A Model Checking Intermediate Language

Initial Proposal

The NSF:CCRI Team

January 16, 2023
Intermediate Language (IL) goals

The IL has been designed so that it

• is a *general enough* intermediate *target language* for MC
• can support a *variety* of user-facing *modeling languages*
• can be *directly supported* by tools or *compiled* to lower level languages
• can leverage SAT/SMT technology
General design principles

IL models are meant to be **produced and processed** mostly by tools

So the IL was designed to have

- simple, easily parsable syntax
- a rich set of data types
- little syntactic sugar, at least initially
- well-understood semantics
- a small but comprehensive set of commands
- simple translations to lower level languages such as Btor2 and Aiger
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Design principles — implications

1. **Little direct support** for many of the **features** offered by
   - hardware modeling languages such as VHDL and Verilog or
   - system modeling languages such as SMV, TLA+, PROMELA, UNITY, Lustre

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Current proposal

Extension the SMT-LIB language with new commands to define and check systems

Each system definition

- defines a transition system via the use of SMT formulas
- generally imposes minimal syntactic restrictions on those formulas
- is parametrized by a state signature, a sequence of typed variables
- partitions state variables into input, output and local variables
- is hierarchical, i.e., may include (instances of) previously defined systems as subsystems
- can encode both synchronous and asynchronous system composition
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Current focus

Finite-state systems

but with an eye to infinite-state systems too
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Technical preliminaries

Formally, a transition system is a pair $S$ of predicates of the form

$$S = \left( I_S[i,o,s], T_S[i,o,s,i',o',s'] \right)$$

where

- $i$ and $i'$ are two tuples of input variables with the same length and type
- $o$ and $o'$ are two tuples of output variables with the same length and type
- $s$ and $s'$ are two tuples of local variables with the same length and type
- $I_S$, the initial state condition is a formula with free vars from $[i,o,s]$
- $T_S$, the transition condition is a formula with free vars from $[i,o,s,i',o',s']$

Note: A (full) state of $S$ is a valuation of $(i,o,s)$
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SMT-LIB commands

As in SMT-LIB

(set-logic \( L \))

(declare-sort \( s \ n \))

(define-sort \( s \ (u_1 \, \ldots \, u_n) \, \tau \))

(declare-fun \( f \ ((x_1 \, \sigma_1) \, \ldots \, (x_n \, \sigma_n)) \, \sigma \))

(define-fun \( f \ ((x_1 \, \sigma_1) \, \ldots \, (x_n \, \sigma_n)) \, \sigma \, t \))

(declare-datatype \( d \) (\ldots ))

(assert \( F \))

(perhaps a few more)
SMT-LIB commands

New

(define-system S ...)  
(check-system S ...)  

(declare-enum-sort S (c₁ ... cₙ))
Logical semantics

A **define-system** command implicitly defines a *model* (i.e., a Kripke structure) of First-Order Linear Temporal Logic (FO-LTL)

An FO-LTL formula $F[f, x, x']$ with

- free (immutable) constants/functions (aka, uninterpreted symbols) from $f$
- free (mutable) variables from $x, x'$

is *satisfiable* in an SMT theory $\mathcal{T}$ if there is

1. a $\mathcal{T}$-interpretation $\mathcal{I}$ of $f$ and
2. an infinite trace $\pi$ over $x$ in $\mathcal{I}$

that satisfy $F$
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that satisfy $F$
Trace semantics

Fix

- an FOL-LTL formula $F[f, x, x']$ over a theory $\mathcal{T}$
- a $\mathcal{T}$-interpretation $\mathcal{I}$ of $f$
- an infinite trace $\pi = s_0, s_1, \ldots$ where $s_i$ is an assignment of $x$ into $\mathcal{I}$ for all $i \geq 0$

Let $\pi^i = s_i, s_{i+1}, \ldots$ for all $i \geq 0$

$(\mathcal{I}, \pi)$ satisfies $F$, written $(\mathcal{I}, \pi) \models F$, iff

1. $\mathcal{I}[x \rightarrow s_0(x), x' \rightarrow s_1(x)]$ satisfies $F$ when $F$ is atomic
2. $(\mathcal{I}, \pi) \not\models G$ when $F$ is $\neg G$
3. $(\mathcal{I}, \pi) \models G_j$ for $j = 1, 2$ when $F$ is $G_1 \land G_2$
4. $(\mathcal{I}, \pi^1) \models G$ when $F$ is next $G$
5. $(\mathcal{I}, \pi^i) \models G$ for all $i = 0, \ldots$ when $F$ is always $G$
6. $(\mathcal{I}, \pi^i) \models G$ for some $i = 0, \ldots$ when $F$ is eventually $G$
7. $\ldots$
Trace semantics

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4. $(\mathcal{I}, \pi^i) \models G$ when $F$ is $\text{next } G$
5. $(\mathcal{I}, \pi^i) \models G$ for all $i = 0, \ldots$, when $F$ is $\text{always } G$
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7. $\ldots
Finite-Trace semantics

Fix

• an FOL-LTL formula $F[f, x, x']$ over a theory $T$
• a $T$-interpretation $I$ of $f$
• an infinite trace $\pi = s_0, s_1, \ldots$ where $s_i$ is an assignment of $x$ into $I$ for all $i \geq 0$

