

**A QUANTITATIVE AND SYSTEMATIC METHODOLOGY TO INVESTIGATE  
ENERGY CONSUMPTION IN MULTIMODAL TRANSPORTATION SYSTEMS**

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**ABSTRACT**

Energy issues in transportation systems have garnered increasing attention in recent years in both the public and private sectors. This study proposes a systematic methodology for policy-makers to improve energy consumption efficiency in multimodal intercity transportation systems considering suppliers' operational constraints and travelers' mobility requirements. A bi-level optimization model is developed for this purpose and considers the air, rail, private auto, and transit modes. The upper level model is a mixed integer nonlinear program that aims to minimize energy consumption subject to transportation suppliers' operational constraints and the traffic demand distribution to paths resulting from the lower level model. The lower level model is a linear program that seeks to maximize the intercity trip utilities of travelers. The interactions between the multimodal transportation suppliers and intercity traffic demand are considered under the goal of minimizing energy consumption at the system level. The proposed bi-level mixed integer nonlinear model is relaxed and transformed into a mathematical program with complementarity constraints, and solved using a customized branch-and-bound algorithm. Numerical experiments, conducted using multimodal travel options between Lafayette, Indiana and Washington, D.C. reiterate that shifting traffic demand from private cars to the transit and rail modes can significantly reduce energy consumption. More importantly, the proposed methodology is able to provide quantitative analyses for system energy consumption and traffic demand distribution among transportation modes under specific policy instruments. The results also illustrate the need to systematically incorporate the interactions between traveler preferences, network structure, and supplier operational schemes to provide policy-makers insights for developing traffic demand shift mechanisms to minimize system energy consumption. Hence, the proposed methodology can provide policy-makers the ability to analyze energy consumption in the transportation sector under different policy instruments.

## **A Quantitative and Systematic Methodology to Investigate Energy Consumption in Multimodal Transportation Systems**

### **BACKGROUND AND MOTIVATION**

The current transportation system relies heavily on non-renewable fuel energy. It accounts for 71 percent of the nation's petroleum use and 30 percent of U.S. greenhouse gas emissions (1, 2). These statistics suggest that reducing the energy consumption in the transportation sector can significantly enhance national energy security and help control greenhouse gas emissions. However, the current transportation system is central to the U.S. societal mobility and commerce and cannot be easily or quickly altered. Therefore, improving the transportation system energy consumption efficiency without sacrificing mobility needs disproportionately is a key imperative, and motivates the current study.

The total energy consumption of the current transportation system is a function of the fuel efficiency of the transportation modes and the intensity of transportation mode usage. Therefore, a natural solution strategy to consider so as to save energy is to develop fuel-efficient vehicles. However, relevant studies (3) suggest otherwise as the improvements resulting from fuel-efficient vehicles have been offset by the higher usage of larger luxury cars and SUVs by travelers. Instead, this study focuses on the commonly-addressed demand-side strategy of shifting traffic demand from low fuel efficiency modes to high fuel efficiency modes. Empirical data indicates that cars and light trucks used for personal travel alone account for the majority of fuel consumption in the transportation sector (2). The fuel efficiency of transportation modes (per passenger per gallon) degrades in the order of rail, road, and air modes (4). Hence, to reduce energy consumption in current transporting systems, the primary focus is on shifting the passenger traffic demand in cars, light-duty trucks, and air to high fuel efficiency and/or high occupancy modes such as rail and public transit. This raises the key question of how to foster such a traffic demand shift.

Strategic policies which influence transportation supplier actions as well as traffic mode choices represent a promising solution paradigm to realize the demand shift from low fuel efficiency modes to higher efficiency ones. Thereby, the three key players including the travelers (who form the traffic demand), transportation suppliers (who provide the traffic supply options), and policy-makers (who design and implement policy instruments), work independently in the short-term, but interactively in long-term to address energy consumption of transportation sector. For example, transportation suppliers, who provide vehicles, fuel and traffic infrastructure, and operate transportation modes and serve commercial freight and passenger, typically focus on profits. Their decisions and actions are significantly impacted by policy instruments such as tax, subsidy, mandatory policies, etc. By contrast, travelers will choose intermodal/multimodal paths based on their preferences and level of service attributes of the modes (such as frequency, fare, travel time, and waiting time) provided by transportation suppliers. The mode choices of the travelers will eventually impact the operational decisions of transportation suppliers. Therefore, the interactions among the three players as well as their operational/behavior characteristics need to be considered so that a successful policy instrument can be implemented.

The conceptual perspectives discussed heretofore are well-recognized. The key gap to successfully implement a policy instrument is the need for a systematic tool to predict the traffic demand shift, the energy consumption reduction, and the actions on transportation suppliers, so that the effects of a designed policy instrument can be holistically captured. However, as identified in the literature review in the next section, most existing studies only provide conceptual suggestions based on historical trends but cannot characterize these effects quantitatively. To address this gap, the proposed study proposes mathematical models to predict the energy consumption effects resulting from traffic demand shift and traffic supply operational actions, which are triggered by the implementation of certain policy instruments. Intercity multimodal transportation networks that include the private car, transit, rail, and air modes are considered in this context. Traveler preferences, transportation supplier operational constraints as well as the interactions among traffic demand, suppliers and policy-makers will be considered in the modeling process. The associated solutions can provide policy-makers in both the transportation and energy

consumption sectors insights so that strategic policy instruments can be designed to realize traffic demand shifts and achieve system energy savings in the long-term.

The rest of the paper is organized as the follows. The next section reviews the past literature in this domain. Then, preliminaries including definitions, assumptions, and notations are provided. The mathematical model and its solution methodology are described. Next, numerical experiments and the associated insights are discussed. The paper concludes with some comments and insights related to the problem context.

## LITERATURE REVIEW

Past studies in multimodal transportation system have addressed traffic demand, supply modeling as well as their interactions. However, holistic studies that integrate these aspects with broader system level objectives such as energy conservation are lacking. Thus, they typically address one of the sub-problems rather than the broader perspective of this study. Garbade and Soss (5) analyze the interactions between traffic demands and suppliers. A dynamic model is developed to capture the relationships between demand, supply, cost and revenue in the market for mass transit services in New York City. Li and Wachs (6) propose several intermodal performance indicators in which service input, service output and service consumption are measured using total cost, revenue capacity, and unlinked passenger trips based on economic principles and evaluation objectives. Aifadopoulou et al. (7) explore the routing problem in multimodal transportation networks. A multi-objective optimum path algorithm, identifying feasible paths according to compatibility of various modes, intermodal stations, and users' preferences, is designed for passenger pre-trip planning. Hamdouch et al. (8) propose a toll pricing framework, and conduct a congestion pricing study for multimodal transportation systems. Todd (9) summarizes basic principles for multimodal transportation planning and evaluation, and states that transportation modeling techniques are being enhanced to consider a wider range of options (such as pricing incentive and multiple modes) and impacts (such as emissions and land use).

