

Prediction of Aerodynamic Tonal Noise from Open Rotors

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Abstract

A numerical approach for predicting tonal aerodynamic noise from “open rotors” is presented. “Open rotor” refers to an engine architecture with a pair of counter-rotating propellers. Typical noise spectra from an open rotor consist of dominant tones, which arise due both to the steady loading/thickness and the aerodynamic interaction between the two bladerows. The proposed prediction approach utilizes Reynolds Averaged Navier Stokes (RANS) Computational Fluid Dynamics (CFD) simulations to obtain near-field description of the noise sources. The near-to-far-field propagation is then carried out by solving the Ffowcs Williams-Hawkings equation. Since the interest of this paper is limited to tone noise, a linearized, frequency domain approach is adopted to solve the wake/vortex-blade interaction problem.

This paper focuses primarily on the speed scaling of the aerodynamic tonal noise from open rotors. Even though there is no theoretical mode cut-

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off due to the absence of nacelle in open rotors, the far-field noise is a strong function of the azimuthal mode order. While the steady loading/thickness noise has circumferential modes of high order, due to the relatively large number of blades ($\approx 10 - 12$), the interaction noise typically has modes of small orders. The high mode orders have very low radiation efficiency and exhibit very strong scaling with Mach number, while the low mode orders show a relatively weaker scaling. The prediction approach is able to capture the speed scaling (observed in experiment) of the overall aerodynamic noise very well.

Keywords: open rotor noise, rotor-rotor interaction, CROR noise

1. Introduction

Single rotation propellers are highly efficient but are restricted to low forward flight speeds and are also limited in the thrust they can generate. A counter-rotating propeller design provides higher thrust and high aerodynamic efficiency at high flight speeds. This is possible because the aft, counter-rotating bladerow takes out the swirl put in by the front rotor. The fuel burn benefit over conventional, ducted fan designs is estimated to be more than 10 percent. A counter-rotating pusher propeller configuration is considered in this report and will henceforth be referred to as “open rotor” (see Fig. 1).

One of the technology roadblocks for the open rotor architecture is the associated aerodynamic noise. The noise spectra from an open rotor appear overwhelmingly tonal however the broadband noise contributes significantly to the overall EPNL (effective perceived noise levels) [1]. The tonal noise is

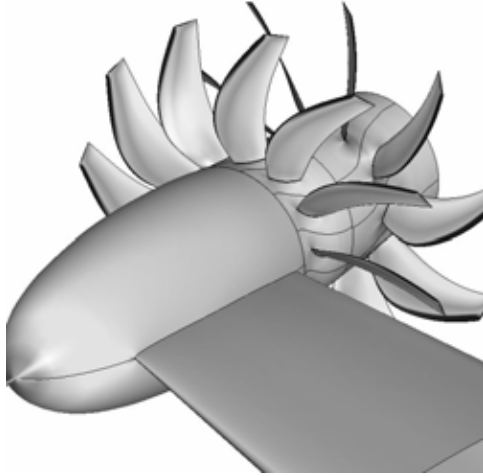


Figure 1: Open rotor configuration considered here for noise assessment.

15 caused by the aerodynamic and aeroacoustic interaction between the rotors,
16 and the interaction between the rotors and the pylon/wing/fuselage. The
17 same interactions also produce broadband noise due to the turbulence in the
18 flow.

19 A methodology for numerical prediction of open rotor aerodynamic tone
20 noise is presented here. The approach employs three-dimensional, RANS (for
21 steady loading and thickness noise) and time-linearized RANS (for interac-
22 tion noise) simulations to characterize noise sources in the near field. Such
23 an approach has previously been successfully used to predict tone noise from
24 fan-OGV interaction in a ducted configuration [2, 3, 4]. For an open rotor, an
25 additional step of near-to-far field radiation is required, which is carried out
26 by solving the Ffowcs Williams-Hawkings (FW-H) equation [5, 6] using the
27 near-field sources defined on a translating, permeable surface. General Elec-
28 tric Company’s proprietary flow solver, TACOMA [2, 7, 8] is used to carry
29 out all the flow solutions used in the present work. A separate, frequency

30 domain, FW-H solver has been developed which has been validated (results
31 in following sections) against analytical solutions of canonical problems.

32 The concept of counter-rotating, un-ducted propellers was seriously in-
33 vestigated first in the early 1980s when oil price was soaring. Significant ad-
34 vances leading to engine flight tests were performed, but the ensuing slump
35 in oil price put the concept on hold. In the last 5-8 years, the concept has
36 been revived and is under serious consideration to be the choice propulsor
37 for the next single-aisle aircraft. Since the concept of an open rotor has been
38 around for a while, and aerodynamic noise has been one of its biggest design
39 challenges, there is a rich history of publications in this field.

40 Peake and Parry [9] nicely summarizes the turbomachinery noise chal-
41 lenges facing modern turbofan engines with a focus on open rotors. The
42 paper also provides a brief summary of the historic and recent progress in
43 predicting and reducing open rotor noise. Hubbard [10] was the first to lay the
44 foundations of counter-rotation propeller noise theory, which Hanson [11, 12]
45 elaborated on and developed formulae for analytically predicting noise due
46 to aerodynamic interference (wake interaction) between the two bladerows of
47 a counter-rotating propeller. Hanson [11] also investigated the phenomenon
48 of acoustic interference between the two rotors and between multiple modes
49 from the same rotor. Several efforts have been devoted also into investigat-
50 ing the effects of angle of attack and the substantial noise increase observed
51 when these machines are operated in non-uniform flow, see e.g., Mani [13]
52 and Hanson [14].

53 Among recent efforts, Carazo *et al.* [15] demonstrated an analytical method
54 for predicting tonal noise from open rotors, wherein the unsteady loading on

55 the aft bladerow due to wake interaction is computed using Amiet's theory.
56 Noise due only to dipole sources was considered and a far-field radiation
57 model was derived from the formulation of a rotating acoustic dipole embed-
58 ded in a uniform meanflow. Blandeau and Joseph [16] have further demon-
59 strated an analytical capability to predict broadband noise in open rotors
60 due to wake interaction between bladerows. The turbulence in the wakes is
61 assumed to be homogeneous and isotropic in their analyses.

62 In recent years, considerable effort has gone into using the 3-D, Unsteady
63 Reynolds Averaged Navier Stokes (U-RANS) approach for noise prediction,
64 see e.g., Spalart [17] and Peters and Spakovszky [18]. Deconinck *et al.* [19]
65 use the nonlinear harmonic approach to predict aerodynamic tonal noise
66 from open rotors. They write the flow solution as a combination of the mean
67 (time-steady) flow and the perturbation (time-unsteady) quantities. The
68 perturbation quantities are represented as complex harmonics for frequencies
69 of interest and solved for in the frequency domain. Significant time savings
70 are achieved by realizing that only a few relevant frequencies are of interest
71 and that for each frequency only a single passage simulation has to be carried
72 out.

73 A recent three-part paper by Colin *et al.* [20, 21, 22] provides a compre-
74 hensive overview of various methods that can be used for open rotor noise
75 evaluation. Their own numerical approach is also based on solving the U-
76 RANS equations. They utilize the chorochronic approach wherein only a
77 single passage of each bladerow is simulated, however time accurate data (of
78 the order of periodicity in the blade row) needs to be accrued in the boundary
79 cells. While theoretically, such direct simulation approaches should resolve

80 all necessary physics of noise generation mechanisms, they all face the chal-
81 lenge of simultaneously resolving both the meanflow hydrodynamic scales
82 and the small acoustic amplitudes. The linearized RANS approach utilized
83 in the current paper isolates the acoustic problem by linearizing about the
84 meanflow and hence permits accurate resolution of acoustics. For tone noise
85 calculations, it is also very cost effective.

86 Parry *et al.* [1] investigated the relative importance of tonal versus broad-
87 band noise from “isolated” open rotors at zero angle of attack (similar config-
88 uration as considered here) and concluded that although there are a plethora
89 of tones with significant protrusion above broadband noise, on a one-third
90 octave level, the broadband noise cannot be ignored. While it is evidently
91 important, no attempt is made here to predict broadband noise. In later
92 sections, comparisons are drawn between measurements and prediction; the
93 test data is decomposed into tonal and broadband components in a manner
94 similar to that described in Parry [1].