Let $\pi^i = s_i, s_{i+1}, \ldots$ for all $i \geq 0$

$(I, \pi)$ $n$-satisfies $F$ for some $n \geq 0$, written $(I, \pi) \models_n F$, iff

1. $I[x \mapsto s_0(x), x' \mapsto s_1(x)]$ satisfies $F$ when $F$ is atomic
2. $(I, \pi) \not\models_n G$ when $F$ is $\neg G$
3. $(I, \pi) \models_n G_j$ for $j = 1, 2$ when $F$ is $G_1 \land G_2$
4. $(I, \pi^1) \models_{n-1} G$ and $n - 1 > 0$ when $F$ is next $G$
5. $(I, \pi^i) \models_{n-i} G$ for all $i = 0, \ldots, n - 1$ when $F$ is always $G$
6. $(I, \pi^i) \models_{n-i} G$ for some $i = 0, \ldots, n - 1$ when $F$ is eventually $G$
7. $\ldots$
System **definition** command (Base case)

```
(define-system S
 :input  ((i₁ δ₁) ⋯ (iₘ δₘ))  ; input vars
 :output ((o₁ τ₁) ⋯ (oₙ τₙ))  ; output vars
 :local  ((s₁ σ₁) ⋯ (sₚ δₚ))  ; local vars
 :init   I  ; initial state formula
 :trans  T  ; transition formula
 :inv    P  ; invariant formula
)
```
System **definition command**  (Base case)

```lisp
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```

where

- each var gets a **primed copy**: \(i₁', ..., o₁', ..., s₁', ...\)
- \(I\) and \(P\) are **one-state** formulas (over unprimed vars only)
- \(T\) is a **two-state** formula (over unprimed and primed vars)
- all attributes are **optional** and their order is immaterial
  - however, :input, :output, :local must occur before :init, :trans, :inv
**System definition command**  (Base case)

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  :output ((o₁ τ₁) ... (oₙ τₙ)) ; output vars
  :local  ((s₁ σ₁) ... (sₚ δₚ)) ; local vars
  :init   I ; initial state formula
  :trans  T ; transition formula
  :inv    P ; invariant formula
)
```

**Semantics**

\[ S = (I_S, T_S) = (I[i, o, s], P[i, o, s] \land T[i, o, s, i', o', s']) \]

where \( i = (i_1, \ldots, i_m), o = (o_1, \ldots, o_n), s = (s_1, \ldots, s_n) \)

\( S \) denotes the set of all infinite traces that satisfy the FO-LTL formula

\[ I_S \land \text{always } T_S \]
System **definition command** (Base case)

\[
\text{(define-system } S \\
\quad \text{:input } ((i_1 \delta_1) \cdots (i_m \delta_m)) \quad ; \text{input vars} \\
\quad \text{:output } ((o_1 \tau_1) \cdots (o_n \tau_n)) \quad ; \text{output vars} \\
\quad \text{:local } ((s_1 \sigma_1) \cdots (s_p \delta_p)) \quad ; \text{local vars} \\
\quad \text{:init } I \quad ; \text{initial state formula} \\
\quad \text{:trans } T \quad ; \text{transition formula} \\
\quad \text{:inv } P \quad ; \text{invariant formula}
\)]

**Note:**

Systems are meant to be *progressive*: every reachable state has a successor wrt $T_S$

However, they may not be because of the generality of $T$ and $P$

(It is possible to define deadlocking systems)
Default values for missing attributes

<table>
<thead>
<tr>
<th>attribute</th>
<th>default</th>
</tr>
</thead>
<tbody>
<tr>
<td>:input</td>
<td>()</td>
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<tr>
<td>:output</td>
<td>()</td>
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<tr>
<td>:local</td>
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<tr>
<td>:init</td>
<td>true</td>
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<td>true</td>
</tr>
<tr>
<td>:inv</td>
<td>true</td>
</tr>
</tbody>
</table>
Examples

; The output of Delay is initially in [0,10] and
; then is the previous input
(define-system Delay
  :input ((in Int))
  :output ((out Int))
  :init (<= 0 out 10) ; more than one possible initial output
  :trans (= out’ in) ; the new output is the old input
)

; A clocked lossless channel, stuttering when clock is false
(define-system StutteringClockedCopy
  :input ((clock Bool) (in Int))
  :output ((out Int))
  :init (=> clock (= out in)) ; out is arbitrary when clock is false
  :trans (ite clock (= out’ in’) (= out’ out))
)
Examples

(declare-datatype Event (par (X) (Absent) (Present (val X))))

; An event-triggered channel that arbitrarily loses its input data
(define-system LossyIntChannel
  :input ((in (Event Int)))
  :output ((out (Event Int)))
  :local ((s Bool))
  ; at all times, whether the input event is transmitted
  ; or not depends on value of s
  :inv (or (= out in) (= out Absent))
)
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)

; Equivalent formulation using unconstrained local state
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  ; or not depends on value of s
  :inv (= out (ite s in Absent))
)
Example: timed light switch

TimedSwitch models a timed light switch where, once on, the light stays on for 10 steps unless it is switched off before

A Boolean input is provided as a toggle signal
Example: timed light switch

(define-enum-sort LightStatus (On Off))

