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Air traffic network optimization via Laplacian energy maximization

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ABSTRACT

Designing robust air traffic network is an ongoing research effort that seeks to improve the network robustness when one or more air routes are added to the existing network. We demonstrate that Laplacian energy is a fair and promising measure of network robustness based on a case study of a real air traffic network and extensive numerical experiments. Therefore, this paper aims at maximizing the Laplacian energy to enhance network robustness. The corresponding weighted Laplacian energy maximization problem is formulated as flight route addition problem to facilitate practical operations. Three methods are proposed to solve the flight route addition problem, including tabu search, greedy algorithm and integer program. Their trade-off between optimality performance and computational efficiency is demonstrated through the numerical results on a scale-free network. A case study on a real air traffic network is also included for further investigation.

1. Introduction

The rapid growth of the air transportation industry is expected to continue in the future decades. The Federal Aviation Administration (FAA) estimated the global air traffic activities will increase by average of 3% every year until 2025 [1]. Moreover, the high economic growth in the Asia-Pacific market drives the passenger growth averaging 5.2% a year during 2008–2025 [2]. The expanding traffic demands will lead to further departure delays, flight congestions and cancellations with the limited airport resources and airspace capacities. Inspired to tackle the expanding air traffic challenges, several projects are underway including Next Generation Air Transportation System (NextGen) and the Single European Sky Air Traffic Management Research (SESAR) [3]. Recently, Singapore also established Air Traffic Management Research Institute (ATMRI) to carry out research seeking solutions for air traffic issues in Asia-Pacific Region [4]. One critical issue of these projects is how to design and maintain a robust air traffic network, which is capable of sustaining the airport and route failure happenings due to severe weather, airspace flow program, equipment shortage, ground delay program and other emergence events [5,6].

1.1. Literature review

In the past decades, an increasing number of studies focused on the air traffic network and its robustness. According to the research objectivity, these works could be grouped into two categories, namely analysis of structural features and topological optimization. The structural analysis of air transportation network applied complex network theory [7], a young and active area of scientific research, to seek and define the most efficient topological structure of air traffic networks. Lin concluded there was a spatial hierarchical structure within China's aviation network by examining weekly flight pattern [8]. Wang et al. explored the network structure and nodal centrality of China's air transport network by degree distribution, clustering coefficient, closeness and betweenness [9]. The structures of seven largest carrier networks in the USA were analyzed [10], where Wuellner et al. found that the networks with dense interconnectivity were extremely resilient to airports and routes attacks. Sun et al. made a deep investigation on the assessment of structural similarity of air navigation route systems in 58 countries [11] and applied seven centrality measures to study the temporal evolution of the European air transportation system, including two network layers: air airport network and the air navigation route network [12]. Recently, an aggregate level of analysis was carried out to analyze the scale-free and small-world characteristic of the world airline networks. Bagler analyzed the India's domestic civil aviation network which could be characterized by a small-world network [13]. Xu and Harriss studied the U.S. domestic intercity passenger air transportation network and found that the network was a small-world network accompanied with assortative mixing patterns and rich-club phenomenon [14]. Li and Cai demonstrated the airport network in China embodied mixture features of small worlds and scare-free networks, and exhibited topological uniqueness features [15]. Zanin and Lillo made

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a short review on the recent use of complex network theory to describe the structure and dynamics of air transport [16].

There were very few research results on the topology optimization. Reggiani et al. analyzed the connectivity and concentration of Lufthansa's network and then proposed a multi-criteria analysis to strategically configure the airline network patterns [17]. Redondi et al. used the module identification techniques to evaluate the influences of new routes on the air traffic network connectivity [18]. Wuellner et al. introduced network rewiring schemes that boosted resilience to node and edge failures [10]. Cai et al. investigated the application computational intelligence to crossing waypoints location problem in the context of real world air route network design in China [19]. Several researches concentrated on the optimization of algebraic connectivity, which was considered as an efficient measure for the robustness of air traffic network. Vargo et al. explored the effectiveness of a tabu search algorithm and semidefinite programming relaxation to increase the algebraic connectivity of U.S. air traffic network [20]. In [21], Wei et al. proposed three approaches to maximize the algebraic connectivity of air transportation. Then they formulated a new air transportation network model and solved the algebraic connectivity optimization problem for small scale network and large scale network through finding both the edges of the graph and weight assignments [22].

1.2. Contribution

Compared with the existing literature, this study for the first time measures and optimizes the robustness of air traffic network by Laplacian energy, which could be considered as one kind of graph entropy [23]. Besides the fact that Laplacian energy is a good global fairness measure of network like algebraic connectivity, it also takes into account local information of network. This is the major motivation why we adopt the measure of Laplacian energy to optimize the air traffic network. The Laplacian energy maximization problem is correspondingly formulated for this weighted air traffic network as a basic flight routes addition problem. In order to efficiently solve the problem, how to choose the most promising edges to add to the graph is demonstrated. Based on the weighted air traffic network, we further develop an integer programming model.

The structure of this paper is outlined as follows: Section 2 illustrates the relationships of Laplacian energy and air traffic network by investigating a real air transportation network of Jetstar Asian Airway. The basic Laplacian energy maximization is then presented based on practical operations. In section 3, three methods including tabu search, greedy algorithm and integer programming are proposed to efficiently solve the basic problem. In section 4, we compare the performance and computational efficiency of three methods via simulations, and the real air traffic networks are also investigated. Section 5 concludes this paper.

