Numerical Prediction of Exhaust Fan Tone Noise from High Bypass Aircraft Engines

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The ability to accurately predict fan noise is important in designing and optimizing aircraft engine turbofans for low noise emissions. In this paper, a prediction methodology for exhaust fan tone noise analysis is described and validated against various canonical test cases and NASA Source Diagnostic Test (SDT) data. The prediction process consists of solving Reynolds-Averaged Navier-Stokes (RANS) equations to compute the fan wake, and calculating the acoustic response of the outlet guide vanes (OGV) to the fan wake using linearized Euler equations. Very good agreement is observed between numerical predictions and semi-analytical results for canonical cases. Detailed comparisons against SDT data are presented for unsteady vane pressure and integrated in-duct exhaust noise power levels. Geometric trends for different OGV configurations at various operating conditions are also analyzed.

Nomenclature

\(a_{m,n}\) Complex amplitude of the \((m, n)\) mode
\(c\) Speed of sound
\(C_p\) Coefficient of pressure
\(j\) \(\sqrt{-1}\)
\(k_z\) Complex wave number in axial direction
\(m\) Circumferential mode number
\(n\) Radial mode number
\(N_B\) Number of rotor blades
\(N_V\) Number of stator vanes
\(r_H\) Hub radius
\(r_T\) Tip radius
\(U\) Column vector of primitive flow variables
BPF Blade passing frequency
\(P_{ref}\) Reference acoustic pressure = 20 \(\mu\) Pa
\(PWL_{ref}\) Reference acoustic power = \(10^{-12}\) watts

Symbols
\(\Omega\) Rotor shaft rotation rate (rad/s)
\(\phi\) Phase of a complex variable
\(\psi_{m,n}\) Eigenvector corresponding to the \((m, n)\) mode
\(\sum\) Sum of radial mode power levels

Superscripts
\((a)\) \(\overline{p}(p' + \overline{p}v_z v'_z)\)
\((a)\) \(\overline{p}(p' + \overline{p}v_z v'_z)\)
\((b)\) \(\overline{p}v'_z + \overline{p}v_z\)
\(\overline{Q}\) Time average of variable \(Q\)

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I. Introduction

ROTOR-stator interaction tone noise, often referred to as “fan tone” noise, is a major source of annoyance in high-bypass ratio aircraft engines. This noise source is especially prominent at low speed conditions when fan noise typically dominates over jet noise. Fan tone noise is a result of periodic interaction of the rotor wake with the downstream stators (outlet guide vane, OGV, or strut). During this interaction, the vortical energy in the wake is converted into acoustic energy which radiates both upstream and downstream. While the upstream traveling acoustic waves are partially blocked (and scattered) by the rotor, the downstream traveling waves propagate down the duct relatively unimpeded and radiate through the exhaust.

The phenomenon of fan tone noise generation falls into the general category of unsteady flows through blade rows of axial flow turbomachinery. There are three major approaches to study such problems; viz., classical semi-analytical methods, linearized computational analyses, and nonlinear time-marching computational aeroacoustics (CAA) methods. Small harmonic perturbation assumptions are made in the linearized analyses to obtain a system of equations in the frequency domain. It is generally accepted\textsuperscript{1,3} that these assumptions are valid for the prediction of fan tone noise. Classical analyses further simplify the problem by assuming a uniform background flow. Nonlinear time-marching CAA methods directly solve for the entire flow and acoustic fields by marching solutions forward in time. These methods are typically very computationally intensive and therefore not suitable for design purposes. Classical analyses are computationally inexpensive but they may not be representative of realistic turbomachinery flow as the background flow in the region between the rotor and the vane is highly non-uniform. Linearized analyses can serve to meet the challenges of detailed blade design by providing a compromise between accuracy and computation time.

Examples of classical methods include the 2-D flow analysis by Smith,\textsuperscript{4} and semi-analytical lifting line and lifting surface methods of Namba\textsuperscript{5,6} and Schulten.\textsuperscript{7,8} Linearized Euler equations were numerically solved for two-dimensional flows in the potential-based, cascade analyses of Whitehead\textsuperscript{9} and Hall and Verdon.\textsuperscript{10} An extension of this approach to 3-D flows was presented by Prasad and Verdon\textsuperscript{11} who analyzed wake-stator interaction for flat plate cascades. This approach has also been used to assess the acoustic benefit of sweep and lean in stator vanes. Envia \textit{et al.}\textsuperscript{12} performed a parametric study involving a strip-wise analysis of wake-stator interaction coupled with a 3-D duct Green’s function to estimate in-duct noise levels. In the present work, 3-D linearized Euler equations are solved for harmonic perturbations about non-uniform background flow to predict fan tone noise for a realistic fan configuration. Three stator geometries are investigated and the results are compared against experimental data.

