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Let us determine the allowable ductile-to-brittle transition temperature of 38KhN3MFA steel in an M170×6.0 stud with a crack-type defect constituting 10% of the gross diameter from the value of t_{lim} for an impact specimen with a defectiveness of 10% of the gross cross section. In this case we first find t_{lim} = +25°C and then $[t_c]$ = −5°C (broken line 1 in Fig. 3 and Eq. (1)). With use of t_{lim} obtained from the test data of type 11 specimens to GOST 9454-78 with a defectiveness of 20% of the gross cross section the allowable ductile-to-brittle transition temperature of 38KhN3MFA steel with t_{lim} = +80°C is $[t_c]$ = −60°C (broken line 2 in Fig. 3).

According to Fig. 3, in comparing the conservative and more accurate approaches to determination of the brittle strength of fasteners the difference in the allowable ductile-to-brittle transition temperatures of the metal is 50°C. With the more accurate evaluation of the allowable ductile-to-brittle transition temperatures of the fastener metal in relation to the technical condition of the steel taking into consideration the level of defectiveness of the fastener and the specified operating life of the thread joint more soundly based requirements may be imposed for effectiveness of the material in the stages of design, production, and service.

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NUMERIC MODELING OF THE STRENGTH AND LONGEVITY OF STRUCTURES WITH ALLOWANCE FOR SCALE EFFECT.

REPORT 1. SUBSTANTIATION OF STRENGTH AND LONGEVITY CRITERIA

N. V. Novikov, V. I. Levitas,
and S. I. Shestakov

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The nonlocal criterion of the static strength of structurally inhomogeneous construction materials, whose basic mechanical properties depend heavily on the loaded volume in a complex stress–strain state, is developed. A statistical interpretation of the proposed criterion is given. The case where the criterion is used for an isotropic material with volume-dependent ultimate tensile, compressive, and torsional strengths is analyzed as an example. A longevity criterion, which makes it possible to account for the different character of the material's rupture strength as the form of the stress state changes, in addition to the scale effect, is proposed for structural components that operate under asymmetric low-cycle loading. This criterion is in accord with the static criterion and is obtained on the assumption of the invariance of the limiting-stress diagrams, which apply to the ultimate strength relative to the form of stress state.

The scale effect is observed in analyzing the physicomechanical properties of many structural materials (sintered cermets, cast iron), which are grown from the melts of brittle crystals, i.e., diamonds and others. In using these materials as structural materials, it is obviously necessary to model the variation in their strength and longevity by computational means, proceeding from different operating conditions. The accounting of the inhomogeneity and form of stress state can be referred to a number of basic methodological factors, and the derivation of criteria of the structural strength and longevity of sintered tungsten-containing hard alloys can be considered, using a minimum amount of experimental data on their physicomechanical properties [1, 2]. Note that these criteria may be applicable in calculating the strength and longevity of a broad circle of other brittle materials.

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To evaluate the effectiveness of the criteria let us calculate the static and fatigue strengths of heavily loaded hard-alloy components of different types of high-pressure apparatus with different effective volumes, which are used to synthesize hard-alloy materials under an external pressure of 45-55 GPa with heating to 1000°C. As will be indicated in subsequent reports, computational data are in good agreement with familiar results of the practical evaluation of the longevity of these components.

1. Let us adopt a static-strength criterion for structural material in general form:

$$\sigma_e = F[\sigma, A_i(V)] \leqslant 1, \qquad i = 1, 2, \ldots, l, \tag{1}$$

where σ_e are dimensionless equivalent stresses, F is a certain function, σ is a stress tensor, and A_i are material constants (ultimate strengths in compression, tension, etc., for anisotropic material, which differ in different directions). As a result of the presence of scale effect, the constants A_i depend on the loaded volume V. In a uniform stress state, the loaded volume corresponds to the total volume of the effective component of the structure.