A few studies have addressed the energy consumption and emission issues jointly in transportation systems since these two problems are usually linked together. Most existing studies rely on examining current polices and their performances, and then propose conceptual strategic policy recommendations. For example, Poudenx (10) conducted a brief survey on twelve major cities with various policies in place to curb private vehicle use and assesses their success in terms of energy consumption and greenhouse gas emissions. Rickwood et al. (11) examine the current state of research in the energy and greenhouse gas emissions attributable directly or indirectly to urban form. Kenworthy (12) reviews transport, urban form, energy use and CO<sub>2</sub> emission patterns in an international sample of 84 cities. Policy recommendations to reduce urban passenger transport energy use and CO<sub>2</sub> emissions are outlined in the aforementioned studies (10, 11, 12); however, no quantitative mechanisms are proposed to predict the consequences of a policy implementation. They all indicate that detailed research is needed to examine the relationships among urban form, traffic demand and energy use in multimodal transportation systems. Zumerchik et.al (13) propose a metric to measure energy consumption in multimodal transportation systems, but the interactions between energy consumption, traffic demand, and transportation suppliers are not explicitly incorporated. Euritt et al. (14) explore strategies to reduce energy consumption and CO<sub>2</sub> emissions in Texas. Four alternative scenarios, reflecting different strategies, are conducted based on Long-Range Alternative Planning/Environmental Data Base (LEAP/EDB). Pedersen et al. (3) identify policy options to reduce energy use and greenhouse gas emissions from the U.S. transportation context, though only strategic aspects and not quantitative analyses are provided. Hence, neither (3) nor (14) perform systematic quantitative analyses. Knittel (2) examines the primary mechanisms through which reductions in U.S. oil consumption might take place, including increased fuel economy of existing vehicles, increased use of non-petroleum-based low-carbon fuels, alternatives to the internal combustion engine, and reduced vehicle-miles travelled. The effects of these mechanisms are compared to using a Pigouvian tax in energy consumption reduction. In summary, extensive policy discussions and other analyses have

been addressed based on historical data, but they mostly focus on fuel economics rather than predict energy consumption resulting from traffic demand shift and transportation supplier operational actions, which are the key factors considered in the proposed study.

The state of the art indicates that though several studies in recent years have broadly analyzed energy consumption issues in the transportation sector, systematic quantitative analyses are lacking for multimodal transportation systems. There are gaps in terms of the need for analytical models that can demonstrate how the designed policy instruments on the energy side will transfer into the corresponding energy consumption output in the transportation sector. The lack of such quantitative tools has become a key barrier for policy-makers to identify holistic solutions addressing energy consumption issues in multimodal transportation systems. Thus, the key contribution of the proposed study is to partly bridge this gap, and develop analytical models to address the issues relevant to the design of energy saving strategies in the transportation sector by integrating transportation mobility, energy consumption, and policy instruments. It can provide a capability for policy-makers in both the transportation and energy sectors to systematically understand the interactions between energy consumption and transportation.

## PRELIMINARIES

The proposed study addresses energy consumption for intercity transportation incorporating interactions among transportation suppliers (mode service characteristics in terms of fare, waiting time, travel time, and frequency), traffic demand (mode choice decisions), and policy-makers (long-term policies to mitigate system energy usage). To formulate the associated analytical model, this section introduces the abstracted Intermodal Network (*IN*) and the associated variables and parameters in this study.

### Intermodal Network

The intercity transportation modes considered are private auto, transit (including metro), rail and air. The passenger traffic demand is divided into business and nonbusiness trips considering their different attitudes to the mode service characteristics. The intermodal transportation network available between an origin  $o$  and destination  $s$  is abstracted as an Intermodal Network (*IN*), in which the node set  $N$  corresponds to the cities or the mode transfer terminals, and the link set  $E$  represents the available connections through the various transportation modes between the terminals or cities. Here, the individual links in the proposed intermodal network are differentiated by the connectivity associated with the mode rather than the specific physical connections on the ground or the departure/arrival schedules, since the proposed study emphasizes mode choice and route selection in an aggregated manner. Correspondingly, the variables or parameters associated with each link in the intermodal network are representative of all possible scheduled service time slots in a day or all available physical links in a network. For example, an *IN* employs only one link with an expected travel time and expected monetary cost to represent various road-based paths by private auto between an origin and a destination. Similarly, an expected travel fare is used to represent the travel cost for a certain transportation mode though in reality fares may vary by departure/arrival times for scheduled service systems. Such an abstraction mechanism is a deliberate approach to enable integration and capture the interactions among the disparate high-level problem characteristics addressed earlier. Since the interactions between policy instruments, transportation system and energy consumption are complex, understanding the trends at a high level can then be translated to specific supply action needs in the transportation system to bridge potential gaps at the ground level; for example, by increasing scheduled service frequency for specific segments or by adding a physical link or an intermodal transfer facility to directly connect two modes.

## Variables and Parameters

This section defines the decision variables and parameters associated with intercity multimodal transportation systems. First, the indices are introduced: (1)  $i$ : index of modes,  $i \in I = \{1, 2, 3, 4\}$  where 1, 2, 3, and 4 represent private auto, transit, rail, and air mode, respectively; (2)  $l$ : index of link,  $l \in \{1, 2, \dots, L\}$  where  $L$  is the total number of links; (3)  $h$ : index of intermodal paths,  $h \in \{1, 2, \dots, H\}$ , where  $H$  is the total number of intermodal paths; and (4)  $\kappa$ : index of traveler class,  $\kappa \in \{1, 2\}$ , where 1 and 2 refer to business trip and nonbusiness trip, respectively;  $K = 2$  is the total number of traveler classes. Next, the variables and parameters associated with each link in the  $IN$  are provided from the supplier, demand, and supply-demand interaction perspectives.

From the transportation supplier side, this study considers that each transportation mode  $i$  has a fixed capacity  $p_{li}^0$ . It consumes  $\delta_{li}$  gallons of gasoline to carry the travelers on the mode across a link  $l$ . In addition, each transportation mode  $i$  has current frequency  $r_{li}^0$  times per day, and travelers need to wait  $w_{li}^0$  units of time, and travel  $t_{li}^0$  units of time to cross link  $l$  with fare/cost  $c_{li}^0$ . Let  $p = \{p_{li}^0\}$ ,  $\delta = \{\delta_{li}\}$ ,  $c^0 = \{c_{li}^0\}$ ,  $r^0 = \{r_{li}^0\}$ ,  $w^0 = \{w_{li}^0\}$ , and  $t = \{t_{li}^0\}$  represent their corresponding sets. The overall network capacity is represented by the product of  $rp$ . To capture the variations in service due to changes in traffic demand, four variables  $c_{li}$ ,  $r_{li}$ ,  $w_{li}$ , and  $t_{li}$  are introduced to denote the variations associated with travel fare/cost, service frequency, waiting time, and travel time, respectively, of mode  $i$  on link  $l$ .  $c = \{c_{li}\}$ ,  $r = \{r_{li}\}$ ,  $w = \{w_{li}\}$ , and  $t = \{t_{li}\}$  are used to represent the corresponding sets. For an available intermodal path in an  $IN$ ,  $T_h$  and  $C_h$  are used to denote travel time and fare, respectively, on path  $h$ ; and  $T$  and  $C$  are used to represent their corresponding sets. The intermodal mode-link-path incidence matrix is defined as  $M = \{m_h\}_H$  with sub-matrix  $m_h = [m_h^{li}]$ , where  $m_h^{li} = 1$  if intermodal path  $h$  uses mode  $i$  on link  $l$ , otherwise  $m_h^{li} = 0$ . For example, matrix  $m_1$  for an intermodal path 1 is:

