Saturation Technique for Optimizing Planned Acceptance Rates in Traffic Management Initiatives

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***Abstract*— Traffic Management Initiatives (TMIs), including Ground Delay Programs (GDP) and Collaborative Trajectory Options Programs (CTOP), are tools that air traffic managers use to balance demand and capacity in congested airports and airspace regions. In the current Collaborative Decision Making (CDM) paradigm, the Federal Aviation Administration (FAA) will set the Planned Acceptance Rates (PARs) for the constrained airspace resources, then run resource allocation algorithms to assign ground delays and/or reroutes to affected flights. In this paper, we have addressed a fundamental question in TMI PAR planning: do there exist optimal PARs which only depend on the physical airport or airspace capacity but not the demand? We show that this conjecture holds true in the deterministic capacity case but not in the general stochastic case. Several critical implications of this conclusion are discussed. We propose a new stochastic model and develop a heuristic saturation technique. We demonstrate that this technique can not only reveal the properties and limiting behaviors of GDP models but also could potentially be used as a robust PAR policy when facing demand uncertainty. We then show this ancillary saturation technique in GDP planning becomes an indispensable tool in CTOP optimization. The findings of this paper provide valuable insights in understanding the TMI rate-planning problem and a more robust algorithm for GDP optimization.**

***Keywords*— GDP; CTOP; Stochastic Model; Demand Uncertainty; Rate Planning**

NOMENCLATURE

Notation Used in SSM and Queueing Version of SSM

- P_t Number of flights planned to arrive at the resource in period t
- G_t Number of flights held on the ground in lieu of arriving in period t
- D_t Demand in period t
- $A_{t,q}$ Number of flights held in the air at the resource from time interval t to $t+1$, under scenario q
- $M_{t,q}$ Max number of flights that can use the resource in period t, under scenario q
- $\bar{P}_t$ Saturated PAR for time interval t
- L_t Number of flights accepted to resource in period t
- $\mathcal{T} = \{1, 2, \cdots, T\}$ Discrete time intervals during which demand greater than capacity at the resource
- $\mathcal{T}^+ = \{1, 2, \cdots, T, T+1\}$ An extra time period is added to $\mathcal{T}$ to ensure all flights can land
- ca Ratio of air and ground delay costs
- p_q Probability scenario q occurs

Notation Used in ESOM

- P_t^r Number of flights planned to arrive at resource r in period t
- $L_{t,q}^r$ Number of flights accepted to resource r in period t, under scenario q
- $A_{t,q}^r$ Number of flights held in the air at resource r from time interval t to $t+1$, under scenario q
- G_t^r Number of flights whose arrival time at $r \in$FCA is adjusted from time interval t to $t+1$ or later using ground delay at their point of origin
- $M_{t,q}^r$ Maximum capacity of resource $r \in$ PCA, in time interval t, under scenario q
- D_t^r Demand at resource r in time interval t
- $f_t^{r,r'}$ Fraction of flights from resource r directed to resource r' in interval t
- CONN Set of ordered pairs of resources. $(r, r') \in$ CONN iff r is connected to r' in the directed graph
- $\Delta^{r,r'}$ Number of time periods needed to travel from resource r to r'. Defined for all pairs $(r, r') \in$ CONN
- q_{ij} Reroute cost for flight i taking assigned route j
- d_i Assigned ground delay for flight i

I. INTRODUCTION

The goal of Air Traffic Flow Management (ATFM) is to alleviate projected demand-capacity imbalances at airports and in en route airspace regions through formulating and applying strategic Traffic Management Initiatives (TMIs). There are three classical TMIs: Ground Delay Programs (GDP), Airspace Flow Programs (AFP), and Reroutes. AFP can be considered as en route version of GDP, since they share the same principles and even software tools. The essence of these TMIs is applying ground delay for flights bound for congested airspace which would otherwise experience costly and unsafe air delay, or assigning flights to longer routes to make them avoid the congested areas. Designed to be a superset of the classical TMIs, Collaborative Trajectory Options Programs (CTOP) allow flight operators to submit a set of desired reroute options (called a Trajectory Options Set or TOS) to express their conditional preference among different route choices for each flight, automatically assign ground delays and/or reroutes around one or more Flow Constrained Areas (FCAs), and thus better balance demand with available capacity [1].