Since the structure operates under a nonuniform stress state, it is necessary, first of all, to define more precisely what to assume as the loaded volume of the material in the case in question in order to evaluate their strength.

Let us assume that for the class of materials investigated, the $A_i(V)$ relationship can be written on the basis of Weibull's theory [3] in the form

$$A_i = \frac{K_i}{V^{1/m_i}}, \quad m_i > 0, \tag{2}$$

where K_i are constants, and m_i are parameters of material homogeneity.

Considering the case of nonuniform uniaxial tension and assigning the normal stress σ_1 as the equivalent stresses, we obtain

$$\frac{\sigma_1}{\sigma_+(V)} \leqslant 1; \quad \sigma_+(V) = \frac{K}{V^{1/m}}, \tag{3}$$

where σ_+ is the ultimate tensile strength. It follows from Weibull's theory [3] that the quantity σ_+ depends on the loaded volume of the material:

$$V_\ell = \int_V \left(\frac{\sigma_1(r)}{\sigma_{1\max}}\right)^m dV, \tag{4}$$

where r is the radius-vector of a point of volume V, and $\sigma_{1\max}$ is the maximum value of σ_1 in the stressed volume of the material. The integration is performed over the entire volume V of the component. According to the equations given in [3]:

$$\sigma_+ = \int_0^\infty \exp\left[-\int\left(\frac{\sigma_1}{\sigma_0}\right)^m dV\right] d\sigma_1,$$

where σ_0 is a constant.

For a uniform distribution of the stresses σ_1 in the volume of the component, we have

$$\sigma_+ = \int_0^\infty \exp\left[-\left(\frac{\sigma_1}{\sigma_0}\right)^m V\right] d\sigma_\ell.$$

Hence follows (3). For a nonuniform distribution of these stresses,

$$\sigma_+ = \int_0^\infty \exp\left[-\left(\frac{\sigma_{1\max}}{\sigma_0}\right)^m \int_V \left(\frac{\sigma_1(r)}{\sigma_{1\max}}\right)^m dV\right] d\sigma_{1\max}.$$

In this case, we obtain Eq. (4) for V_ℓ.

The purpose of the study is to find the simplest noncontradictory generalization of Eq. (4) for the loaded volume of the material in the case of a complex stress state, arbitrary strength criterion (1), and the relationship between the homogeneity parameter and the type of stress state.

Let us assume

$$V_\ell = \int_V \left(\frac{\sigma_e(r)}{\sigma_{e\max}}\right)^{m(n)} dV, \tag{5}$$

where $\sigma_{e\max}$ is the maximum equivalent stress in the effective volume of the material. The parameter m depends on the unit vector $n = \frac{\sigma}{|\sigma|}$ $(|\sigma| = (\sigma\cdot\sigma)^{1/2})$ of vector σ in stress space, i.e., on the form of the stress state.

The replacement of σ_1 by σ_e is natural and noncontradictory. Note that in the special case considered by Weibull, $\sigma_1 = \sigma_e$. If, however, the initial stress state is nonuniform, and the distribution of σ_e is uniform, $V_\ell = V$ follows from Eq. (5). This implies that all points of volume V are of equal strength.

Let us substantiate the selection of the function $m(\mathbf{n})$. We can assume that relationship (2) is derived for a stress state characterized by unit vectors n_i. We should then have $m(n_i) = m_i$ when $n = n_i$. In effect, if the quantity m, which is obtained in experiments under uniform tension, exists in Eq. (4) for nonuniform tension, the parameter m_i, as determined in experiments for uniform compression, should be in Eq. (6) for the case of nonuniform compression, and so forth. Moreover, since there is only a small number of points m_i for the function $m(\mathbf{n})$, it is expedient to assume the condition $m_i^{\min} \leq m(\mathbf{n}) \leq m_i^{\max}$ ($m_i^{\min}$ and $m_i^{\max}$ are the minimum and maximum values of the parameters m_i). In the opposite case, the maximum and minimum values of the function $m(\mathbf{n})$ are uncontrolled.