95 Shielding of aerodynamic noise is one way to mitigate the noise challenge
96 posed by the open rotor architecture. Towards this, Stephens and Envia [23]
97 reported the experimental findings of an acoustic shielding experiment car-
98 ried out in the 9” x 15” low-speed wind tunnel (LSWT) at NASA Glenn.
99 They tested acoustic shielding from two (long and short) plates that are rep-
100 resentative of an airplane wing or a horizontal/vertical stabilizer. They [23]
101 also mention that the spatial resolution of the microphones is not enough to
102 accurately resolve tonal noise directivity, as it can be very peaky. Installa-
103 tion effects on scattering of noise have also been investigated analytically -
104 scattering by the aircraft fuselage treated as a hard infinitely long cylinder

105 in [24] and scattering by the centerbody in [25].

106 The present paper focuses on a time-linearized, RANS-based numerical
107 approach for open rotor tone noise prediction. While the methodology applies
108 to any flight condition, the validation effort and focus is directed towards
109 community noise at take-off condition. The following section describes the
110 prediction process followed by validation against analytical solutions and
111 comparisons against experimental data.

112 **2. Prediction Process**

113 The proposed open rotor aerodynamic noise prediction process involves
114 multiple steps, which are summarized below. A flowchart illustrating the
115 process flow is also provided in Fig. 2.

- 116 1. Multi-stage, RANS calculations are performed using TACOMA [7, 8]
117 to compute meanflow solutions. One passage of each bladerow is sim-
118 ulated with periodic boundary conditions across passage boundaries
119 (see Fig. 3). For each rotor, the simulation is performed in its frame
120 of reference enabling steady state simulation for meanflow calculation.
121 For validation cases, where measured aerodynamic performance data
122 is available, the blade pitch is iteratively changed in CFD until shaft
123 horse power (SHP) between the CFD and data are matched. This was
124 required since the use of measured (when the blades were not running)
125 pitch angles resulted in differences in predicted versus measured SHP
126 of about a fraction of a percent. These differences can arise due to two
127 reasons: (1) flexing of blades under aerodynamic and centrifugal loads,
128 thus changing the blade pitch/twist during operation, and (2) errors in

129 the CFD method used in predicting aerodynamic loads (hence power).
130 The shaft power differences can also be minimized by adjusting the
131 shaft rotation speed in the simulations. However, changing the rota-
132 tion speed will dramatically alter the radiation efficiencies of the tones
133 (modes) and hence the predicted acoustic power in the farfield. There-
134 fore, the choice of scaling by using pitch rather than rotor speed is
135 preferable and is employed here.

136 2. Rotor alone noise sources (that due to blade thickness and steady load-
137 ing) are obtained directly from RANS simulations described in step 1.
138 Primitive flow variables are extracted on surfaces in front of, above, and
139 aft of the simulated blade, which are then replicated (as many times as
140 the number of blades) to form a full annulus surface enclosing all the
141 blades of a rotor (see Fig. 4). This is the FW-H surface over which a
142 boundary integral is evaluated for far-field noise prediction. Such a sur-
143 face is also referred to as “permeable” surface as it allows flow through
144 it. One of the benefits of using such a surface is that it only translates
145 with the engine hence making the FW-H surface solver simpler; a sur-
146 face on or around the individual blades (that rotates with the blades)
147 will accelerate because of rotation. Time history for rotor alone (steady
148 in rotor frame) field is obtained simply by rotating the flow variables on
149 the FW-H surface with the shaft rotation rate. This is achieved cheaply
150 by using uniform grid distribution in the circumferential direction and
151 using the CSHIFT routine in Fortran 90.

152 3. For rotor-rotor interaction noise, an additional RANS simulation is car-
153 ried out in the gap region between the two bladerows. This is performed

154 on a wake-tracking grid, to allow better resolution of the velocity gradi-
155 ents in the wake and hence minimize numerical errors. This procedure
156 has previously been demonstrated by the authors [26] for ducted fans.
157 From this solution, the front rotor wake is extracted at the inlet bound-
158 ary of the CFD domain of the aft rotor and decomposed into front rotor
159 blade passing frequency harmonics. Frequency domain, linearized un-
160 steady Navier-Stokes analyses are then carried out independently for
161 each harmonic. Only a single passage of the aft bladerow has to be sim-
162 ulated by applying the phase lag condition on the domain boundaries
163 in the circumferential direction. Each rotor wake harmonic scatters
164 into multiple frequencies (frequency scattering) as it interacts with the
165 spinning aft rotor and produces what are often referred to as “sum”
166 and “difference” tones. Unsteady primitive flow variables are extracted
167 from the single-passage unsteady calculations and processed (using the
168 phase lag boundary condition) to generate data on the full-annulus
169 FW-H surface. The FW-H solver uses time-accurate, primitive flow
170 variables on the permeable surface as input. The frequency domain
171 solution is thus converted to the time domain by performing an inverse
172 Fourier transform.

173 4. The last step involves solving the FW-H equation using time-dependent
174 flow information on the FW-H surface. This step is the same for rotor
175 alone and interaction noise prediction. Radiated sound power level can
176 be obtained by integrating the sound intensity flux through a sphere
177 surrounding the open rotor (sound source). The microphones in the
178 experiments used for validation are on a sideline (parallel to the engine

179 centerline) arc (see Fig. 5). Sound intensity flux through the cylindri-
180 cal surface formed by the revolving the arc by 360° is therefore used as
181 the sound power metric to compare predictions to measurements. Axi-
182 symmetric sound field is therefore assumed, which holds true when each
183 tone has only one azimuthal (circumferential) mode. When multiple
184 azimuthal modes are present, constructive and destructive interference
185 in the azimuthal direction determines the azimuthal directivity. This
186 assumption however should be true for most of the tones under con-
187 sideration if the model is at perfectly zero angle of attack. One of the
188 tones for a 12x10 configuration, for example, that will have multiple
189 azimuthal modes is the tone at frequency 70Ω (Ω being the shaft ro-
190 tation rate) as it arises from the combination (sum) of 5^{th} harmonic of
191 the front rotor with the 1^{st} harmonic of the aft rotor ($(5 \times 12 + 1 \times 10)\Omega$)
192 as well as the 7^{th} harmonic ($7 \times 10\Omega$) of the aft rotor.
193 Furthermore, the sound power radiated at very shallow angles, not
194 covered by the microphones in the experiments, is ignored in the com-
195 parisons.

196 **3. Results**

197 Results from a recent test campaign [27, 23] conducted at the NASA
198 9'x15' low speed wind tunnel (LSWT) are used to verify the accuracy of the
199 proposed prediction process. Elliott [27] describes in detail the LSWT test
200 facility, the open rotor propulsion rig (ORPR), as well as the procedure for
201 gathering far-field acoustic data in this facility. One of the many configura-
202 tions tested in this campaign was designated as the F31A31 historical baseline

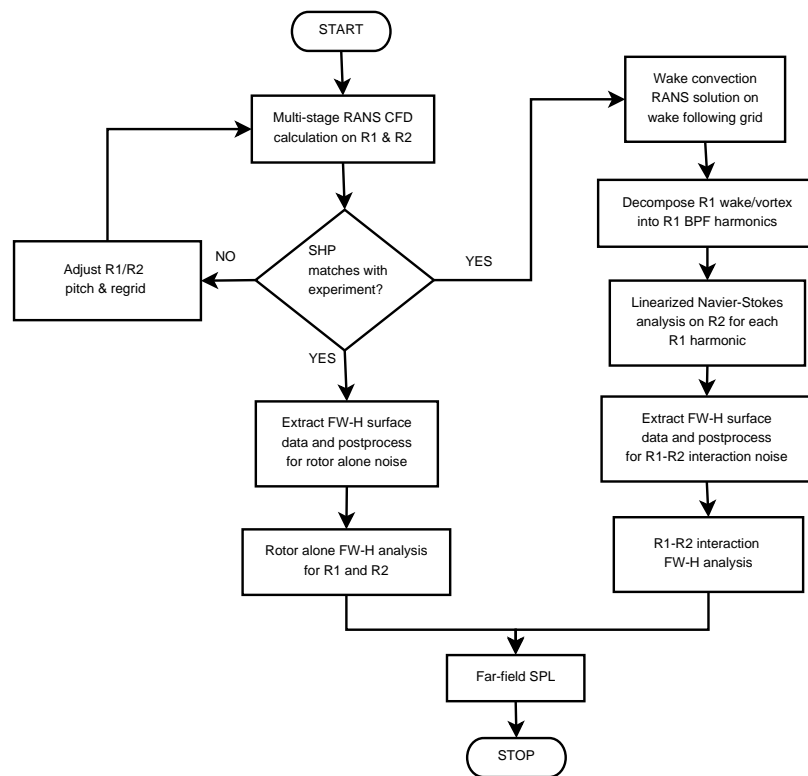


Figure 2: Flowchart of the open rotor noise prediction process.