Maximizing airspace utilization and preserving equity among competing airspace users are two objectives of AT-

FM. In the current Collaborative Decision Making (CDM) paradigm, the Federal Aviation Administration (FAA) will set the Planned Acceptance Rates (PARs) for the constrained resources at discrete time intervals and then run resource allocation algorithms to assign ground delays and/or reroutes to affected flights. Since there is inherent uncertainty in weather forecasts, and the demand can also be stochastic, we need to deal with a sequential Decision Making Under Uncertainty (DMU) problem. Various DMU frameworks have been explored by researchers, including Markov Decision Process (MDP) [2][3], Chance Constrained Programming (CCP) [4][5] and Simulation-based Optimization [6][7]. The dominant approach in TMI optimization has been stochastic programming. In the stochastic programming framework, we assume the weather evolution can be modeled using a scenario tree, which is an input to the model. Most of the literature is focused on capacity uncertainty and the Single Airport Ground Holding Problem (SAGHP).

Two pioneering works on applying two-stage and multistage stochastic programming on SAGHP were done by Richetta et al. in the early 1990s [8][9]. These two models were proposed before CDM and assume the control of individual flights, therefore are not CDM-compatible. The first stochastic model that conforms to the current operating procedure was published by Ball et al. [10], referred to as the Static Stochastic Model (SSM). SSM is a two-stage high aggregate model that directly computes PARs for a weather-impacted airport. Kotnyek et al. [11] showed that if the ground delay cost is marginally increasing, the slot assignments from [8] are consistent with the CDM First-Scheduled-First-Served (FSFS) principle. The aforementioned models assume that once a ground-delay decision is made, it will not be revised, even if the flight is still on the ground. Mukherjee formulated a flight-level multistage model that allows a flight to take ground delays multiple times based on the latest capacity information and the scenario tree structure [12]. His model gives the theoretical lower bound for the scenario-based SAGHP. Estes et al. proposed an aggregate version of Mukherjee's model and showed that it is more computationally tractable [13]. Both flight-level and aggregate versions of Mukherjee's type of models are not compatible with current CDM software.

CTOP rate planning is substantially more challenging than for GDPs due to two reasons: multiple constrained resources and the fact that demands shift as PARs change. There are three references in literature on the CTOP rate-planning problem up to the present: in [14], we present a highly aggregate CDM-compatible stochastic model, called Enhanced Stochastic Optimization Model (ESOM), which is an extension to SSM and could directly produce PARs for the FCAs in a CTOP program; in [15] and [16], from mediumly aggregate level to flight level, we formulate nine muti-commodity stochastic models to benchmark against the ESOM.

To summarize, the static model of Richetta et al. (under very mild conditions) and the SSM model of Ball et al. could give us CDM-compatible PARs for the SAGHP, and ESOM can generate CDM-compatible PARs for a CTOP. It is not clear how to extend Richetta's result to solve a CTOP problem yet. Given the importance of SSM and ESOM, and their inheritance relationship, these two models will be the focus of this paper.

This paper is organized as follows: in section II we discuss several key conjectures, properties, and new model related to SSM, and propose the saturation technique (a heuristic approach). We validate the usefulness of the saturation technique using a realistic GDP test case. In section III, we explain why CTOP naturally has a demand uncertainty problem and how a saturation technique can help alleviate it in ESOM; in section IV, we summarize the findings of this paper and point out future work.

II. Saturation Technique, Properties, and Queueing Version of Single-Resource Static Stochastic Model (SSM)

For ease of reference, SSM is listed below:

$$\min \quad \sum_{t=1}^{T} G_t + \sum_{t=1}^{T}\sum_{q=1}^{Q} c_a p_q A_{t,q} \tag{1}$$

$$\text{s.t.} \quad P_t = D_t - (G_t - G_{t-1}) \tag{2}$$

$$M_{t,q} \geq P_t - (A_{t,q} - A_{t-1,q}) \tag{3}$$

$$G_t, P_t, A_{t,q} \geq 0 \tag{4}$$

$$G_0 = G_{T+1} = A_{0,q} = A_{T+1,q} = 0 \tag{5}$$

We assume $c_a > 1$ and $M_{T+1,q}$ is sufficiently large for all scenarios. One important characteristic of SSM (and also Richetta's static model) is that the airport PARs (we will use PARs and PAR vector interchangeably in this paper) proposed by SSM may be set lower than necessary simply because there was not sufficient demand to warrant a higher PAR. As an extreme example, the optimal PAR in a given time period could be zero, because no flights were requesting to use the airport. This does not mean that all flights should be banned from entering during that time period. A natural question would be: given capacity scenarios, is there a set of PARs which are optimal with respect to any demand profiles? In other words, can capacity information alone determine the optimal PARs? This simple question has deep implications: if such optimal PARs exist, this will eliminate the need to consider demand uncertainties caused by arrival time drift, cancellations, and pop-ups [17][18]. A further deduction is: if such demand-independent optimal PARs exist, they should be the optimal solution to the case in which we have sufficiently high demand, because as demand increases we expect the optimal PARs to stabilize to a constant value. This motivates us to flood the resource with artificially large demand to find the ideal demand-independent optimal PARs. We call this approach *the saturation technique*.