; Guarded-transitions-style definition
(define-system TimedSwitch :input ((press Bool)) :output ((sig Bool))
:local ((s LightStatus) (n Int))
:inv (= sig (= s On))
:init (and
    (= n 0)
    (ite press (= s On) (= s Off))
)
:trans (and
    (=> (and (= s Off) (not press')) ; Off ->
        (and (= s' Off) (= n' n))) ; Off
    (=> (and (= s Off) press') ; Off ->
        (and (= s' On) (= n' n))) ; On
    (=> (and (= s On) (not press') (< 10 n)) ; On ->
        (and (= s' On) (= n' (+ n 1)))) ; On
    (=> (and (= s On) (or press' (>= n 10)) ; On ->
        (and (= s' Off) (= n' 0))) ; Off
)
)
Example: timed light switch

(define-enum-sort LightStatus (On Off))

; Set-of-transitions-style definition
(define-system TimedSwitch2 :input ((press Bool)) :output ((sig Bool))
  :local ((s LightStatus) (n Int))
  :inv (= sig (= s On))
  :init (and
    (= n 0)
    (ite press (= s On) (= s Off))
  )
  :trans
    (let (; Transitions
      (stay-off (and (= s Off) (not press') (= s' Off) (= n' n)))
      (turn-on (and (= s Off) press' (= s' On) (= n' n)))
      (stay-on (and (= s On) (not press') (< n 10) (= s' On)
        (= n' (+ n 1))))
      (turn-off (and (= s On) (or press' (>= n 10))
        (= s' Off) (= n' 0)))
    )
    (or stay-off turn-on turn-off stay-on)
  )
)
Example: timed light switch

(define-enum-sort LightStatus (On Off))

; Equational-style definition
(define-system TimedSwitch3 :input ((press Bool)) :output ((sig Bool))
 :local ((s LightStatus) (n Int))
 :inv (= sig (= s On))
 :init (and
    (= n 0)
    (= s (ite press On Off))
  )
 :trans (and
    (= s’ (ite press’ (flip s)
      (ite (or (= s Off) (>= n 10)) Off
           On)))
    (= n’ (ite (or (= s Off) (s’ Off)) 0
                  (+ n 1)))
  )
)

(define-fun flip ((s LightStatus)) LightStatus (ite (= s Off) On Off))
Special predicate: **OnlyChange**

For every system \( S = (I_S[i, o, s], T_S[i, o, s, i', o', s']) \)

**OnlyChange** is a *multi-arity* predicate over \( o, s, o', s' \):

\[
\text{OnlyChange}(x_1, \ldots, x_n) \equiv \bigwedge \{ y' = y \mid y \in (o \cup s \cup o' \cup s') \setminus \{x_1, \ldots, x_n\} \}
\]

**Fixes** the value of all output and local variables not in \( (x_1, \ldots, x_n) \)

It is a useful abbreviation in transition conditions to express transitions that leave many state variables unchanged.

**Note:** **OnlyChange**\((x_1, \ldots, x_n)\) does not actually constrain the \( x_i \)'s in any way.
**Special predicate: OnlyChange**

For every system $S = (I_S[i, o, s], T_S[i, o, s, i', o', s'])$

*OnlyChange* is a *multi-arity* predicate over $o, s, o', s'$:

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It is a useful *abbreviation* in transition conditions to express transitions that leave many state variables unchanged

**Note:** *OnlyChange*(x₁, ..., xₙ) does not actually constrain the xᵢ's in any way
Special predicate: **OnlyChange**

For every system $S = (I_S[i, o, s], T_S[i, o, s, i', o', s'])$

*OnlyChange* is a *multi-arity* predicate over $o, s, o', s'$:

$$\text{OnlyChange}(x_1, \ldots, x_n) \equiv \bigwedge \{ y' = y \mid y \in (o \cup s \cup o' \cup s') \setminus \{x_1, \ldots, x_n\} \}$$

**Fixes** the value of all output and local variables not in $(x_1, \ldots, x_n)$

It is a useful *abbreviation* in transition conditions to express transitions that leave many state variables unchanged

**Note:** *OnlyChange*$(x_1, \ldots, x_n)$ does not actually constrain the $x_i$’s in any way
Example

; increment $n_i$ iff $n = i$; $n_i$ is 0 initially if not incremented

(define-system Increment :input ((i Int))
  :output ((inc Bool) (n1 Int) (n2 Int) ... (n5 Int))
  :inv (= inc (<= 1 i 5))
  :init (and
    (=> (= n 1) (and (= n1 1) (= n2 n3 n4 n5 0)))
    ...
    (=> (= n 5) (and (= n5 1) (= n1 n2 n3 n4 0)))
    (=> (not (<= 1 n 5)) (= n1 n2 n3 n4 n5 0))
  )
  :trans (and
    (=> (= n’ 1) (and (= n1’ (+ n1 1)) (OnlyChange inc n1)))
    ...
    (=> (= n’ 5) (and (= n5’ (+ n5 1)) (OnlyChange inc n5)))
    (=> (not (<= 1 n’ 5)) (OnlyChange inc))
  )
)
System definition — Synchronous composition