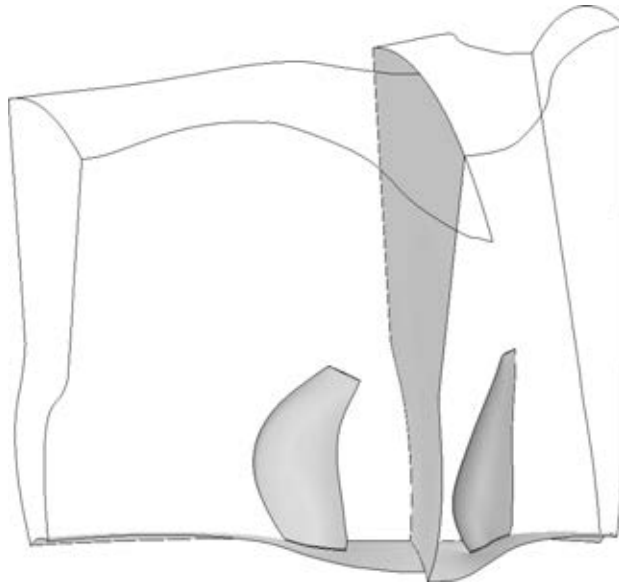
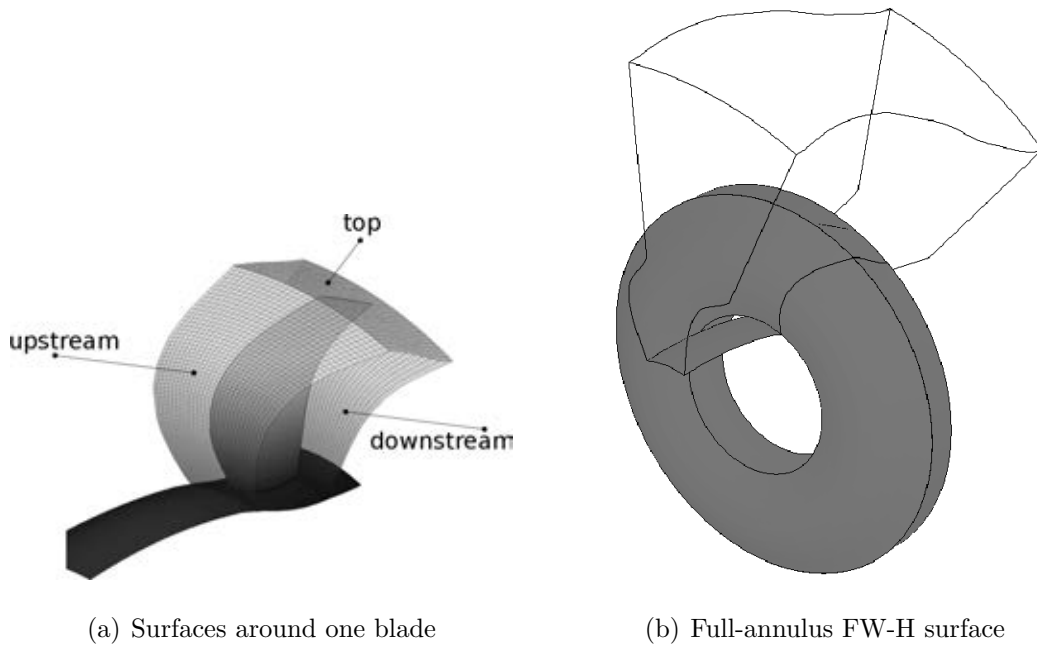
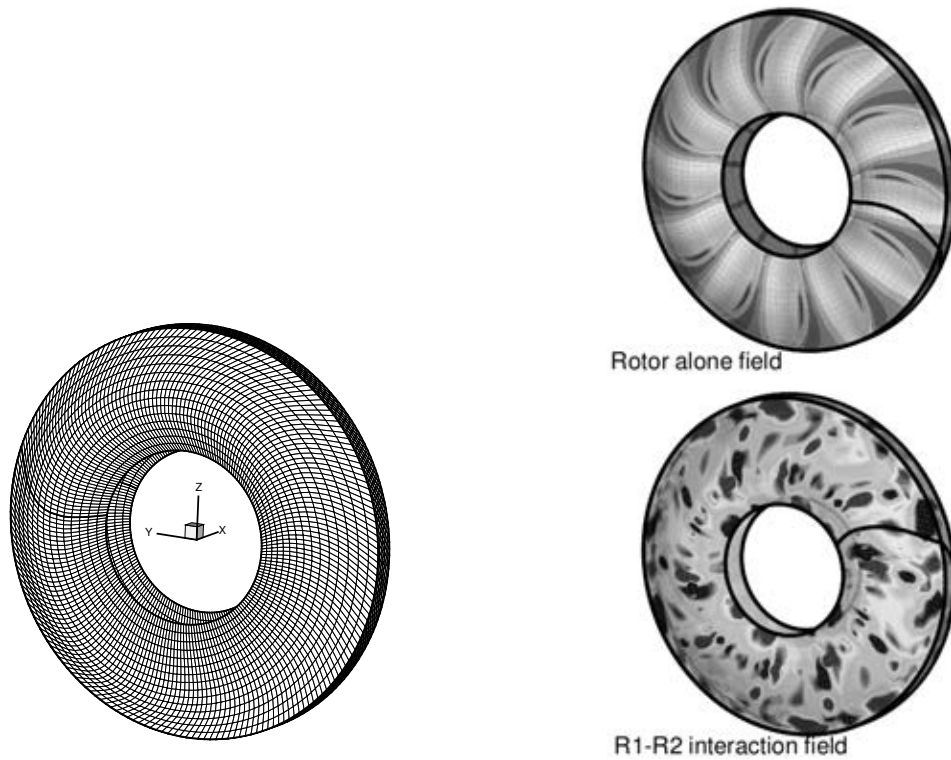


Figure 3: Multi-stage analysis configuration showing one blade each of the two bladerows of the F31A31 design and the interface plane. The front bladerow is referred to as R_1 and the aft, R_2 .



(a) Surfaces around one blade

(b) Full-annulus FW-H surface



(c) Grid on FW-H surface

(d) Pressure contours

Figure 4: A description of the process of creating the FW-H surface: (a) surfaces in front (upstream), aft (downstream), and on top of (top) a single blade, (b) single passage to full annulus extension, (c) grid on the full FW-H surface, and (d) pressure contours on the FW-H surface for rotor alone and interaction noise computation. The two plots in (d) are on different scales.