We pose three conjectures about SSM:

1) If we increase the demand to the SSM, we will get the saturated PARs, meaning if we further increase the demand, the SSM PARs do not change.

2) Saturated PARs are the upper bounds of the optimal PARs under all demand circumstances.
3) Under any demand case, if we perform ground delay planning according to saturated PARs, we will get the optimal objective value.

Remarks:

1) The first conjecture says once traffic demand levels are sufficiently high, the optimal solution to SSM will stabilize, and no subsequent increase in demand level will change the optimal solution (indifference to further pain), which we refer to as saturated PARs.
2) Given a demand profile, there will be a corresponding optimal PAR vector computed from SSM. The second conjecture claims all optimal PARs will be bounded by saturated PARs ($P_t \leq \bar{P}_t$). Put it in another way, the saturated PAR is the greatest number of flights that should ever be admitted in each time period.
3) Conjecture 3 is a very strong statement and says the saturated PARs are the demand-independent optimal PARs.

Conjecture 3 needs further elaboration. To plan according to saturated PARs, we need the following *Queueing Version of SSM*:

$$\begin{aligned}
\min \quad & \sum_{t=1}^{T} G_t + \sum_{t=1}^{T}\sum_{q=1}^{Q} c_a p_q A_{t,q} && (6)\\
\text{s.t.} \quad & L_t = D_t - (G_t - G_{t-1}) && (7)\\
& y_t \mathbb{M} \geq \bar{P}_t - L_t \geq 0 && (8)\\
& (1-y_t)\mathbb{M} \geq G_t && (9)\\
& M_{t,q} \geq L_t - (A_{t,q} - A_{t-1,q}) && (10)\\
& G_t, L_t, A_{t,q} \geq 0 && (11)\\
& G_0 = G_{T+1} = A_{0,q} = A_{T+1,q} = 0 && (12)\\
& y_t \in \{0,1\}, A_{t,q}, G_t, L_t \in \mathbb{Z}_+ && (13)
\end{aligned}$$

where saturated PARs $\bar{P}_t$ are used as parameters and upper bounds for the actual number of accepted flights L_t, $\mathbb{M}$ is a sufficiently large constant, and y_t are ancillary binary variables. Constraints (8) and (9) mean that we will create $|\bar{P}_t|$ slots at time period t irrespective to demand; it doesn't matter if some slots turn out to be unused ($\bar{P}_t > L_t$), and if so it implies $G_t = 0$.

We will first investigate the three conjectures for the simple case where the capacity information is deterministic, i.e. there is only one capacity scenario.

A. Deterministic Capacity Case

For ease of reference, the deterministic version of SSM is shown below:

$$\begin{aligned}
\min \quad & \sum_{t=1}^{T} (G_t + c_a A_t) && (14)\\
\text{s.t.} \quad & P_t = D_t - (G_t - G_{t-1}) && (15)\\
& M_t \geq P_t - (A_t - A_{t-1}) && (16)\\
& G_t, P_t, A_t \geq 0 && (17)\\
& G_0 = G_{T+1} = A_0 = A_{T+1} = 0 && (18)
\end{aligned}$$

Conjecture 1: We claim as long as cumulative demand is not less than cumulative capacity $\sum_{i=1}^{t} D_i \geq \sum_{i=1}^{t} M_t$, the deterministic SSM will be saturated. The optimal PARs $P_t = M_t$. Note that in the deterministic case we not only provide a sufficient condition for which we can get saturated PARs, but we also pinpoint the values of saturated PARs. *Proof:*

Proof by contradiction. Suppose P_t, G_t, A_t are the optimal solutions to deterministic SSM. Suppose $\exists t'$ such that optimal $P_{t'} > M_{t'}$. Then:

$$A_{t'} \geq P_{t'} - M_{t'} + A_{t'-1} \geq 1$$

We can modify P_t, G_t, A_t and get a new solution P_t^*, G_t^*, A_t^*. Let:

$$\begin{aligned}
P_{t'}^* &:= P_{t'} - 1\\
P_{t'+1}^* &:= P_{t'+1} + 1\\
G_{t'}^* &:= G_{t'} + 1\\
A_{t'}^* &:= A_{t'} - 1
\end{aligned}$$

This new solution is also feasible and has a smaller objective value:

$$\begin{aligned}
\sum_{t=1}^{T}(G_t^* + c_a A_t^*) &= \sum_{t=1}^{T}(G_t + c_a A_t) + (1 - c_a)\\
&< \sum_{t=1}^{T}(G_t + c_a A_t)
\end{aligned}$$

which contradicts the assumption that P_t, G_t, A_t are optimal solutions. Therefore, independent of demand profile D_t, P_t should be always smaller than M_t.

