An Interval-based TOS Allocation Model for Collaborative Trajectory Options Program (CTOP)

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Encoding flight operator's preference over desired route options, Trajectory Options Set (TOS) is the core element of Collaborative Trajectory Options Program (CTOP). An important research question in CTOP is how to assign trajectory options and delays to the impacted flights to minimize total system delay costs while maintaining equality across airspace users. The main goal of our work is to develop a centralized optimization algorithm which can be an alternative solution to the current CTOP Ration-by-Schedule (RBS) allocation scheme and can be used in decision support tools for air traffic managers as well as airline dispatchers. A mixed integer linear model is formulated and important issues including TOS route restrictions, intra-airline cancellation and substitution are discussed in details in the paper. The model is tested on a realistic CTOP use case. The preliminary results are very promising in terms of computational efficiency, implying practical viability. The proposed model is not limited to be used in CTOP, but can be applied to solve the very general multiple constrained airspace with flights having multiple route options optimization problem as well.

Nomenclature

N	Number of flights
A	List of airlines
f_i	i-th flight, $i = 1, \cdots, N$
F_a	set of indices of flights of airline a
N_i	Number of route options for flight f_i
r_{ij}	j-th route of flight f_i, $j = 1, \cdots, N_i$
q_{ij}	Cost of the route r_{ij}
δ_{ij}	Binary indicator whether flight i will take route j
d_{ij}	Ground delay of flight i following route r_{ij}
a_{ij}^k	Air delay of the flight i following route r_{ij} before entering FCA k
Z	Number of FCAs
M_t^k	Acceptance rate of FCA k at time period t
t_{ij}^k	Time period at which route r_{ij} is planned to cross FCA k
τ_i^k	Time period at which flight i crosses FCA k
Ω_{ij}	An ordered set of indices of the FCAs which route r_{ij} crosses
$\tilde{\Omega}_i$	A set of indices of the FCAs which flight f_i crosses at any route
Φ^k	A set of indices of the routes which are planned to cross FCA k
$\tilde{\Phi}^k$	A set of indices of flights which might cross FCA k at any of their routes
$B_{i,t}^k$	Binary indicator whether flight i passes FCA k at interval t
$B_{i,\lambda,\mu}$	Binary indicator whether flight i taking λ time periods ground delay and μ time periods air delay
$c(i,\lambda,\mu)$	Cost for flight i to take λ time periods ground delay and μ time periods air delay
$v_{a,t}^k$	Number of slots owned by airline a for FCA k during time period t

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I. Introduction

As a key tool in the NextGen portfolio, Collaborative Trajectory Options Program (CTOP) is a new Traffic Management Initiative (TMI) which is used to balance demand and capacity in the en route and terminal airspace. CTOP combines many features of its preceding TMIs including Ground Delay Program (GDP), Airspace Flow Program (AFP) and Reroutes, and has the ability handle multiply Flow Constrained Areas (FCAs) within a single program. CTOP could automatically assign delays and/or reroutes to flights to dynamically manage the demand through the congested regions as conditions change. A new concept in CTOP is Trajectory Options Set (TOS), which is set of desired route options submitted by flight operators, with each option associated with Relative Trajectory Cost (RTC) and some usage restrictions. A TOS encodes flight operator's conditional preference over different route choices and brings greater flexibility and efficiency to the airspace users [1].

Since the first GDP model proposed by Odoni in 1987 [2], there has been moderate research on TMI optimization problems in the past three decades. A TMI has airline side and the Federal Aviation Administration (FAA) side research problems. The most classical airline side problem is how to do slot substitutions and flight cancellations to best response to a TMI [3] [4] [5]. The two FAA side problems which have attracted most attentions are resource allocation problem and airport or FCA rate planning problem. In the current Collaborative Decision Making (CDM) paradigm, Ration-by-Schedule (RBS) is accepted as the standard principle for equitable resource allocation [6] [7]. However, in some applications and under certain mild assumptions, other ration scheme has been proved to be superior [8] [9]. Most of the literature on rate planning problem has been focused on single airport GDP planning under uncertainty. Several stochastic optimization models with varying degree to which traffic managers can modify or revise flights' controlled departure times have been proposed. In static stochastic model and semi-dynamic multistage stochastic model, ground delay decisions are made at the beginning of the planning horizon [10] [11] and at flights' scheduled departure time [12], respectively. In these two types of models, once a ground delay decision has been made, it cannot be revised. In a dynamic multistage stochastic model, a flight can be ground delayed multiple times [13]. From static model to semi-dynamic model to dynamic model, more information about weather scenario tree and flights are utilized, which bring the benefit of lower system delay cost at the cost of less departure predictability and loss of CDM compatibility. Other decision making under uncertainty techniques including Markov Decision Process and chance constraints programming have also been explored by researchers, which assume the weather evolves according to a Markov Chain or assume the capacity estimation conforms to log-concave distribution [14] [15] [16].

