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SOME RELATIONS FOR FINITE INELASTIC DEFORMATION OF MICROHETEROGENEOUS MATERIALS WITH MOVING DISCONTINUITY SURFACES

VALERY I. LEVITAS
Institut für Baumechanik und Numerische Mechanik, Universität Hannover, Appelstrasse 9A, 30167 Hannover, Deutschland;
Institute for Superhard Materials of the Ukrainian SSR Academy of Sciences, Avtozavodska 2, 254074 Kiev, Ukraine

1. Introduction

Consider in reference configuration V_τ the representative volume v of an inelastic material bounded by a surface S and made up by vectors $\tilde{\boldsymbol{r}}_\tau$ at time $t = \tau$ ($\sim$ means local values of parameters). Describe movement by the function $\tilde{\boldsymbol{r}}\,(\tilde{\boldsymbol{r}}_\tau\,,\ t)$, where $\tilde{\boldsymbol{r}}$ is the position vector in actual configuration V_t. The functions $\tilde{\boldsymbol{r}}\,(\tilde{\boldsymbol{r}}_\tau\,,\ t)$ and $\dot{\tilde{\boldsymbol{r}}}\,(\tilde{\boldsymbol{r}}_\tau\,,\ t)$ undergo discontinuities on moving surfaces Σ. The surface Σ can be a surface of noncoherent phase transition (PT) or a slip surface. If only velocities are discontinues on Σ, and a jump $[\tilde{\boldsymbol{r}}] = 0$, then the PT is a coherent one. If $[\tilde{\boldsymbol{r}}] \neq 0$ PT is a noncoherent one.

Micro–to–macro transition for the representative volume of elastoplastic materials without discontinuity surfaces at finite strain was developed by Hill (see [1] and references) and Rice [2] (similar works for materials with discontinuities are unknown to us). It has been shown that when a associated flow rule is satisfied on the micro–level it is valid on the macro–level as well. This result is rather sensitive to the decomposition of the total strain into elastic and plastic parts. It has been deduced in [3]–[5] that the "best" measures of elastic and plastic strains (these notions are defined strictly) should be based on Lee decomposition $\boldsymbol{F} = \boldsymbol{F}_e \cdot \boldsymbol{F}_i$ of the deformation gradient into elastic and plastic parts. Works [1, 2] use other decompositions. Here it is demonstrated that the measure of plastic strain rate in [1] depends on $\dot{\boldsymbol{F}}_e$ and the elasticity law in [2] depends on $\boldsymbol{F}_p$ in a special unrealistic way. One of the results of the present paper is a derivation of a relation between macroscopic $\dot{\boldsymbol{F}}_i$ and microscopic $\dot{\tilde{\boldsymbol{F}}}_i$ inelastic gradients. It is shown that the macroscopic associated flow rule is only valid at small elastic strains.

For materials with moving discontinuity surfaces of the velocity and the displacement vector the macroscopic variables have been introduced, and energy relations as well as an equation for the rate of inelastic macrodeformation have been derived. It is shown in particular, that the relation between the macroscopic Cauchy and Piola–Kirchhoff stresses, as well as an equation for the power of external stresses in terms of macroscopic stress and velocity gradient tensors have the same form, as for a volume without discontinuities. Some preliminary results have been presented in [6].

2. Macroscopic variables and energy identities

Define macrotensors as in Hill's work [1]

$$\boldsymbol{F} = \frac{1}{v}\int \tilde{\boldsymbol{r}}\,\boldsymbol{n}\,dS\,; \qquad \dot{\boldsymbol{F}} = \frac{1}{v}\int \dot{\tilde{\boldsymbol{r}}}\,\boldsymbol{n}\,dS\,;$$
$$\boldsymbol{P}^t = \frac{1}{v}\int \tilde{\boldsymbol{r}}_\tau\,\tilde{\boldsymbol{P}}\cdot\boldsymbol{n}\,dS\,; \qquad \dot{\boldsymbol{P}}^t = \frac{1}{v}\int \tilde{\boldsymbol{r}}_\tau\,\dot{\tilde{\boldsymbol{P}}}\cdot\boldsymbol{n}\,dS\,, \tag{1}$$

where $\boldsymbol{n}$ is the unit outward normal, $\tilde{\boldsymbol{F}} = \boldsymbol{\nabla}\tilde{\boldsymbol{r}}$, $\tilde{\boldsymbol{P}}$ the Piola–Kirchhoff stress tensor, superscript "t" denotes transposition.