The following section describes the noise prediction process used in this work. Section III presents a validation of the process against semi-analytical results for canonical problems. Section IV presents comparisons of the numerical predictions against SDT experimental data. Comparisons are made for vane unsteady surface pressure and modal exhaust sound power levels, as well as geometric trends for the three different OGV configurations.

II. Noise Prediction Process

Figure 1 shows a flowchart of the numerical prediction process used in the present work. It consists of three steps: (1) calculation of rotor wake by solving the viscous meanflow over the rotor, (2) calculation of background meanflow over the OGV, and (3) simulation of wake impinging on the OGV.

The first step of the process involves the solution of Reynolds Averaged Navier-Stokes (RANS) equations (in rotor frame of reference) to obtain the velocity defect in the wake of the rotor. A two-equation $k-\omega$ turbulence model is employed to correctly capture the mean velocity defect in the wake. Numerically computed wakes are used for the results presented in this paper although experimentally obtained wake profiles can also be employed with the procedure outlined.

In the second step, the viscous meanflow over the OGV is calculated to ensure correct operating conditions in the simulation. An equivalent inviscid meanflow, approximately matching the vane loading with the viscous solution, is then obtained by tuning the inlet boundary conditions. The parameters typically modified to achieve this load matching are inlet total pressure and incidence angle, and typically a few iterations are...
required to obtain a suitable solution. It is important to accurately capture the steady loading on the vane in the inviscid meanflow simulation because of the coupling between steady and unsteady loading on the vane. Figure 2 shows a sample comparison of a load-matched inviscid solution versus the original viscous solution. Note that the coefficient of pressure ($C_p$) profiles for the inviscid solution are shifted down with respect to the viscous solution, but the loading ($\Delta C_p$) matches well.

The third step of the process requires a gust-response simulation using the linearized, unsteady, Euler equations (see Appendix A). The problem is formulated and solved in the frequency domain with focus on the harmonics of the blade passing frequency (BPF). A modified formulation of the three-dimensional non-reflecting boundary conditions developed by Verdon is used in this analysis; the details are available in Appendix A. Appendix A also presents a validation of the numerical solution of the eigensystem.

An inviscid representation of the wake is sought as input to the gust-response calculation. The end-wall boundary layer and tip vortex effects are removed from the wake profile by smoothly extrapolating it in the radial direction near the hub and tip. The inviscid wake is referred to as the “idealized” wake. The
wake defect is calculated by subtracting the circumferentially-averaged meanflow from the idealized wake. A spatial Fourier transformation in the circumferential direction is then performed at each radial grid location to decompose the wake into BPF harmonics. For a constant rotor rotation rate, the circumferential Fourier transform in rotor reference frame is equivalent to a temporal Fourier transform in the stationary frame. A sample comparison of the first BPF harmonic of the viscous and the idealized wake profiles is provided in figure 3.

![Figure 3. Spanwise variation of wake gust velocity components.](image)

The harmonic representation of the idealized wake profile is applied as a perturbation (gust) at the inlet boundary for the linear unsteady calculation. This perturbation is convected downstream by the mean flow where it interacts with the OGV generating upstream and downstream propagating acoustic waves.

Finally, a 3D acoustic mode decomposition is performed at the exit boundary of the computational domain and the sound pressure and power level for each downstream propagating acoustic mode are calculated. The non-reflecting boundary condition decomposes the perturbation field at the exit boundary into incoming and outgoing waves. The outgoing and non-decaying acoustic modes are then used along with equations (1) and (2) to calculate the integrated sound pressure (SPL) and power (PWL) levels:

\[
\text{SPL} = 10 \log_{10} \left( \frac{1}{r_T^2 - r_H^2} \sum_{m,n} |a_{m,n}|^2 \int_{r_H}^{r_T} |\psi_{m,n}|^2 r \, dr \right) - 10 \log_{10}(P_{\text{ref}}^2),
\]