It directly follows that $A_t \equiv 0 \ \forall t \in \mathcal{T}^+$, because it satisfies constraint (16) and it is the smallest possible value of the second part of the objective function. Therefore the deterministic SSM can actually be reduced to the following model:

$$\begin{aligned}
\min \quad & \sum_{t=1}^{T} G_t \\
\text{s.t.} \quad & P_t = D_t - (G_t - G_{t-1}) \\
& M_t \geq P_t \\
& G_t, P_t \geq 0 \\
& G_0 = G_{T+1} = 0
\end{aligned} \qquad (19)$$

We have proved that $P_t \leq M_t$. Now we proceed to prove the inequality relation is actually equality. If $\exists t'$ such that optimal solution $P_{t'} < M_{t'}$, then following the cumulative demand condition we have

$$G_t = \sum_{i=1}^{t}(D_t - P_t) \geq \sum_{i=1}^{t}(M_t - P_t) > 1 \quad T \geq t \geq t'$$

Again we can construct a new solution. Let

$$\begin{aligned}
P_{t'}^* &:= P_{t'} + 1\\
P_t^* &:= P_t \quad t \neq t'\\
G_t^* &:= G_t - 1 \quad T \geq t \geq t'\\
G_t^* &:= G_t \quad t' > t
\end{aligned}$$

The new solution P^*_t, G^*_t has a smaller objective value:

$$\sum_{t=1}^{t'-1} G_t + \sum_{t=t'}^{T} G^*_t = \sum_{t=1}^{t'-1} G_t + \sum_{t=t'}^{T} G_t - (T - t' + 1)$$
$$< \sum_{t=1}^{t'-1} G_t + \sum_{t=t'}^{T} G_t$$

which contradicts the assumption that P_t, G_t are optimal solutions. Therefore saturated PARs exist and they are $P_t = M_t \ \forall t \in \mathcal{T}$. It's easy to know the cumulative demand condition is also a necessary condition for $P_t = M_t$: $\sum_{i=1}^{t} D_i = \sum_{i=1}^{t} P_i + G_t \geq \sum_{i=1}^{t} M_i$. ■

Conjecture 2: *Proof:* While proving conjecture 1, we have already shown in the deterministic case, for any demand profile, the optimal PAR $P_t \leq M_t$. Note that the right-hand side is just the saturated PARs. ■

Conjecture 3: We need to show the optimal L_t, G_t we get by solving the queueing version of SSM (6)-(13) ($q = 1$, replace $\bar{P}_t$ by M_t and set $A_t = 0$) are equal to the optimal P_t and G_t obtained from solving the reduced version of SSM (19) for any demand profile.

Proof: Since we have more constraints in the queueing version of SSM than in SSM, we only need to show the optimal solution from SSM is also feasible for the queueing version of SSM, through checking whether P_t satisfies the extra constraint $M_t > P_t \Rightarrow G_t = 0$. To show this set of constraints hold true, we will prove by contradiction. Suppose $\exists t'$ such that $M_{t'} > P_{t'}$ and $G_{t'} > 0$, let t'' be the first time period after t' such that $G_t = 0$. Since $G_{T+1} = 0$, t'' will always exist. For t'' we must have $P_{t''} = D_{t''} + G_{t''-1} \geq 1$. Let

$$\begin{aligned}
P^*_t &:= P_t \quad t \neq t', t'' \\
P^*_{t'} &:= P_{t'} + 1 \\
P^*_{t''} &:= P_{t''} - 1 \\
G^*_t &:= G_t - 1 \quad T \geq t'' - 1 \geq t \geq t' \\
G^*_t &:= G_t \quad \text{for other } t
\end{aligned}$$

we will have a smaller objective value with P^* and G^*, which contradicts the assumption that P_t and G_t are optimal to (19). ■

To summarize, if the capacity of a single resource is known deterministically, all three conjectures hold true and we should always create the same number of slots as the known capacity irrespective of demand.

B. Stochastic Capacity Case

Now we will prove the general and practical case, where the future capacity realizations are represented by a finite set of scenarios. Different from the deterministic case, here we can only provide a sufficient condition and prove conjecture 1, which is the existence of saturated PARs.