$$\begin{array}{l}
 \begin{array}{c} 1 \quad 2 \quad 3 \quad 4 \\
 1: o - 1 \\
 2: 1 - 2 \\
 3: o - 3 \\
 4: 3 - 4 \\
 5: 4 - s \\
 6: 2 - s \end{array} \begin{array}{c} \left[ \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right],
 \end{array}
 \end{array}$$

where the rows/columns represent the corresponding links/modes included in the path. Then, path 1 in the  $IN$  implies that a traveler starts from origin (o) and takes transit system link ( $m_1^{12} = 1$ ) to the first terminal 1, transfers to the rail system and arrives at terminal 2 ( $m_1^{23} = 1$ ), and finally chooses the auto mode (for example, a taxi) ( $m_1^{61} = 1$ ) to reach the destination  $s$ . Using travel fare as an example, the fare for an individual intermodal path  $h$  can then be computed as  $C_h = \sum_{(i,l)} c_{li} * m_h^{li}$ , where the defined rule of the matrix operation is that the element  $C_h$  in the array  $C$  is equal to the sum of the products of the elements  $c_{li}$  and  $m_h^{li}$  as  $i = 1, 2, \dots, I$  and  $l = 1, 2, \dots, L$ .

From the traffic demand side, the intercity trips are grouped into two classes, business trips (B) and nonbusiness trips (NB). The corresponding traffic demand is denoted by  $D^\kappa$ ,  $\kappa = 1, 2$ . A linear utility function factoring travel time, waiting time, travel fare, and service frequency is used to quantify the satisfaction of an individual traveler in the context of choosing an intermodal path  $h$ . Accordingly,  $a_t^\kappa$ ,  $a_w^\kappa$ ,  $a_c^\kappa$ , and  $a_r^\kappa$  are the parameters to represent the weights of travel time, waiting time, travel fare, and service frequency for traffic demand of class  $\kappa$  in the utility function. The traffic demand of class  $\kappa$  on path  $h$  is represented by variable  $x_h^\kappa$ .  $x = \{x_h = \sum_{\kappa=1}^K x_h^\kappa\}_{h=1}^H$  represents the corresponding path flow set. Accordingly,  $y_{li}$  is used to represent the traffic flow of mode  $i$  on link  $l$ , and  $y = \{y_{li}\}$  represents its set. This study assumes a deterministic O-D demand ( $D = \sum_{\kappa=1}^2 D^\kappa$ ).

To capture the supply-demand (S-D) interactions, this study considers that if the traffic demand for mode  $i$  (only for transit, rail, and air) on link  $l$  is greater than  $\varepsilon \times 100\%$  of the seat capacity, then traffic demand is sufficient and the binary variable  $z_{li} = 1$ ; otherwise it is set to 0. Similarly, if the traffic

demand is less than  $\epsilon \times 100\%$  of the seat capacity, then traffic demand is insufficient and the binary variables  $s_{li} = 1$ , otherwise 0. For a given link,  $z_{li}$  and  $s_{li}$  cannot be 1 at the same time but it is possible that both  $s_{li}$  and  $z_{li}$  are equal to zero, which means the traffic demand is neither clearly sufficient nor clearly insufficient. According to the state of S-D relationship, transportation suppliers (for transit, rail, and air modes) may adjust their travel fare and service frequency to sustain their profit requirements or service level. Accordingly, this study defines variables  $\alpha_{li}^1$  and  $\alpha_{li}^2$  to represent the incremental and decremental rate of travel fare for mode  $i$  on link  $l$ , respectively. Similarly,  $\beta_{li}^1$  and  $\beta_{li}^2$  are used to represent the incremental and decremental rate of service frequency for mode  $i$  on link  $l$ , respectively. From the demand side, service level adjustments will lead to ridership changes.  $\zeta_{li}$  and  $\eta_{li}$  are used to represent the increasing or decreasing elasticity of travel fare for mode  $i$  on link  $l$ , respectively;  $\zeta$  and  $\eta$  represent their corresponding sets. Similarly,  $\theta_{li}$  and  $\vartheta_{li}$  are used to represent the increasing or decreasing elasticity of service frequency for mode  $i$  on link  $l$ , respectively;  $\theta$  and  $\vartheta$  represent their corresponding sets. From a supply perspective, the service adjustment will impact profits;  $\pi_{li}^r$  and  $\pi_{li}^c$  are used to represent the profit thresholds associated with service frequency and travel fare adjustments, respectively. Note that the profit thresholds partially reflect the impacts of policy instrument on traffic supply. For example, the profit threshold of transit mode is usually low or even zero since policy-makers usually provide subsidy to transit so that it can sustain its normal operation with no profit.

## MATHEMATICAL MODEL

### Interaction Analyses

The relationships among travelers, transportation suppliers, and policy-makers are first analyzed at the conceptual level. Individual travelers are assumed to determine their paths before departure based on the service characteristics of intermodal paths and their budget limits. This pre-determined trip plan is assumed to not change during the travel. Transportation suppliers supply the services. Their operational decisions are made mainly under the need to sustain their operational profit requirements (such as for rail and air) or service level (such as for public transit). The policy-makers aim to develop a long-term plan/policy that emphasizes the mitigation of energy consumption in the intercity multimodal transportation network. The decision mechanisms of the three players are independent in the short-term in that individual traveler trip plans and transportation supplier operational strategies may likely not consider network energy consumption. Thereby, in the short-term, the actions of policy-makers may not cause shifts in individual traveler mode/route choices or directly influence the operations of transportation suppliers. However, consistent with systems perspectives in transportation, the three players may interact in the long-term through a feedback-loop process. Thereby, the status of energy consumption can serve as a feedback/input to improve previous system policy implementation on the transportation network, and change the multimodal transportation system as a long-term effect. Incorporating the aforementioned interactions, the mathematical model discussed hereafter predicts the minimum system energy consumption under a policy instrument given that the traffic demand is optimally distributed among the transportation modes consistent with the modal utilities of travelers, and the transportation suppliers take collaborative actions to foster the energy consumption efficiency objective under acceptable profits or service levels.

### Bi-level Optimization Model

This section proposes a bi-level decision framework  $M$ . The upper level model aims to minimize the system energy consumption subject to transportation suppliers' operational constraints related to frequency and travel fare, and the traffic demand distribution to paths resulting from the lower level model, which seeks to maximize the intercity trip utilities of travelers computed using a utility function. Hence, the interactions between the multimodal transportation suppliers and intercity traffic demand are

considered under the goal of minimizing energy consumption at the system level. The optimal solution provides a traffic demand distribution to paths that results in the minimum system energy consumption under the specific policy implementation.