(1)

where \( r_H \) and \( r_T \) are the hub and tip radii of the annular duct, \( m \) and \( n \) are respectively the circumferential and radial numbers of the propagating acoustic modes, \( \psi_{m,n} \) and \( a_{m,n} \) are the eigenmode and the mode amplitude, respectively, corresponding to the \((m,n)\) mode, and \( P_{\text{ref}} = 20 \) µPa is the reference acoustic pressure.

\[
\text{PWL} = 10 \log_{10} \left( \pi Re \left( \sum_{m,n} |a_{m,n}|^2 \int_{r_H}^{r_T} \psi_{m,n}^a(\psi_{m,n}^b)^* r \, dr \right) \right) - 10 \log_{10}(\text{PWL}_{\text{ref}}),
\]

(2)

where \( \psi_{m,n}^a \) and \( \psi_{m,n}^b \) are the eigenmodes corresponding to the quantities \( \bar{p}(p' + \bar{p}\bar{v}_z v'_z) \) and \( \bar{p}v'_z + \rho'\bar{v}_z \) respectively, and \( \text{PWL}_{\text{ref}} = 10^{-12} \) watts is the reference power level. Note that the summation in equations (1) and (2) is performed over the propagating acoustics modes only.

**III. Validation against Semi-Analytical Results**

In this section, validation of the prediction process against semi-analytical results for two benchmark problems\textsuperscript{13,14} is presented. The interaction of a vortical gust with a flat plate cascade is solved in two and three spatial dimensions to predict the unsteady loading on the blades.

![Figure 3. Spanwise variation of wake gust velocity components.](image)
The gust may be represented generally by

$$\mathbf{v}' = \sum_{n=1}^{\infty} v_n \exp \{ i n N_B (-\Omega t + k_z z + \theta + \phi) \},$$

(3)

where $N_B$ is the number of rotor blades, $\Omega$ is the shaft rotation rate, $\phi$ is the phase, and $n$ is the harmonic number. For a gust convecting with a uniform meanflow velocity $\bar{v}_z$, the axial wave number, $k_z = \Omega/\bar{v}_z$. The radial dependence of the gust must satisfy the solenoidal constraint $\nabla \cdot \mathbf{v}' = 0$. Following Prasad and Verdon,\textsuperscript{11} the gust is defined as

$$(v_{n_r}, v_{n_\theta}, v_{n_z})^T = a_n \bar{v}_z \left( 0, 1, -\frac{\bar{v}_z}{\Omega r} \right)^T,$$

(4)

where $a_n$ is the complex amplitude of the gust. For the validation results presented here, $a_n = 1$, $\bar{v}_z = 1$, and $\Omega$ is varied by changing the tip Mach number, $M_t = \Omega r_T/c$ of the rotor, where $r_T$ is the tip radius and $c$ is the speed of sound.

A. Gust Response of a 2-D Flat Plate Cascade

A narrow annulus flat plate cascade is used to approximately simulate a 2-D case for comparison against results from Smith’s\textsuperscript{4} analysis. The grid used for the simulations is shown in Figure 4. Figure 5 compares the imaginary part of unsteady pressure in the cascade and Figure 6 compares the unsteady lift on the cascade. The agreement between the numerical results and Smith’s results\textsuperscript{4} is excellent.

Figure 4. Grid used to calculate the gust response of a narrow-annulus flat-plate cascade. Every other point is omitted for clarity.

(a) (b)

Figure 5. Contours of imaginary part of unsteady pressure for $M_t = 0.65$ case: (a) Smith’s\textsuperscript{4} results, and (b) numerical results.

B. Gust Response of a 3-D Flat Plate Annular Cascade

Figure 7 shows the grid used for the numerical simulations of the 3-D flat plate annular cascade. Semi-analytical results for this case were presented by Namba and Schulten.\textsuperscript{6} Figure 8 presents a comparison of the unsteady lift on the blade at three spanwise locations. The results are in good agreement with the semi-analytical results. More variation is observed at the tip and hub than at the mid-span section.
Figure 6. Unsteady lift on a 2-D flat plate cascade for (a) $M_t = 0.3897$, (b) $M_t = 0.433$, (c) $M_t = 0.4763$, and (d) $M_t = 0.6495$. Lines are predictions and symbols are classical results by Smith.\textsuperscript{5}