The simplest alternative to the function, which satisfies the two enumerated conditions, will be

$$m(\mathbf{n}) = \frac{\sum_{i=1}^{l} a_i m_i}{\sum_{i=1}^{l} a_i} \tag{6}$$

where $a_i(n) \geq 0$, $a_i(n_j) = \delta_{ij}$ (δ_{ij} is the Kronecker symbol

If for $a_i(n)$ we assume

$$a_i(n) = \frac{(1 - n\cdot n_1)(1 - n\cdot n_2)\ldots \ldots(1 - n\cdot n_{i-1})(1 - n\cdot n_{i+1})\ldots(1 - n\cdot n_l)}{(1 - n_i n_1)(1 - n_i n_2)\ldots \ldots(1 - n_i n_{i-1})(1 - n_i n_{+1})\ldots(1 - n_i n_l)}, \tag{7}$$

these conditions will be satisfied. Equation (7) is an ℓ-th-order polynomial in terms of the components of unit vector $\mathbf{n}$; in this case, each of the factors is nonnegative. Conversion from the vector $\mathbf{n}$ to scalar quantities is accomplished using scalar multiplication of the unit vectors. Thus, the difference in the form of the stress state is evaluated from the angle between corresponding unit vectors. The denominator in Eqs. (7) is not equal to zero, since $1 - n_i\cdot n_k = 0$ only when $i = k$, and this factor is absent in (7).

Let us consider the calculation of these parameters for an isotropic material. For uniform tension, the tensor σ in the principle axes assumes the form $\sigma \equiv \{\sigma_1;\ 0;\ 0\}$. Then, $n_1 = \{1;\ 0;\ 0\}$. Similarly, we have $n_2 = \{0;\ 0;\ -1\}$ for comparison, and $-n_3 \equiv \left\{\frac{\sqrt{2}}{2};\ 0;\ -\frac{\sqrt{2}}{2}\right\}$, $n_1\cdot n_2 = 0$; $n_1\cdot n_3 = \frac{\sqrt{2}}{2}$; $n_2\cdot n_3 = \frac{\sqrt{2}}{2}$ for pure shear. Using the ultimate tensile, compressive, and torsional strengths of the material as parameters A_i, we obtain

$$\left.\begin{aligned} a_1 &= 3{,}41\,(1 + n^3)\,[1 - 0.707\,(n^1 - n^3)];\\ a_2 &= 3{,}41\,(1 - n^1)\,[1 - 0.707\,(n^1 - n^3)];\\ a_3 &= 11{,}65\,(1 - n^1)\,(1 + n^3), \end{aligned}\right\} \tag{8}$$

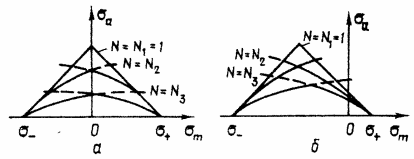

Fig. 1. Limiting-stress diagrams of materials that resist failure similarly (a) and differently (b) in tension and compression for different number of load cycles N.

when $\ell = 3$, where n^k ($k = 1, 2, 3$) are the principal components of the unit vector $\mathbf{n}$.

The stress state of the isotropic material is independent of the orientation of the principal axes, and the principal axes of the vectors n and n_i can be assumed coincident. Note that Eqs. (8) are valid for $n_1 \geq n_2 \geq n_3$, since the condition that the function $m(\mathbf{n})$ should be invariant with respect to the rearrangement of n^k for different k was not considered in the derivation of these equations.

Using the ultimate tensile and compressive strengths as parameters A_i, we can write for $\ell = 2$

$$a_1 = 1 + n^3; \quad a_2 = 1 - n^1; \tag{9}$$

$$m(n) = \frac{(1+n^3)\,m_1 + (1-n^1)\,m_2}{2+n^3-n^1}. \tag{10}$$

In the tension, compression, and shear tests, $\sigma_2 = 0$; the component n^2 is therefore absent in expressions (8)-(10).