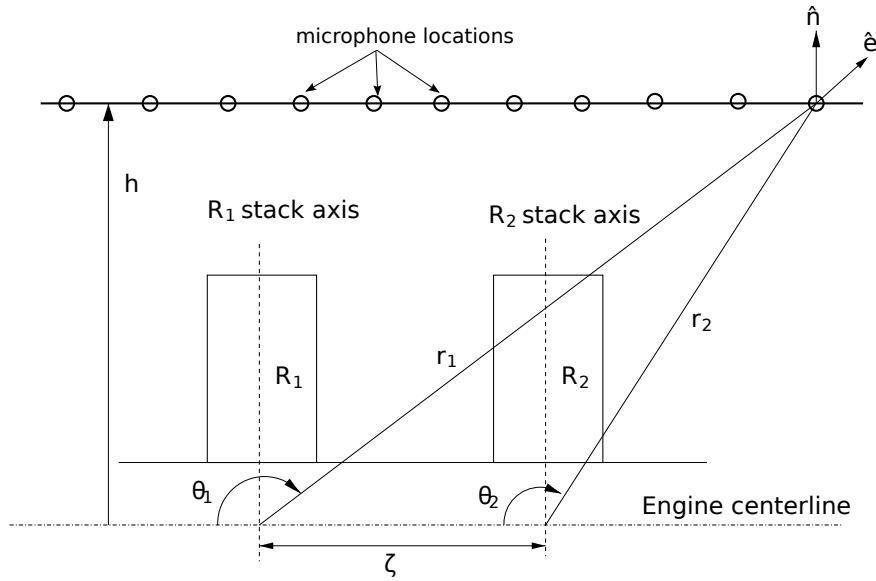


Figure 5: Schematic illustrating the sideline microphone locations.

203 design. This geometry has a 12-bladed front rotor and a 10-bladed aft rotor.
 204 Around the speed/thrust of interest (takeoff condition), the interaction tones
 205 dominate over the rotor-alone tones (arising from finite blade thickness and
 206 steady loading) and hence the focus here is on comparing interaction tones
 207 between data and predictions.

208 3.1. Ffowcs Williams-Hawkings Equation Solver

209 The Ffowcs Williams-Hawkings (FW-H) equation is a re-formulation of
 210 the linearized Euler equations using the Lighthill's acoustic analogy. A fre-
 211 quency domain formulation[6] of the FW-H equation is used here and the
 212 equations are provided in Appendix A.

213 A frequency domain FW-H equation solver is developed and validated
 214 against analytical solutions for point sources (monopole, dipole, and quadrupole)

215 in a quiescent medium. A cube is defined around the point source at which
216 the complete flow-field (density, pressure, and velocities) due to the source
217 are computed analytically. The information on the six faces of the cube is
218 then used by the FW-H solver to compute the sound pressure outside of the
219 cube. Far-field directivities are compared for the three sources in Fig. 6,
220 where excellent agreement can be observed.

221 Predictions are also made in the near field of the source, although it
222 should be borne in mind that the derivation of the FW-H equation itself
223 makes the approximation that the observer is in the far field. Hence the near-
224 field solution cannot be expected to be exact. Comparisons are nevertheless
225 made (see Fig. 7) in the near field as well, and are found to be reasonable
226 except very near the surface. In Fig. 7, the nearest surface point is located
227 at a distance of 2.12 units from the origin (shown by the arrow in the figure).
228 The near field of the dipole and the quadrupole source is reasonably well
229 captured, while the far-field prediction is excellent.

230 Since the interest is in predicting open rotor noise in flight condition (non-
231 zero forward velocity), the FW-H code is also verified against the analytical
232 solution of a point source in a moving medium. Three different flight speeds
233 are considered, namely, flow Mach number equal to 0.25, 0.5, and 0.75. This
234 adequately covers the range of flight speeds of interest although the focus of
235 this paper is on noise during take-off, when the flight Mach number is around
236 0.25. Directivity comparisons in the far-field showing excellent agreement are
237 plotted in Fig. 8.

238 These canonical validation cases provide sufficient confidence in the ac-
239 curacy of the FW-H solver to attempt the open rotor noise prediction.

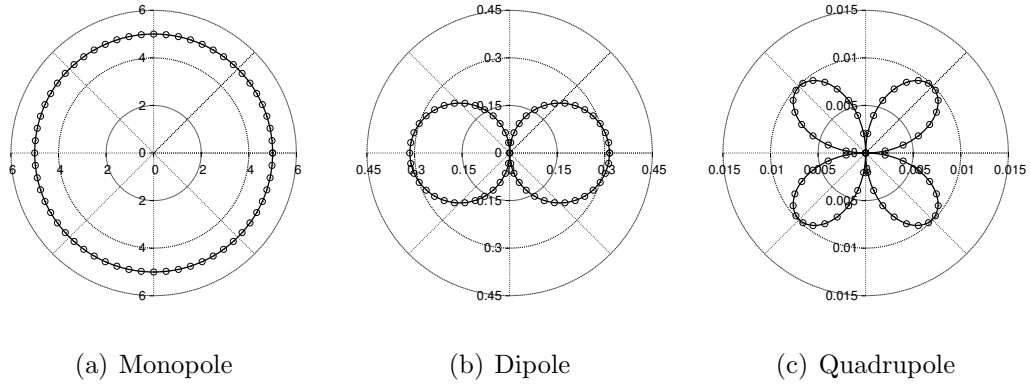


Figure 6: Directivity comparisons of pressure amplitude for a point source radiating in a quiescent medium between analytical solution (solid lines) and FW-H predictions (open circles). Pressure amplitudes are plotted in this polar plot.

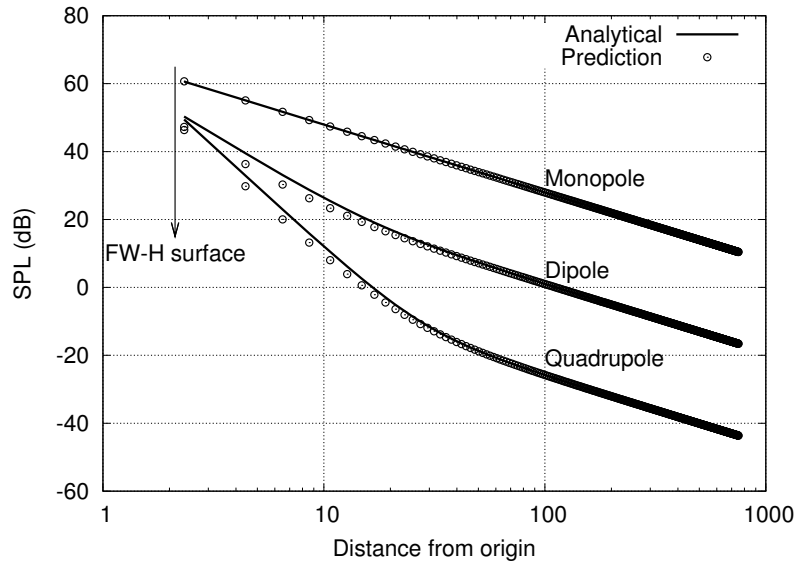


Figure 7: Near- and far-field comparisons of sound pressure levels (SPLs) between analytical solutions (solid lines) and FW-H predictions (open circles).

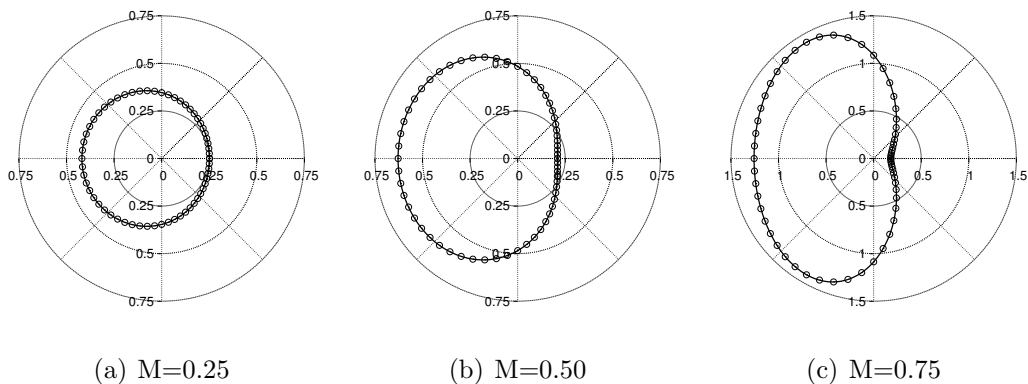


Figure 8: Directivity comparisons of pressure amplitudes for a point source radiating in a moving medium between analytical solution (solid lines) and FW-H predictions (open circles). The axial flow Mach numbers considered are: (a) $M = 0.25$, (b) $M = 0.50$, and (c) $M = 0.75$.