2. Problem formulation

2.1. Principles of Laplacian energy

An air traffic network can be described as a graph $G = (V, E, W)$, where the node set $V(G) = \{v_1, v_2, \ldots, v_n\}$ represents all the n distinct airports, the edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$ denotes all the m direct flight routes between pairs of airports. Each edge $e = (v_i, v_j)$ is attached with a weight $w_{i,j}$. If there is no edge between v_i and v_j, $w_{i,j} = 0$. Since if a flight route exists from node v_i to v_j, a return flight route also exists from v_j to v_i, G can be constructed as an undirected graph.

We define

$$W(G) = \begin{pmatrix} 0 & w_{1,2} & \ldots & w_{1,n} \\ w_{2,1} & 0 & \ldots & w_{2,n} \\ . & . & . & . \\ w_{n,1} & w_{n,2} & \ldots & 0 \end{pmatrix}$$

and

$$X(G) = \begin{pmatrix} x_1 & 0 & \ldots & 0 \\ 0 & x_2 & \ldots & 0 \\ . & . & . & . \\ 0 & 0 & \ldots & x_n \end{pmatrix}$$

with $x_i = \sum_{j=1}^{n} w_{i,j} = \sum_{u \in N(v_i)}^{n} w_{v_i,u}$, and x_i is the sum-weight of all the edges connecting to node v_i, where $N(v_i)$ is the neighborhood of v_i.

Definition 1. The matrix $L(G) = X(G) - W(G)$ is called the Laplacian matrix of the weighted network G.

Definition 2. The eigenvalues of $L(G)$ are denoted by $\lambda_1 \le \lambda_2 \le \ldots \lambda_n$. The Laplacian energy of G is defined as the invariant:

$$E_L(G) = \sum_{i=1}^{n} \lambda_i^2. \quad (1)$$

2.2. Laplacian energy and air traffic network robustness

In order to illustrate the relationship between Laplacian energy and air traffic network robustness, a real air traffic network of Jetstar Asia Airway among Indonesia, Australia, and New Zealand is investigated (shown in Fig. 1). Algebraic connectivity λ_2, which is proved to be a fair robustness measure of air traffic network [22], is used to show that the Laplacian energy is capable of reflecting the global tightness of network. Meanwhile, a case study is used to illustrate the advantages of Laplacian energy over algebraic connectivity.

The route map of Jetstar includes 26 airports and 68 routes. These 26 airports include Adelaide (ADL), Auckland (AKL), Avalon (AVV), Ayer Rock (AYQ), Brisbane (BNE), Ballina (BNK), Christchurch (CHC), Cairns (CNS), Denpasar (DPS), Darwin (DRW), Dunedin (DUD), Hobart (HBA), Hamilton Island (HTI), Praya (LOP), Launceston (LST), Maroochydore (MCY), Melbourne (MEL), Mackay (MKY), Newcastle (NTL), Coolangatta (OOL), Perth (PER), Proserpine (PPP), Sydney (SYD), Townsville (TSV), Wellington (WLG), and Queenstown (ZQN). The comparative experiment begins with creating 3 different weighted air traffic networks with the same topology. These networks are randomly assigned with one of three types of weights, $w_{i,j} = \{1, 2, 3\}$, which represent the link strengths. A larger weight assignment indicates the route is more resilient and a smaller weight means it's easier to breakdown. Then the links are randomly removed till the network is not connected, that is, at least one pair of nodes cannot reach each other through any one or multiple routes.

The simulation results shown in Fig. 2 indicate both values of Laplacian energy and algebraic connectivity are monotonically nonincreasing with respect to the number of removed edges. The deletion process terminates with minimal values of Laplacian energy and algebraic connectivity. The similar nonincreasing trend of the two curves shows that Laplacian energy can be also used to represent the global robustness of network like algebraic connectivity. Moreover, the value of Laplacian energy gradually changes when removing (or adding) edges for a network, which is more effective in differentiating and optimizing the network performance than the measure of algebraic connectivity.

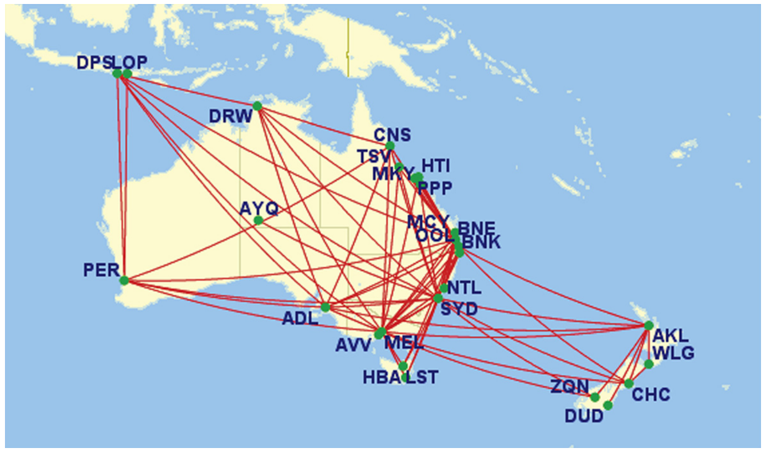

Fig. 1. Air traffic network route map for Jetstar Asia Airway (Indonesia, Australia, and New Zealand) in 2015.