In the general case, if the vectors n_i are arranged in some space Ω, the dimension of which are smaller than the stress space under consideration, the orthogonal Ω component of the unit vector n will not affect a_i and m, according to Eqs. (7). If the unit vector n is orthogonal to Ω, Eq. (7) will be unity in the denominator, and the quantity a_i will be independent of $\mathbf{n}$.

For anisotropic materials, it is necessary to consider the orientation of the principal axes with respect to the coordinate system. It is therefore expedient to use not three-dimensional, but six-dimensional unit vectors n and n_i in the space of the coordinate stresses.

Thus, eqs. (5)-(7) proposed for calculation of the loaded volume V_ℓ for a complex stress state, an arbitrary strength criterion, and the presence of several homogeneity parameters are noncontradictory and the simplest of the possible equations.

The set of Eqs. (1), (2), and (5)-(7) is a nonlocal criterion of structural strength. The nonlocal nature of the criterion is dictated by the fact that the equivalent stresses at a given point depend not only on the stress tensor in the criterion, but also on the stress distribution over the entire region under investigation. Note that the integral is taken over the entire effective volume in Eq. (5).

Let us make a statistical interpretation of the proposed strength criterion. With this aim, we can replace σ_1 by σ_e in the expression for the Weibull failure probability S

$$S = 1 - \exp\left[-\int_V \left(\frac{\sigma_{1\max}}{\sigma_0}\right)^m dV\right] =$$

$$= 1 - \exp\left[-\left(\frac{\sigma_{1\max}}{\sigma_0}\right)^m V_\ell\right], \quad V_\ell = \int_V \left(\frac{\sigma_1}{\sigma_{1\max}}\right) dV$$

and assume $m = m(\mathbf{n})$ and $\sigma_0 = \sigma_0(\mathbf{n})$, i.e.,

$$S = 1 - \exp\left[-\left(\frac{\sigma_{e\max}}{\sigma_0(n)}\right)^{m(n)} V_\ell\right] \tag{11}$$

$$V_\ell = \int_V \left(\frac{\sigma_e}{\sigma_{e\max}}\right)^{m(n)} dV.$$

This replacement implies that the equivalent stresses for a representative microvolume agree with the equivalent stresses for a microspecimen, i.e., the failure processes are similar on the micro- and macrolevels. The assumption concerning similitude (self-similarity) of the failure process at different scale levels is used widely in theories of continual failure.

The most probable value of σ_e during failure, which is, according to (1), equal to unity, is defined in the following manner:

$$\sigma_e = 1 = \int_0^\infty \sigma_{e\,max} dS = \int_0^\infty \exp\left[-\left(\frac{\sigma_{e\,max}}{\sigma_0(n)}\right)^{m(n)} V_\ell\right] d\sigma_{e\,max} = \frac{I_{m(n)}\sigma_0^{(n)}}{V_\ell^{1/m(n)}},$$

where

$$I_{m(n)} = \int_0^\infty \exp[-z^{m(n)}]\, dz \quad [1],$$

hence,

$$\sigma_0(n) = \frac{V_\ell^{1/m(n)}}{I_{m(n)}}. \tag{12}$$

For known m(**n**) and V_ℓ values, consequently, we can find σ_0(**n**). Equation (11) is now closed for determination of S:

$$S = 1 - \exp[\sigma_{e\,max} I_{m(n)}]^{m(n)}. \tag{13}$$

Although the volume V_ℓ does not explicitly figure into Eq. (13), it enters into σ_{emax} in terms of the constants $A_i(V)$; here, $\sigma_e = \sigma_1$ in the special case. Knowing the probability S of the material's brittle failure, therefore, we can readily determine the dispersion of the limiting value of σ_e as a function of the form of stress state. For a macroinhomogeneous material, the dependence of the material constants on the coordinates of the microvolume under consideration is substituted in the resultant equations.