Using the Gauss theorem we derive

$$\boldsymbol{F} = \langle\tilde{\boldsymbol{F}}\rangle_\tau - \frac{1}{v}\int[\tilde{\boldsymbol{r}}]\,\boldsymbol{n}\,d\Sigma\,; \qquad \boldsymbol{P}^t = \langle\tilde{\boldsymbol{P}}^t\rangle_\tau - \frac{1}{v}\int[\tilde{\boldsymbol{r}}_\tau\,\tilde{\boldsymbol{P}}]\cdot\boldsymbol{n}\,d\Sigma\,;$$
$$\dot{\boldsymbol{F}} = \langle\dot{\tilde{\boldsymbol{F}}}\rangle_\tau - \frac{1}{v}\int[\dot{\tilde{\boldsymbol{r}}}]\,\boldsymbol{n}\,d\Sigma\,; \qquad \dot{\boldsymbol{P}}^t = \langle\dot{\tilde{\boldsymbol{P}}}^t\rangle_\tau - \frac{1}{v}\int[\tilde{\boldsymbol{r}}_\tau\,\dot{\tilde{\boldsymbol{P}}}]\cdot\boldsymbol{n}\,d\Sigma\,; \tag{2}$$

where $\langle\tilde{\boldsymbol{a}}\rangle_\tau := \frac{1}{v}\int\tilde{\boldsymbol{a}}\,d\bar{v}\,,\ \bar{v} = v - \Sigma$. We assume $[\tilde{\boldsymbol{r}}_\tau] = 0$. We will consider a small $[\tilde{\boldsymbol{r}}]$ and neglect difference in geometry of points $\tilde{\boldsymbol{r}}^+$ and $\tilde{\boldsymbol{r}}^-$. As $[\tilde{\boldsymbol{P}}]\cdot\boldsymbol{n} = [\dot{\tilde{\boldsymbol{P}}}]\cdot\boldsymbol{n} = 0$ and $[\tilde{\boldsymbol{r}}_\tau] = 0$, then $\boldsymbol{P} = \langle\tilde{\boldsymbol{P}}\rangle_\tau\,;$ $\dot{\boldsymbol{P}} = \langle\dot{\tilde{\boldsymbol{P}}}\rangle_\tau$, i.e. the same when $[\tilde{\boldsymbol{r}}] = 0$ [6]. Considering actual configuration V_t as reference one we obtain

$$\boldsymbol{T}^t = \frac{1}{v_t}\int\tilde{\boldsymbol{r}}\tilde{\boldsymbol{T}}\cdot\boldsymbol{n}_t\,dS_t = \langle\tilde{\boldsymbol{T}}^t\rangle_t - \frac{1}{v_t}\int[\tilde{\boldsymbol{r}}\tilde{\boldsymbol{T}}]\cdot\boldsymbol{n}_t\,d\Sigma_t\,; \quad \langle\tilde{\boldsymbol{a}}\rangle_t := \frac{1}{v_t}\int\tilde{\boldsymbol{a}}\,d\bar{v}_t\,,$$

where $\boldsymbol{T}$ is the Cauchy stress tensor and subscript t denotes the parameters in the actual configuration. As $[\tilde{\boldsymbol{T}}]\cdot\boldsymbol{n}_t = 0$ and $\tilde{\boldsymbol{T}}^t = \tilde{\boldsymbol{T}}$, we have

$$\boldsymbol{T}^t = \langle\tilde{\boldsymbol{T}}\rangle_t - \frac{1}{v_t}\int[\tilde{\boldsymbol{r}}]\,\tilde{\boldsymbol{T}}\cdot\boldsymbol{n}_t\,d\Sigma_t\,. \tag{3}$$

Due to second term in this formula we cannot affirm that $\boldsymbol{T}$ is symmetrical tensor. Only at $[\tilde{\boldsymbol{r}}] = 0$ we obtain $\boldsymbol{T}^t = \langle\tilde{\boldsymbol{T}}\rangle_t = \boldsymbol{T}$.

