will start by calculating the Mach number immediately behind the shock. Using the normal shock tables with M₁ = 1.64 we find that M₂ = 0.686. Next, we find the sonic reference area behind the shock using the area-Mach relation:

$$(A_2^*)^2 = A_s^2 M_2^2 \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M_2^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}} \quad (2.11)$$

where A_s is the nozzle area at the shock, in this case at tap 11. For the given problem, A₂^{*} = 0.5574 sq. inches. The above table now becomes:

Tap No.	A/A*	Mach #	P/P _t	P	P _g
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post-shock	1.105	0.69	0.7274		
13					
15					

We can complete the first three columns by using the isentropic flow part of the tables to find pressure ratios and Mach numbers for the various area ratios. The first three columns are shown completed in the following table

Tap No.	A/A*	Mach #	P/P _t	P	P _g
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post-shock	1.105	0.69	0.7274		
13	1.125	0.66	0.7465		
15	1.137	0.65	0.7528	14.7	

In the previous table we have entered the exit pressure in pounds per square inch. In this case we take the exit pressure to be sea-level standard pressure. We now calculate the total pressure behind the shock using this value of exit pressure and the pressure ratio at the exit:

$$P_{t2} = \frac{P_t}{P} P = \left(\frac{1}{0.7528} \right) 14.7 = 19.53 \quad (2.12)$$

Using this value of total pressure, the other static pressures behind the shock can be calculated and tabulated using the pressure ratios in column 4 and $P_{t2} = 19.53$:

Tap No.	A/A*	Mach #	P/P _t	P	P _g
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post-shock	1.105	0.69	0.7274	14.21	
13	1.125	0.66	0.7465	14.58	
15	1.137	0.65	0.7528	14.7	

Our last major task is to find the total pressure ahead of the shock, P_{t1} . These values can be calculated from:

$$P_{t1} = \frac{P_{t1}}{P_1} \frac{P_1}{P_2} \frac{P_2}{P_{t2}} P_{t2} \quad (2.13)$$

The terms of $\frac{P_{t1}}{P_1}$ and $\frac{P_2}{P_{t2}}$ in (2.13) are listed in the table. The pressure ratio $\frac{P_1}{P_2}$ can be found using the Normal Shock tables and a value of $M_1 = 1.64$. For the current case, the total pressure ahead of the shock was found to be $P_{t1} = 21.57$ psi. With the total pressure known, the only task remaining is to finish filling out column 5 of the above table. This can be done using the equation

$$P = \frac{P}{P_t} P_t \quad (2.14)$$

where everything on the right side of (2.14) is known. Finally, since we will be measuring gauge pressure in pounds per square inch in the lab, we convert absolute pressure in psi using

$$P_g = P - P_{atm} \quad (2.15)$$

where P_g is the gauge pressure. The following table completes the procedure:

Tap No.	A/A*	Mach #	P/P_t	P	P_g
1	1.681	0.37	0.9098	19.6	4.9
2	1.111	0.67	0.7401	16	1.3
3	1.008	0.97	0.5469	11.8	-2.9
5	1.000	1.00	0.5283	11.4	-3.3
7	1.088	1.35	0.3370	7.27	-7.43
9	1.176	1.50	0.2724	5.88	-8.82
11	1.258	1.61	0.2318	5	-9.7
pre-shock	1.294	1.64	0.2217	4.78	-9.92
post-shock	1.105	0.69	0.7274	14.21	-0.49
13	1.125	0.66	0.7465	14.58	-0.12
15	1.137	0.65	0.7528	14.7	0

We have now found the pressure distribution at discrete points throughout the nozzle. Note that when analyzing a large variety of nozzles or exit conditions, this is a very time consuming process. In general, a better method is to solve the equations numerically using a computer program. It should be clear from the above example how this method can be used to calculate pressure for shocks at different locations or for first, second or third critical.

AerE344 Lab Exercise #10: Pressure Measurements in a de Laval Nozzle

Writeup Guidelines

This lab writeup will be a typical formal lab report. You will need to make the required plots (as listed below) and provide discussion about the measurement results as a part of your lab report.

You will measure the wall pressure along a de Laval nozzle. The experiment should be run over a wide range of operating conditions. At the least, these conditions should be included:

- 1) Under-expanded flow
- 2) 3rd critical
- 3) Over-expanded flow with oblique shocks
- 4) 2nd critical
- 5) Normal shock existing inside the nozzle
- 6) 1st critical

The time at which each of the above cases occurs will be determined by observing the shock patterns in the Schlieren image. For each case, record the total pressure (tank pressure) and the wall pressure (see nozzle diagram). The wall pressures will be recorded by using a DSA system.

Required Plots:

- Plots of the measured pressure (static and total pressure) as a function of distance along the nozzle axis for the cases 2, 4, 5 and 6.
- Plots of the theoretically predicted pressure (static and total pressure) as a function of distance along the nozzle axis for the cases 2, 4, 5 and 6.
- Plots with the measured and predicted values of the wall pressure distribution on the same graphs for the cases 2, 4, 5 and 6 for comparison.
- Plots of the theoretically predicted and measured Mach number as a function of distance along the nozzle axis for the cases 2, 4, 5 and 6.

Questions that should be answered:

- What can you say about the predicted pressures in comparison to the measured pressures?
- That is, does the theory under- or over-predict the wall pressure?
- Give some possible reasons for the differences.
- What might this mean for the prediction of other flow quantities such as density, temperature, etc.?
- Point out any interesting anomalies you might see in the measured data.
- Present your results in formal report format.