Before describing the mathematical model, two important considerations incorporated in the modeling process are discussed. First, instead of maximizing the profits of the transportation mode suppliers, the proposed model limits them to within acceptable ranges (represented by constraints (8) and (12) in the formulation below) since making profit is not a requirement in some (public sector) transportation modes, such as public transit systems. By contrast, limiting the energy consumption or greenhouse emissions may be more appealing to policy-makers. Hence, some policy options such as subsidies are usually provided to those traffic modes so that they can operate without being overly concerned about profits. Second, the proposed model implicitly incorporates the effects of policy instruments due to the lack of well-defined models to represent the relationships between the transportation mode services and the policy instruments. Thereby, instead of directly factoring the policy instruments as decision variables, this study explores the effect of a policy instrument on the system energy consumption through pertinent parameters on the transportation suppliers' side such as travel fare and profit threshold. The evolution of a policy instrument will change the associated parameters and lead to a trajectory that reflects optimal energy consumption solutions. Hence, the impacts of the policy instrument on the network energy consumption and the traffic demand distribution to paths are captured. The mathematical model is as follows:

$$\begin{aligned}
 \text{M} \quad & \text{Min} \sum_l \sum_i r_{li} \delta_{li} & (1) \\
 \text{s.t} \quad & \epsilon r_{li} p_{li} - y_{li} \geq -\sum_{\kappa=1}^K D^\kappa z_{li}, i \in \{2, 3, 4\}, l \in L & (2) \\
 & \epsilon r_{li} p_{li} - y_{li} \leq \sum_{\kappa=1}^K D^\kappa s_{li}, i \in \{2, 3, 4\}, l \in L & (3) \\
 & z_{li} + s_{li} \leq 1, i \in \{2, 3, 4\}, l \in L & (4) \\
 & c_{l1} = c_{l1}^0, l \in L & (5) \\
 & c_{li} = (1 + \alpha_{li}^1 z_{li} - \alpha_{li}^2 s_{li}) c_{li}^0, i \in \{2, 3, 4\}, l \in L & (6) \\
 & b_{li}^c \leq c_{li} \leq u_{li}^c, i \in I, l \in L & (7) \\
 & \zeta_{li} \alpha_{li}^1 z_{li} + \eta_{li} \alpha_{li}^2 s_{li} \geq (z_{li} + s_{li}) \pi_{li}^c, i \in \{2, 3, 4\}, l \in L & (8) \\
 & r_{l1} = y_{l1}, l \in L & (9) \\
 & r_{li} = (1 + \beta_i^1 z_{li} - \beta_i^2 s_{li}) r_{li}^0, i \in \{2, 3, 4\}, l \in L & (10) \\
 & b_{li}^r \leq r_{li} \leq u_{li}^r, i \in I, l \in L & (11) \\
 & \theta_{li} \beta_i^1 z_{li} + \vartheta_{li} \beta_i^2 s_{li} \geq (z_{li} + s_{li}) \pi_{li}^r, i \in \{2, 3, 4\}, l \in L & (12) \\
 & \alpha_{li}^1 \geq 0, i \in I, l \in L & (13) \\
 & \alpha_{li}^2 \geq 0, i \in I, l \in L & (14) \\
 & \beta_{li}^1 \geq 0, i \in I, l \in L & (15) \\
 & \beta_{li}^2 \geq 0, i \in I, l \in L & (16) \\
 & z_{li} \in \{0, 1\} \text{ and } s_{li} \in \{0, 1\}, l \in L, i \in I & (17) \\
 & y = \sum_{\kappa=E}^K m_h x_h^\kappa & (18) \\
 & \rho_h^\kappa = a_c^\kappa C_h + a_w^\kappa W_h + a_t^\kappa T_h + a_r^\kappa R_h, h \in H, \kappa \in K & (19) \\
 & x \in \text{argmax} \{ \sum_{h=1}^H \sum_{\kappa=1}^K \rho_h^\kappa x_h^\kappa \} & (20) \\
 \text{s.t} \quad & \sum_{h=1}^H x_h^\kappa = D^\kappa, \kappa \in K & (21) \\
 & \sum_{h=1}^H (m_h^{li} \sum_{\kappa=1}^K x_h^\kappa) \leq r_{li} p_{li} & (22)
 \end{aligned}$$



$$x_h^\kappa \geq 0, h \in H, \kappa \in K \quad (23)$$

$$t_{li} = t_{li}^0, i \in I, l \in L \quad (24)$$

$$w_{li} = w_{li}^0, i \in I, l \in L \quad (25)$$

In the formulation of M, Equation (1) represents the objective function of the optimization model in the upper level model, which seeks to minimize the energy consumption over the multimodal transportation network subject to the constraints from Equation (2) to Equation (25), among which Equation (2) to Equation (19) represents the upper level constraints, and Equation (20) to Equation (25) represents the model in lower level.

The constraints from Equations (2) to (4) indicate whether the traffic demand of mode  $i$  on link  $l$  is sufficient or insufficient, according to its traffic capacity. Three possible cases are captured. Case 1,  $z_{li} = 1$ , and  $s_{li} = 0$ , indicates that traffic demand for mode  $i$  on link  $l$  is sufficient. Correspondingly, the transportation suppliers will either increase travel fare to moderate traffic demand or increase service frequency (please see Equation (6) and Equation (10), respectively) so that transportation suppliers can sustain current service level and profit. Case 2,  $z_{li} = 0$ , and  $s_{li} = 1$ , indicates that the traffic demand for mode  $i$  on link  $l$  is insufficient. Then, transportation suppliers will appropriately decrease travel fare or service frequency to sustain their profits (please refer to Equation (6) and Equation (10), respectively). Case 3,  $z_{li} = 0$ , and  $s_{li} = 0$ , indicates a state where the traffic demand is neither clearly sufficient nor insufficient. Thereby, transportation suppliers may apply a flexible strategy; the optimal strategy is determined by the bi-level optimization model itself. In addition, it is assumed that the operational decisions of the transportation suppliers of transit, rail and air modes need to sustain their profit requirements in all of the above three cases. The profit thresholds of different transportation modes are influenced by the policy instruments. For example, a public transit mode may have zero profit due to the subsidy provided by policy-makers. This leads to two more constraints: Equation (8) and Equation (12). Correspondingly, Equation (7) and Equation (11) provide the feasible ranges of the travel fare and service frequency.

Equation (5) indicates that the travel fare of private auto is not changed by traffic demand, but its “service frequency” can be easily increased to satisfy all traffic demand, as indicated by Equation (9). That is, the “service frequency” for private auto is simply its flow on that link. Equations (13) to (17) specify the feasible regions for the decision variables in the first level. Equation (18) maps the path flows to the link flows. Equation (19) measures the utility of path  $h$  for the travelers in class  $\kappa$ , which also denotes the preference of those travelers for that path.

Equation (20) represents the objective function of the lower level model. It aims to maximize the utilities of all travelers in the intercity trips. Equation (21) and Equation (22) represent the traffic flow conservation constraint and the capacity constraint, respectively. Equation (23) indicates that the feasible path flow is nonnegative. Equation (24) and Equation (25) identify the travel time and waiting time, respectively, for each mode on an individual link. As this study addresses a long-term planning and policy context, its focus is on determining the system optimal solution within the capacity of each transportation mode. Hence, the traffic congestion associated with the traffic demand assignment is not considered. Correspondingly, free flow travel times for the private auto and transit, and static waiting times for transit, rail, and air modes, are employed.

## SOLUTION METHOD

The proposed bi-level optimization model is characterized by a mixed integer nonlinear program at the upper level, and a linear program at the lower level. A bi-level optimization model is generally NP problem, precluding a polynomial time algorithm to solve the proposed model. Based on the characteristics of the bi-level optimization model, this study develops an efficient customized branch-and-bound solution methodology in this section.