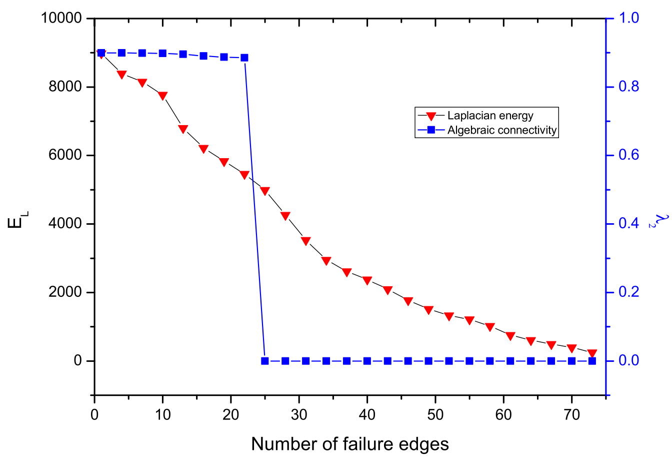

Fig. 2. E_L, λ_2 in terms of random edge failure for Jetstar Asia Airway (Indonesia, Australia, and New Zealand).

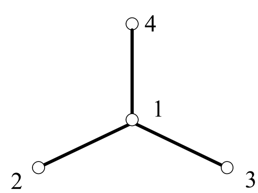

Fig. 3. The minimum spanning tree topology with four nodes.

The measure of Laplacian energy is more "sensitive" than the measure of algebraic connectivity because Laplacian energy cannot only reflect the global feature but also capture the local characteristics. We can demonstrate this effect through the following simple and insightful case as shown in Fig. 3, in which an extra link will be added to test the two measures. Let $\Delta\lambda_2$ and ΔE_L denote the change of algebraic connectivity and Laplacian energy after adding an edge to the network in Fig. 3. After a single edge $e_{2,3}$ with $w_{2,3} = 3$ is added between nodes 2 and 3, $\Delta\lambda_2 = 0$ and $\Delta E_L = 48$. Moreover, $\Delta\lambda_2$ is independent of the edge weight $w_{2,3}$ and still remains zero even if $w_{2,3} = \infty$. However, the network has been actually improved as the flow can be directly transferred between nodes 2 and 3, which is ignored by the measure of algebraic connectivity. To capture this local improvement, the measure of Laplacian energy is more promising and effective.

We further demonstrate this effectiveness of Laplacian energy over algebraic connectivity through the two extensive numerical experiments below. In experiment I, we randomly generate 10 000 scale-free networks with random network size range from 10 to 100, each edge is assigned with a random weight selecting from $\{1, 2, 3\}$. We count the number of network instances whose values of Laplacian energy (algebraic connectivity) will change after adding a random edge. In experiment II, we generate three scale-free networks with $n = \{10, 50, 100\}$ nodes. Then random deletion process is conducted till the network is separated. We count the times of value changes in terms of Laplacian energy and algebraic connectivity. The results are shown in Table 1. It can be easily observed that algebraic connectivity fails to indicate the robustness degradation of network in much more instances than Laplacian energy. Thus, Laplacian energy is a more effective measure in differentiating network robustness.

Table 1
Results of two experiments.

Experiment	Description	E_L	λ_2
I	total change times in 10 000 trails	10 000	2753
II	total change times with $n = 10$	33	25
	total change times with $n = 50$	1125	615
	total change times with $n = 100$	4561	1539

2.3. Basic Laplacian energy optimization formulation

In real world, many flights already serve in local and world regions, it is needless to create a new optimal air traffic network from scratch. Directly adding a few routes into the existing air traffic network is the most easiest and efficient way to improve its robustness. In the paper, we would like to find the optimal strategy on the selection of potential edges. As noted in previous section, the existing weighted air traffic network is represented by $G = (V, E_0)$, where V denotes the set of airports and $E_0 \subset E$ contains the existing routes between the airports. Without loss of generality, at most one edge (flight route) is allowed to be connected between any pair of nodes (airports). The size of V is n and the size of E_0 is m. The objective is to maximize the Laplacian energy of resulted network after adding $\mathbf{k}$ edges selected from the set $E_P = \{x : x \in E, x \notin E_0\}$ to the existing network. All the weights of edges in E are non-negative integer values between α and β. Let ΔE denote the set of edges to be added in the existing air transportation network. We aim at solving the following problem:

$$\max E_L(G(V, E_0 + \Delta E))$$

$$\text{s.t.} \begin{cases} |\Delta E| = \mathbf{k} \\ \Delta E \subseteq P \\ w_{i,j} \in \{0, \{\alpha, \beta\}\} \end{cases} \quad (2)$$