Like its forerunners, CTOP also has both sides research problems and also several unique problems owing to its new functionalities. For example, two new airline side problems are how to generate candidate routes and how to set RTC values [17] [18] [19] [20]. The traditional FAA side research problems continue to exist, including airspace resources allocation [21] [22] and FCA acceptance rate planning [23] [24]. However, these problems become more challenging due to the need to coordinate multiple FCAs and the TOS-induced demand variability [25]. Some new FAA side problem include how to predict TOS submission, when to do the automatic revision to response TOS update and pop-up flights etc.

This paper endeavors to solve selected problems from both sides. For the FAA side, we want to take advantage of reroute flexibility offered by the TOS set to find efficient solution in terms of total system cost, and at the same time pursue equality among airlines. The motivation is that the current CTOP algorithm does not explicitly consider system efficiency. We believe an model that can easily trade off efficiency and equality is useful in revealing how best can we achieve in terms of efficiency and the cost of fairness, and in helping air traffic managers understand, initiate, and perform post-analysis for CTOP programs. For the airline side, we propose a slot substitution model which could account for nonlinearity of flight delay cost. It can show the benefit of allowing airlines to reoptimize their internal delay cost functions. This work is closely related to our collaborators' work [26]. The key difference is that in [26], the two consecutive flights are scheduled to maintain a certain distance or time separation in order to satisfy the capacity constraints (named space-based), whereas in our work we require number of flights that are scheduled to arrive at a FCA in a time interval should be within its acceptance rate (named interval-based). The former formulation ambitiously solves TOS allocation problem and flight separation problem in a single optimization model and therefore can be computational challenging even for small test case with 30 flights. Our formulation is more in line with air traffic flow management practice and is more computationally efficient. The latter allows our formulation to be extended to an efficient stochastic version that account for capacity uncertainties in the future.

This paper is organized as follows: to give the reader the basic background knowledge, in section II we will first briefly introduce the TOS composition and current CTOP algorithm. In section III, we will discuss in detail the model assumption, model formulation and related issues including equality, and intra-airline cancellation and substitution. In section IV we will introduce the experiment setup and discuss the main results. In section V, we summarize the findings of this paper and point out future work.

II. CTOP In a Nutshell

Fig 1 shows an example of a TOS. A TOS consists of a flight's ID, origin and destination airports, Initial Gate Time of Departure (IGTD, the departure time when the flight was first created.), Earliest Runway Time of Departure (ERTD, the earliest time the flight can depart) and candidate routes information.

RTCs are values submitted by the flight operator to express his/her preference over route options. There are three optional requirements for each route that can be provided by flight operator: Required Minimum Notification Time (RMNT) which allows for needed preparation time, such as adding fuel; Trajectory Valid Start Time (TVST) and Trajectory Valid End Time (TVET) which are the earliest and latest acceptable take-off times for that TOS option, respectively.