First, the binary integer variables (such as  $z$  and  $s$ ) are relaxed as continuous variables with the lower bound equal to zero and the upper bound equal to one. Next, the relaxed bi-level optimization model is transformed into a mathematical program with complementarity constraints (MPCC) by substituting the linear program at the lower level using its KKT conditions (which represent the necessary and sufficient optimality conditions). Then, the optimal solution of the relaxed bi-level optimization model can be obtained by solving the MPCC model. A customized branch-and-bound algorithm is developed for this purpose; it selectively enumerates the possible combinations of the binary integer variables in the upper level to search for the optimal solution of the bi-level optimization model. Under each branch of branch-and-bound algorithm, a standard MPCC model is solved.

The transformation of the mixed integer nonlinear bi-level optimization model to the equivalent MPCC model is as follows. The Lagrangian function of the linear program at the lower level is written as Equation (26), where  $\lambda$ ,  $\mu_{li}$ , and  $\gamma_h^k$  are the Lagrangian coefficients.

$$\mathcal{L}(x, \lambda, \mu, \gamma) = \sum_{h=1}^H \sum_{\kappa=1}^K \rho_h^{\kappa} x_h^{\kappa} - \sum_{\kappa=1}^K \lambda^{\kappa} (\sum_{h=1}^H x_h^{\kappa} - D^{\kappa}) - \sum_{l=1}^L \sum_{i=1}^I \mu_{li} (\sum_{h=1}^H (m_h^{li} \sum_{\kappa=1}^K x_h^{\kappa}) - r_{li} p_{li}) + \sum_{h=1}^H \sum_{\kappa=1}^K \gamma_h^{\kappa} x_h^{\kappa} \quad (26)$$

Through the Lagrangian function  $\mathcal{L}$ , the KKT conditions of the linear program are derived and given in Equations (29) to (31). The bi-level optimization model  $M$  is then re-written as a MPCC model by substituting the linear program at the lower level by its KKT conditions, subject to the integer variables in Equation (17) being relaxed to the continuous variables shown in Equations (27) and (28). The formulation of the MPCC model, where the unchanged constraints in the upper level of  $M$  are denoted by  $M^1$ , is as follows.

$$\text{MPCC} \quad \text{Min} \sum_l \sum_i r_{li} \delta_{li}$$

$$\text{s.t} \quad M^1$$

$$0 \leq z_{li} \leq 1, l \in L, i \in I \quad (27)$$

$$0 \leq s_{li} \leq 1, l \in L, i \in I \quad (28)$$

$$\rho_h^{\kappa} - \lambda^{\kappa} - \sum_{l=1}^L \sum_{i=1}^I \mu_{li} m_h^{li} + \gamma_h^{\kappa} = 0, h \in H, \kappa \in K \quad (29)$$

$$\sum_{h=1}^H x_h^{\kappa} = D^{\kappa}, \kappa \in K \quad (30)$$

$$0 \leq \gamma_h^{\kappa} \perp x_h^{\kappa} \geq 0, h \in H, \kappa \in K \quad (31)$$

$$0 \leq \mu_{li}^l \perp r_{li} p_{li} - \sum_{h=1}^H (m_h^{li} \sum_{\kappa=1}^K x_h^{\kappa}) \geq 0, i \in I, l \in L \quad (32)$$

## CASE STUDY

This section presents a case study to illustrate the applicability of the proposed methodology, and then discusses the associated insights.

### Experiment Setup

An intercity trip from Lafayette, Indiana to Washington, D.C. is used to illustrate the applicability of the proposed mathematical model. As shown in Figure 1, the corresponding  $IN$  includes the nodes labeled by the cities Indianapolis, Pittsburgh and Washington, D.C., and the airports IND, BWI, IAD and DCA where travelers can switch transportation modes, to complete the trip. 22 intermodal/multimodal paths are included in the experiments. Each path is presented as a chain of links and a chain of modes. For example, path 1 is presented as the chain of links (1-4-8) with the chain of modes (private auto, air, transit).

Associated with the *IN* in Figure 1, input data (travel time, travel fare, waiting time, etc.) were obtained online for each link through websites such as Greyhound, CDA, airports, Wikipedia, and the existing literature. Table 1a and Table 1b summarize the input data for this case study. The utility function in Table 2 is based on the traffic demand models by Koppelman (15).

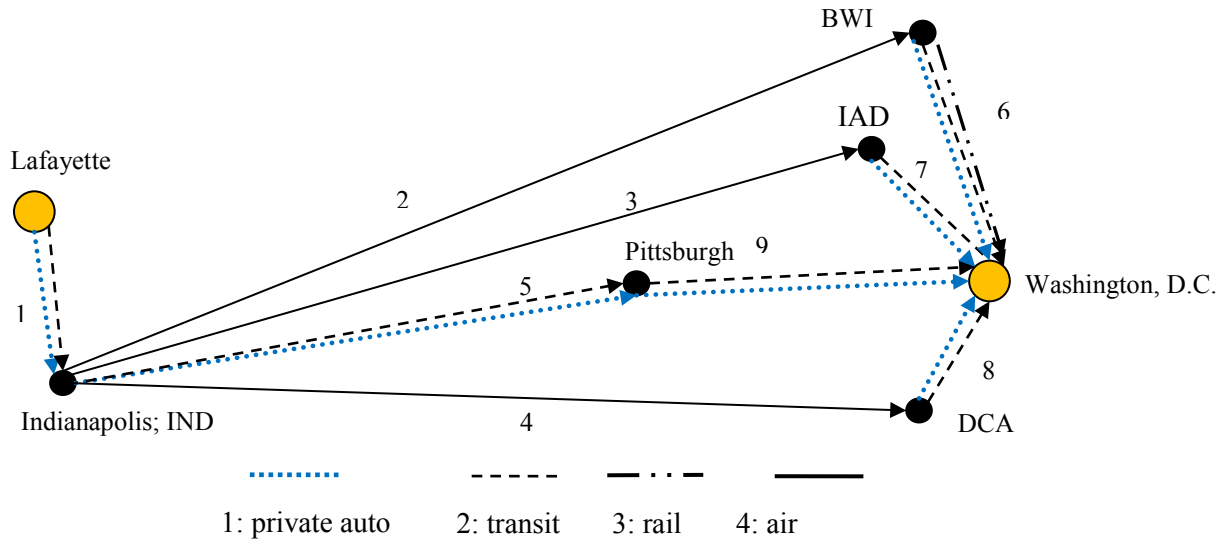


FIGURE 1 . The intermodal network from Lafayette, Indiana to Washington, D.C.