Since $E_L(G)$ is actually computed from the weighted Laplacian matrix L corresponding to the graph G, we interchangeably use $E_L(G)$ and $E_L(L)$ for the value of Laplacian energy in the rest of paper. The augmented Laplacian matrix L can be expressed as the dot product summation of edge vectors [21]:

$$L = L_0 + \sum_{e=1}^{|P|} y_e w_e h_e h_e^T$$

L_0 is the Laplacian matrix of the existing network. y_e is a Boolean variable to indicate whether edge e is selected. For any edge e between two nodes i and j, the corresponding edge vector $h_e \in \mathbf{R}^n$ is defined as $h_e(i) = 1$, $h_e(j) = -1$. $|P|$ is the size of pre-determined candidate edge set E_P. The air traffic network optimization problem can be rewritten as:

$$\max E_L(L_0 + \sum_{e=1}^{|P|} y_e w_e h_e h_e^T)$$

$$\text{s.t.} \begin{cases} \sum_{e=1}^{|P|} y_e = \mathbf{k} \\ y_e \in \{0, 1\} \\ w_{i,j} \in \{0, \{\alpha, \beta\}\} \end{cases} \quad (3)$$

3. Solutions

Given the air traffic network $G(V, E_0)$, the optimization problem is finding $\mathbf{k}$ edges from set E_P with maximal $E_L(G(V, E_0 + \Delta E))$. The brute-force search or exhaustive search computes $C_{|P|}^{\mathbf{k}}$ different E_L, and select the biggest one as optimal solution. In general, $C_{|P|}^{\mathbf{k}}$ is so large that running time is extremely long.

3.1. Tabu search

In this paper, we develop a specialized tabu search algorithm to solve the problem. Tabu search [24,25] is a metaheuristic search method which employs a local search method to tackle the optimization problem. The local search procedures iteratively move from one potential solution to another improved solution till certain termination criteria satisfies. The motivation behind the use of a tabu search is to prevent the solutions stuck into local optimum region. In order to avoid the pitfall and access unexplored solutions region left by local search procedures, a tabu list T is constructed to exclude moves which would bring the solution back to the previous worse solutions. A successful tabu search requires a good balance between intensification and diversification. The intensification makes a detailed exploration of some region of solution space, usually in the vicinity of a good solution. The diversification forces the search towards the unexplored promising regions. In the following subsections, we elaborate the details of the tabu search algorithm developed in this work.

3.1.1. Neighbor

Tabu search starts with an initial solution s_0 and then iteratively searches neighborhood of the current solution. $N(s)$ denotes the most promising neighbor to move to. The definition of $N(s)$ is critical to the efficiency of the tabu search. In our problem, the solution s contains the $\mathbf{k}$ edges to be added. Each edge $e_{i,j}$ in solution s connects two nodes v_i and v_j. The sub-neighbor $N(s, t)$ of t edge is formed by the edges in E_P that are directly linked to v_i and v_j. To prevent the size of $N(s, t)$ equals to 0, a random jump inside E_P is employed to generate the candidates of $N(s, t)$. The neighbor of current solution s is the union of the sub-neighbors of all edges in s, $N(s) = \bigcup_{t=1}^{\mathbf{k}} N(s, t)$.

3.1.2. Tabu list and aspiration criteria

In [24,25], several general methods to generate tabu list were proposed. In our case, the tabu list T records the set of solutions that have been visited in the recent past iterations. For each edge t in current solutions, all the candidate solutions that move from edge t to its neighbor $N(s, t)$ are visited. If the candidate solution has better performance and is not included in T, the solution is selected and added into T; otherwise this candidate is not selected. Tabus are sometimes so powerful that they might prohibit attractive moves or they might lead to an overall stagnation of searching process. In order to prevent this phenomenon, aspiration criteria are designed to allow one to ignore the tabus. The simplest and commonly used aspiration criterion allows a move, even the solution is in T, when the solution is changed to with an objective function better than that of the current best know solution. The best observed value E_L^* should be recorded in each step. When a move finds the solution s with a better E_L than the current best solution E_L^*, the aspiration criterion is applied.

3.1.3. The complete tabu search algorithm

Algorithm 1 shows the complete tabu search. Line 1 initializes the input variables. Line 2 shows the optimization progress terminates after Φ iterations. Lines 3–5 give the procedures that construct the sub-neighbors of the current solution s. Line 7 constructs s' from $N(s, t)$. Lines 8–11 check the solution with aspiration criteria. Lines 12–14 confirm whether s' is in tabu list T.

Algorithm 1 Tabu Search Algorithm.

```
1:  Set the iteration counter Φ and randomly generate an initial solution s_0. Set
    this solution as the current solution as well as the best solutions*. T is set to
    an empty queue with the pre-fixed size |T|.
2:  for iteration = 1 to Φ do
3:    for t = 1 to k do
4:      construct N(s, t) of the tth edge in s
5:    end for
6:    while 1 do
7:      pick one edge t' from N(s, t) to construct s'
8:      if E_L(s') > E_L*(s)
9:        s = s', update T
10:       E_L* = E_L(s), s* = s
11:     end if
12:     if s' is not in T then
13:         s = s', update T
14:     end if
15:   end while
16: end for
17: output E_L* and s*
```

3.2. Greedy algorithm

The greedy algorithm tries to find a global optimum with the strategy of making the locally optimal choice at each step. Although the greedy algorithm strategy does not guarantee a global optimal solution, this heuristic yields a solution that approximates the optimal solution in a reasonable time. Here we describe a greedy local heuristic to solve problem (2). We add one edge at a time, each time choosing the edge $e_{i,j}$ which gives the largest predicted increase in $E_L(G)$. The iteration terminates till k edges are selected. The local optimal condition is proved as follows.