Figure 7. Grid used to calculate the gust response of a three-dimensional flat-plate cascade. Every other point is omitted for clarity.
Figure 8. Unsteady lift at three spanwise locations of a 3-D flat plate cascade. Lines are predictions, filled circles are results from Namba, and hollow circles are results from Schulten.6
IV. Comparison Against NASA Source Diagnostics Test Data

The Source Diagnostic Test (SDT) is a comprehensive experimental study of the aerodynamics and acoustics of a representative high-bypass ratio fan stage. One of the goals of the SDT study was to determine the factors affecting the generation of fan tone noise due to the interaction of the fan rotor wake with the outlet guide vanes. Aerodynamic and acoustic measurements were taken for various rotor and stator geometry configurations and operational speeds. Two rotor and three stator geometries were tested in the SDT. The three stator geometries were nominal (NOM), low vane count (LVC), and low noise (LN). The design parameters for the rotor and stator are outlined in Table 1 and Table 2 respectively. The rotor speed parameters are outlined in Table 3.

<table>
<thead>
<tr>
<th>Rotor</th>
<th>No. Blades</th>
<th>L.E. Sweep</th>
<th>Design Tip Speed (ft/sec)</th>
<th>Pressure Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>R4</td>
<td>22</td>
<td>0</td>
<td>1215</td>
<td>1.47</td>
</tr>
<tr>
<td>M5</td>
<td>22</td>
<td>0</td>
<td>1350</td>
<td>1.50</td>
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</tbody>
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Table 2. Stator design parameters

<table>
<thead>
<tr>
<th>Stator</th>
<th>No. Vanes</th>
<th>L.E. Sweep</th>
<th>Aspect Ratio</th>
<th>Solidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal (NOM)</td>
<td>54</td>
<td>0</td>
<td>3.51</td>
<td>1.52</td>
</tr>
<tr>
<td>Low Vane Count (LVC)</td>
<td>26</td>
<td>0</td>
<td>1.67</td>
<td>1.51</td>
</tr>
<tr>
<td>Low Noise (LN)</td>
<td>26</td>
<td>30</td>
<td>1.67</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Table 3. Rotor speed parameters

<table>
<thead>
<tr>
<th>Condition</th>
<th>RPM (Corrected)</th>
<th>Percent Speed</th>
<th>Tangential Tip Speed (Corrected) (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach</td>
<td>7809</td>
<td>61.7</td>
<td>750</td>
</tr>
<tr>
<td>Cutback</td>
<td>11075</td>
<td>87.5</td>
<td>1063</td>
</tr>
<tr>
<td>Takeoff</td>
<td>12657</td>
<td>100</td>
<td>1215</td>
</tr>
</tbody>
</table>

A subset of the experimental results was selected to validate the proposed prediction method. This included the R4 rotor with all three stator configurations. For each stator configuration, calculations were performed for approach, cutback and takeoff rotor speeds and at the first and second blade passing frequencies. This gives a total of 16 cases (the nominal stator is cut-off at first BPF except at takeoff condition). These are listed in Table 4.

Figure 9 shows the grids used to perform the acoustic calculation. The typical grid size used for the acoustic simulations is of the order of 4 million cells. Further refinement of the grid had insignificant effect on the acoustic results. The grid is clustered near the leading edge to accurately capture the wake interaction with the vane. The inlet and the exit boundaries are chosen to be very close to the blade because of two reasons. Firstly, the wake from the rotor calculation is obtained at the inlet of the OGV computation domain. If this plane is far away from the vane leading edge, the viscous dissipation of the wake, which is not captured in the linearized inviscid analysis, will introduce large error. Secondly, a smaller domain allows densely-packed grids to improve spatial resolution. The proximity of the inlet and exit boundaries can lead to spurious reflections, but the high fidelity, 3-D non-reflecting boundary condition is expected to minimize that error. A possible approach to eliminate the error from the inviscid wake evolution is to decompose the wake at the leading edge of the vane and then “unwind” the gust to the inlet of the computational domain in an inviscid fashion to ensure that the vane effectively sees the desired “viscous” gust in the linearized calculation. This approach is however not employed here.