TABLE 1 Input Data Set

Link	Mode	$p^0$	$\delta$	$c^0(\$)$	$t^0(\text{h})$	$r^0$	$w^0(\text{h})$	
1	1	1	5	15	1	-	0	
1	2	40	3	20	1.9	6	0.333	
2	4	100	240	100	1.667	1	1.5	
3	4	120	230	110	1.1	2	1.5	
4	4	120	235	120	1.333	2	1.5	
5	1	1	25	70	8.333	-	0	
5	2	80	18	100	11.667	1	0.5	
6	1	1	8	35	0.667	N	0	
6	2	40	7	20	0.833	10	0.25	
6	3	90	4	10	0.5	12	0.333	
7	1	1	6	25	0.583	-	0	
7	2	40	5	15	0.667	8	0.25	
8	1	1	5	10	0.333	-	0	
8	2	60	3	5	0.333	8	0.333	
9	1	1	20	35	5	-	0	
9	2	60	10	50	6.667	1	0.333	
Link	Mode	$\theta \& \beta$	$\zeta$	$\eta$	$b^r$	$u^r$	$b^c$	$u^c$
1	1	-	-	-	0.001	-	10	40
1	2	0.4	0.2	0.2	1	8	15	35
2	4	0.4	0.9	0.9	1	2	80	150
3	4	0.4	0.9	0.9	1	3	90	150
4	4	0.4	0.9	0.9	1	3	100	160

5	1	-	-	-	0	-	40	90
5	2	0.6	0.2	0.2	0	2	90	150
6	1	-	-	-	0	-	30	40
6	2	0.4	0.2	0.2	1	12	18	35
6	3	0.5	0.5	0.5	1	16	5	20
7	1	-	-	-	0	-	20	35
7	2	0.2	0.2	0.2	1	10	10	20
8	1	-	-	-	0	-	5	20
8	2	0.2	0.2	0.2	1	12	1	10
9	1	-	-	-	0	-	30	60
9	2	0.6	0.2	0.2	0	2	45	60

“-”: not applicable.

TABLE 2 Coefficients in Utility Function

Coefficients	Business trip $\kappa = 1$	Nonbusiness Trip $\kappa = 2$
$a_r^\kappa$	0.0006	0.0399
$a_c^\kappa$	-0.00256	-0.046
$a_t^\kappa$	-0.046	-0.00193
$a_w^\kappa$	-0.0157	-0.0066

### Effect of Energy Consideration

The effect of energy consumption consideration on the optimal traffic demand distribution among multimodal paths is analyzed. Accordingly, two experimental scenarios are addressed. In the first scenario, without considering the system energy consumption, traffic demand is assigned so as to maximize the utilities of travelers' trips. That is, the optimal traffic demand distribution is obtained by solving the lower level of the bi-level optimization model  $M$ . Figure 2a illustrates the associated results. In the second scenario, the traffic demand is distributed by considering the system energy consumption, the suppliers' operation, and the utility of travelers' trips. Accordingly, the system optimal solution is obtained by solving the bi-level optimization model  $M$ . Figure 2b shows the experiment results. All of the experiments are conducted under different combinations of business (B) and nonbusiness (NB) trips.

Figure 2 indicates that when energy consumption is not considered, traffic demand is primarily distributed to paths that include private auto for both business trip and non-business trip (see Figure 2(a)). As energy consumption becomes one of the considerations (that is, a policy instrument is implemented on the suppliers, which will trigger changes on service and profit levels), intermodal paths that include transit and rail are highly recommended for all traffic demand combinations (see Figure 2(b)). Based on this traffic demand shift when energy consumption becomes a consideration (EC\_opt), the energy consumption is significantly reduced (see Figure 2(c)). It also illustrates that when energy consumption is not considered (EC\_org), NB trips skew towards private auto usage (see Figure 2(a)) leading to increased energy consumption. To realize this shift, the travel fares of airline and rail links need to be increased so that the suppliers can sustain acceptable profits with fewer customers. Accordingly, most transit links are recommended to cut their current travel fares to attract more customers. These travel fare changes indicate the required actions from transportation supplier side (see Figure 2(d)).

The above results first illustrate the capability of the proposed methodology to predict how the designed policy instruments related to energy consumption will translate to the corresponding traffic demand shift, transportation suppliers' actions as well as energy consumption output in the transportation sector. In addition, the systematic analyses suggest that shifting traffic demand from private auto to public

transit or rail represents the key approach to mitigate system energy consumption, which is consistent with the commonly suggested solutions to this problem. Further, the results reinforce a well-known key deficiency related to current national energy conservation strategies. That is, most of the energy conservation strategies are focused on transportation suppliers (such as mandates for “fuel-efficient” vehicles), while the private auto travel mode is relatively untouched for multiple reasons. Thereby, many travelers use private auto instead of the energy-conserving public transportation alternative. It points to the need to explore policy instruments (such as increasing gasoline tax, emission tax, etc.) through systematic quantitative analyses that capture the system-level interactions and emergent phenomena to develop robust strategies that are deployable and effective. The results also indicate that the intermodal trips that include the air mode are strongly recommended by the model in both scenarios, though current studies (13) indicate that the air mode has relatively low energy efficiency in shipping passengers. This interesting finding implies that the energy efficiency of intermodal paths may perform differently compared to the energy efficiency of individual modes. Therefore, it may be more meaningful to consider using the energy efficiency characteristics of intermodal paths instead of those of individual transportation modes as the basis to formulate system policies to serve the traffic demand under acceptable service levels.

### **Impact of Highway Travel Time on System Energy Consumption**

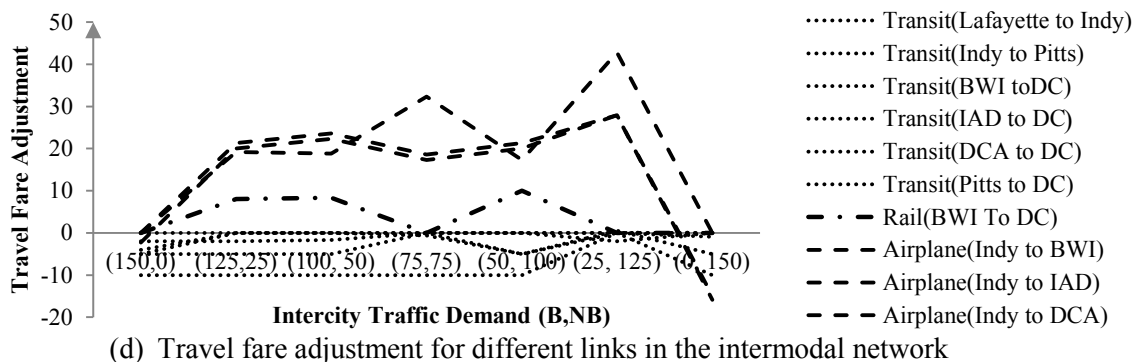
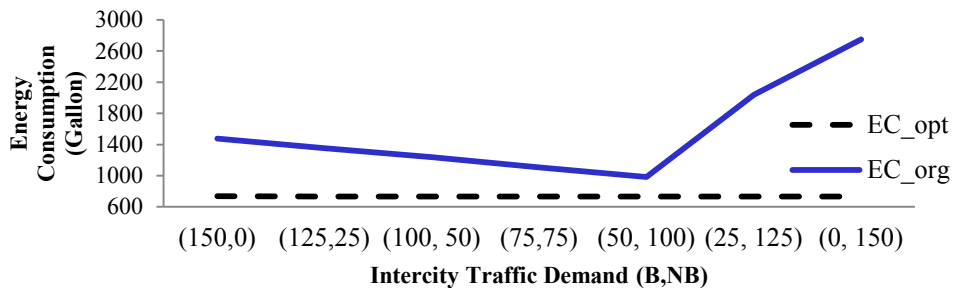
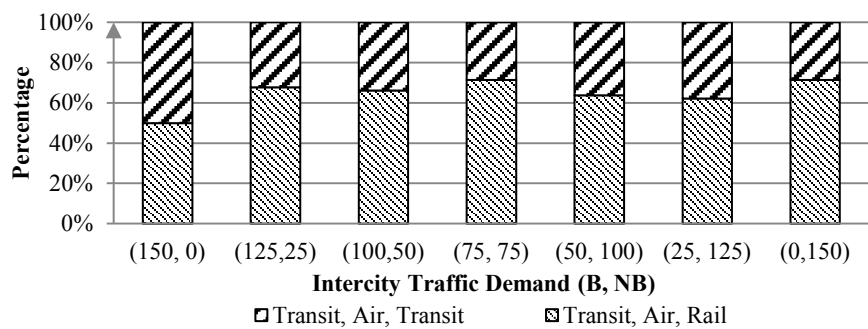
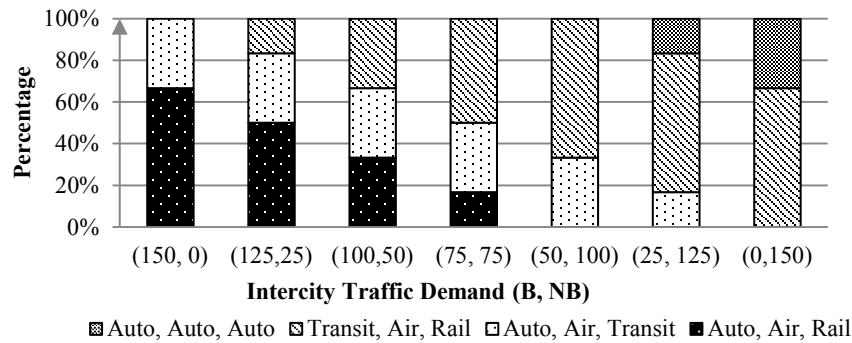
The next set of experiments investigates the impact of highway congestion on the system traffic demand distribution under the goal of energy saving. They also demonstrate the capability of the proposed methodology to track the effect of traveler preferences, network structure, and supplier operational scheme on system energy consumption.