Lemma 1. *For any network $G = (V, E, W)$, the Laplacian energy can be express as [23]*

$$E_L(G) = \sum_{i=1}^{n} x_i^2 + 2\sum_{i<j} w_{i,j}^2 \tag{4}$$

with $x_i = \sum_{j=1}^{n} w_{i,j} = \sum_{u\in N(v_i)}^{n} w_{v_i,u}$, where $N(v_i)$ is the neighborhood of v_i. x_i is termed as the sum-weight of a vertex v_i.

Theorem 1. *$G = (V, E, W)$ is a weighted network with m edges $\{e_1, e_2, \ldots, e_m\}$. Let H be the network obtained by adding edge $e_{i,j}$ from E_P, the increase of Laplacian energy with respect to $e_{i,j}$ is:*

$$(\Delta E_L)_{i,j} = E_L(H) - E_L(G) = 2w_{i,j}(x(v_i) + x(v_j)) + 4w_{i,j}^2 \tag{5}$$

Proof. Without loss of generality, assume $H = G + e_{p,q}$ and $p < q$. Let $N(e_{p,q})$ be the nodes connected by $e_{p,q}$ in G and $x^*(v_i)$ be the corresponding sum-weight of vertex v_i in H. Thus we have:

$$x^*(v_i) = \begin{cases} x(v_{p\cup q}) + w_{p,q}, & \text{if } v_{i\cup j} \in N(e_{p,q}); \\ x(v_i), & \text{otherwise;} \end{cases} \tag{6}$$

so, by Lemma 1 and Eq. (4),

$$\begin{aligned} E_L(H) = & \sum_{v_i\in N(e_{p,q})} (x(v_i) + w_{p,q})^2 + \sum_{v_i\notin N(e_{p,q})} x^2(v_i) \\ & + 2\sum_{i<j, i\notin N(e_{p,q}), j\notin N(e_{p,q})} w_{i,j}^2 + 2w_{p,q}^2 \end{aligned} \tag{7}$$

thus the increase of Laplacian energy with respect to $e_{p,q}$ is

$$\begin{aligned} (\Delta E_L)_{p,q} &= E_L(H) - E_L(G) \\ &= \sum_{v_i\in N(e_{p,q})} [(x(v_i) + w_{p,q})^2 - x^2(v_i)] + 2w_{p,q}^2 \\ &= \sum_{v_i\in N(e_{p,q})} (2x(v_i)w_{p,q} + w_{p,q}^2) + 2w_{p,q}^2 \\ &= 2w_{p,q}(x(v_p) + x(v_q)) + 4w_{p,q}^2 \qquad \square \end{aligned}$$

Using the local optimal condition, we design a greedy method which selects the edge $e_{i,j}$ from E_P with maximal $2w_{i,j}(x(v_i) + x(v_j)) + 4w_{i,j}^2$ at each step, where $x(v_i)$ and $x(v_j)$ are the sum-weight of vertices v_i and v_j. $w_{i,j}$ is the weight associated with edge $e_{i,j}$. Lines 1–2 initialize the set of candidate edges E_P and initial solution. Line 4 selects the edge from E_P which makes the maximal improvement of E_L. Lines 6–7 make addition and deletion operations on E and E_P respectively.

Algorithm 2 Greedy Algorithm.

```
1. given graph G = (V, E_0, W), candidate edge set E_P
2. let E = E_0
3.   for 1 to k do
4.     e_{i,j} = argmax_{e_{i,j}∈P} 2w_{i,j}(x(v_i) + x(v_j)) + 4w_{i,j}^2
5.     E = E + e_{i,j}
6.     E_P = E_P − e_{i,j}
7.   end for
8. output G = (V, E)
```

3.3. Binary integer programming

Basic Laplacian energy optimization formulation can be converted into a binary integer program, which provides exact optimal solution to compare with two proposed heuristic algorithms. Assume E_C is the edge set of a complete undirected graph, with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$, in which every pair of distinct vertices is connected by a unique edge. Defined E_0 as the existing edges, E_P as the set of pre-defined edges for addition, and $E_Q = \{x : x \in E_C, x \notin E_0, x \notin E_P\}$. W_0, W_P, W_Q are the corresponding sets of edge weights. Let W denote the weight matrix of E_C, $w_{i,j}$ be the weight of edge $e_{i,j}$

$$w_{i,j} = \begin{cases} 0, & \text{if } e_{i,j} \in E_Q \text{ or } i = j; \\ w_{j,i}, & \text{otherwise;} \end{cases} \tag{8}$$

and Y denote the decision variable matrix

$$y_{i,j} = \begin{cases} 0, & \text{if } e_{i,j} \in E_Q \text{ or } i = j; \\ 1, & \text{if } e_{i,j} \in E_0; \\ y_{j,i}, & \text{if } e_{i,j} \in E_P; \end{cases} \tag{9}$$

The weighted matrix can be derived as:

$$\begin{aligned} W(E_0 + E_P) &= W \cdot Y \\ &= \begin{pmatrix} w_{1,1}y_{1,1} & w_{1,2}y_{1,2} & \ldots & w_{1,n}y_{1,n} \\ w_{2,1}y_{2,1} & w_{2,2}y_{2,2} & \ldots & w_{2,n}y_{2,n} \\ . & . & . & . \\ w_{n,1}y_{n,1} & w_{n,2}y_{n,2} & \ldots & w_{n,n}y_{n,n} \end{pmatrix} \end{aligned} \tag{10}$$