Flight ID

ACID	ORIG	DEST	IGTD	TYPE	ERTD
ABC123	LAX	ATL	05/1945	LJ60	05/1945

Trajectory Option Set

RTC	RMNT	TVST	TVET	Route	ALT	SPEED
0				TRM PKE DRK J6 IRW FSM MEM ERLIN9	350	435
30			2045	TRM PKE DRK J134 LBL SGF BNA RMG4	350	435
50		2045		TRM PKE DRK J134 BUM FAM BNA RMG4	350	430
60		1945	2145	TRM BLH J169 TFD J50 SSO J4 EWM J66 ABI J4 MEI LGC2	350	425
70	45	1745	2200	TRM BLH J169 TFD ELP J2 JCT J86 IAH J2 LCH J590 GCV LGC2	310	430

Figure 1: TOS Example of a Flight from LAX to ATL [27]

FAA allocate the routes to flights on a flight by flight basis according to their earliest Initial Arrival Times (IATs). A flight's IAT is the earliest ETA (Estimated Time of Arrival) at any of a CTOP's FCAs using any of the flight's TOS options. We can consider IAT as a flight's CTOP capture time. This is the CTOP version of Ration by Schedule (RBS). For a given flight, CTOP allocation algorithm will calculate the

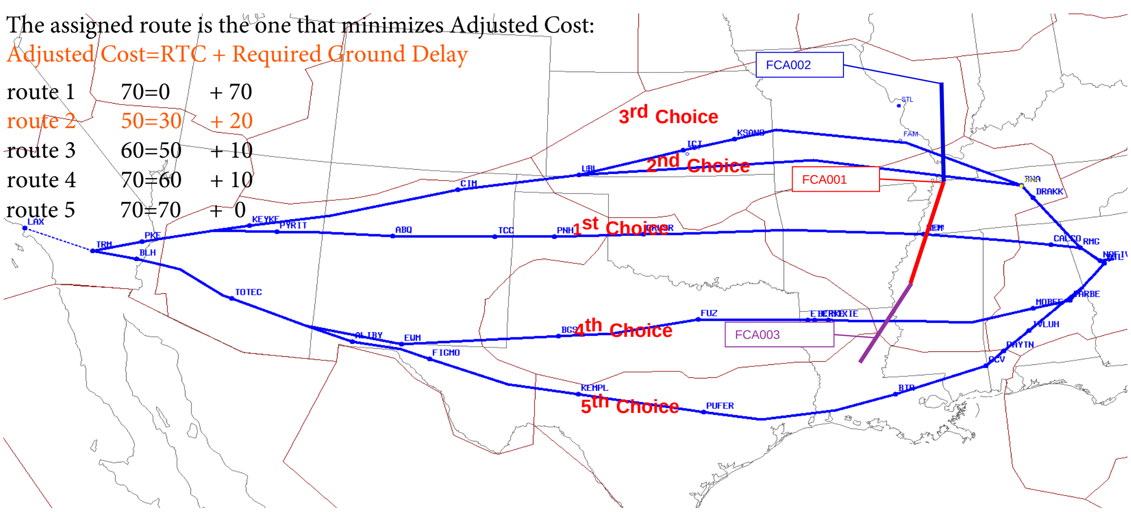

Figure 2: Flight Routes in the TOS and the Adjusted Cost [27]

adjusted cost for each candidate route and assign the route with the minimum adjusted cost to this flight. The key equation here is:

$$\text{Adjusted Cost} = \text{RTC} + \text{Required Ground Delay}$$

Required ground delay is calculated by the CTOP algorithm given current available slots, which is the ground hold time this flight will need to bear in order to take a specific route. Assume no route restriction is violated, as shown in Fig 2, this flight will be allocated with route 2, which has the smallest adjusted cost among all route options.

The three optional route restrictions may also affect the route assignment. The scheduled the departure time of this flight is 19:45. Assume the current time is 19:10. The RMNT dictates that if route 5 if chosen, then this flight cannot depart until 19:10 + 45 mins = 19:55. In this case the required ground delay is actually 10 minutes and the adjusted cost is 80 minutes. It can be seen that apart from available slots, the route restrictions can be another source for required ground delay. For route 3, the flight needs to take 60 minutes ground delay in order to meet the TVST requirement: 19:45 + 60 mins = 20:45. Therefore if the route restriction is considered, the adjusted cost for route 3 is 110 minutes. Route 2 still has the smallest the adjusted cost and satisfies TVET: 19:45 + 20 mins = 20:05 ≤ 20:45.

For a more detailed introduction to CTOP algorithm, the readers are referred to [7] or [28].