Comparisons against the SDT experimental data are presented in three parts. First, the unsteady vane
Table 4. SDT cases chosen for validation.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Rotor</th>
<th>OGV</th>
<th>Operating Condition</th>
<th>Harmonic (× BPF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>R4</td>
<td>NOM</td>
<td>Takeoff</td>
<td>1</td>
</tr>
<tr>
<td>1b</td>
<td>R4</td>
<td>NOM</td>
<td>Takeoff</td>
<td>2</td>
</tr>
<tr>
<td>2a</td>
<td>R4</td>
<td>NOM</td>
<td>Cutback</td>
<td>1</td>
</tr>
<tr>
<td>2b</td>
<td>R4</td>
<td>NOM</td>
<td>Cutback</td>
<td>2</td>
</tr>
<tr>
<td>3a</td>
<td>R4</td>
<td>NOM</td>
<td>Approach</td>
<td>1</td>
</tr>
<tr>
<td>3b</td>
<td>R4</td>
<td>NOM</td>
<td>Approach</td>
<td>2</td>
</tr>
<tr>
<td>4a</td>
<td>R4</td>
<td>LVC</td>
<td>Takeoff</td>
<td>1</td>
</tr>
<tr>
<td>4b</td>
<td>R4</td>
<td>LVC</td>
<td>Takeoff</td>
<td>2</td>
</tr>
<tr>
<td>5a</td>
<td>R4</td>
<td>LVC</td>
<td>Cutback</td>
<td>1</td>
</tr>
<tr>
<td>5b</td>
<td>R4</td>
<td>LVC</td>
<td>Cutback</td>
<td>2</td>
</tr>
<tr>
<td>6a</td>
<td>R4</td>
<td>LVC</td>
<td>Approach</td>
<td>1</td>
</tr>
<tr>
<td>6b</td>
<td>R4</td>
<td>LVC</td>
<td>Approach</td>
<td>2</td>
</tr>
<tr>
<td>7a</td>
<td>R4</td>
<td>LN</td>
<td>Takeoff</td>
<td>1</td>
</tr>
<tr>
<td>7b</td>
<td>R4</td>
<td>LN</td>
<td>Takeoff</td>
<td>2</td>
</tr>
<tr>
<td>8a</td>
<td>R4</td>
<td>LN</td>
<td>Cutback</td>
<td>1</td>
</tr>
<tr>
<td>8b</td>
<td>R4</td>
<td>LN</td>
<td>Cutback</td>
<td>2</td>
</tr>
<tr>
<td>9a</td>
<td>R4</td>
<td>LN</td>
<td>Approach</td>
<td>1</td>
</tr>
<tr>
<td>9b</td>
<td>R4</td>
<td>LN</td>
<td>Approach</td>
<td>2</td>
</tr>
</tbody>
</table>

surface pressure is compared for one of the cases (case number 6a in Table 4). Second, the exhaust modal power levels are analyzed, and lastly, geometric trends in the exhaust power levels are examined.

A. Vane Unsteady Pressure

The SDT vane unsteady pressure measurements were reported by Envia. Measurements were made for seven fan tip speeds for the LVC and LN vanes. Pressure transducers were embedded inside the vanes and were exposed to both sides of the airfoil to measure the pressure difference between the suction and pressure sides. The suction side was taped in another set of measurements to measure the pressure fluctuations on the pressure side alone. In the discussion below, ∆p refers to the pressure difference between the pressure and suction sides.

Figure 10 compares the spanwise distribution of the unsteady pressure magnitude and phase at 20% chord for case 6a. A very good qualitative match is observed although the magnitude is under-predicted. It is encouraging to see that the phase variation along the span is captured accurately. It is important to capture the radial phasing correctly because it determines the coupling between the aerodynamic excitation and the acoustic duct modes. The agreement with the data is in general better for pressure side comparisons than for ∆p comparisons.

Figure 11 shows chordwise distributions of the vane unsteady pressure at three spanwise locations - 80%, 67%, and 20% for case 6a. The predictions are qualitatively in agreement but low in magnitude in comparison to data. Also, the agreement appears to be better near the tip than at the hub. This is partially due to the fact that the noise level is much lower near the hub.

B. Exhaust Tone Noise

The perturbation field at the exit boundary of the acoustic computation domain is decomposed into upstream and downstream travelling acoustic modes and convected modes as described in Appendix A. The downstream propagating (non-decaying) acoustic modes are assumed to propagate to the duct exit without any scattering (i.e., uniform cross-section, co-annular duct assumption). The in-duct sound power levels are compared against SDT measurement using this assumption.
Figure 9. Grids used for acoustics calculations for the SDT cases.