Conjecture 1: We claim as long as demand $D_t > \max_q M_{t,q} \ t \in \mathcal{T}$, stochastic SSM will be saturated and optimal PARs will stabilize. We denote the corresponding solution as $\bar{P}_t$. We can define any such demand *saturated demand*. We further claim that $\max_q M_{t,q} \geq \bar{P}_t$ and in realistic $\bar{P}_t$ are unique. We first prove the second statement by contradiction. If $\exists t'$ such that $\bar{P}_{t'} > \max_q M_{t',q}$, then

$$A_{t',q} \geq \bar{P}_{t'} - M_{t',q} + A_{t'-1,q} \geq 1, \quad \forall q$$

Similar to what we did in section II-A, by letting

$$\begin{aligned}
\bar{P}^*_{t'} &:= \bar{P}_{t'} - 1 \\
\bar{P}^*_{t'+1} &:= \bar{P}_{t'+1} + 1 \\
\bar{G}^*_{t'} &:= \bar{G}_{t'} + 1 \\
\bar{A}^*_{t',q} &:= \bar{A}_{t',q} - 1
\end{aligned}$$

we can construct a new solution which has a smaller objective value. Thus we reach a contradiction to the assumption that $\bar{P}_t$ are optimal.

It is not difficult to construct an example that has multiple optimal solutions, e.g. Table I. Therefore in general SSM may not have a unique solution. In the case of Table I, the non-uniqueness is caused by the fact that we can reduce ground delay cost by paying more air delay cost. However, in practice, we usually can at best estimate p_q to the second decimal. Instead of choosing c_a as an integer, we can pick a number like 2.0001, which has a negligible effect on the objective function but will exclude the possibility of exchanging ground delay for air delay. We haven't mathematically proven that this will guarantee the uniqueness of the optimal solution, but no counterexample was found through extensive random generated GDP simulations. We can argue in realistic the uniqueness of staturation PARs can be achieved.

Time Interval	1	2	3	4	5 (T+1)
Scen 1 Capacity Prob 0.4	33	57	49	53	∞
Scen 2 Capacity Prob 0.5	23	26	20	25	∞
Scen 3 Capacity Prob 0.1	8	57	7	47	∞
Saturated Demand	100	100	100	100	0
Saturated PARs	23	42	4	47	284
Saturated PARs	23	41	5	47	284

TABLE I
AN EXAMPLE WITH NON-UNIQUE OPTIMAL SOLUTIONS ($c_a = 2$).

We continue to prove the first point: once demands satisfy the saturation condition, adding more demand will not change the optimal PARs $\bar{P}_t \ t \in \mathcal{T}$. For any $t \in \mathcal{T}$, denote $\delta D_t \geq 0 \ \exists t \in \mathcal{T}$ as the amount of extra demand for interval t and denote the new optimal solution to the new demand $D_t + \delta D_t$ as $\tilde{P}_t, \tilde{G}_t, \tilde{A}_{t,q}$. We will show that $\tilde{P}_t$ is also optimal to original SSM with demand D_t. If $\bar{P}_t$ is the unique optimal solution, then $\tilde{P}_t = \bar{P}_t$.

Proof: Since $\tilde{P}_t, \tilde{G}_t, \tilde{A}_{t,q}$ are optimal, the objective value of SSM should be no greater than the scenario in which we accept $\bar{P}_t$ flights at each time period.

$$\sum_{t=1}^{T}\tilde{G}_t + \sum_{t=1}^{T}\sum_{q=1}^{Q} c_a p_q \tilde{A}_{t,q} \leq$$
$$\sum_{t=1}^{T}(\bar{G}_t + \sum_{j=1}^{t}\delta D_j) + \sum_{t=1}^{T}\sum_{q=1}^{Q} c_a p_q \bar{A}_{t,q}$$

Because $\tilde{P}_t \leq \max_q M_{t,q}$, therefore we must have $\tilde{G}_t \geq \sum_{j=1}^{t}\delta D_j$. Denote $G_t = \tilde{G}_t - \sum_{j=1}^{t}\delta D_j$ and rewrite constraints $\tilde{P}_t = D_t + \delta D_t - (\tilde{G}_t - \tilde{G}_{t-1})$ as

$$\tilde{P}_t = (D_t + \delta D_t) - (G_t + \sum_{j=1}^{t}\delta D_j - G_{t-1} - \sum_{j=1}^{t-1}\delta D_j)$$
$$= D_t - (G_t - G_{t-1})$$

that is to say $\tilde{P}_t, G_t, \tilde{A}_{t,q}$ are also optimal with respect to the original problem with demand D_t. ■

Conjecture 2 and Conjecture 3: We construct a counterexample in Table II to show conjectures 2 and 3 don't hold. In this example, we have four planning time periods and three capacity scenarios, shown in row 1 and rows 2-4, respectively. If we run SSM with saturated demand, we can get saturated PARs in row 6, which turns out to be the unique optimal solution for this saturated GDP problem. If we feed SSM the demand shown in row 7, we can get corresponding optimal PARs (row 8) and optimal value (row 9). The last row shows the objective value if we run the queueing version of SSM with saturated PARs.