III. Mathematical Formulation

A. Model Assumptions

1. We assume the flights captured by CTOP are not controlled by other TMIs at the same time. In other words, we do not consider the the TMI interactions
2. We assume the TOS data, FCA entry times, CTOP planning horizon and FCA acceptance rates are known. Note here we assume the rates are given and can accurate reflect the airspace capacity constraints. There is no uncertainty in any of the parameters
3. We assume all flights are required to exit the FCA network by the end of the planning horizon
4. We assume we can only delay a flight by integer multiples of the size of a time period. In this paper, we use the bin size of 15 minutes. Then the possible delay times are 0 minutes, 15 minutes, etc
5. We assume we can air delay flights in a proactive way

B. Model Formulation

The objective function minimizes total system delay costs, which is composed of reroute costs, ground delay and air delay costs. α and β are weighting coefficients. Unless otherwise stated, we use $\alpha = 1$ and $\beta = 2$ in this paper, which means one unit of air delay is twice expensive as one unit of ground delay.

$$\min \quad \sum_{i=1}^{N}\sum_{j=1}^{N_i} q_{ij}\delta_{ij} + \alpha d_{ij} + \beta \sum_{j=1}^{N_i}\sum_{k\in\Omega_{ij}} a_{ij}^{k} \tag{1}$$

It worths pointing out that because air delay is more expensive than ground delay, therefore $a_{ij}^k \equiv 0$ if k is the first FCA that route r_{ij} crosses ($k \in \Omega_{ij}$ and $\mathrm{id}(k) = 1$).

Only if we choose route r_{ij} for flight f_i, can d_{ij} and a_{ij}^k be nonzeros. $\mathbb{M}_i$ is a constant, which is the maximum allowable delay for flight i.

$$0 \le d_{ij} + \sum_{k\in\Omega_{ij}} a_{ij}^{k} \le \delta_{ij}\mathbb{M}_i \quad \forall i, j \tag{2}$$

One and only route should be chosen for each flight f_i:

$$\sum_{j=1}^{N_i} \delta_{ij} = 1, \text{ for } i = 1, \cdots, N \tag{3}$$

The ETA of flight i to FCA k along route j is:

$$\tau_i^k = \sum_{\substack{j=1\dots N_i\\(i,j)\in\Phi^k}} t_{ij}^k\delta_{ij} + d_{ij} + \sum_{\mathrm{k}\in\Omega_{ij};2\le\mathrm{id(k)}\le\mathrm{id}(k)} a_{ij}^{\mathrm{k}} \quad \forall i, k, i \in \tilde{\Phi}^k \tag{4}$$

Parameter t_{ij} in this paper is also in unit of time periods, which can be obtained by data preprocessing. The third term is the cumulative air delays this flight take before reaching FCA k.

We introduce auxiliary binary variable $B^t_{k,i}$, which relates the flight arrival time with flight count. $B^t_{k,i} = 1$ means flight i will cross FCA k using one of its route options. There are three constraints that $B^t_{k,i}$ need to meet. A flight can only arrive at a FCA once:

$$\sum_{t=1}^{T} B^t_{k,i} \leq 1 \tag{5}$$

If $\tau^k_i = t$, then $B^t_{k,i} = 1$:

$$\sum_{t=1}^{T} tB^t_{k,i} = \tau^k_i \tag{6}$$

Since we know the earliest unimpeded travel time for flight i to reach FCA k, we can actually reduce the coefficients of left hand sides in (5) and (6) by summing t from $\min_j t^k_{ij}$ instead of 1.

Finally, we have capacity constraints. The number of flights that are scheduled to arrive at FCA k at time t should be no greater than its acceptance rate:

$$\sum_{i=1}^{N} B^t_{k,i} \leq M^t_k \quad \forall t, k \tag{7}$$

Note that if the route assigned to flight i doesn't pass FCA k, then $\tau^k_i = 0$ and thus $B^t_{k,i} = 0$ for all t. Therefore flight i will not be counted when calculating the demand for FCA k. If a route-out option is chosen for a flight ($i \notin \tilde{\Phi}^k \ \forall k$), then this flight will not be considered as demand at any FCA.