From Lemma 1 and the definition of sum-weight, we have

$$\begin{aligned} E_L(G) &= \sum_{i=1}^{n} x_i^2 + \sum_{i\neq j} w_{i,j}^2 \\ &= \sum_{i=1}^{n} \left(\sum_{j=1}^{n} w_{i,j}y_{i,j} \right)^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} (w_{i,j}y_{i,j})^2 \end{aligned} \tag{11}$$

Thus, the objective function of the optimization problem can be expressed as:

$$\begin{aligned} \max E_L(G) &= \sum_{i=1}^{n}\sum_{j=1}^{n} (w_{i,j}y_{i,j})^2 + \sum_{i=1}^{n}\left(\sum_{j=1}^{n} w_{i,j}y_{i,j}\right)^2 \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} (w_{i,j}^2 y_{i,j}^2) + \left(\sum_{i=1}^{n}\sum_{j=1}^{n} (w_{i,j}^2 y_{i,j}^2) \right. \\ &\quad \left. + 2\sum_{i=1}^{n}\left(\sum_{j\neq k} w_{i,j}w_{i,k}y_{i,j}y_{i,k}\right)\right) \\ &= 2\sum_{i=1}^{n}\sum_{j=1}^{n} (w_{i,j}^2 y_{i,j}^2) + 2\sum_{i=1}^{n}\sum_{j\neq k} w_{i,j}w_{i,k}y_{i,j}y_{i,k} \end{aligned}$$

which is equivalent to:

$$\min -2\sum_{i=1}^{n}\sum_{j=1}^{n} \left(w_{i,j}^2 y_{i,j}^2\right) - 2\sum_{i=1}^{n}\sum_{j\neq k} w_{i,j}w_{i,k}y_{i,j}y_{i,k} \tag{12}$$

Note that the following equation satisfies,

$$y_{i,j}y_{i,k} = y_{i,j}, \text{ for } j = k$$
$$y_{i,j}y_{i,k} = \min\{y_{i,j}, y_{i,k}\}, \text{ for } j \neq k$$

After introducing variable $z_{i,j,k} = y_{i,j}y_{i,k} = \min\{y_{i,j}, y_{i,k}\}$, the basic problem can then be reduced to the following integer linear programming problem.

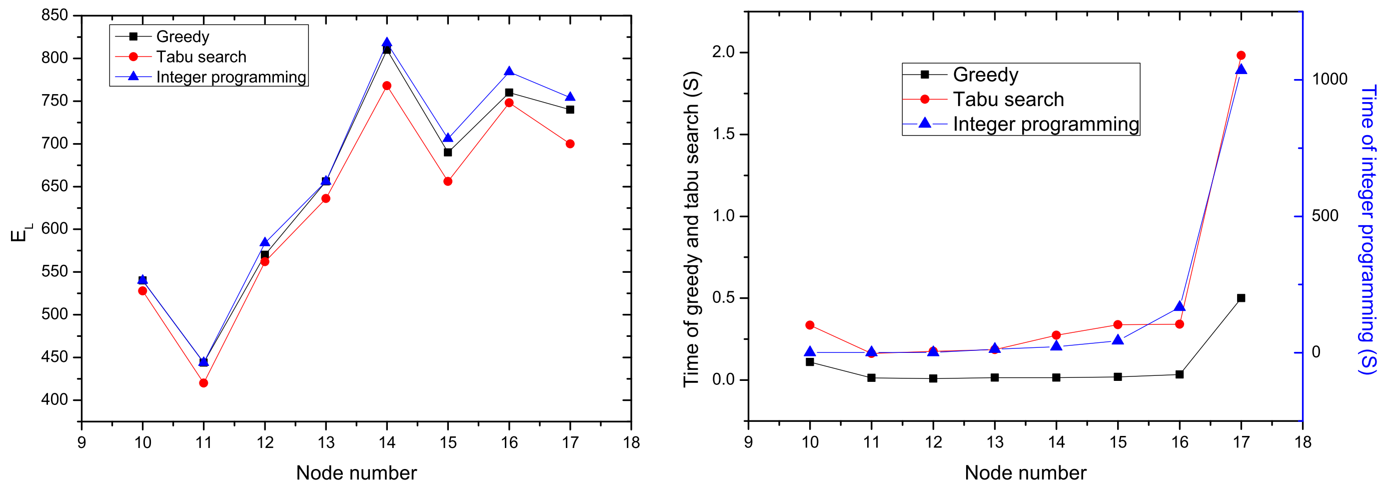

Fig. 4. The impact of network size n.

$$\min -2\sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_{i,j}{}^2 y_{i,j}\right) - 2\sum_{i=1}^{n}\sum_{j\neq k} w_{i,j}w_{i,k}z_{i,j,k}$$

s.t.