Three scenarios of highway congestion are considered; the highway travel time  $t$  is increased by 1.25, 1.5, or 1.75 times its original value  $t^0$ . The associated optimal distributions of traffic demand to paths are summarized in Figure 3, where the  $x$ -axis in each figure represents the traffic demand under different combinations of business and nonbusiness trips (B, NB), and the  $y$ -axis denotes the share of the corresponding intermodal travel option (path). Four intermodal paths cover all the traffic demand in each scenario: path 5 (node chain: 1-2-6; mode chain: transit, air, and transit), path 6 (node chain: 1-2-6; mode chain: transit, air, and rail), path 10 (node chain: 1-3-7; mode chain: transit, air, and transit), and path 14 (node chain: 1-4-8; transit, air, and transit). Typically, Paths 6 and 14 entail significant ridership, and Paths 5 and Path 10 have lesser ridership.

As the highway becomes more congested (from the scenario in Figure 3(a) to the scenario Figure 3(c)), the ridership of Path 14 decreases while that of Path 6 increases over different traffic demand compositions. It indicates that some traffic demand shifts from path 14 (transit, air, and transit) to Path 6 (transit, air, and rail) due to the highway traffic congestion. Also, when the highway traffic is moderately congested (when highway travel time is 1.25 or 1.5 times  $t^0$ ) and the business trip represents the main traffic demand, Path 14 has higher ridership than Path 6 (see Figures 3(a) and (3b)). However, when highway becomes even more congested (Figure 3(c)) and nonbusiness trips are represent the main traffic demand, Path 6 is more preferred. These two observations indicate that when the highway becomes more congested, policies to improve rail service can attract travelers to the intermodal path that includes the rail mode. It would potentially represent a good solution to satisfy the requirements related to both traffic demand mobility and system energy consumption.

Figure 4 compares the four intermodal paths in terms of travel time, travel fare, and energy consumption. While Paths 5, 10, and 14 have the same transportation mode chain (transit, air, and transit), their characteristics vary. Figure 4(a) illustrates that Paths 14, 10, and 6 have similar travel times, and perform better than Path 5. Figure 4(b) indicates that Path 6 requires a lower travel fare than Paths 5, 14, and 10 (the most expensive one). Thereby, Paths 5 and 10 have apparent disadvantages in travel time and travel fare, respectively. Therefore, to balance the benefits in terms of travel time and fare, travelers may prefer Path 6. Figure 4(c) shows that Paths 10 and 14 are attractive from the energy savings perspective,

with Path 10 having the least energy consumption. Hence, there are tradeoffs involved when all three factors are considered.



**FIGURE 2 Effect of energy consideration**

Neither Path 6 nor Path 10 can individually lead to the optimal solution of the proposed bi-level optimization model when the tradeoffs between traveler preferences (represented by trip utility) and system energy consumption are systematically considered in an integrated framework. Consequently, the optimal traffic demand distribution to intermodal paths resulting from the bi-level model assigns more traffic demand to Paths 6 and 14 rather than Paths 5 and 10, for all scenarios. Furthermore, since Path 6 has a travel fare advantage over Path 14 as the proportion of nonbusiness traffic demand increases, some traffic demand is shifted to Path 6 as non-business trips are more sensitive to travel fare (see the utility function in Table 2). Also, as the traffic on the highway becomes more congested, the advantage of Path 14 relative to travel time decreases; correspondingly, more traffic is assigned to Path 6.

The detailed analyses indicate the difficulty of developing energy saving strategies for multimodal transportation networks. They also illustrate the capability of the proposed mathematical model to enable policy-makers to track the evolution of system energy consumption along with the variation of different factors such as traffic mode energy consumption efficiency, network structure, highway traffic conditions, travel fare, etc. The model can also aid policy-makers to identify interaction effects, enabling them to develop more robust strategies for energy conservation in the transportation sector.