C. Equality Issue

Equality is a key issue that has to be considered when designing any resource allocation algorithm which involves competing users. The total cost for an airline is $\sum_{i \in F_a} c_i$, where $c_i = \sum_{j=1}^{N_i} q_{ij}\delta_{ij} + \alpha d_{ij} + \beta \sum_{j=1}^{N_i} \sum_{k \in \Omega_{ij}} a^k_{ij}$. In [26], the authors incorporate the fairness measure by minimize the weighted system delay costs and maximum average airline costs:

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} c_i + \gamma y \\ & y \geq \frac{1}{|F_a|} \sum_{i \in F_a} c_i \quad \forall a \in A \end{aligned} \tag{8}$$

Note that the implicit assumption here is that the RTC of the most preferred route $q_{10} \equiv 0$. If RTC represents entire route cost rather than the extra cost of rerouting, then the above formulation will favor airlines with many long haul flights.

An airline company may try to game the algorithm by submitting large unfaithful RTC values for reroute options, hoping that if a reroute option is chosen then less delays will be assigned to this airline in general. This practice will run the risk of losing reroute opportunity and experiencing long ground delays.

D. Constraints on Delays and Nonlinearity of Delay Cost

The delay is modeled at flight by flight level, thus it is straightforward to add constraint on ground delay or the amount of air delay at any FCA.

The cost function of a flight on delay times is typically nonlinear. This is especially so if we allow both ground delay and air delay. We can show that by using binary variable $B_{i,\lambda,\mu}$, we have the ability to model arbitrary complex cost functions:

$$\sum_{\lambda,\mu} B_{i,\lambda,\mu} = 1 \tag{9}$$

$$\sum_{\lambda} \lambda B_{i,\lambda,\mu} = \sum_{j=1}^{N_i} d_{ij} \tag{10}$$

$$\sum_{\mu} \mu B_{i,\lambda,\mu} = \sum_{j=1}^{N_i} \sum_{k\in\Omega_{ij}} a_{ij}^k \tag{11}$$

$$c_i = \sum_{\lambda,\mu} c(i,\lambda,\mu) B_{i,\lambda,\mu} + \sum_{j=1}^{N_i} q_{ij}\delta_{ij} \tag{12}$$

Constraint (9) says that a flight cannot take (λ, μ) ground and air delays, and a different ground and air delay combination at the same time. Constraint (10) and (11) relate the $B_{i,\lambda,\mu}$ with the number of ground delay and air delay period. Formula 12 gives the total cost for flight i. $c(i, \lambda, \mu)$ are given parameters, therefore the expression is still linear. Because we only allow flight to take delay in unit of bin size, therefore the number of $B_{i,\lambda,\mu}$ will not be very large.

When solving FAA side problem, a linear mapping from time to cost is preferred, because we do not want an airline to overstate its own delay costs. Nonlinear cost function will be used in section F.

E. TOS Route Restrictions

Until now, we haven't considered the route restrictions yet. It can be easily shown that the route restriction can be translated into required ground delay. If we know the current time, flight's scheduled departure time and RMNT, we can directly calculate the minimum ground delay needed for flight i to take route j $\mathrm{RMNT\_GD}_{ij}$:

$$d_{ij} \geq \delta_{ij}\mathrm{RMNT\_GD}_{ij} \tag{13}$$

Similarly, TVET and TVST impose upper and lower constraints on the required ground delay time.

F. Intra-airline Cancellation and Substitutions

In this section, we show that by slightly tweaking the TOS allocation model, we can enable collaborative decision making by allowing airlines to do intra-airline substitutions.

Two important parameters here are $v_{a,t}^k$ and $c(i, \tau, \mu)$. $v_{a,t}^k$ can be determined from the solutions of previous TOS allocation model. $c(i, \tau, \mu)$ is flight specific and can be nonlinear to capture downstream miss-connection effect of crews, passengers, and the airframe, etc. Each airline can minimize its internal delay and reroute cost:

$$\begin{aligned} \min \quad & \sum_{i\in F_a}\{\sum_{\lambda,\mu} c(i,\lambda,\mu)B_{i,\lambda,\mu} + \sum_{j=1}^{N_i} q_{ij}\delta_{ij}\} \\ \text{s.t.} \quad & (2)-(6) \quad \forall i \in F_a \\ & \sum_{i\in F_a} B_{k,i}^t \leq v_k^t \quad \forall t, k \end{aligned} \tag{14}$$

It is common that an airline is willing to cancel some of its flights and use the vacant slots to reduce delays of other flights. Good computational performance of model (14) enables the airline to do fast what-if analysis when making cancellation decisions.