240 3.2. Validation Against Test Data

241 For comparisons against test data, we focus our attention on the F31A31
 242 geometry, a 12×10 configuration. The present investigation is further lim-
 243 ited to studying the variation of noise with blade tip speed (RPM), while
 244 keeping the blade stagger angle fixed - the engine thrust is therefore not held
 245 constant. A number of changes occur with increasing rotational speed that
 246 all contribute to noise increase in an open rotor. These are - (1) increase in
 247 radiation efficiencies of the acoustic modes, (2) increase in rotor blade wake
 248 deficit (due to increased blade incidence), and (3) increased unsteady lift on
 249 the aft rotor due to (a) high relative velocity, and (b) high mean loading.
 250 The scaling with Mach number of different tones is determined by which of
 251 these dominate.

252 The proposed procedure for open rotor noise prediction does remarkably
 253 well in predicting the speed scaling of the rotor-rotor interaction tones, as is

254 evident from Figs. 9 and 10, even though the absolute noise levels are slightly
 255 over-predicted. Linear curve fits (on a log-log scale) are plotted in the figures.
 256 The following nomenclature is used to represent the tones: $[a, b]$ refers to the
 257 tone at frequency $a \times R1 \text{ BPF} + b \times R2 \text{ BPF}$. In the cases considered here,
 258 both rotors (R1 and R2) rotate at the same shaft rotation rate, Ω . The
 259 sum tone $[a, b]$ therefore has a frequency of $(a \times N1 + b \times N2)\Omega$, where N1
 260 and N2 are R1 and R2 blade counts respectively. Appendix B provides
 261 a mathematical reasoning for why the “sum” and “difference” tones appear
 262 in such interactions and shows the relationship between the interaction tone
 263 frequency and its azimuthal mode number.

264 Figure 9 compares the overall tone power level variation with blade speed
 265 between prediction and data, which is obtained by adding (log sum) the
 266 acoustic power in the dominant tones. The frequency domain analyses is car-
 267 ried out for the first four harmonics of R1, which implies that the simulations
 268 (theoretically) should predict the following tones: $[1, (1 \dots \infty)]$, $[2, (1 \dots \infty)]$,
 269 $[3, (1 \dots \infty)]$ and $[4, (1 \dots \infty)]$. Since the geometric resolution (mesh) of the
 270 aft rotor is finite, only a finite number of “scattered” modes can be cap-
 271 tured in the linearized runs. Finite spatial order accuracy and artificial dis-
 272 sipation in the numerical scheme determine the grid resolution (number of
 273 points per wavelength) required to accurately resolve the higher order spa-
 274 tial modes. Only the first four scattered modes are therefore retained in the
 275 post-processing and used to compute the overall tonal power level. Similar
 276 filtering is applied to the experimental data as well to make a one-to-one
 277 comparison.

278 Figure 10 shows the speed trend comparison for four groups of tones.

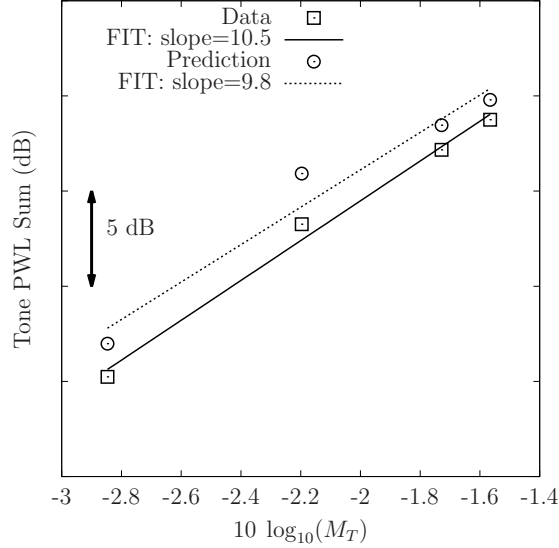
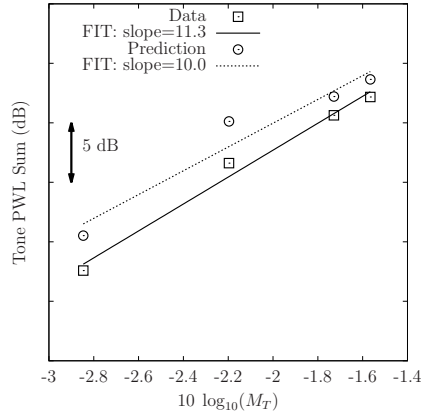


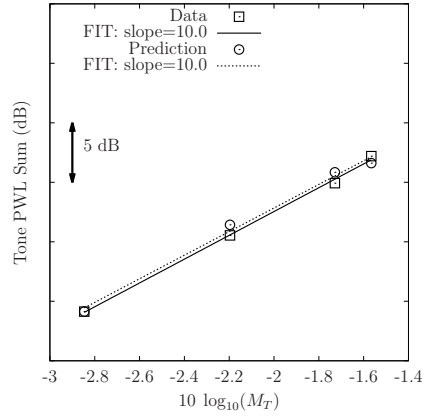
Figure 9: Comparison of measured and predicted sound power level sum of the rotor-rotor interaction tones.

279 These are grouped based on the wake harmonic of the front rotor. For exam-
 280 ple, in the figure, $(1, \sum_1^4)$ refers to the sum of $[1, 1]$, $[1, 2]$, $[1, 3]$ & $[1, 4]$ tones.
 281 Analyzing the results in such groups is useful as it identifies the contribution
 282 of noise by a specific wake harmonic of the front rotor. Good agreement
 283 is observed for these sets of comparisons as well. It is also noted that the
 284 overall tone power level (in Fig. 9) is very much governed by the interaction
 285 of the first wake/vortex harmonic of R1 with R2 (i.e., by the $[1, \sum_1^4]$ tones).
 286 While this is true for the cases considered here, it may not always hold true
 287 (e.g., at other blade pitch and speed settings).

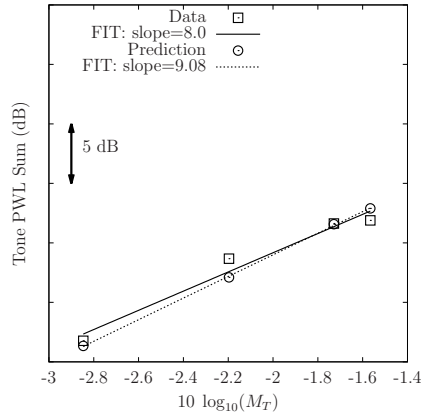
288 Figure 11 compares the acoustic power in each tone between data and
 289 prediction. The agreement in general is good; the largest discrepancy is ob-
 290 served for tones with two properties: first, they are relatively low in noise



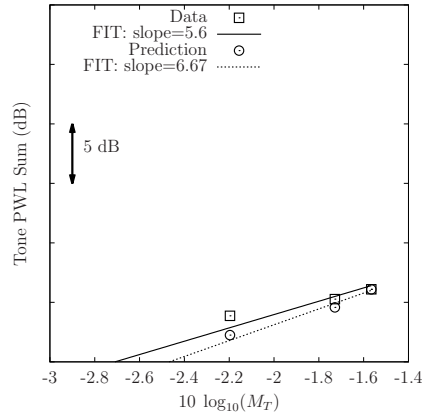
(a) $(1, \sum_1^4)$ tone sum



(b) $(2, \sum_1^4)$ tone sum



(c) $(3, \sum_1^4)$ tone sum



(d) $(4, \sum_1^4)$ tone sum

Figure 10: Comparison of measured and predicted sound power level sum grouped as blade passing harmonics of the front rotor.