(define-system S
  :input  ((i₁ δ₁) ⋅⋅⋅ (iₘ δₘ)) ; input vars
  :output ((o₁ τ₁) ⋅⋅⋅ (oₙ τₙ)) ; output vars
  :local  ((s₁ σ₁) ⋅⋅⋅ (sₚ δₚ)) ; local vars
  :subsys (N₁ (S₁ x₁ y₁)) ; component subsystem
  ⋯
  :subsys (Nₚ (Sₚ xₚ yₚ)) ; component subsystem
)

where

1. q > 0 and each Sᵢ is the name of a system other than S
2. S₁, ⋯, Sₚ need not be all distinct
3. each Nᵢ is a local synonym for Sᵢ, with N₁, ⋯, Nₚ distinct
4. each xᵢ consists of S’s variables of the same type as Sᵢ’s input
5. each yᵢ consists of S’s local/output variables of the same type as Sᵢ’s output
6. the directed subsystem graph rooted at S is acyclic
System definition — Synchronous composition

(define-system S
  :input  ((i₁ δ₁) ⋯ (iₘ δₘ)) ; input vars
  :output ((o₁ τ₁) ⋯ (oₙ τₙ)) ; output vars
  :local  ((s₁ σ₁) ⋯ (sₚ δₚ)) ; local vars
  :subsys (N₁ (S₁ x₁ y₁)) ; component subsystem
          ...                     
  :subsys (Nₚ (Sₚ xₚ yₚ)) ; component subsystem)

Semantics

Let \( S_k = (I_k[i_k, o_k, s_k], T_k[i_k, o_k, s_k, i'_k, o'_k, s'_k]) \) for \( k = 1, \ldots, q \), with \( s_1, \ldots, s_q \) all distinct

Let \( i = (i₁, \ldots, iₘ) \), \( o = (o₁, \ldots, oₙ) \), \( s = s_1, \ldots, s_q, s_1, \ldots, s_q \)

\[
S = (I_S[i, o, s], T_S[i, o, s, i', o', s'])
\]

with \( I_S = \bigwedge_{k=1}^{q} I_k[x_k, y_k, s_k] \) \( T_S = \bigwedge_{k=1}^{q} T_k[x_k, y_k, s_k, x'_k, y'_k, s'_k] \)
System definition — Synchr. composition extended

\begin{align*}
(\text{define-system } & S \\
: \text{input} & \quad ((i_1 \ \delta_1) \ \cdots \ (i_m \ \delta_m)) \quad ; \text{input vars} \\
: \text{output} & \quad ((o_1 \ \tau_1) \ \cdots \ (o_n \ \tau_n)) \quad ; \text{output vars} \\
: \text{local} & \quad ((s_1 \ \sigma_1) \ \cdots \ (s_p \ \delta_p)) \quad ; \text{local vars} \\
: \text{subsys} & \quad (N_1 \ (S_1 \ x_1 \ y_1)) \quad ; \text{component subsystem} \\
\cdots & \quad \cdots \\
: \text{subsys} & \quad (N_q \ (S_q \ x_q \ y_q)) \quad ; \text{component subsystem} \\
: \text{init} & \quad I \quad ; \text{initial state formula} \\
: \text{trans} & \quad T \quad ; \text{transition formula} \\
: \text{inv} & \quad P \quad ; \text{invariant formula} \\
) \\
\end{align*}

Semantics

\[ S = (I_S[i, o, s], T_S[i, o, s, i', o', s']) \]

with \[ I_S = I \wedge \bigwedge_{k=1}^q I_k[x_k, y_k, s_k] \]
\[ T_S = T \wedge P \wedge \bigwedge_{k=1}^q T_k[x_k, y_k, s_k, x'_k, y'_k, s'_k] \]
Examples

(define-system Latch  :input ((s Bool) (r Bool))  :output ((o Bool))
 :local ((b Bool))
 :trans (= o’ (or (and s (or (not r) b))
  (and (not s) (not r) o)))
)

(define-system OneBitCounter  :input ((inc Bool) (start Bool))
 :output ((out Bool) (carry Bool))
 :local ((set Bool) (reset Bool))
 :subsys (L (Latch set reset out))
 :inv (and (= set (and inc (not reset)))
  (= reset (or carry start))
  (= carry (and inc out)))
)

(define-system ThreeBitCounter
 :input ((inc Bool) (start Bool))
 :output ((out0 Bool) (out1 Bool) (out2 Bool))
 :local ((car0 Bool) (car1 Bool) (car2 Bool))
 :subsys (C1 (OneBitCounter inc start out0 car0))
 :subsys (C2 (OneBitCounter car0 start out1 car1))
 :subsys (C3 (OneBitCounter car1 start out2 car2))
)
Define-system + SMT-LIB commands and types appear sufficient to allow faithful reductions from (full or large fragment of) Moore and Mealy machines, I/O automata, SMV and nuXMV, UNITY, TLA+, Reactive Modules, Lustre, and SAL.
System **checking** command