IV. Experimental Results

To demonstrate the performance of proposed models, we create an operational use case based on actual events from July 15, 2016[29].

A. Southern ZDC and EWR with Convective Activity

This use case primarily addresses convective weather activity in southern Washington Center (ZDC). Figure 3 shows the pattern of convective weather activity for that day. It can be seen that southern ZDC is adversely impacted by the weather. We further assume there is demand-capacity imbalance at EWR airport. In principle, the EWR imbalance could be addressed by an isolated GDP. However, much of the traffic bound for EWR is passing through southern ZDC; therefore, we show how the EWR arrival traffic can be folded into the same CTOP that addresses southern ZDC. Note that the traffic congestion at southern ZDC is comparable to an AFP with two wing FCAs added, shown in Figure 4. The FCA network is shown in Figure 5. We assume there is a four-hour capacity reduction in ZDC/EWR from 2000Z to 2359Z.

Figure 3: Weather Forecast for 2210z, Taken at 1522z on July 15, 2016

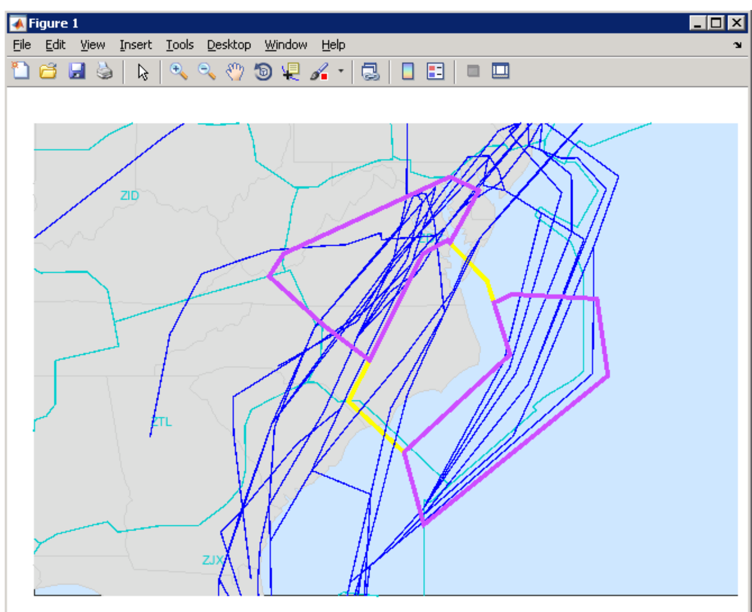

Figure 4: Traffic Routing Around the Original FCA

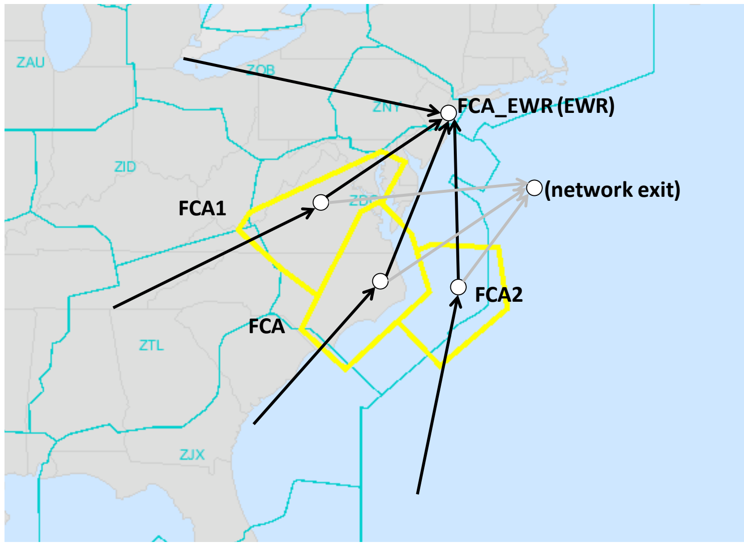

Figure 5: Geographical Display of the FCA Network

Resource/Time Bin	20:00	15	30	45	21:00	15	30	45	22:00	15	30	45	23:00	15	30	45	00:00	15	30	45
FCA	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	25	25	25	25
FCA1	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	50	50	50	50
FCA2	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
EWR	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10	10	10	10