291 amplitude (and hence less relevant to the overall tone noise level), and sec-
 292 ond, these tones should have a large azimuthal mode number if rotor-rotor
 293 interaction is the sole noise generation mechanism. As an example, consider
 294 the $[4, 1]$ tone. The predicted tone power level is more than 20 dB lower than
 295 measured data. The frequency of this tone is $(4 \times 12 + 1 \times 10)\Omega = 58\Omega$ while
 296 its circumferential mode number is $(4 \times 12 - 1 \times 10) = 38$. The radiation
 297 efficiency of this mode is very low as explained below. Radiation efficiency
 298 of each tone is given by a Bessel function of order equal to the azimuthal
 299 mode number and the argument given by the radial wave number multiplied
 300 by radius. The radial wave number is proportional to the frequency of the
 301 tone. Asymptotic behaviour of Bessel functions (as the argument becomes
 302 smaller than the order) is given by

$$J_n(x) \sim \frac{1}{n!} \left(\frac{x}{2}\right)^n \quad (1)$$

303 For relatively small speeds (Ω) considered here, the frequencies and hence
 304 the argument of the Bessel function becomes smaller than the order for a few
 305 tones (e.g., $[3, 1]$, $[4, 1]$, and $[4, 2]$) and hence their radiation efficiency plum-
 306 mets. Radiation efficiency of acoustic modes can also be explained using
 307 the concept of “sonic” or “Mach” radius introduced by Parry [28]. For a
 308 given observer location, the sonic radius is defined as the radius at which the
 309 source moves towards the observer at sonic speed. The sonic radius deter-
 310 mines the dominant noise producing region. For modes where the argument
 311 of the Bessel function is smaller than the order (i.e., where Eq. 1 holds),
 312 the sonic radius lies outboard of the tip radius. These modes therefore have
 313 poor radiation efficiencies. This is further illustrated in Fig. 12 where far-
 314 field noise from a point source (as calculated using Hanson’s noise radiation

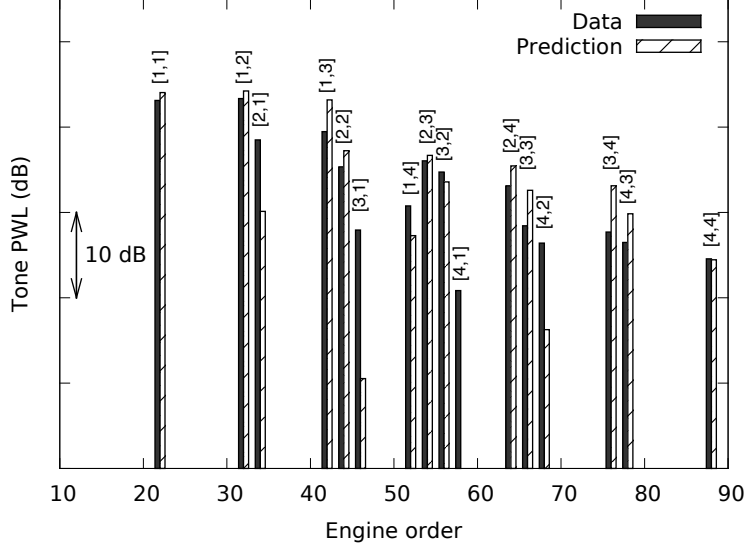


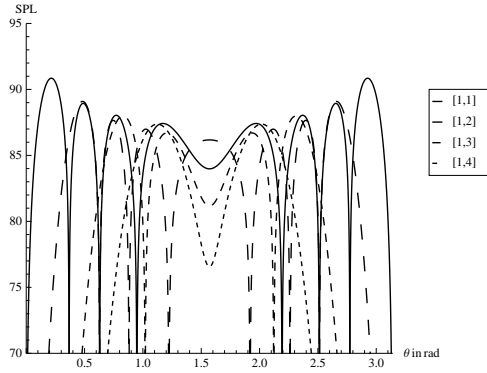
Figure 11: Interaction tone PWL spectra comparison between data and prediction at one sample operating point.

315 formula [11]) for different interaction tones are compared. Figure 12 demon-
 316 strates the variation of radiation efficiency with azimuthal mode number for
 317 a few tones. Plots (a) and (b) in Fig. 12 show the directivity of sets of
 318 tones [1, 1], [1, 2], [1, 3], [1, 4] and [4, 1], [4, 2], [4, 3], [4, 4] respectively. The
 319 reader is reminded that the azimuthal mode number of each tone is unique
 320 (theoretically) and is given by $(a \times N1 - b \times N2)$ for the tone $[a, b]$. The
 321 azimuthal mode numbers for these tones are also listed in parentheses in
 322 plots (c) and (d) of Fig. 12, which integrate the directivity and show the
 323 sound power levels (relative to the power in [1, 1] tone). As the azimuthal
 324 mode order increases, the sound radiation starts to concentrate in the plane
 325 of rotation and the radiation patterns look much like that of rotor alone noise
 326 (see e.g. directivity of [4, 1] tone in plot (b)). Integrated sound power levels

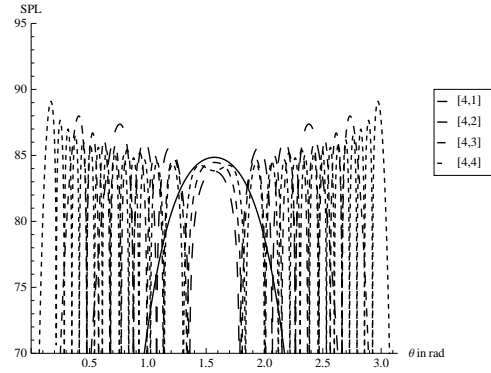
327 confirm that increasing azimuthal mode order leads to drop in the sound
328 power. Since the source amplitude in this canonical example is unity for all
329 tones, the reduction in power is completely due to the reduction in radiation
330 efficiency.

331 The predicted reduced levels of noise for tones $[3, 1]$, $[4, 1]$, and $[4, 2]$ in
332 Fig. 11 therefore are expected due to the reduced radiation efficiencies of
333 these modes. The relatively large power in the measured data for these tones
334 may be explained by the following. It is conjectured that the origin of these
335 tones in experiment is not simply due to R1-R2 interaction but perhaps due
336 to the interaction of a “spatially modulated” R1 wake with R2. Such a
337 modulation occurring for example if the open rotor operates at a slightly
338 non-zero angle of attack. The interaction of such spatially modulated wake
339 would then produce the same time spectral content but the azimuthal order
340 of the modes would be lower, enhancing the radiation efficiency of these
341 tones. In such cases, the directivity of the tones would show a variation
342 with azimuthal angle. The current test campaign however did not include
343 azimuthal directivity measurements, and hence it is not possible to verify
344 this hypothesis.

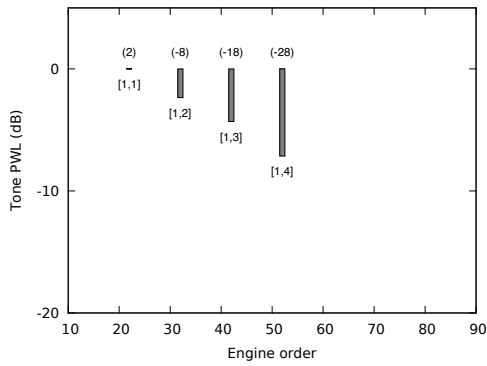
345 Another evidence of “unsuspected” noise radiation in the open rotor ex-
346 periments is observed (see Fig. 13) in the spectral decay of rotor alone tones,
347 e.g., consider R1 alone tones: $[n, 0]$, where $n = 12, 24, 36, \dots$ etc. Analytical
348 theories e.g., due to Gutin [29] as well as the predictions made herein suggest
349 a sharp dropoff with higher harmonics of noise due to thickness and steady
350 loading, due again to rapid reduction in radiation efficiency (through increase
351 in the order of the Bessel function). Similar results (not shown here) were ob-



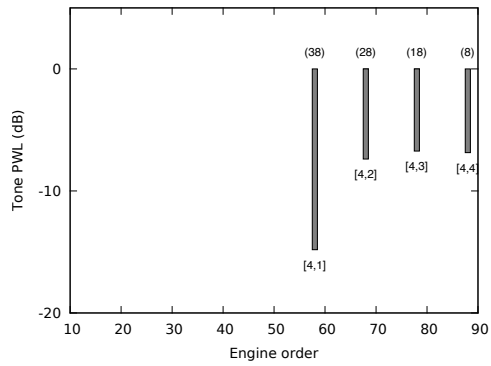
(a) Directivity of tones: (1,1...4)



(b) Directivity of tones: (4,1...4)



(c) PWL of tones: (1,1...4)



(d) PWL of tones: (4,1...4)

Figure 12: Directivity and sound power levels of selected interaction tones and for a model point source problem. Sound power is normalized so that [1,1] tone has PWL=0. In (c) and (d) the number in parentheses is the azimuthal mode order of the tone.