(\texttt{check-system } S
  \textbf{:input} \ ((i_1 \ \delta_1) \ \ldots \ (i_m \ \delta_m)) \ ; \text{renaming of } S\text{'s input vars}
  \textbf{:output} \ ((o_1 \ \tau_1) \ \ldots \ (o_n \ \tau_n)) \ ; \text{renaming of } S\text{'s output vars}
  \textbf{:local} \ ((s_1 \ \sigma_1) \ \ldots \ (s_p \ \delta_p)) \ ; \text{renaming of } S\text{'s local vars}
  \textbf{:assumption} \ (a \ A) \ ; \text{environmental assumption}
  \textbf{:reachable} \ (r \ R) \ ; \text{reachability condition}
  \textbf{:fairness} \ (f \ F) \ ; \text{fairness condition}
  \textbf{:current} \ (c \ C) \ ; \text{initiality condition}
  \textbf{:query} \ (q \ (g_1 \ \ldots \ g_q)) \ ; \text{trace query to be checked}
)
System checking command

\[(\text{check-system } S \\
: \text{input} \ ( (i_1 \ \delta_1) \ \cdots \ (i_m \ \delta_m)) \ \text{; renaming of } S\text{'s input vars} \\
: \text{output} \ ( (o_1 \ \tau_1) \ \cdots \ (o_n \ \tau_n)) \ \text{; renaming of } S\text{'s output vars} \\
: \text{local} \ ( (s_1 \ \sigma_1) \ \cdots \ (s_p \ \delta_p)) \ \text{; renaming of } S\text{'s local vars} \\
: \text{assumption} \ (a \ A) \ \text{; environmental assumption} \\
: \text{reachable} \ (r \ R) \ \text{; reachability condition} \\
: \text{fairness} \ (f \ F) \ \text{; fairness condition} \\
: \text{current} \ (c \ C) \ \text{; initiality condition} \\
: \text{query} \ (q \ (g_1 \ \cdots \ g_q)) \ \text{; trace query to be checked} \\
)\]

where

- \(a, r, f, c, q\) are identifiers; each \(g_i\) ranges over \(\{a, r, f, c\}\)
- \(C\) is a one-state (non-temporal) formula over the given vars
- \(A, R, F\) are one- or two-state (non-temporal) formulas over the given vars
- all attributes are optional and their order is immaterial
- all attributes but the first three are repeatable
System checking command

(\text{check-system } S  \\
:\text{input} \quad ((i_1 \delta_1) \cdots (i_m \delta_m)) \quad \text{; renaming of } S\text{'s input vars}  \\
:\text{output} \quad ((o_1 \tau_1) \cdots (o_n \tau_n)) \quad \text{; renaming of } S\text{'s output vars}  \\
:\text{local} \quad ((s_1 \sigma_1) \cdots (s_p \delta_p)) \quad \text{; renaming of } S\text{'s local vars}  \\
:\text{assumption} \quad (a A) \quad \text{; environmental assumption}  \\
:\text{reachable} \quad (r R) \quad \text{; reachability condition}  \\
:\text{fairness} \quad (f F) \quad \text{; fairness condition}  \\
:\text{current} \quad (c C) \quad \text{; initiality condition}  \\
:\text{query} \quad (q (g_1 \cdots g_q)) \quad \text{; trace query to be checked}  
)
System **checking** command semantics

\[
\text{(check-system } S \\
:\text{input} \quad \left( (i_1 \; \delta_1) \; \ldots \; (i_m \; \delta_m) \right) \); \text{renaming of } S\text{’s input vars} \\
:\text{output} \quad \left( (o_1 \; \tau_1) \; \ldots \; (o_n \; \tau_n) \right) \); \text{renaming of } S\text{’s output vars} \\
:\text{local} \quad \left( (s_1 \; \sigma_1) \; \ldots \; (s_p \; \delta_p) \right) \); \text{renaming of } S\text{’s local vars} \\
:\text{assumption} \quad (a \; A) \); \text{environmental assumption} \\
:\text{reachable} \quad (r \; R) \); \text{reachability condition} \\
:\text{fairness} \quad (f \; F) \); \text{fairness condition} \\
:\text{current} \quad (c \; C) \); \text{initiality condition} \\
:\text{query} \quad (q \; (a \; r))
\]

Query \(q\) succeeds iff the formula below is \(n\)-satisfiable in LTL for some \(n > 0\)

\[
I_S \land \text{always } T_S \land \text{always } A \land \text{eventually } R \land \text{always eventually } F
\]

where \(I_S\) and \(T_S\) are the initial state and transition predicate of \(S\) **modulo** the renamings above
System **checking** command semantics

(check-system $S$
 :input $((i_1 \delta_1) \cdots (i_m \delta_m))$ ; renaming of $S$’s input vars
 :output $((o_1 \tau_1) \cdots (o_n \tau_n))$ ; renaming of $S$’s output vars
 :local $((s_1 \sigma_1) \cdots (s_p \delta_p))$ ; renaming of $S$’s local vars
 :assumption $(a A)$ ; environmental assumption
 :reachable $(r R)$ ; reachability condition
 :fairness $(f F)$ ; fairness condition
 :current $(c C)$ ; initiality condition
 :query $(q (a f r))$
)

Query $q$ succeeds iff the formula below is satisfiable in LTL

$$I_S \land \text{always } T_S \land \text{always } A \land \text{eventually } R \land \text{always eventually } F$$

where $I_S$ and $T_S$ are the initial state and transition predicate of $S$ modulo the renamings above
System **checking** command semantics

(check-system \( S \))
  :input \( (((i_1 \delta_1) \cdots (i_m \delta_m)) \); renaming of \( S \)'s input vars
  :output \( (((o_1 \tau_1) \cdots (o_n \tau_n)) \); renaming of \( S \)'s output vars
  :local \( (((s_1 \sigma_1) \cdots (s_p \delta_p)) \); renaming of \( S \)'s local vars
  :assumption \( (a A) \); environmental assumption
  :reachable \( (r R) \); reachability condition
  :fairness \( (f F) \); fairness condition
  :current \( (c C) \); initiality condition
  :query \( (q (a c r)) \)