After analyzing the static strength, let us examine the longevity criterion for the components in a complex stress state, which takes into account the scale effect and nonuniformity of the stress state, the latter conforming to proposed static-strength criterion (1), (2), and (5)-(7). Its use should be substantiated for evaluation of the fatigue strength of materials with a different rupture strength when the form of stress state is varied. Let us assume that there are no macroscopic plastic deformations of the material.

In the case of uniform tension—compression, the relation between the amplitude $\sigma_a > 0$ and mean σ_m stresses and the number of cycles N to failure (limiting-stress diagram) can be adopted as

$$\frac{\sigma_a}{\sigma_+} = f\left(\frac{\sigma_m}{\sigma_+},\ N\right), \tag{14}$$

In this case, $f\left(\frac{\sigma_m}{\sigma_+},\ 1\right) = 1 - \frac{\sigma_m}{\sigma_+}$, i.e., the condition of failure under static load $\sigma_m + \sigma_a = \sigma_+$ is satisfied when N = 1.

Graphic interpretation of Eq. (14) for different N = const is shown in Fig. 1a. If the material resists tension and compression similarly, the diagram is symmetric about the σ_a axis and the function f is even. It is impossible, however, to approximate the diagram of some smooth even function for both positive and negative σ_m. But the derivative $\frac{\partial\sigma_a}{\partial\sigma_m} \neq 0$ experiences a jump when $\sigma_m = 0$ (Fig. 1a). When $N = 1$ $\sigma_a = \sigma_+ - \sigma_m$ and $\frac{\partial\sigma_a}{\partial\sigma_m} = -1$.

The symmetry condition of the diagram can be satisfied in the following manner: Eq. (14) can be used when $\sigma_m > 0$, and the equation

$$\frac{\sigma_a}{\sigma_+} = f\left(-\frac{\sigma_m}{\sigma_+},\ N\right). \tag{15}$$

when $\sigma_m < 0$. The function $f\left(\frac{\sigma_m}{\sigma_+},\ N\right)$ increases monotonically with decreasing N and σ_m.

The following interpretation is possible as a result of the monotonicity of the function f: for any (positive or negative) σ_m and known σ_a, the two equations (14) and (15) can be employed directly, and the minimum of the two values of N obtained taken, i.e.,

$$\frac{\sigma_a}{\sigma_+} = f\left(\frac{\sigma_m}{\sigma_+},\ N_1\right);\quad \frac{\sigma_a}{\sigma_+} = f\left(-\frac{\sigma_m}{\sigma_+},\ N_2\right);\qquad N = \min(N_1,\ N_2). \tag{16}$$

For materials that resist tension and compression differently (an ultimate compressive strength $\sigma_- \neq \sigma_+$), we can assume

$$\frac{\sigma_a}{\sigma_+} = f\left(\frac{\sigma_m}{\sigma_+},\ N_1\right);\quad \frac{\sigma_a}{\sigma_-} = f\left(-\frac{\sigma_m}{\sigma_-},\ N_2\right);\qquad N = \min(N_1,\ N_2). \tag{17}$$

For alternating shear, we can assume

$$\frac{\tau_a}{\tau_+} = f\left(\frac{\tau_m}{\tau_+},\ N_1\right);\quad \frac{\tau_a}{\tau_-} = f\left(-\frac{\tau_m}{\tau_-},\ N_2\right);\qquad N = \min(N_1,\ N_2), \tag{18}$$

where $\tau_a > 0$ and τ_m are the amplitude and mean values of the tangential stresses, and τ_+ and τ_- are the limiting shear strengths in the two different directions.