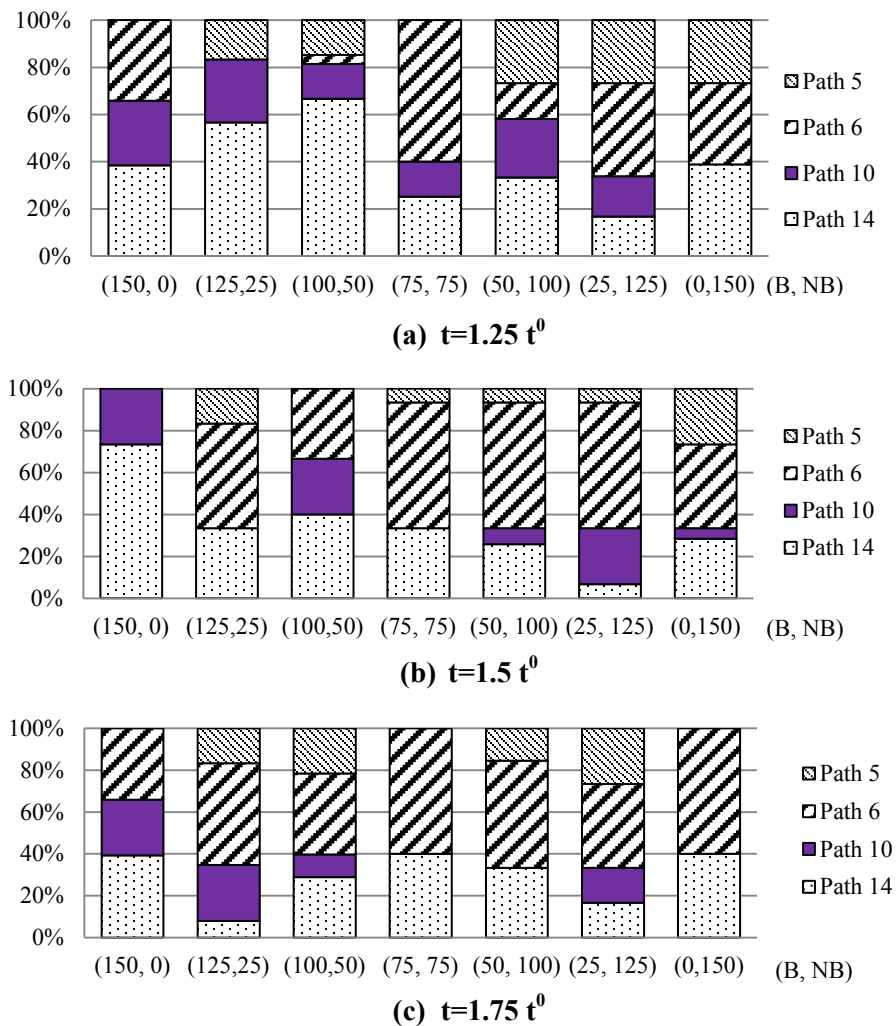
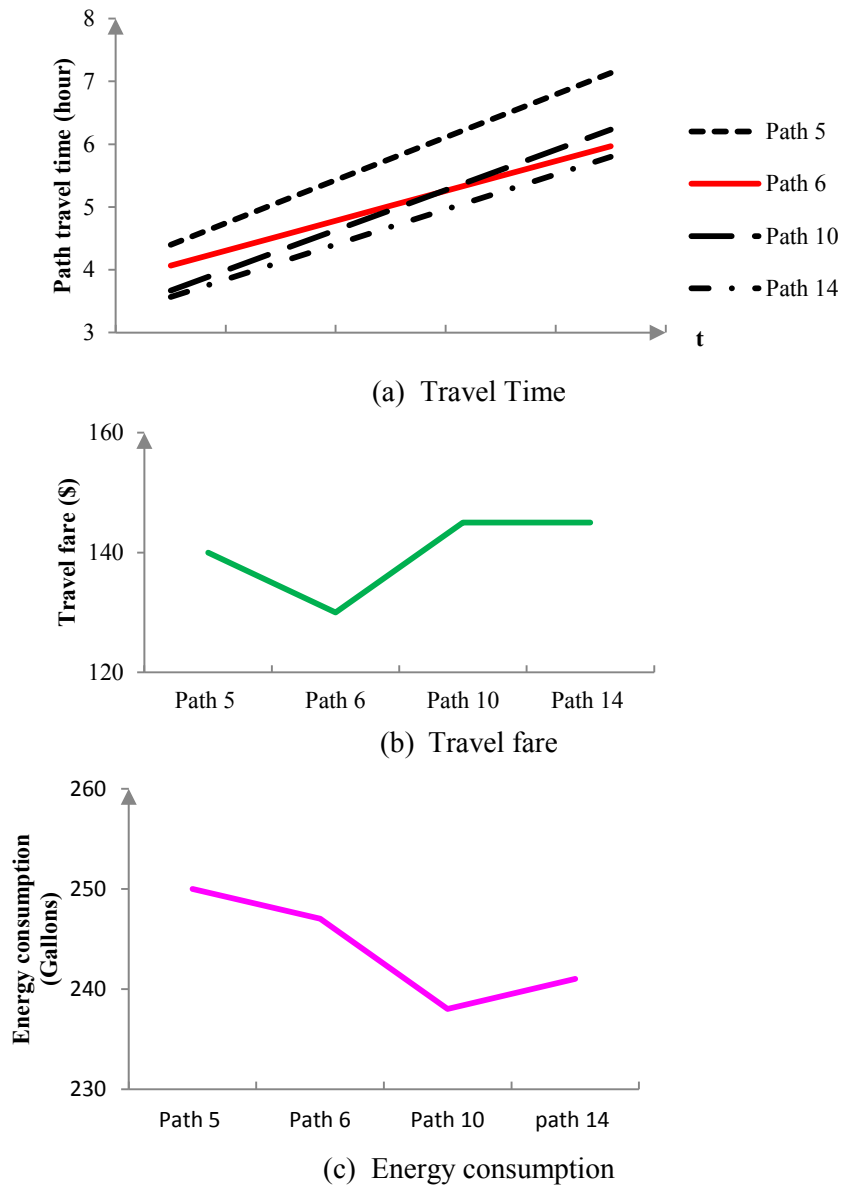


FIGURE 3 Traffic flow distribution among paths



**FIGURE 4** Travel time, travel fare, and energy consumption of intermodal paths 5, 6, 10, and 14

**CONCLUDING COMMENTS**

This study proposes a quantitative and systematic methodology for policy-makers to enhance energy consumption efficiency in multimodal intercity transportation systems. A bi-level optimization model is proposed that considers suppliers’ operational constraints and travelers’ mobility requirements. Traveler preferences, transportation suppliers’ operational constraints, as well as the interactions between travelers, suppliers and policy-makers, are incorporated in the modeling process. Multiple transportation modes including private auto, transit, rail, and air are considered, and traffic demand is differentiated into business and nonbusiness trips. The bi-level model is solved using a customized branch-and-bound algorithm with a MPCC model embedded at each branch.

Numerical experiments are conducted using a multimodal traffic network covering some intermodal routes from Lafayette, IN to Washington, D.C. The results reiterate that partly shifting traffic demand



from private auto to transit and rail will significantly reduce energy consumption. The increment of travel fares in most transportation modes along with this shift indicates that the whole community needs to share the costs associated with the energy savings objective. The results also indicate that systematically considering traffic mode energy efficiency, traveler preferences, and network structure will lead to a better energy saving strategy than factoring only the energy efficiency of individual modes. As highway traffic congestion increases, policy instruments which shift more traffic demand to intermodal paths that include the rail mode can potentially satisfy the traffic demand needs and also mitigate system energy consumption.

More importantly, the experiments illustrate that the proposed methodology is able to provide quantitative analyses for system energy consumption and traffic demand distribution among transportation modes under specific policy instruments. It enables policy-makers in both the transportation and energy sectors to analyze the trajectories of system energy consumption and traffic demand shifts along with the evolution of the policy instruments or traffic infrastructure and supply (such as traffic mode services, traffic dynamics, and network structure changes). Thus, the proposed systematic and integrated analytical methodology bridges a key gap related to the ability to study the effects of policy instruments on the corresponding energy consumption output in the transportation sector. It also suggests the perspectives that focus on intermodal path characteristics rather than individual modes in isolation are more meaningful in developing energy-efficient policies.

The proposed methodology provides a platform to study other energy-related issues in multimodal transportation networks. A potential extension is to vary the gas prices in the current mathematical model to explore how gas prices impact the system energy consumption in the transportation sector. Further, the proposed mathematical model can be enhanced to include policy instrument variables so that the interactions between policy-makers and transportation suppliers can be explicitly captured in the mathematical model.

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