Figure 10. Span-wise distribution of unsteady surface pressure magnitude and phase at 20% chord for case 6a. $\Delta p$ represents pressure difference between the suction and pressure sides.
Figure 11. Chord-wise distribution of unsteady pressure and phase at three span locations - (a,b) 87%, (c,d) 60%, and (e,f) 20% for case 6a. Solid lines with circles are experiments, and dashed lines with triangles are predictions.
The following points should be borne in mind when analyzing these comparisons. First, the NASA SDT in-duct modes are assumed to be Tyler-Sofrin\textsuperscript{15} modes. This is true only when the meanflow is uniform. On the contrary, the acoustic modes in the predictions are calculated over non-uniform meanflow that is present at the exit boundary of the acoustic domain. The difference due to this discrepancy should be small because the OGV removes most of the flow swirl, and the flow behind the OGV is fairly uniform. This is exemplified in Figure 12 where the acoustic modes predicted by the two methods described above are compared. Second, the cross sectional area of the duct continuously varies from the OGV trailing edge to the exit of the nozzle, which alters the acoustic modes by both change in duct geometry as well as the resulting change in meanflow. This alteration can be described as scattering of energy into different modes, which is not accounted for in the present work. This effect is also expected to be small because the duct area changes only moderately over this distance.

Figure 12. First four cut-on acoustic radial modes for case 3b. Solid lines represent modes calculated using non-uniform meanflow and dashed lines represent modes calculated by assuming uniform meanflow.

The modal power in the test data is obtained by a least-squares fit of the measured pressure projected onto the cut-on acoustic modes predicted by the Tyler-Sofrin theory.\textsuperscript{15} In the predictions, an inner product of the perturbation field with the left eigenvector of a mode gives the amplitude of that mode. The radially integrated sound pressure and power levels are then calculated using this amplitude with equations (1) and (2).

The circumferential modes expected for a given rotor-stator configuration are given by the relation \( m = nN_B + kN_V \), where \( N_B \) and \( N_V \) are the numbers of rotor blades and stator vanes respectively, \( n \) is the BPF harmonic number, and \( k \) is any integer. For example, for nominal vane at second BPF, circumferential modes that can be present are: \( 2 \times 22 + (-1) \times 54 = -10, \ 2 \times 22 + (0) \times 54 = 44 \) and so on. The mode corresponding to \( k = 0 \) is a rotor-locked mode but is still due to the rotor-stator interaction. It is not part of the rotor-alone “self” noise, although the rotor-alone field can co-exist with this tone, especially upstream of the rotor.

Figures 13, 14, and 15 compare the predicted total and modal in-duct acoustic power levels with the SDT measurements for all the cases listed in Table 4. The total exhaust acoustic power is compared in the last two bars of the histograms. The BPF tone for the nominal geometry is cut-off at cutback and approach condition therefore Figure 13 presents only second BPF results for these cases. Several acoustic modes are cut on for some cases. Therefore, for clarity, the radial modes corresponding to the higher circumferential modes for such cases are summed up and plotted using the \( \sum \) symbol in the histogram plots.

The predictions capture the modal distribution quite well. The total power levels seem to be consistently under-predicted for all cases and the average error is observed to be about 5 dB. The reason for consistent under-prediction is uncertain. Erroneously low wake harmonic amplitudes could be one reason why this is observed. In general, the predictions are better for the first BPF harmonic than for the second. Also, the predictions are better for the nominal vane geometry than for the other two. Interestingly, the error seems to correlate with the number of cut-on modes in the problem. Higher error is observed for cases where the number of cut-on modes is large.
Figure 13. Modal results for Nominal OGV configuration at Takeoff, Cutback, and Approach conditions.
Figure 14. Modal results for Low Vane Count OGV configuration at Takeoff, Cutback, and Approach conditions.
Figure 15. Modal results for Low Noise OGV configuration at Takeoff, Cutback, and Approach conditions.
C. Geometric Trends

Quantitative comparisons are a good measure of successful prediction. However, trend predictions are of particular interest for design capability. In the SDT, there are two geometric trends - (1) effect of sweep (from LVC to LN), and (2) effect of reduced number of vanes (from NOM to LVC). Figure 16 presents a comparison between experiments and predictions of geometric trends at takeoff, cutback, and approach conditions. Comparison is made for individual harmonics as well as total exhaust noise levels.