Table 1: FCA Acceptance Rates

B. FCA Acceptance Rates

The FCA acceptance rates are listed in Table 1. From 2000Z to 2359Z, the rate of FCA reduced by nearly half from 25 to 13. FCA1 and EWR airport capacity are also adversely affected by the weather. In this paper, we assume the rate information is given. In reality, this information can be obtained from deterministic weather translation technique, or computed from stochastic rate planning algorithm which uses probabilistic capacity information [30] [31] [32].

Note in GDP optimization, we usually add one extra time period to make sure all flights will land at the end of the planning horizon. Because CTOP has multiple constrained resources, we need to add more than one time period depending on the topology of the FCA network. In this use case, we add four extra time periods, because the maximum average travel time between the three en route PCAs and KEWR is around 1 hour (4 time periods). For any time periods outside the CTOP start-end time, e.g. the extra four time periods in Table 1, we assumed nominal acceptance rate.

C. Traffic Demand Modeling

For flight data, we used historical flight data pulled from September 8, 2016 as a representative clear weather day for traffic demand. We avoided using the actual flight data from July 15, 2016 because flight plans and airline operational schedules were likely influenced by weather forecasts and related ATFM events. We only keep flights which pass through one of the 3 PCAs created in ZDC plus all EWR arrivals. The resulting set contained 1098 flights. To form the base (preferred) route for each flight, we drew historical filed flight plans (from Sept. 8, 2016) from the System Wide Information Management (SWIM) data.

A typical TOS package that might be submitted for this day would have one route for each FCA, and one route-out route to avoid all FCAs. To model the TOS sets that airlines might submit in response to a CTOP, we can draw from a combination of reroute TMIs from SWIM database and routes in the Coded Departure Route (CDR) database. There are in total 1368 TOS options for 890 flights, on average 1.55 options per flight.

D. Numerical Results

The optimization models are solved using Gurobi 7.5.2 on a workstation with 2.6GHz processors and 16 GB RAM. There are in total 26767 integer decision variables and 5563 constraints for the formulation without equality term; 26767 integer variables, 1 real variable and 5642 constraints for the formulation with equality term.

1. Benefit of Rerouting

We first show the benefit of allowing TOS submission in Table 2. Here the cost is in unit of ground delay (15 minutes), and the airline equality term is not added. We can see that with more reroute options, the total system cost decreases by more than half (62.6%). In both cases, no flight will take expensive air delay.

	Reroute Cost	Ground Delay Cost	Air Delay Cost	System Cost	Running Time (Seconds)
With TOS	34.93	148	0	182.93	4.07
Without TOS	0	489	0	489	3.30

Table 2: System Cost With and Without TOS Submissions

2. The Price of Fairness

We pick 5 values for the weighting coefficient of fairness measure γ: 0, 50, 100, 200, 500. It can be seen from Fig 6 that, increasing γ from 0 to 50 will decrease maximum average airline costs (increase fairness) without incurring any extra system cost. Increasing γ from 50 to 500 will help to further improve fairness, at the expense of increasing system cost. For the first data points, the exact optimal solution can be found in around 5 seconds. For the latter two data points, the optimality gap is within 1% if we early stop at 30 seconds.