It is convenient to decompose $[\dot{\tilde{\boldsymbol{r}}}] = [\dot{\tilde{\boldsymbol{r}}}]_1 + [\dot{\tilde{\boldsymbol{r}}}]_2$, where $[\dot{\tilde{\boldsymbol{r}}}]_1$ is the jump when $[\tilde{\boldsymbol{r}}] = 0$ and $[\dot{\tilde{\boldsymbol{r}}}]_2 = [\dot{\tilde{\boldsymbol{r}}}] - [\dot{\tilde{\boldsymbol{r}}}]_1$ (i.e. $[\dot{\tilde{\boldsymbol{r}}}]_2$ is the jump when Σ is fixed, $v_n = 0$). The compatibility condition $[\dot{\tilde{\boldsymbol{r}}}]_1 \boldsymbol{n} = -[\tilde{\boldsymbol{F}}]\, v_n$ is valid for $[\dot{\tilde{\boldsymbol{r}}}]_1$, where v_n is normal velocity of Σ in configuration V_τ. In this case

$$\dot{\boldsymbol{F}} = \langle \dot{\tilde{\boldsymbol{F}}} \rangle_\tau + \frac{1}{v} \int \Big([\tilde{\boldsymbol{F}}]\, v_n - [\dot{\tilde{\boldsymbol{r}}}]_2\, \boldsymbol{n} \Big)\, d\Sigma . \tag{4}$$

Direct calculations using Eqs. (2), (4) and $\boldsymbol{\nabla} \cdot \tilde{\boldsymbol{P}} = 0$ prove the identities

$$\frac{1}{v} \int (\tilde{\boldsymbol{r}} - \boldsymbol{F} \cdot \tilde{\boldsymbol{r}}_\tau)(\tilde{\boldsymbol{P}} - \boldsymbol{P}) \cdot \boldsymbol{n}\, dS = \langle \tilde{\boldsymbol{F}} \cdot \tilde{\boldsymbol{P}}^t \rangle_\tau - \boldsymbol{F} \cdot \boldsymbol{P}^t - \frac{1}{v} \int [\tilde{\boldsymbol{r}}] \tilde{\boldsymbol{P}} \cdot \boldsymbol{n}\, d\Sigma\,; \tag{5}$$

$$\frac{1}{v} \int \Big(\dot{\tilde{\boldsymbol{r}}} - \dot{\boldsymbol{F}} \cdot \tilde{\boldsymbol{r}}_\tau \Big) \cdot \Big(\tilde{\boldsymbol{P}} - \boldsymbol{P} \Big) \cdot \boldsymbol{n}\, dS =$$
$$= \langle \dot{\tilde{\boldsymbol{F}}} : \tilde{\boldsymbol{P}}^t \rangle_\tau - \dot{\boldsymbol{F}} : \boldsymbol{P}^t + \frac{1}{v} \int \Big([\tilde{\boldsymbol{F}}] : \tilde{\boldsymbol{P}}^t v_n - [\dot{\tilde{\boldsymbol{r}}}]_2 \cdot \tilde{\boldsymbol{P}} \cdot \boldsymbol{n} \Big)\, d\Sigma\,; \tag{6}$$

$$\frac{1}{v} \int \Big(\dot{\tilde{\boldsymbol{r}}} - \dot{\boldsymbol{F}} \cdot \tilde{\boldsymbol{r}}_\tau \Big) \cdot \Big(\dot{\tilde{\boldsymbol{P}}} - \dot{\boldsymbol{P}} \Big) \cdot \boldsymbol{n}\, dS =$$
$$= \langle \dot{\tilde{\boldsymbol{F}}} : \dot{\tilde{\boldsymbol{P}}}^t \rangle_\tau - \dot{\boldsymbol{F}} : \dot{\boldsymbol{P}}^t + \frac{1}{v} \int \Big([\tilde{\boldsymbol{F}}] : \dot{\tilde{\boldsymbol{P}}}^t v_n - [\dot{\tilde{\boldsymbol{r}}}]_2 \cdot \dot{\tilde{\boldsymbol{P}}} \cdot \boldsymbol{n} \Big)\, d\Sigma . \tag{7}$$