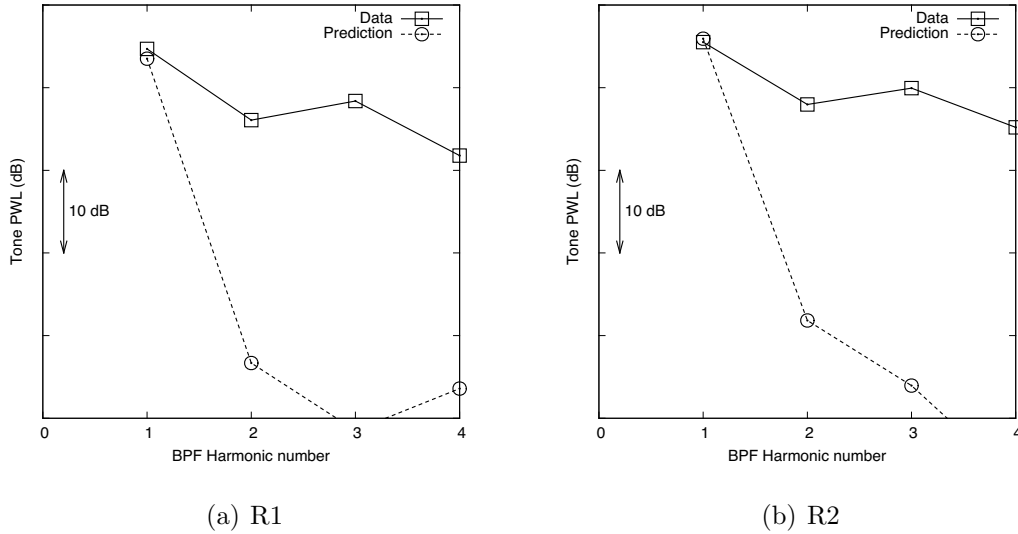


Figure 13: Variation of rotor alone acoustic power with (a) R1 and (b) R2 harmonics.

352 served with other semi-analytical prediction methods [30, 31]. Measured data
 353 shows some reduction but it is not as large and also it plateaus out around the
 354 second blade passing frequency. Note that this level is still above the mea-
 355 sured broadband noise. Again, it is suspected that the measured noise here
 356 is due to a different source, e.g., inlet distortion. While there are turbulence
 357 screens employed in the experiment to minimize the inlet turbulence levels,
 358 there is still a possibility of having coherent turbulence structures chopped
 359 by the blades to produce tones at blade passing frequency. The azimuthal
 360 order of the pattern due to the interaction of these distortions with the rotor
 361 bladerows may be much lower than that for steady loading (thickness) noise
 362 source, making them highly efficient at radiating. It is suspected that noise
 363 due to such interaction masquerades as “rotor alone” tones especially at high
 364 frequencies.

365 **4. Conclusion**

366 A new prediction methodology utilizing linearized RANS analysis in com-
367 bination with an integral method approach (Ffowcs Williams-Hawkings equa-
368 tion solution) to predict aerodynamic tonal noise from open rotors is pre-
369 sented. A frequency domain FW-H solver is developed and validated against
370 analytical solutions of point sources (mono-, di-, and quadru-pole) in a qui-
371 escent medium as well as for a point monopole in a moving medium. The
372 prediction process is then applied to the historic F31A31 open rotor baseline
373 geometry recently tested at the NASA 9' x 15' low-speed wind tunnel. Noise
374 trends with blade tip Mach number are compared to show the validity of
375 the proposed prediction process. Very good agreement between prediction
376 and data is observed in noise trends with blade tip speed. Absolute levels
377 are slightly over-predicted (around 2-4 dB). Greatest mismatch between data
378 and prediction (data being higher) is observed for tones which are expected
379 to have very high circumferential mode number and therefore very low radia-
380 tion efficiency. It is conjectured that the high acoustic power levels measured
381 in such modes arise from "non-ideal" R1-R2 interaction such as would occur
382 if the R1 wake is spatially modulated.

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 392 on the validation test cases.

393 **Appendix A. FW-H Formulation**

394 The permeable surface Ffowcs Williams-Hawkings equation, upon ignor-
 395 ing the volume integral term, can be written as

$$4\pi |\mathbf{x}| p'(\mathbf{x}, t) = \frac{x_i}{c |\mathbf{x}|} \frac{\partial}{\partial t} \int [p' n_i + \rho u_i (u_j - U_j) n_j] d\Sigma + \frac{\partial}{\partial t} \int [\rho_0 u_i + \rho' (u_i - U_i)] n_i d\Sigma, \quad (\text{A.1})$$

396 where Σ denotes the surface enclosing all the sound sources for the given
 397 problem. The sound emitted by the source located at \mathbf{x}_s at time τ is received
 398 by the observer located at \mathbf{x} at time t . The relation between the source time,
 399 τ and the observer time, t is

$$c(t - \tau) = |\mathbf{x} - \mathbf{x}_s|, \quad (\text{A.2})$$

400 where c is the speed of sound. For an observer in the farfield ($|\mathbf{x}| \gg |\mathbf{x}_s|$)
 401 Eq. A.2 can be approximated as

$$c(t - \tau) \approx |\mathbf{x}| - \frac{\mathbf{x}_s \cdot \mathbf{x}}{|\mathbf{x}|}. \quad (\text{A.3})$$

402 Recognizing that the source \mathbf{x}_s is located at $\mathbf{x}_s = \mathbf{y}$ at time $\tau = 0$ and moves
 403 with the velocity \mathbf{U} (i.e., $\mathbf{x}_s = \mathbf{y} + \mathbf{U}\tau$), Eq. A.3 can be further expanded as

$$\begin{aligned}
 c(t - \tau) &\approx |\mathbf{x}| - \frac{\mathbf{x} \cdot \mathbf{y}}{|\mathbf{x}|} - \frac{\tau \mathbf{U} \cdot \mathbf{x}}{|\mathbf{x}|}, \text{ or,} \\
 t - \tau &\approx \frac{|\mathbf{x}|}{c} - \frac{\mathbf{x} \cdot \mathbf{y}}{c|\mathbf{x}|} - \frac{\tau \mathbf{U} \cdot \mathbf{x}}{c|\mathbf{x}|}, \text{ or,} \\
 (1 - M_r)\tau &\approx t - \frac{|\mathbf{x}|}{c} + \frac{\mathbf{x} \cdot \mathbf{y}}{c|\mathbf{x}|}. \tag{A.4}
 \end{aligned}$$

404 where M_r is the source Mach number in the direction of the observer. Taking
 405 the derivative of Eq. A.4 w.r.t. τ gives

$$\begin{aligned}
 (1 - M_r) \frac{d\tau}{dt} &= 1, \text{ or,} \\
 \frac{d\tau}{dt} &= \frac{1}{1 - M_r}, \tag{A.5}
 \end{aligned}$$

406 which is the Doppler frequency shift. The source angular frequency, ω is
 407 perceived by the observer to be $\omega/(1 - M_r)$. Fourier transform Eq. A.1 to
 408 write the observer sound pressure at the frequency, $\omega/(1 - M_r)$ as

$$\begin{aligned}
 4\pi |\mathbf{x}| \int_{-\infty}^{\infty} p'(\mathbf{x}, t) e^{-\frac{i\omega t}{1 - M_r}} dt &= \frac{x_i}{c|\mathbf{x}|} \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial t} \int [p' n_i + \rho u_i (u_j - U_j) n_j] d\Sigma \right\} e^{-\frac{i\omega t}{1 - M_r}} dt \\
 &+ \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial t} \int [\rho_0 u_i + \rho' (u_i - U_i)] n_i d\Sigma \right\} e^{-\frac{i\omega t}{1 - M_r}} dt. \tag{A.6}
 \end{aligned}$$