The correspondence between the functions f in Eqs. (17) and (18) is a basic physical assumption that significantly lowers the required volume of experimental investigations. This assumption implies the invariance of the limiting-stress diagrams as applies to the corresponding ultimate static strength relative to the form of stress state; this ensues from the following fact pointed out in [2, 3]: the ratio of endurance limits for repeated tension and shear is approximately equal to the ratio of the corresponding ultimate strengths. Actually, it follows from Eqs. (16)-(18) that for equal N and

$$\frac{\sigma_m}{\sigma_+} = -\frac{\sigma_m}{\sigma_-} = \frac{\tau_m}{\tau_+} = -\frac{\tau_m}{\tau_-}$$

we obtain

$$\frac{\sigma_a}{\sigma_+} = \frac{\sigma_a}{\sigma_-} = \frac{\tau_a}{\tau_+} = \frac{\tau_a}{\tau_-}. \tag{19}$$

Let us examine the graphic interpretation of relationships (14) and (15). When N = 1, two straight lines $\frac{\sigma_a}{\sigma_+} + \frac{\sigma_m}{\sigma_+} = 1$ and $\frac{\sigma_a}{\sigma_-} - \frac{\sigma_m}{\sigma_-} = 1$, which are equally inclined to the σ_a and σ_m axes and which pass through the points σ_+ and σ_-, respectively, when $\sigma_a = 0$, correspond to these relationships (Fig. 1b). They intersect when $\sigma_m = \frac{\sigma_+ - \sigma_-}{2}$. N > 1, the value of σ_m at the point of intersection is determined from the condi-

tion $\sigma_a = \sigma_+ f\left(\frac{\sigma_m}{\sigma_+}, N\right) = \sigma - f\left(-\frac{\sigma_m}{\sigma_+}, N\right)$. Note that when $\sigma_+ = \sigma_-$, the function f should describe the experiments well only if $\sigma_m > 0$ (in view of its evenness), $\sigma_+ \neq \sigma_-$, and $\sigma_m < 0$. This suggests that many of the familiar approximations [4, 5] are unacceptable for describing the diagrams of materials that resist tension and compression differently.

As an additional assumption, let us set

$$f\left(\frac{\sigma_m}{\sigma_+}, N\right) = \varphi\left(\frac{\sigma_m}{\sigma_+}\right) Q(N), \tag{20}$$

in this case, $\varphi(0) = 1$. Here,

$$\frac{\sigma_a}{\sigma_+} = \varphi\left(\frac{\sigma_m}{\sigma_+}\right) Q(N_1); \quad \frac{\sigma_a}{\sigma_-} = \varphi\left(-\frac{\sigma_m}{\sigma_-}\right) Q(N_2); \quad N = \min(N_1, N_2) \tag{21}$$

follows from Eqs. (17).

Let $\sigma_- > \sigma_+$. When $\sigma_m = 0$ (symmetric cycle), then $N_1 = N$ and $\sigma_a = \sigma_+ Q(N) = \sigma_{-1}(N)$, where $\sigma_{-1}(N)$ is the material's endurance limit for a symmetric cycle. Of course, $Q(N) = \frac{\sigma_{-1}(N)}{\sigma_+}$, and $Q(1) = 1$. Similarly, $Q(N) = \frac{\tau_{-1}(N)}{\tau_+}$, where $\tau_+ < \tau_-$, $\tau_-(N)$ is the endurance limit for symmetric shear.

Equations (21) and (18) can be rewritten as

$$\frac{\sigma_a}{\sigma_{-1}(N_1)} = \varphi\left(\frac{\sigma_m}{\sigma_+}\right); \quad \frac{\sigma_a}{\sigma_{-1}(N_2)} \frac{\sigma_+}{\sigma_-} = \varphi\left(-\frac{\sigma_m}{\sigma_-}\right); \quad N = \min(N_1, N_2); \tag{22}$$

$$\frac{\tau_a}{\tau_{-1}(N_1)} = \varphi\left(\frac{\tau_m}{\tau_+}\right); \quad \frac{\tau_a}{\tau_{-1}(N_2)} \frac{\tau_+}{\tau_-} = \varphi\left(-\frac{\tau_m}{\tau_-}\right); \quad N = \min(N_1, N_2). \tag{23}$$

Let us examine the possibility of using a longevity criterion in the case of a complex stress state with allowance for adopted static-strength criterion (1), (2), and (5)-(7).