Time Interval	1	2	3	4	5 (T+1)
Scen 1 Capacity Prob 0.3	12	54	41	40	∞
Scen 2 Capacity Prob 0.5	4	10	19	60	∞
Scen 3 Capacity Prob 0.2	17	1	7	33	∞
Saturated Demand	100	100	100	100	0
Saturated PARs	17	1	35	40	307
Demand	16	61	18	18	0
Optimal PARs	16	1	36	40	20
Optimal Obj Value	185.4				
Optimal Obj if Planning Using Saturated PARs	185.6				

TABLE II

COUNTEREXAMPLE FOR SINGLE RESOURCE STATIC STOCHASTIC MODEL (SSM) WITH $c_a = 2$.

There are three key observations from this table:

- The saturated PARs are different from any capacity scenario.
- In the third time period, the optimal PAR (36 flights) is one unit higher than the saturated PAR (35 flights). Therefore the saturated PAR is not an upper bound for any optimal PARs. Conjecture 2 doesn't hold.
- If we plan according to saturated PARs, we will get a worse objective value (185.6) than the value (185.4) that we get by planning according to the real demand. Conjecture 3 doesn't hold.

	Conject. 1	Conject. 2	Conject. 3
Deterministic Case	True	True	True
Stochastic Case	True	False	False

TABLE III

RESULTS OF SINGLE AIRPORT GDP PLANNING PROBLEM

In summary, for the stochastic capacity case, in practice we could compute a unique saturated PAR vector. However, this saturated PAR vector is neither the upper bounds of all possible optimal PARs nor it is the ideal demand-independent optimal PARs.

There is an important implication of the above conclusion: Even in single-resource planning like GDP, the optimal PAR vector is in general dependent on demand information. If we believe the weather will evolve as a scenario tree, then any rate calculation technique based solely on capacity information, for example a certain quantile/reduction factor from weather translation model or a certain way to combine the scenarios, or based on capacity and inaccurate demand estimation, generally will not be optimal for the actual demand profile.

C. Saturated PARs as an Approximation

Although the saturated PARs are not ideal, they still could be a good robust rate policy with respect to demand uncertainty. In Table II, even though the saturated PARs are not optimal with respect to the real demand, the optimal value we get by running queueing SSM with saturated PARs is very close to the optimal value we get from running SSM with actual demand (0.1% optimality gap). We will examine the practicability of saturated PARs using a realistic GDP test case at San Francisco International Airport (SFO).

1) Experiment Setup: A well-known problem with SFO is that the low-altitude cloud layer that develops overnight precludes simultaneous arrival operations on closely spaced parallel runways, reducing the arrival capacity from 60 to 30 flights per hour. A time line of discrete probability distribution of marine stratus layer burnoff time is shown in Table IV. We select a representative GDP day at SFO and get the data from the FAA Aviation System Performance Metrics (ASPM) database [19]. The GDP planning horizon is from 0900 PST to 1600 PST.

2) Models Comparison: The number of scheduled flights, PARs calculated using SSM and saturated PARs are drawn in Figure 1. Both PARs give us the same optimal objective value 52.48.

We assume during the GDP active period the demand could be perturbed by maximum +/- 4 flights at each hour. In this case we are also facing demand uncertainty. Figure 2 shows the cost distributions of 400 sampled scenarios if we implement two candidate PARs respectively. It is clear that saturated PARs perform much better than SSM PARs in terms of total delay costs. In fact the former dominates the latter in every scenario! This example shows saturated PARs are more robust than SSM PARs in the face of both

Fog Clearance Time	0700	0800	0900	1000	1100	1200	1300	1400	1500
Probability	0.03	0.05	0.11	0.22	0.25	0.20	0.12	0.01	0.01

TABLE IV

DISTRIBUTION OF MARINE STRATUS LAYER BURNS OFF TIME (PST)