409 Convert $\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial \tau}$ and $dt \rightarrow d\tau$ in the above using Eq. A.5 to get

$$\begin{aligned}
 4\pi |\mathbf{x}| \hat{p}(\mathbf{x}, \frac{\omega}{1 - M_r}) &= \frac{x_i}{c(1 - M_r)|\mathbf{x}|} \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial \tau} \int [p' n_i + \rho u_i (u_j - U_j) n_j] d\Sigma \right\} e^{-\frac{i\omega \tau}{1 - M_r}} (1 - M_r) d\tau \\
 &+ \frac{1}{(1 - M_r)} \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial \tau} \int [\rho_0 u_i + \rho' (u_i - U_i)] n_i d\Sigma \right\} e^{-\frac{i\omega \tau}{1 - M_r}} (1 - M_r) d\tau. \tag{A.7}
 \end{aligned}$$

410 The hat ($\hat{}$) denotes a Fourier transformed quantity. Using Eq. A.4 to
 411 express t in terms of the source time τ in the exponent gives

$$\begin{aligned}
 4\pi |\mathbf{x}| \hat{p}(\mathbf{x}, \frac{\omega}{1 - M_r}) &= \frac{x_i}{c |\mathbf{x}|} \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial \tau} \int [p' n_i + \rho u_i (u_j - U_j) n_j] d\Sigma \right\} e^{-i\omega \tau} d\tau e^{-\frac{i\omega}{1 - M_r} \left(\frac{|\mathbf{x}|}{c} - \frac{\mathbf{x} \cdot \mathbf{y}}{c |\mathbf{x}|} \right)} \\
 &+ \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial \tau} \int [\rho_0 u_i + \rho' (u_i - U_i)] n_i d\Sigma \right\} e^{-i\omega \tau} d\tau e^{-\frac{i\omega}{1 - M_r} \left(\frac{|\mathbf{x}|}{c} - \frac{\mathbf{x} \cdot \mathbf{y}}{c |\mathbf{x}|} \right)} \quad (\text{A.8})
 \end{aligned}$$

412 The constant phase shift, $\exp\left(-\frac{i\omega}{1 - M_r} \frac{|\mathbf{x}|}{c}\right)$, which represents the time
 413 delay for the sound to reach the observer, can be dropped from the above to
 414 write

$$\begin{aligned}
 4\pi |\mathbf{x}| \hat{p}(\mathbf{x}, \frac{\omega}{1 - M_r}) &= \frac{x_i}{c |\mathbf{x}|} \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial \tau} \int [p' n_i + \rho u_i (u_j - U_j) n_j] d\Sigma \right\} e^{-i\omega \tau} d\tau e^{-\frac{i\omega}{1 - M_r} \left(-\frac{\mathbf{x} \cdot \mathbf{y}}{c |\mathbf{x}|} \right)} \\
 &+ \int_{-\infty}^{\infty} \left\{ \frac{\partial}{\partial \tau} \int [\rho_0 u_i + \rho' (u_i - U_i)] n_i d\Sigma \right\} e^{-i\omega \tau} d\tau e^{-\frac{i\omega}{1 - M_r} \left(-\frac{\mathbf{x} \cdot \mathbf{y}}{c |\mathbf{x}|} \right)} \quad (\text{A.9})
 \end{aligned}$$

415 The partial derivative operator, $\partial/\partial\tau$ can be taken inside the Σ integral as
 416 it is independent of τ . Further, realizing that

$$\int_{-\infty}^{\infty} \frac{\partial \psi(\tau)}{\partial \tau} \exp(-i\omega \tau) d\tau = i\omega \int_{-\infty}^{\infty} \psi(\tau) \exp(-i\omega \tau) d\tau, \quad (\text{A.10})$$

417 Eq. A.9 can be rewritten as

$$\begin{aligned}
 4\pi |\mathbf{x}| \hat{p}(\mathbf{x}, \frac{\omega}{1 - M_r}) &= i\omega \frac{x_i}{c |\mathbf{x}|} \int \left[p' n_i + \widehat{\rho u_i (u_j - U_j) n_j} \right] \exp \left\{ -\frac{i\omega}{1 - M_r} \left(-\frac{\mathbf{x} \cdot \mathbf{y}}{c |\mathbf{x}|} \right) \right\} d\Sigma \\
 &+ i\omega \int \left[(\rho_0 u_i + \widehat{\rho' (u_i - U_i)}) n_i \right] \exp \left\{ -\frac{i\omega}{1 - M_r} \left(-\frac{\mathbf{x} \cdot \mathbf{y}}{c |\mathbf{x}|} \right) \right\} d\Sigma \quad (\text{A.11})
 \end{aligned}$$

418 which is the form of the integral equation used here.

419 **Appendix B. R1-R2 Interaction Noise**

420 A mathematical reasoning for the generation of sum and difference tones
 421 due to rotor-rotor (R1-R2) interaction is given below. In the stationary,
 422 cylindrical frame of reference (x, r, θ, t) , the R1 wake can be represented by

$$V_g = \sum_{n=0}^{\infty} \hat{V}_g(x, r) \exp \{i n N_{R_1} (-\Omega_1 t + \theta)\}, \quad (\text{B.1})$$

423 where Ω_1 is the angular velocity of R1. In the frame of reference attached to
 424 R2, (x', r', θ', t') where

$$x' = x, r' = r, t' = t, \ \& \ \theta' = \theta + \Omega_2 t,$$

425 the wake/gust appears as

$$V_g = \sum_{n=0}^{\infty} \hat{V}_g(x', r') \exp \{i n N_{R_1} (-(\Omega_1 + \Omega_2)t + \theta')\}. \quad (\text{B.2})$$

426 Hence, the frequency of the gust in the R2 frame of reference is $\omega'_g =$
 427 $nN_{R_1}(\Omega_1 + \Omega_2)$. This is the frequency at which the forced response cal-
 428 culation using linearized RANS is carried out. The solution of the linearized
 429 RANS equations yields near-field pressure in the R2 frame of reference, which
 430 can be written as

$$p = \sum_{n=0}^{\infty} \sum_{k=-\infty}^{\infty} \hat{p}(x', r') \exp \{i(-\omega t + m'\theta')\}, \quad (\text{B.3})$$

431 where $m' = nN_{R_1} - kN_{R_2}$ and k is an integer, as given by the Tyler-Sofrin
 432 theory [32]. Writing the above expression in the ground frame of reference
 433 gives

$$\begin{aligned} p &= \sum_{n=0}^{\infty} \sum_{k=-\infty}^{\infty} \hat{p}(x', r') \exp \{i(-nN_{R_1}(\Omega_1 + \Omega_2)t + (nN_{R_1} - kN_{R_2})(\theta + \Omega_2 t))\} \\ &= \sum_{n=0}^{\infty} \sum_{k=-\infty}^{\infty} \hat{p}(x, r) \exp \{i(-(nN_{R_1}\Omega_1 + kN_{R_2}\Omega_2)t + (nN_{R_1} - kN_{R_2})\theta)\} \end{aligned} \quad (\text{B.4})$$

434 Equation B.4 suggests that the frequencies of the R1-R2 interaction tones,
435 and the corresponding circumferential modes are given by

$$\omega_p = (nN_{R_1}\Omega_1 + kN_{R_2}\Omega_2) \text{ and } m = nN_{R_1} - kN_{R_2} \text{ respectively.}$$

436 Note that Ω_1 and Ω_2 are magnitudes of the shaft rotation rates; the direction
437 of rotation is taken into account in relating θ' to θ . For the case when the
438 shaft rotation rates of the two rotors are equal ($\Omega_1 = \Omega_2 = \Omega$), the expression
439 for interaction frequencies reduces to

$$\omega_p = (nN_{R_1} + kN_{R_2})\Omega, \quad \text{where } -\infty < k < \infty,$$

440 and hence the expression “sum” and “difference” tones is used to refer to
441 rotor-rotor interaction tones.

442 Note that while the “sum” tones are easily observed in experiments, the
443 “difference” tones hardly are. This is primarily because the circumferential
444 mode number corresponding to a “difference” tone is much higher (which
445 corresponds to the order of the Bessel function) while the frequency (which
446 corresponds to the argument of the Bessel function) is much lower, thus
447 rendering the radiation efficiency of “difference” tones to be very low.

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