Query \( q \) succeeds iff the formula below is \( n \)-satisfiable in LTL for some \( n > 0 \)

\[
C \land \text{always } T_S \land \text{always } A \land \text{eventually } R \land \text{always eventually } F
\]

where \( l_S \) and \( T_S \) are the initial state and transition predicate of \( S \) modulo the renamings above.
System **checking** command semantics

```
(check-system S
 :input  (((i₁ δ₁) ⋅⋅⋅ (iₘ δₘ))) ; renaming of S’s input vars
 :output (((o₁ τ₁) ⋅⋅⋅ (oₙ τₙ))) ; renaming of S’s output vars
 :local  (((s₁ σ₁) ⋅⋅⋅ (sₚ δₚ))) ; renaming of S’s local vars
 :assumption (a A) ; environmental assumption
 :reachable  (r R) ; reachability condition
 :fairness  (f F) ; fairness condition
 :current  (c C) ; initiality condition
 :query  (q (g₁ ⋅⋅⋅ gₗ)) ; trace query to be checked)
```

For each successful query, the model checker is expected to produce

- a $\mathcal{T}$-interpretation $\mathcal{I}$ (of the free immutable symbols) and
- a **witnessing** trace in $\mathcal{I}$
System **checking** command semantics

\[
\text{(check-system } S \\
: \text{input} \quad \((i_1 \delta_1) \cdots (i_m \delta_m)) \quad ; \text{renaming of } S\text{’s input vars} \\
: \text{output} \quad \((o_1 \tau_1) \cdots (o_n \tau_n)) \quad ; \text{renaming of } S\text{’s output vars} \\
: \text{local} \quad \((s_1 \sigma_1) \cdots (s_p \delta_p)) \quad ; \text{renaming of } S\text{’s local vars} \\
: \text{assumption} \quad (a A) \quad ; \text{environmental assumption} \\
: \text{reachable} \quad (r R) \quad ; \text{reachability condition} \\
: \text{fairness} \quad (f F) \quad ; \text{fairness condition} \\
: \text{current} \quad (c C) \quad ; \text{initiality condition} \\
: \text{query} \quad (q (g_1 \cdots g_q)) \quad ; \text{trace query to be checked} 
\]

For each successful query, the model checker is expected to produce

- a $\mathcal{T}$-interpretation $\mathcal{I}$ (of the free immutable symbols) and
- a **witnessing** trace in $\mathcal{I}$

*Different queries may be given different interpretations and traces*
Example 1

(check-system NonDetArbiter
  :input ((req1 Bool) (req2 Bool))
  :output ((gr1 Bool) (gr2 Bool))

; There are never concurrent requests
:assumption (a1 (not (and req1 req2)))

; The same request is never issued twice in a row
:assumption (a2 (and (=> req1 (not req1'))
  (=> req2 (not req2')))))

; Neg of: Every request is immediately granted
:reachable (r (not (and (=> req1 gr1) (=> req2 gr2)))))

; check the reachability of r under assumptions a1 and a2
:query (q (a1 a2 r))
)
Example 2 — Temporal queries

(define-system Historically :input ((b Bool)) :output ((hb Bool))
 :init (= hb b) :trans (= hb' (and b' hb)))

(define-system Before :input ((b Bool)) :output ((bb Bool))
 :init (= bb' false) :trans (= bb' b))

(define-system Count :input ((b Bool)) :output ((c Int))
 :init (= c (ite b 1 0)) :trans (= c' (+ c (ite b 0 1))))

(define-system Monitor :input ((r1 Bool) (r2 Bool)) :output ((g1 Bool) (g2 Bool))
 :local ((a1 Bool) (a2 Bool) (b0 Bool) (b1 Bool) (b2 Bool)
 (h1 Bool) (h2 Bool) (c Int) (bf Bool))
 :subsys (A (NonDetArbiter r1 r2 g1 g2))
 :subsys (H1 (Historically a1 h1))
 :subsys (H2 (Historically a2 h2))
 :subsys (C (Count g1 c))
 :subsys (B (Before b0 bf))
 :inv (and
 (= a1 (and (not r1) (not r2))) (= a2 (and (not g1) (not g2))) (= b0 (= c 4))
 (= b1 (=> h1 h2)) ; b1 = if there have been no requests, there have been no grants
 (= b2 (=> bf (not g1)))) ; b2 = a request is granted at most 4 times

(check-system Monitor :input ((r1 Bool) (r2 Bool))
 :output ((g1 Bool) (g2 Bool))
 :local (_ _ _ (b1 Bool) (b2 Bool) _ _ _ _)
 :assumption (A (not (and r1 r2))) :reachable (P (not (and b1 b2)))
 :query (Q (A P))
)
Example 3 — Multiple queries

(check-system NonDetArbiter :input ((r1 Bool) (r2 Bool))
  :output ((g1 Bool) (g2 Bool))
  :assumption (a (not (and r1 r2)))
; Neg of: Every request is (immediately) granted
 :reachable (p1 (not (and (=> r1 g1) (=> r2 g2))))
; Neg of: In the absence of other requests, every request is granted
 :reachable (p2 (not (=> (!= r1 r2) (and (=> r1 g1) (=> r2 g2))))))
; Neg of: A request is granted only if present
 :reachable (p3 (not (and (=> g1 r1) (=> g2 r2))))
; Neg of: At most one request is granted at any one time
 :reachable (p4 (not (not (and g1 g2))))
; Neg of: In case of concurrent requests, one of them is always granted
 :reachable (p5 (not (=> (and r1 r2) (or g1 g2))))
 :query (q1 (a p1)) :query (q2 (a p2)) :query (q3 (a p3))
 :query (q4 (a p4)) :query (q5 (a p5))
)