The benefit of sweep in reducing interaction noise is well known.\textsuperscript{5,16} Sweep introduces additional spanwise phase variation in the gust and consequently in the unsteady loading on the vane. This reduces the total acoustic power when pressure perturbation is integrated over the span. Also, higher spanwise phasing enhances the coupling of unsteady lift on the blade with higher radial order modes (which are more cut-off) thereby reducing noise. The benefit of sweep for noise reduction can be clearly seen in Figure 16 by focusing on trend lines from LVC to LN. The effect of sweep is well captured by the numerical results.

The effect of reduced vane count (NOM to LVC trend) on fan tone noise is more complicated. Any change in blade count has to be accompanied by a change in the blade chord to maintain the blade solidity. This is required to maintain the same aerodynamic performance of the machine. The LVC OGV in SDT, which has only 26 blades, has a wider chord than the NOM vane which has 54 blades. The reduction in vane count is a direct reduction in the number of noise sources, which is beneficial for noise reduction. Also, the wide chord blade effectively sees a lower reduced frequency than the nominal blade which again reduces noise (Sear's function\textsuperscript{17}). However, with the reduction in vane count the unsteady loading on each vane goes up, which generates more noise. Furthermore, the reduction in the number of vanes permits lower-order circumferential modes in the duct which are more efficient at radiating acoustic energy. The net radiated noise is a sum of all these effects and the resulting trends can be confounding.

The biggest penalty of reducing the number of vanes below a threshold (two times the number of rotor blades) is that the first BPF gets cut-on for cutback and approach conditions also. The effect of this on overall noise can be seen in Figure 16 (f,i). The large increase in noise from NOM to LVC is due to the first BPF tone which is only present for the LVC geometry. The vane count reduction is however beneficial in reducing the rotor-stator interaction broadband noise.

V. Conclusion

A prediction methodology for fan tone noise simulations has been described. The process has been validated against canonical cases where the agreement with semi-analytical results is found to be very good. A detailed comparison against the NASA Source Diagnostic Test data has been performed for vane unsteady pressure and in-duct exhaust noise levels. The capability of the process to predict geometric trends has also been verified against SDT data. The average error has been observed to be about 5 dB with the numerical results being consistently lower than the data.

VI. Acknowledgment

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Figure 16. Comparison of Geometry trends.
A. Formulation of the Three-Dimensional Non-Reflecting Boundary Conditions

The linearized Euler equations in cylindrical coordiantes are

\[
\frac{\partial U'}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \hat{A} U'\right) + \frac{1}{r} \hat{B} \frac{\partial U'}{\partial \theta} + \hat{C} \frac{\partial U'}{\partial z} - \hat{D} U' = 0, \tag{5}
\]

where,

\[
U' = \begin{bmatrix}
\rho' \\
v' \\
v' \\
p'
\end{bmatrix},
\]

and the matrices \(\hat{A}, \hat{B}, \hat{C},\) and \(\hat{D}\) expressed in primitive variables are

\[
\hat{A} = \begin{pmatrix}
\bar{v}_r & \bar{p} & 0 & 0 & 0 \\
0 & \bar{v}_r & 0 & 0 & \frac{1}{\bar{r}} \\
0 & 0 & \bar{v}_r & 0 & 0 \\
0 & 0 & 0 & \bar{v}_r & 0 \\
0 & \gamma \bar{p} & 0 & 0 & \bar{v}_r
\end{pmatrix},
\]

\[
\hat{B} = \begin{pmatrix}
0 & \bar{p} & 0 & 0 \\
0 & \bar{v}_\theta & 0 & 0 & \frac{1}{\bar{r}} \\
0 & 0 & \bar{v}_\theta & 0 & 0 \\
0 & 0 & 0 & \bar{v}_\theta & 0 \\
0 & 0 & \gamma \bar{p} & 0 & \bar{v}_\theta
\end{pmatrix},
\]

\[
\hat{C} = \begin{pmatrix}
0 & 0 & \bar{p} & 0 \\
0 & \bar{v}_z & 0 & 0 & 0 \\
0 & 0 & \bar{v}_z & 0 & 0 \\
0 & 0 & 0 & \bar{v}_z & \frac{1}{\bar{r}} \\
0 & 0 & 0 & \gamma \bar{p} & \bar{v}_z
\end{pmatrix}, \quad \text{and}
\]