Let the tensor of the stresses at the point under investigation in the loaded volume of the material vary cyclically between the values σ_1 and σ_2. We can assume that $\sigma = \sigma_1 + (\sigma_2 - \sigma_1)\lambda$, where λ is a parameter that varies cyclically from zero to unity. The effect of the frequency λ of the variation can be considered insignificant.

Two static-strength surfaces (1), which are distinguished one from the other by different volumes V_ℓ^1 and V_ℓ^2 under the loads σ_1 and σ_2 are shown in Fig. 2a. Let us denote the amplitude and mean values of the tensor σ by $\sigma'_a = \frac{\sigma_1 - \sigma_2}{2}$ and $\sigma'_m = \frac{\sigma_1 + \sigma_2}{2}$, respectively.

Let us examine the special case when straight line ML passes through the origin of coordinates. The cases of tension—compression, variable-sign shear, and cyclic loading with an arbitrary form of stress state are presented in Fig. 2b. Using the above assumption concerning the invariance of the limiting-stress diagrams with respect to the form of stress state, we obtain

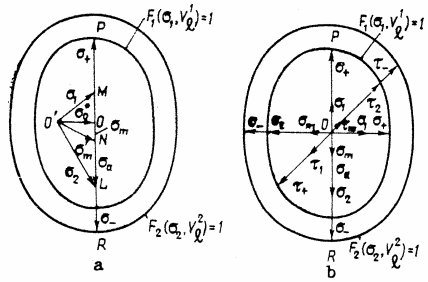

Fig. 2. For calculation of longevity of structures formed from brittle material in arbitrary stress state ($\sigma^*_0 \neq 0$) (a), and in case where stress vector passes through origin of coordinates ($\sigma^*_0 = 0$) (b). ($F_1(\sigma_1, V^1_\ell) = 1$ AND $F_2(\sigma_2, V^2_\ell) = 1$ are the limiting strengths corresponding to the two cases of loading.)

$$\frac{|\sigma_a|}{|\sigma_+|} = f\left(\frac{|\sigma_m|}{|\sigma_+|}\eta,\ N_1\right);$$
$$\frac{|\sigma_a|}{|\sigma_-|} = f\left(-\frac{|\sigma_m|}{|\sigma_-|}\eta,\ N_2\right),\ \eta\frac{(\sigma_+\sigma_m)}{|\sigma_+\sigma_m|}; \tag{24}$$
$$N = \min(N_1,\ N_2),$$

where σ_+ and σ_- are vectors that originate at point O', and terminate at the points of intersection between line ML and the surfaces described by the static-strength criterion; the parameter $\eta = 1$ when the directions of the vectors σ_+ and σ_m coincide; in the opposite case, $\eta = -1$ and is introduced to account for the sign (direction) of σ_m. It is obvious that (17) and (18) are a special case of Eq. (24), and $|\sigma_+|$ and $|\sigma_-|$ are the ultimate strengths for the given form of stress state.

Let line ML not pass through the origin of coordinates. Since in this case, cyclic variation of the stresses occurs, as before, with respect to a certain line in stress space, we can attempt to reduce this case to the one previously analyzed. For this purpose, it is sufficient to find the zero point on line ML. Point O, which is near the origin of coordinates O', can be adopted as this point, i.e., the vector $\sigma^*_0 = O'O$ is orthogonal to line ML (Fig. 2a). Then, setting $\sigma_m = ON = \sigma'_m - \sigma^*_0$, where

$$\sigma_0^* = \sigma_1 - \frac{\sigma_1(\sigma_2 - \sigma_1)}{|\sigma_2 - \sigma_1|^2}(\sigma_2 - \sigma_1), \tag{25}$$

$\sigma_a = NL$, $\sigma_+ = OP$, and $\sigma_- = OR$, and substituting these vectors in Eq. (24), we obtain a longevity criterion for the case under consideration.