$z_{i,j,k} \le y_{i,j}, z_{i,j,k} \le y_{i,k}$, for $i=1,\ldots n, j=1,\ldots n, i<j, i<k, j\neq k$;

$\sum_{i=1}^{|P|}\sum_{i<j}^{|P|} y_{i,j} = \mathbf{k}$, for $i=1,\ldots n, j=1,\ldots n, e_{i,j}\in E_P$;

$y_{i,j}=0$, for $i=1,\ldots n, j=1,\ldots n, e_{i,j}\in E_Q$;

$y_{i,j}=1$, for $i=1,\ldots n, j=1,\ldots n, e_{i,j}\in E_0$;

$y_{i,j}=y_{j,i}$, for $i=1,\ldots n, j=1,\ldots n$;

$y_{i,j}\in\{0,1\}$, for $i=1,\ldots n, j=1,\ldots n$;

$z_{i,j,k}\in\{0,1\}$, for $i=1,\ldots n, j=1,\ldots n, i<j, i<k, j\neq k$;

4. Results and discussions

In this section, we use simulation to compare the performance and computation time of tabu search, greedy algorithm and integer programming for the Laplacian energy maximization problem. A scale-free network with nodes $n=20$ is generated and then used as the current existing networks $G(V, E_0)$. Three types of weights $w_{i,j}=\{1,2,3\}$ are randomly assigned to the edges in E_0 and E_P. We should note that all models and algorithms in this paper are coded in Matlab platform and the binary integer programming problem is solved by CPLEX 12.2 with all solver options set to default. All computational results in this section are implemented on a HP PC with Interl Core i5 processor running at 3.2 GHz and 4 GB of RAM under a 64 bit Windows 7 operating system.

4.1. The impact of n

In this simulation, we investigate the performance and computation time of the three proposed methods for different size networks. In each run, we fix $k=10$ and vary the network size n. The left subfigure of Fig. 4 shows the integer programming has the best performance and the tabu search has the bottom performance. The integer programming always severs as the exact solution with the variation of network size. The right subfigure of Fig. 4 illustrates that the computation times of all approaches increase with the network size. The integer programming is so slow when the network size becomes larger, it sacrifices its running time to obtain the best performance. Among the three methods, tabu search takes the shortest running time, which is slightly faster than greedy method.

The trade-off is that no matter what k is, the integer programming provides the best solution than the other two methods, but its computation time is unacceptable when the network size is huge. The greedy algorithm could find a satisfactory solution within such a short time. According to the simulation results, it is quite convincing that integer programming should be selected for the exact optimal solution if the scale of air traffic network is small; the greedy method can be adopted to obtain a near optimal solution with high computational efficiency when the network size becomes large.

4.2. The impact of k

In this simulation, we study the impact of number of edges k, to be added into the existing network by three different approaches. The parameter settings are $n=20$, $|T|=15$ and the simulation results are illustrated in Fig. 5. From Fig. 5 we observe that when edges are added to the network G, the Laplacian energy increases monotonically. The integer programming always provides the best performance, but the computation speed is slowest. The greedy method has the second best performance with the best computation speed. Therefore, the greedy method should be chosen when we prefer computation efficiency rather than performance. The tabu search is the poorest in terms of performance, but it has the second best computation speed.

In summary, the trade-off is that the number k increases, integer programming always find a better solution than others do. However, its computation speed is unacceptable when the network size is huge. When k is small, the greedy method gives the similar performance with integer programming. If we want to select a small number of edges from E_Q, the integer programming method should be selected; if k is a big number, greedy methods should be adopted to provide the efficient computation speed with an acceptable performance.

4.3. Case study

A real air traffic network of Jetstar Asian Airway is studied. The current air transportation network is weighted based on the historical data of flight cancellation rate from [26], a popular website providing air traffic data collected from a large number of sources, such as governments, airlines, airports, reservation systems, and so on. The corresponding data of cancellation rates is listed in Table 2. It can be observed that the flight cancellation rate ranges from 0% to 9%. The existing edges with cancellation rate [0, 3%) is assigned with weight 3, the ones with cancellation rate [3%, 6%) is assigned with weight 2, and the ones with cancellation rate [6%, 9%] is assigned with weight 1 in this experiment. The design of mechanism that maps cancellation rate into different types of weights is motivated by the practical operation in aviation industry. One flight route with high weight indicates that there are more flights or the aircraft are less influenced by unexpected conditions, such as aircraft mechanical issue, severe weather, etc. It is noted that if there