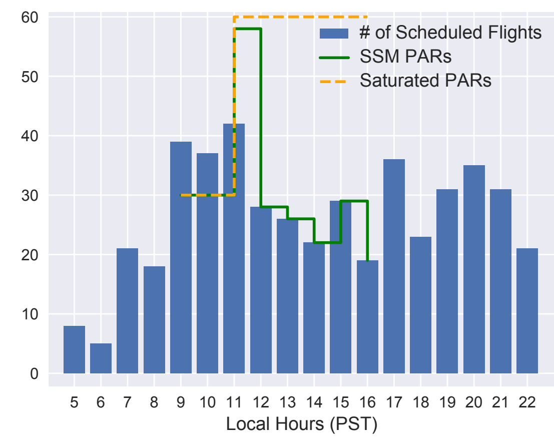

Fig. 1. Scheduled Flights to SFO and Two Candidate PARs

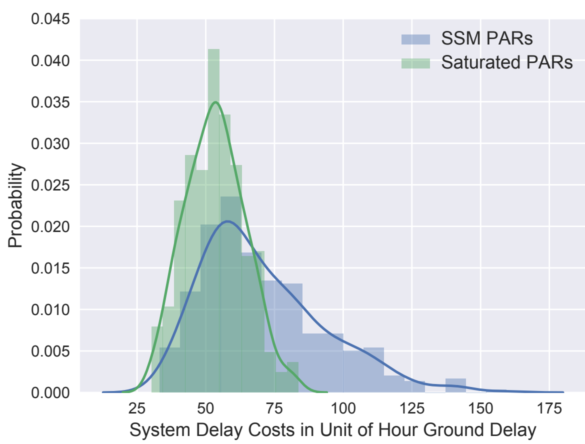

Fig. 2. System Delay Costs Under Demand Uncertainties

scenario tree based capacity uncertainty and unstructured demand uncertainty.

III. Saturation Technique in CTOP, Properties of Enhanced Stochastic Model (ESOM)

In previous sections, we discussed the properties and application of the saturation technique for the single-resource rate-planning problem, of which a GDP is an example. In this section we will see the saturation technique is more needed in multi-resource rate-planning problems like CTOP.

In CTOP, the introduction of TOS brings extra difficulties: firstly, one flight now may have more than one route option, therefore it is not straightforward to estimate the demand to the FCAs; secondly, for a given demand estimation, if we do the planning accordingly, after we run the the CTOP allocation algorithm, which is a multi-resource RBS algorithm [20], the demand may shift and invalidate the proposed PARs. We call this problem *TOS-induced demand variability*. To address it, we proposed an iterative algorithm in [14], shown in Figure 4. There is a key component in this computation loop called ESOM, which takes demand and stochastic capacity information as input and outputs PARs for FCAs that minimize total system ground and air delays. In order for readers to have a basic knowledge of the model, here are the three most important features of ESOM:

- The model is an extension of the single-resource SSM, which means it is also a two-stage static stochastic model and the computation of PAR is also dependent on the demand estimation.
- The model differentiates FCAs, which serve like a valve to control the traffic flows, and Potentially Constrained Areas (PCAs), which are the actual troubled areas and whose future capacity realizations are represented by a finite set of scenarios (Figure 3).
- The model accounts for the travel time between FCAs and PCAs and uses the flow ratios to model the traffic flow split from one element to downstream elements.

The reader can refer to [14] for a detailed derivation of the model.

$$
\begin{aligned}
\min \quad & \sum_{r\in\text{FCA}}\sum_{t=1}^{T} G_t^r + \sum_{r\in\text{PCA}}\sum_{t=1}^{T}\sum_{s=1}^{Q} c_a p_s A_{t,s}^r \\
\text{s.t.} \quad & P_t^r = \text{UpFCA}_t^r + D_t^r - (G_t^r - G_{t-1}^r) \quad \forall r\in\text{FCA}, \forall t \\
& L_{t,s}^r = \text{UpStream}_{t,s}^r - (A_{t,s}^r - A_{t-1,s}^r) \quad \forall r\in\text{PCA}, s, t \\
& M_{t,s}^r \geq L_{t,s}^r \quad \forall r\in\text{PCA}, \forall s, \forall t \\
& \sum_{(r,r')\in\text{CONN}} f_t^r = 1 \\
& G_t^r, P_t^r, L_{t,s}^r, A_{t,s}^r \geq 0 \quad \forall r, \forall s, \forall t \\
& \text{UpFCA}_t^r = \sum_{r',r\in\text{CONN}} f_{t-\Delta^{r',r}}^{r',r} \cdot P_{t-\Delta^{r',r}}^{r'} \\
& \text{UpPCA}_{t,s}^r = \sum_{r',r\in\text{CONN}} f_{t-\Delta^{r',r}}^{r',r} \cdot L_{t-\Delta^{r',r},s}^{r'} \\
& \text{UpStream}_{t,s}^r = \text{UpFCA}_t^r + \text{UpPCA}_{t,s}^r
\end{aligned}
\tag{20}
$$

For the iterative algorithm, in the first iteration we will use flights' most preferred routes as the initial demand estimation. Based on this demand estimation and capacity information, we will run the rate optimization model, i.e. ESOM, and compute the PARs for the FCAs. Then we