Note that the assumption that this is the point at which σ_e assumes a minimum value from criterion (1) may be an alternative scheme for determining point O. In this case, however, line ML may be located on the surface of criterion (1) (e.g., in the case of Coulomb's criterion) and the selection of point O is no longer single-valued. Moreover, it is not altogether clear which of the volumes V^1_ℓ or V^2_ℓ (or which of their combinations) must be used in this case.

Using the expansion of (20), we obtain

$$\frac{|\sigma_a|}{|\sigma_+|}Q^{-1}(N_1) = \frac{|\sigma_a|}{\sigma_{-1}(N_1)|\sigma_+|}\sigma_+ =$$
$$= \varphi\left(\frac{|\sigma_m|}{\sigma_+}\eta\right);\quad \frac{|\sigma_a|}{|\sigma_-|}Q^{-1}(N_2) =$$
$$= \frac{|\sigma_a|}{\sigma_{-1}(N_2)}\frac{\sigma_+}{|\sigma_-|} = \varphi\left(-\frac{|\sigma_m|}{|\sigma_-|}\eta\right) \tag{26}$$
$$N = \min(N_1,\ N_2).$$

As a result of the dependence of the static-strength criterion on V_1, the parameters $|\sigma_+|$ and $|\sigma_-|$ are also functions of V^1_ℓ and V^2_ℓ, respectively. The functions φ and Q (or f) may depend on V_ℓ; in this case, however, it must be determined additionally what combination of V^1_ℓ and V^2_ℓ should be substituted in these relationships for V_ℓ (e.g., $(V^1_\ell + V^2_\ell)/2$).

Note that in real structures, the trajectory of the variation in the vector $\sigma(t)$ (t is time) between the values σ_1 and σ_2 may or may not be rectilinear. The effect of the trajectory of $\sigma(t)$ is difficult to account for. The independence of the longevity criterion on the trajectory of $\sigma(t)$ can therefore be proposed as a first approximation, and criteria in the form of (24) or (26), which yield the minimum number of cycles for σ_1 and σ_2, can be used.

To use longevity criterion (26), it is therefore sufficient to determine the following experimentally: a) the static-strength criterion in the form of (1); b) the $\sigma_{-1}(N)$ fatigue curve for the material under consideration for a symmetric cycle and simple loading; and, c) the function φ, i.e., the limiting-stress diagram for a simple loading and any number of cycles N.

To concretize resultant relationships (26), let us select the function φ in the form

$$\varphi\left(\frac{|\sigma_m|}{|\sigma_+|}\eta\right) = 1 - \frac{|\sigma_m|}{|\sigma_+|}\eta. \tag{27}$$

Then, expressions (26) can finally be written as:

$$\begin{gathered}\frac{|\sigma_a|}{|\sigma_+|} = \left(1 - \frac{|\sigma_m|}{|\sigma_+|}\eta\right) Q(N_1);\\ \frac{|\sigma_a|}{|\sigma_-|} = \left(1 + \frac{|\sigma_m|}{|\sigma_-|}\eta\right) Q(N_2);\\ N = \min(N_1, N_2).\end{gathered} \tag{28}$$

The generalized Pisarenko—Lebedev criterion [4], which according to numerous experimental data [6, 7] most completely describes the limiting state of structurally inhomogeneous hard tungsten—cobalt alloys with strikingly different ultimate tensile and compressive strengths can be recommended as a static-strength criterion of the form of (1) for the class of materials investigated.

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