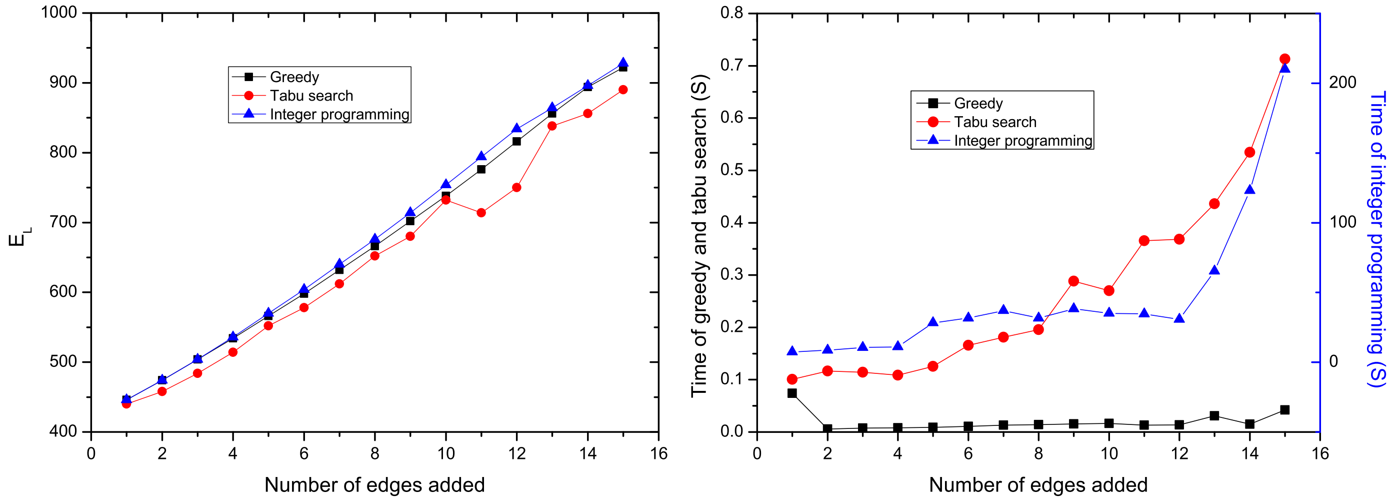

Fig. 5. The impact of number of edges added k.

Table 2
The total 68 routes and their cancellation rates (as of Aug 2015. Source: http://www.flightstats.com/).

Route name	Cancellation rate (%)	Route name	Cancellation rate (%)	Route name	Cancellation rate (%)
ADL-AKL	0	ADL-BNE	0	ADL-CNS	0
ADL-DPS	9	ADL-DRW	0	ADL-MEL	1
ADL-OOL	1	ADL-PER	1	ADL-SYD	1
AKL-CHC	1	AKL-DUD	0	AKL-MEL	1
AKL-OOL	0	AKL-SYD	1	AKL-WLG	0
AKL-ZQN	3	AVV-SYD	0	AYQ-SYD	0
BNE-CNS	0	BNE-DPS	9	BNE-DRW	0
BNE-HBA	0	BNE-HTI	0	BNE-LST	0
BNE-MEL	2	BNE-MKY	1	BNE-NTL	0
BNE-PER	0	BNE-PPP	0	BNE-SYD	2
BNE-TSV	1	BNK-MEL	0	BNK-SYD	0
CHC-MEL	0	CHC-OOL	0	CHC-SYD	0
CHC-WLG	0	CNS-DRW	0	CNS-MEL	0
CNS-OOL	0	CNS-PER	3	CNS-SYD	0
DPS-DRW	6	DPS-MEL	9	DPS-PER	4
DPS-SYD	9	DRW-MEL	0	DRW-SYD	0
HBA-MEL	1	HBA-SYD	0	HTI-MEL	0
HTI-SYD	0	LOP-PER	0	LST-MEL	4
LST-SYD	0	MCY-MEL	0	MCY-SYD	0
MEL-NTL	0	MEL-OOL	1	MEL-PER	1
MEL-SYD	4	MEL-TSV	0	MEL-ZQN	2
NTL-OOL	0	OOL-SYD	1	PER-SYD	0
SYD-TSV	0	SYD-ZQN	2		

are more than one flights assigned to an O–D pair, the cancellation rate is calculated by the joint probability of all the flights in this route and then the edge weights are assigned referring to the mapping mechanism above.

The cancellation rate of candidate edges can be estimated from the historical recorded routes information published by FAA and the current air traffic status. Then we can obtain the weight of each edge through these estimated cancellation rates. In this case study, we assume all the candidate edges have the medium link strength with weight 2 and then the greedy algorithm and the integer programming are applied to select the top 5 and top 10 routes. These results are illustrated in Table 3 and Table 4. When $k = 5$, these two methods select the same routes set; when $k = 10$, the integer programming intends to add more routes to SYD, the greedy selects more scattered routes.

5. Conclusion

In this paper, an experiment on a real air traffic network is conducted to show that the Laplacian energy is a fair and effective measure of demonstrating both global and local robustness of air traffic network. The flight route addition problem is for-

Table 3
Top 5 flight routes to be added to the Jetstar network.

Greedy algorithm	Integer programming
BNE-AKL	SYD-LOP
OOL-BNE	BNE-AKL
SYD-LOP	SYD-NTL
SYD-NTL	OOL-BNE
WLG-SYD	SYD-WLG

Table 4
Top 10 flight routes to be added to the Jetstar network.

Greedy algorithm	Integer programming
BNE-AKL	MEL-AVV
OOL-BNE	MEL-AYQ
SYD-LOP	MEL-DUD
SYD-NTL	MEL-LOP
WLG-SYD	MKY-MEL
SYD-DUD	PPP-MEL
MEL-LOP	WLG-MEL
WLG-MEL	SYD-NTL
SYD-MKY	SYD-PPP
SYD-PPP	WLG-SYD

mulated to maximize the Laplacian energy. Three methods are proposed to solve the basic optimization problem with different performance and computational efficiency. Two heuristic methods including greedy algorithm and tabu search are developed to find the maximal value of Laplacian energy. Furthermore, the basic optimization problem is formulated as integer programming which provides the exact global optimum. Finally, numerical simulations on scale-free network have been implemented to study the trade-off among the tree methods and the real air traffic network of Jetstar is also tested for further verification. We conclude that integer programming and greedy algorithm should be selected to tackle with small scale and large scale network optimization problem respectively. These conclusions could be provided to decision maker on how to select an appropriate algorithm according to trade-off analysis.

For the future direction of this work, we plan to consider the robustness enhancement for large scale problems. We should also consider the constraints of operating cost, traffic congestion and aviation policy when selecting additional edges.

Conflict of interest statement

The authors declare that there are no conflicts of interest.

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