Hence, at macrohomogeneous boundary conditions on $S - \tilde{\boldsymbol{r}} = \boldsymbol{F} \cdot \tilde{\boldsymbol{r}}_\tau$ or $\tilde{\boldsymbol{P}} \cdot \boldsymbol{n} = \boldsymbol{P} \cdot \boldsymbol{n}$ (or $\dot{\tilde{\boldsymbol{P}}} \cdot \boldsymbol{n} = \dot{\boldsymbol{P}} \cdot \tilde{\boldsymbol{r}}_\tau$) – we get zero in the left–hand part, e. g.

$$\langle \dot{\tilde{\boldsymbol{F}}} : \tilde{\boldsymbol{P}}^t \rangle_\tau = \dot{\boldsymbol{F}} : \boldsymbol{P}^t - \frac{1}{v} \int \Big([\tilde{\boldsymbol{F}}] : \tilde{\boldsymbol{P}}^t v_n - [\dot{\tilde{\boldsymbol{r}}}]_2 \cdot \tilde{\boldsymbol{P}} \cdot \boldsymbol{n} \Big)\, d\Sigma . \tag{8}$$

Using the expressions $\tilde{\boldsymbol{T}} = \tilde{\rho}_t\, \tilde{\rho}_\tau^{-1}\, \tilde{\boldsymbol{F}} \cdot \tilde{\boldsymbol{P}}^t$, $\tilde{\boldsymbol{P}} \cdot \boldsymbol{n}\, d\Sigma = \tilde{\boldsymbol{T}} \cdot \boldsymbol{n}_t\, d\Sigma_t$ we have $\langle \tilde{\boldsymbol{F}} \cdot \tilde{\boldsymbol{P}}^t \rangle_\tau = \langle \tilde{\rho}_t^{-1}\, \tilde{\rho}_\tau \tilde{\boldsymbol{T}} \rangle_\tau = \rho_\tau\, \rho_t^{-1} \langle \tilde{\boldsymbol{T}} \rangle_t$, and from Eq. (5) at homogeneous boundary data it follows

$$\frac{\rho_t}{\rho_\tau} \boldsymbol{F} \cdot \boldsymbol{P}^t = \langle \tilde{\boldsymbol{T}} \rangle_t - \frac{1}{v_t} \int [\tilde{\boldsymbol{r}}] \cdot \tilde{\boldsymbol{T}} \cdot \boldsymbol{n}_t\, d\Sigma_t\,, \tag{9}$$

where $\tilde{\rho}_t$ and $\tilde{\rho}_\tau$, ρ_t and ρ_τ are local and overall mass densities in V_t and V_τ. Comparison of Eqs. (3) and (9) gives $\boldsymbol{T}^t = \rho_t\, \rho_\tau^{-1}\, \boldsymbol{F} \cdot \boldsymbol{P}^t$, i.e. the same equation as for material point. The same calculation as in derivation of Eq. (6) but with $\dot{\boldsymbol{F}} = \boldsymbol{P}^t = 0$, will result in the expression

$$\frac{1}{v} \int \dot{\tilde{\boldsymbol{r}}} \cdot \tilde{\boldsymbol{P}} \cdot \boldsymbol{n}\, dS = \langle \dot{\tilde{\boldsymbol{F}}} : \tilde{\boldsymbol{P}}^t \rangle_\tau + \frac{1}{v} \int \Big([\tilde{\boldsymbol{F}}] : \tilde{\boldsymbol{P}}^t v_n - [\dot{\tilde{\boldsymbol{r}}}]_2 \cdot \tilde{\boldsymbol{P}} \cdot \boldsymbol{n} \Big)\, d\Sigma . \tag{10}$$

Comparison with Eq. (8) yields $\frac{1}{v}\int \dot{\tilde{\boldsymbol{r}}}\cdot\tilde{\boldsymbol{P}}\cdot\boldsymbol{n}\,dS = \dot{\boldsymbol{F}}:\boldsymbol{P}^t$.