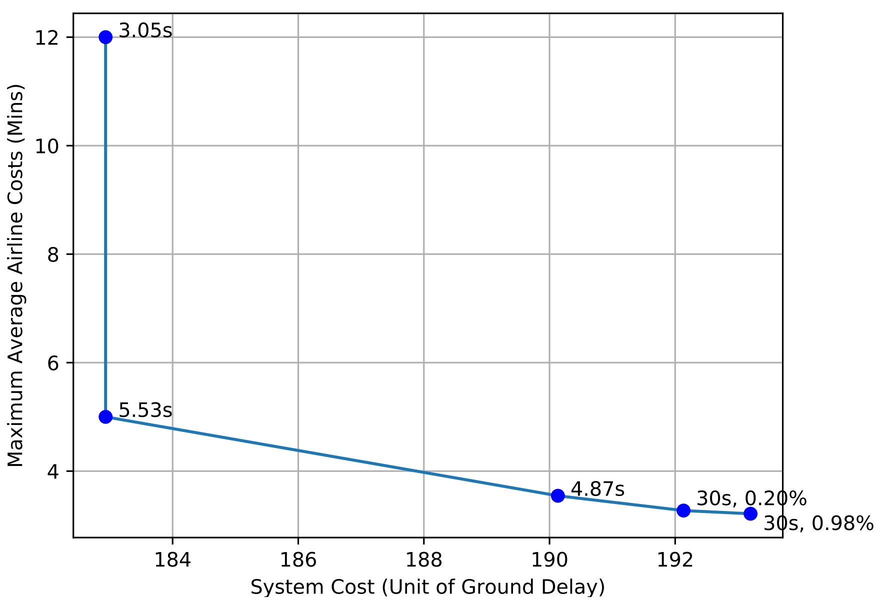

Figure 6: Tradeoff Between System Cost and Equality Measure

3. Benefit of Intra-airline Slot Substitutions

In this section, $\gamma = 50$ is chosen to first solve the TOS allocation model, because it gives us good trade-off between efficiency and equality. We can obtain the number of slots owned by each airline $v_{a,t}^k$ by post-processing the solution of decision variable $B_{k,i}^t$: $v_{a,t}^k = \sum_{i \in F_a} B_{k,i}^t$. We pick the 4 air carriers that have the largest number of CTOP captured flights. For each flight, we randomly generate ground and air delay unit costs which are uniformly distributed within [0.5, 1.5] and [1.5, 2.5], respectively. The intra-airline slot substitution model is applied to 200 different realizations of ground and air delay costs for each flight. The result is summarized in Table 3.

		Airline Cost Reduction		
Airline	Number of Captured Flights	Expected Value	Standard Deviation	Expected Running Time (Seconds)
Hub carrier	110	6.2%	4.0%	0.34
Non-hub carrier 1	117	25.6%	7.8%	0.39
Non-hub carrier 2	85	6.8%	4.2%	0.35
Non-hub carrier 3	73	18.4%	6.2%	0.24

Table 3: Expect Cost Reduction from Intra-Airline Substitutions

In this use case, all four carries have more than 5% expected cost reduction and two carriers have a substantial more than 18% cost reduction.

4. Some Remarks

1. It can be seen from the above numerical results that both TOS allocation model and intra-airline substitution models are quite computationally efficient
2. We have shown the benefit of having more rerouting options. To encourage airlines to submit TOSs, a possible way is to modify the right hand side of $y \geq \lambda_a \frac{1}{|F_a|} \sum_{i \in F_a} c_i$, and let λ_a slightly larger than 1 if the candidate routes per flight is large for airline a
3. A drawback of this formulation is that it is not straightforward to tell how many flights are taking air delay before a FCA, which is related to the workload of air traffic controllers. We will address this problem in [33]

V. Conclusion

In this paper, we proposed a deterministic interval-based TOS allocation model and discussed several related practical issues. Preliminary model performance results are obtained by testing on a realistic 4 hours CTOP use case that involving around 900 flights. We have demonstrated that the TOS allocation model is computationally efficient; by allowing TOS submission, the system cost can drop more than 50%; the tradeoff between system efficiency and equality among air carriers can be conveniently made; intra-airline substitution can help reduce the airline cost by around 5% or more. The ongoing work include extending the formulation to incorporate capacity uncertainties[33] and a detailed comparison with RBS results. Apart from CTOP, This work can also be potentially applied to related research problems including Integrated Demand Management (IDM)[34] [35] and airport arrival/departure fix balancing problem.

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