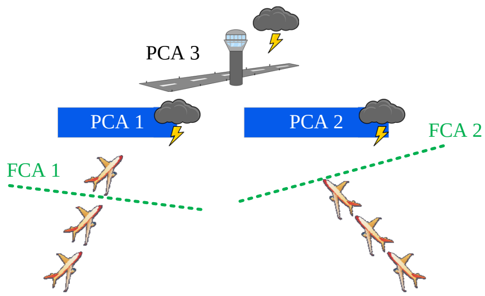

Fig. 3. PCAs and Candidate FCAs

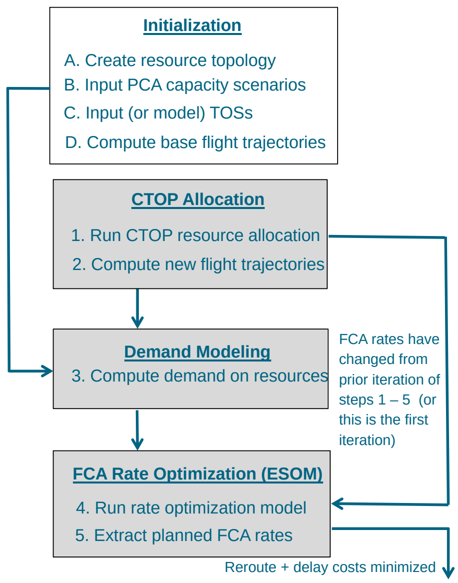

Fig. 4. Rate Computation Loop (RCL)

will run the CTOP resource allocation algorithm, get the new route and ground delay for each flight, and compute the new demand on the resources. Now it is clear that we need to have a computation loop, since the demand shift necessitates the recalculation of PARs and a new PAR vector may result in new demand distribution for the FCAs. We will continue running the computation loop and exit if the following rerouting and delay costs are minimized:

$$\sum_{i=1}^{N}(q_{ij}+\alpha d_i)+\beta\sum_{r\in \text{PCA}}\sum_{t=1}^{T}\sum_{s=1}^{Q}p_s A^r_{t,s} \tag{21}$$

where α and β are weighting parameters.

There is one critical issue with the above approach: if very few or no flights are scheduled to traverse a certain FCA, then the PARs for this FCA computed from ESOM would be near or equal to zero. The CTOP allocation algorithm will consider this FCA being heavily restricted/fully blocked and will allocate very few/no slots to flights. Then in the next iteration there will be very few/no flights rerouted to this FCA even though there may be plenty of airspace capacity available behind this FCA. The crux of this problem is again the demand dependency of the stochastic model. Like we did in GDP, we could use a saturation technique to alleviate this problem:

- Increase demands to FCAs proportionally to sufficiently large numbers to get the saturated PARs
- If there is no demand to an FCA, multiplying the demand values (zeros) by a large constant will not help. We will use $\min_s M^r_{t,s}$ as the PAR, where r is the neighboring PCA. This is because even in the worse scenario we are still able to accept $\min_s M^r_{t,s}$ number of flights

As we concluded in the last section, saturated PARs are not ideal even for the single-resource planning problem. In CTOP, we also shouldn't expect the saturated PARs obtained from each iteration to be the optimal upper bound; rather, we can only treat it as a good approximated bound. The key differences between GDP and CTOP are: in GDP, we have the ability to compute the optimal PARs given nominal demands. The saturated technique is used to reveal the properties and limiting behaviors of the model, and could potentially be used as a robust PAR vector when facing demand uncertainties; whereas in CTOP, even for nominal demands (no flight cancellations, pop-ups, etc.), we have to rely on saturated PARs as a heuristic to get a suboptimal solution.

IV. Conclusions

In this paper, we investigated a key conjecture for GDP: the existence of demand-independent optimal PARs. We proposed the queueing version of SSM to evaluate the performance of PAR policies. We showed that a by-product, discovered while proving the existence conjecture, called saturation technique turns out to be a valuable tool in revealing the properties of GDP models and potentially in designing a robust PAR policy under demand uncertainty. We also explained why CTOP in nature suffers from demand uncertainty and saturation technique is a key tool for finding a good suboptimal solution.

The ongoing work includes further investigating the uniqueness of saturated PARs to stochastic SSM, testing saturated PARs in other typical GDP use cases, giving theoretical analysis to the robustness of saturated PARs, and finding alternative ways to compute CDM-compatible PARs for CTOP.

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VI. Appendix

A. Acronyms

ATFM	Air Traffic Flow Management
FAA	Federal Aviation Administration
TMI	Traffic Management Initiative
CDM	Collaborative Decision Making
GDP	Ground Delay Program

AFP	Airspace Flow Program
FCA	Flow Constrained Area
PCA	Potential Constrained Area
PAR	Planned Acceptance Rate
CTOP	Collaborative Trajectory Options Program
TOS	Trajectory Options Set
RTC	Relative Trajectory Cost
RCL	Rate Computation Loop
RBS	Ration by Schedule
SSM	(Single Resource) Static Stochastic Model
ESOM	Enhanced Stochastic Optimization Model
SAGHP	Single Airport Ground Holding Problem
DMU	Decision Making Under Uncertainty

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