3. Rate of the overall inelastic deformation

Let the representative volume of a microinhomogeneous elastoplastic material v without PT is in the conditions of macroscopically homogeneous strained or stressed state. At all points of the volume v the decomposition $\tilde{\boldsymbol{F}} = \tilde{\boldsymbol{F}}_e\cdot\tilde{\boldsymbol{F}}_i$ is assumed, where $\tilde{\boldsymbol{F}}_e$ and $\tilde{\boldsymbol{F}}_i$ are the gradients of elastic and inelastic deformations. Let us introduce the Piola–Kirchhoff stress tensor with respect to locally unloaded configuration $\tilde{V}_i:\tilde{\boldsymbol{P}}_e = \tilde{\rho}_i\,\tilde{\rho}_t^{-1}\,\tilde{\boldsymbol{T}}\cdot\tilde{\boldsymbol{F}}_e^{t-1} = \tilde{J}\,\tilde{\boldsymbol{P}}\cdot\tilde{\boldsymbol{F}}_i^t$, where $\tilde{J} = \tilde{\rho}_i/\tilde{\rho}_\tau$ and $\tilde{\rho}_i$ is the mass density in configuration $\tilde{V}_i$. Similar expressions are valid for the macrovariables $\boldsymbol{F} = \boldsymbol{F}_e\cdot\boldsymbol{F}_i$, $\boldsymbol{F}_i = \boldsymbol{F}$ at $\boldsymbol{T} = 0$, $\boldsymbol{P}_e = \rho_i\,\rho_t^{-1}\,\boldsymbol{T}\cdot\boldsymbol{F}_e^{t-1} = J\,\boldsymbol{P}\cdot\boldsymbol{F}_i^t$, where $J = \rho_i/\rho_\tau$. Our aim is to determine the relationship between $\dot{\boldsymbol{F}}_i$ and $\dot{\tilde{\boldsymbol{F}}}_i$.

Let us consider the main body and the comparison body. For the comparison body the deformation gradient $\boldsymbol{Q} = \boldsymbol{Q}_e\cdot\boldsymbol{F}_i$, where $\boldsymbol{Q}_e$ is the gradient of the elastic deformation, while Piola–Kirchhoff tensors with respect to configurations V and V_i can be expressed as $\boldsymbol{\tau}$ and $\boldsymbol{\tau}_e = J\,\boldsymbol{\tau}\cdot\boldsymbol{F}_i^t$ (similarly, $\tilde{\boldsymbol{Q}} = \tilde{\boldsymbol{Q}}_e\cdot\tilde{\boldsymbol{F}}_i$, $\tilde{\boldsymbol{\tau}}_e = \tilde{J}\,\tilde{\boldsymbol{\tau}}\cdot\tilde{\boldsymbol{F}}_i^t$). In configurations $\tilde{V}_\tau$, $\tilde{V}_i$ and $\tilde{V}_t$ the both bodies are identical, i.e. $\tilde{\boldsymbol{Q}} = \tilde{\boldsymbol{F}}$, $\boldsymbol{Q}_e = \boldsymbol{F}_e$, $\tilde{\boldsymbol{\tau}} = \tilde{\boldsymbol{P}}$, $\tilde{\boldsymbol{\tau}}_e = \tilde{\boldsymbol{P}}_e$ etc., however the comparison body in contrast to the main body is strained elastically at arbitrary $\dot{\tilde{\boldsymbol{\tau}}}$, if $\dot{\tilde{\boldsymbol{F}}} = \dot{\tilde{\boldsymbol{F}}}_e\cdot\tilde{\boldsymbol{F}}_i + \tilde{\boldsymbol{F}}_e\cdot\dot{\tilde{\boldsymbol{F}}}_i$, then $\dot{\tilde{\boldsymbol{Q}}} = \dot{\tilde{\boldsymbol{Q}}}_e\cdot\tilde{\boldsymbol{F}}_i$ ($\dot{\boldsymbol{F}} = \dot{\boldsymbol{F}}_e\cdot\boldsymbol{F}_i + \boldsymbol{F}_e\cdot\dot{\boldsymbol{F}}_i$, $\dot{\boldsymbol{Q}} = \dot{\boldsymbol{Q}}_e\cdot\boldsymbol{F}_i$).