Each query can be witnessed by a different $T$-interpretation and trace in it
Output format for check-system

(define-system A :input ((i σA)) :output ((o τA)) :local ((s θA)) ... )

(define-system B :input ((i σB)) :output ((o τB)) :local ((s θB))
 :subsys ( ... (S (A ...)) ...) ... )

(check-system B ... :fairness (f ...) :reachable (r ...) ...
 :query (q (r f ...)) ... )

Output:

(response
 :result ((q sat) ...) ; result is sat or unsat for each query
 :model ( ... ) ; SMT-LIB interpretation of free symbols
 :trail (p (; state sequence
      ((i i0) (o o0) (s s0) (S::i iS,0) (S::o oS,0) (S::s sS,0) (r r0) (f f0) ...) 
      ...
      ((i i_k) (o o_k) (s s_k) (S::i iS,k) (S::o oS,k) (S::s sS,k) (r r_k) (f f_k) ...) 
   )

   )
 :trail (l ( ... ))
 ...
 :trace (q :prefix p :lasso l) ; witness trace for query q is pl^ω ...
)
Special predicate: **Deadlock**

For every system \( S = (I_S[i, o, s], T_S[i, o, s, i', o', s']) \)

**Deadlock** is a predicate (implicitly) over \( i, o, s \)

A state \( \{ i \mapsto i_0, o \mapsto o_0, s \mapsto s_0 \} \) satisfies **Deadlock**, or **is deadlocked**, iff

it satisfies the formula \( \exists i' \forall o' \forall s' \neg T_S[i, o, s, i', o', s'] \)
Special predicate: **Deadlock**

For every system $S = (l_s[i, o, s], T_s[i, o, s, i', o', s'])$

**Deadlock** is a predicate (implicitly) over $i, o, s$

A state $\{i \mapsto i_0, o \mapsto o_0, s \mapsto s_0\}$ satisfies **Deadlock**, or **is deadlocked**, iff

it satisfies the formula $\exists i' \forall o' \forall s' \neg T_s[i, o, s, i', o', s']$
Uses of **Deadlock**

Examples

- **(check-system** $S$ ...  
  
  :assumption ($a \ A$) :current ($d$ Deadlock) :query ($a \ d$))

  checks the existence of deadlocked states under assumption $A$

- **(check-system** $S$ ...  
  
  :assumption ($a \ A$) :reachable ($d$ Deadlock) :query ($a \ d$))

  checks the reachability of deadlocked states under assumption $A$

- **(check-system** $S$ ...  
  
  :fairness ($f$ true) :reachable ($r \ R$) :query ($f \ r$))

  checks the reachability of $R$ on infinite (hence deadlock-free) traces
Uses of **Deadlock**

Examples

- \((\text{check-system } S \ldots \text{:assumption } (a A) \text{:current } (d \text{ Deadlock}) \text{:query } (a d))\)
  checks the existence of deadlocked states under assumption \(A\).

- \((\text{check-system } S \ldots \text{:assumption } (a A) \text{:reachable } (d \text{ Deadlock}) \text{:query } (a d))\)
  checks the reachability of deadlocked states under assumption \(A\).

- \((\text{check-system } S \ldots \text{:fairness } (f \text{ true}) \text{:reachable } (r \text{ R}) \text{:query } (f r))\)
  checks the reachability of \(R\) on infinite (hence deadlock-free) traces.
Uses of **Deadlock**

**Examples**

- `(check-system S ... :assumption (a A) :current (d Deadlock) :query (a d))` checks the existence of deadlocked states under assumption `A`.

- `(check-system S ... :assumption (a A) :reachable (d Deadlock) :query (a d))` checks the reachability of deadlocked states under assumption `A`.

- `(check-system S ... :fairness (f true) :reachable (r R) :query (f r))` checks the reachability of `R` on infinite (hence deadlock-free) traces.
What’s intentionally missing (and why)

- Restrictions to just bit vector types
  Other types are useful!

- Stronger syntactic restrictions for :init and :trans formulas
  Should be enforced in the user-facing language

- Direct support for LTL, or your favorite temporal logic, in check-system
  Generality, mostly

- Global (mutable) variables a la SAL
  Tricky to get right

- Parametric components as in SMV or SAL
  Some support. The rest is better provided in the user-facing language

- Compositional reasoning features (i.e., assume-guarantee contracts)
  Too many different approaches out there
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- Compositional reasoning features (i.e., assume-guarantee contracts)
  Too many different approaches out there
Discussion

What currently, intentionally or unintentionally, missing features would be imperative to have?
Possible Extensions
Multiqueries

(check-system $S$
  :input $((i_1 \delta_1) \cdots (i_m \delta_m))$
  :output $((o_1 \tau_1) \cdots (o_n \tau_n))$
  :
  :queries $((q_1 (g_{1,1} \cdots g_{1,n_1})) \cdots (q_k (g_{k,1} \cdots g_{k,n_k})))$
)