\[
\hat{D} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & \frac{(\bar{v}_\theta + \Omega r)^2}{\bar{r}} & -\frac{(\bar{v}_\theta + 2\Omega r)}{\bar{r}} & -\frac{2(\bar{v}_\theta + \Omega r)}{\bar{r}} & 0 & \frac{1}{\bar{r}} \\
-\frac{\bar{v}_r(\bar{v}_\theta + 2\Omega r)}{\bar{r}} - \frac{\bar{v}_r(\bar{v}_\theta + \Omega r)}{\bar{r}} & 0 & \bar{v}_r & 0 & 0 \\
0 & -\frac{(\bar{v}_z + \Omega r)^2}{\bar{r}} & \frac{(\bar{v}_z + \Omega r)^2}{\bar{r}} & 0 & \frac{1}{\bar{r}} \\
0 & 0 & 0 & \frac{(\gamma - 1)\bar{p}(\bar{v}_\theta + \Omega r)}{\bar{r}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{(\gamma - 1)\bar{p}(\bar{v}_\theta + 2\Omega r)}{\bar{r}} & 0 & 0
\end{pmatrix}.
\]

In above, the overbar (\(\bar{\cdot}\)) denotes a meanflow quantity, and the prime (\(\prime\)) denotes a perturbation quantity.

Assuming a wave solution of the following form

\[
U'(r, \theta, z, t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \hat{U}'_{mn}(r) e^{i(\omega t - m\theta - kz)} \tag{7}
\]

and substituting into Eq. (5) gives

\[
j\omega \hat{U}'_{mn} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \hat{A} \hat{U}'_{mn}\right) - \frac{j}{r} \hat{B} \hat{U}'_{mn} - jk_z \hat{C} \hat{U}'_{mn} - \hat{D} \hat{U}'_{mn} = 0. \tag{8}
\]
The radial derivative term in the above equation can be represented using a numerical differential operator (using either spectral or finite differencing) as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \mathbf{\hat{A}} U_{mn}^\prime \right) = \mathbf{L}_r \hat{U}_{mn}^\prime.
\]  

(9)

Substituting into Eq. (8) yields the following eigensystem -

\[
\left[ j\omega I - \mathbf{L}_r - j \frac{M}{r} \mathbf{B} - j k_z \mathbf{C} - \mathbf{D} \right] \hat{U}_{mn}^\prime = 0.
\]  

(10)

Because of the numerical representation of the radial operator, this eigensystem must be solved numerically. The numerical solution of the system generally contains some spurious modes in addition to acoustic and convective modes. A convected mode simply convects downstream with the mean flow and can be identified by comparing its group velocity \( V_g,mn \), with the local meanflow velocity. The group velocity can be calculated using the relation

\[
V_g,mn = \frac{\partial \omega}{\partial k_z} = \frac{\langle \mathbf{L}_{mn}, \mathbf{C} \mathbf{R}_{mn} \rangle}{\langle \mathbf{L}_{mn}, \mathbf{R}_{mn} \rangle},
\]  

(11)

where \( k_z \) is the eigenvalue, and \( L_{mn} \) and \( R_{mn} \) are the left and right eigenvectors of Eq. (10). After filtering out the spurious and the convected modes from the solution, the acoustic perturbation solution can be written as

\[
U'(r, \theta, z, t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{mn} R_{mn}(r) e^{j(\omega t - m\theta - k_z z)}.
\]  

(12)

The eigensystem solution is verified by comparing the eigenvalues and eigenvectors against analytical solution for acoustic modes in an annular duct with uniform meanflow. A sample comparison is provided in figure 17.

![Figure 17](https://via.placeholder.com/150)

(a) Eigenvalues  
(b) Eigenvectors

**Figure 17.** Validation of the numerical eigensystem solution against analytical results: (a) eigenvalues, and (b) eigenvectors. In (b), lines are analytical results and circles are numerically computed results.

The acoustic waves can be grouped into left- and right-running waves. The direction of evanescent waves (non-zero imaginary value of \( k_z \)) is determined by the sign of \( \text{Im}\{k_z\} \) (the magnitude of \( \text{Im}\{k_z\} \) determines the decay rate), and that of propagating (non-decaying) waves is determined by the direction of the group velocity. The amplitudes of the incoming/outgoing acoustic modes from a boundary are obtained by taking an inner product of the perturbation field at the boundary with \( U' \) given by Eq. (12) for the incoming/outgoing direction. At the non-reflecting boundaries, the incoming mode amplitudes are set to zero.
References