From the laws of thermodynamics, in the case of the laws of micro– and macroelasticity being independent of the inelastic deformation, it follows $\dot{\tilde{\boldsymbol{F}}}_e = \tilde{\boldsymbol{\Lambda}}:\dot{\tilde{\boldsymbol{P}}}_e^t = \dot{\tilde{\boldsymbol{P}}}_e^t:\tilde{\boldsymbol{\Lambda}}$; $\dot{\tilde{\boldsymbol{Q}}}_e = \tilde{\boldsymbol{\Lambda}}:\dot{\tilde{\boldsymbol{\tau}}}_e^t$; $\dot{\boldsymbol{F}}_e = \boldsymbol{\Lambda}:\dot{\boldsymbol{P}}_e^t$; $\dot{\boldsymbol{Q}}_e = \boldsymbol{\Lambda}:\dot{\boldsymbol{\tau}}_e^t$; where $\tilde{\boldsymbol{\Lambda}} = \tilde{\rho}_i\,\dfrac{\partial^2\tilde{\Phi}}{\partial\tilde{\boldsymbol{P}}_e^2}$, $\boldsymbol{\Lambda} = \rho_i\,\dfrac{\partial^2\Phi}{\partial\boldsymbol{P}_e^2}$, $\tilde{\Phi}$ is the Gibbs potential, $\rho_\tau\,\Phi = \langle\tilde{\rho}_\tau\,\tilde{\Phi}\rangle_\tau$. Taking into account the quasi–linearity of the of rate–problem of plasticity theory we assume that there is a quasi–linear relationship between $\dot{\tilde{\boldsymbol{\tau}}}^t$ and $\dot{\boldsymbol{\tau}}^t$: $\dot{\tilde{\boldsymbol{\tau}}}^t = \tilde{\boldsymbol{A}}:\dot{\boldsymbol{\tau}}^t = \dot{\boldsymbol{\tau}}^t:\tilde{\boldsymbol{A}}^t$; $\tilde{\boldsymbol{A}} = \tilde{\boldsymbol{A}}\left(\tilde{\boldsymbol{P}},\,\tilde{\boldsymbol{F}}_i,\,\tilde{\boldsymbol{r}}_\tau\right)$, where the concentration (localization) tensor $\tilde{\boldsymbol{A}}$ is determined from the solution of boundary–value problem. If $\boldsymbol{\nabla}\cdot\dot{\tilde{\boldsymbol{\tau}}} = 0$ and $\dot{\tilde{\boldsymbol{F}}} = \boldsymbol{\nabla}\dot{\tilde{\boldsymbol{r}}}$, then for the homogeneous boundary conditions we have from Eq. (7)

$$\dot{\boldsymbol{\tau}}^t:\dot{\boldsymbol{F}} = \langle\dot{\tilde{\boldsymbol{\tau}}}^t:\dot{\tilde{\boldsymbol{F}}}\rangle_\tau\,. \tag{11}$$

Let us calculate the left part of the equality (11), taking into consideration that in the formulas, at first the operation $<\cdot>$ is done and then