- Each query $q_i$ can be witnessed by a different trace
- However, each free immutable symbol has the same interpretation across all queries
**Executable system definitions**

Local and output variables are defined exclusively equationally

```lisp
(define-system TimedSwitch :input ((press Bool)) :output ((sig Bool))
  :local ((s LightStatus) (n Int))
  :inv-def (sig (= s On))
  :init-def (n 0) (s (ite press On Off))
  :next-def (s' (ite press' (ite (= s Off) On Off))
    (ite (= s Off) Off (ite (< n 10) On Off)))
    (n' (ite (or (= s Off) (s' Off)) 0 (+ n 1)))
)
```

**Restrictions:** (guaranteeing progressiveness and executability)

- Each local or output variable must be listed in :inv-def or in both :init-def and :next-def
- No definitional cycles
- No uninterpreted symbols
Parametric definitions — Part I

(define-system Delay :param ((V Type) (d V) (n Int)) :input ((in V)) :output ((out V)) :local ((a (Array Int V))) :inv (and (= in (select a 0)) (= out (select a n))) :init (forall ((i Int)) (=> (<= 1 i n) (= (select a i) d))) :trans (forall ((i Int)) (=> (<= 1 i n) (= (select a' i) (select a (- i 1))))) )

(check-system Delay :param ((V String) (d "") (n 4)) :input ((in String)) :output ((out String)) :local ((a (Array Int String))) ... )

Restrictions: parameters are immutable (rigid)
Parametric definitions — Part II

New binders:

\[
\text{(foreach } ((i_1 \ l_1 \ h_1) \ \cdots \ (i_n \ l_n \ h_n)) \ F) \\
\text{(forsome } ((i_1 \ l_1 \ h_1) \ \cdots \ (i_n \ l_n \ h_n)) \ F)
\]

where

- \( i_1, \ldots, i_n \) are (integer) identifiers, the bound vars
- \( l_k \) and \( h_k \) are integer expressions that can eventually be evaluated statically
- \( F \) is a formula with free occurrences of \( i_1, \ldots, i_n \)
Parametric definitions — Part II

New binders:

(\texttt{foreach ((i_1 l_1 h_1) \cdots (i_n l_n h_n)) \textit{F}})

(\texttt{forsome ((i_1 l_1 h_1) \cdots (i_n l_n h_n)) \textit{F}})

where

- \(i_1, \ldots, i_n\) are (integer) identifiers, the bound vars
- \(l_k\) and \(h_k\) are integer expressions that can eventually be evaluated statically
- \(\textit{F}\) is a formula with free occurrences of \(i_1, \ldots, i_n\)

Semantics

\[
\texttt{(foreach ((i \ l \ h)) \textit{F}) } \equiv \ (\text{and \ \textit{F}[l/i] \ \textit{F}[(l+1)/i] \ \cdots \ \textit{F}[l/i]})
\]

\[
\texttt{(forsome ((i \ l \ h)) \textit{F}) } \equiv \ (\text{or \ \textit{F}[l/i] \ \textit{F}[(l+1)/i] \ \cdots \ \textit{F}[l/i]})
\]

\[
\texttt{(foreach (b_1 \ \cdots \ b_n) \textit{F}) } \equiv \ (\texttt{foreach (b_1) (foreach (b_2 \ \cdots \ b_n) \textit{F}}))
\]

\[
\texttt{(forsome (b_1 \ \cdots \ b_n) \textit{F}) } \equiv \ (\texttt{forsome (b_1) (forsome (b_2 \ \cdots \ b_n) \textit{F}}))
\]
Parametric definitions — Part II

New binders:

\[
\text{(foreach ((} i_1 \ l_1 \ h_1 \text{) \cdots (} i_n \ l_n \ h_n \text{)) } F) \\
\text{(forsome ((} i_1 \ l_1 \ h_1 \text{) \cdots (} i_n \ l_n \ h_n \text{)) } F)
\]

where

- \( i_1, \ldots, i_n \) are (integer) identifiers, the bound vars
- \( l_k \) and \( h_k \) are integer expressions that can eventually be evaluated statically
- \( F \) is a formula with free occurrences of \( i_1, \ldots, i_n \)

Note

- \( \text{(foreach ((} i \ l \ h \text{)) } F) \equiv \text{true} \text{ when } l > h \)
- \( \text{(forsome ((} i \ l \ h \text{)) } F) \equiv \text{false} \text{ when } l > h \)
- \( \text{(foreach ((} i \ l \ h \text{)) } F) \equiv F \equiv \text{(forsome ((} i \ l \ h \text{)) } F) \text{ when } l = h \)
Examples

\[
\text{(define-system } A :\text{input } ((i \, \tau)) :\text{output } ((o \, \tau)) \ldots )
\]

; synchronous composition of A with itself n times
\[
\text{(define-system } C :\text{param } ((n \, \text{Int}))
\text{:input } ((i \, \tau))
\text{:output } ((o \, \tau))
\text{:local } ((s \, (\text{Array Int } \tau))
\text{:inv } (\text{and}
\quad (= i (\text{select s } 0))
\quad (= o (\text{select s } n))
\quad (\text{foreach } ((k \, 1 \, n))
\quad \quad (A (\text{select s } (- k 1)) (\text{select s } k)))
\quad )
\quad )
\text{)}
\]