$<\boldsymbol{:}>$; for example $\boldsymbol{A}\boldsymbol{:}\boldsymbol{B}\cdot\boldsymbol{K} = \boldsymbol{A}\boldsymbol{:}(\boldsymbol{B}\cdot\boldsymbol{K})$. We have $\dot{\boldsymbol{\tau}}^t\boldsymbol{:}\dot{\boldsymbol{F}} =$

$$= \dot{\boldsymbol{\tau}}^t\boldsymbol{:}(\dot{\boldsymbol{F}}_e\cdot\boldsymbol{F}_i + \boldsymbol{F}_e\cdot\dot{\boldsymbol{F}}_i)\,; \qquad \dot{\boldsymbol{\tau}}^t\boldsymbol{:}\dot{\boldsymbol{F}}_e\cdot\boldsymbol{F}_i = \boldsymbol{F}_i\cdot\dot{\boldsymbol{\tau}}^t\boldsymbol{:}\dot{\boldsymbol{F}}_e = J^{-1}\,\dot{\boldsymbol{\tau}}_e^t\boldsymbol{:}\dot{\boldsymbol{F}}_e =$$

$$= J^{-1}\,\dot{\boldsymbol{\tau}}_e^t\boldsymbol{:}\boldsymbol{\Lambda}\boldsymbol{:}\dot{\boldsymbol{P}}_e^t = J^{-1}\,\dot{\boldsymbol{Q}}_e\boldsymbol{:}\dot{\boldsymbol{P}}_e^t = \left(\dot{\boldsymbol{Q}}_e\cdot\boldsymbol{F}_i\right)\boldsymbol{:}\left(J^{-1}\,\boldsymbol{F}_i^{-1}\cdot\dot{\boldsymbol{P}}_e^t\right) =$$

$$= \dot{\boldsymbol{Q}}\boldsymbol{:}\left(\dot{\overline{J^{-1}\,\boldsymbol{F}_p^{-1}\cdot\boldsymbol{P}_e^t}} - J^{-1}\,\dot{\boldsymbol{F}}_i^{-1}\cdot\boldsymbol{P}_e^t - J^{-1}\left(\dot{\boldsymbol{F}}_i\boldsymbol{:}\boldsymbol{F}_i^{-1}\right)\boldsymbol{F}_i^{-1}\cdot\boldsymbol{P}_e^t\right) =$$

$$= \dot{\boldsymbol{Q}}\boldsymbol{:}\dot{\boldsymbol{P}}^t + \dot{\boldsymbol{Q}}\boldsymbol{:}J^{-1}\,\boldsymbol{F}_i^{-1}\cdot\dot{\boldsymbol{F}}_i\cdot\boldsymbol{F}_i^{-1}\cdot\boldsymbol{P}_e^t - \dot{\boldsymbol{Q}}\boldsymbol{:}J^{-1}\left(\dot{\boldsymbol{F}}_i\boldsymbol{:}\boldsymbol{F}_i^{-1}\right)\boldsymbol{F}_i^{-1}\cdot\boldsymbol{P}_e^t\,.$$

Then
$$J^{-1}\,\dot{\boldsymbol{Q}}\boldsymbol{:}\boldsymbol{F}_i^{-1}\cdot\dot{\boldsymbol{F}}_i\cdot\boldsymbol{F}_i^{-1}\cdot\boldsymbol{P}_e^t = \dot{\boldsymbol{Q}}_e\boldsymbol{:}\dot{\boldsymbol{F}}_i\cdot\boldsymbol{P}^t = \dot{\boldsymbol{\tau}}_e^t\boldsymbol{:}\boldsymbol{\Lambda}\boldsymbol{:}\dot{\boldsymbol{F}}_i\cdot\boldsymbol{P}^t =$$

$$= J\,\boldsymbol{F}_i\cdot\dot{\boldsymbol{\tau}}^t\boldsymbol{:}\boldsymbol{\Lambda}\boldsymbol{:}\dot{\boldsymbol{F}}_i\cdot\boldsymbol{P}^t = J\,\dot{\boldsymbol{\tau}}^t\boldsymbol{:}\left(\boldsymbol{\Lambda}\boldsymbol{:}\dot{\boldsymbol{F}}_i\cdot\boldsymbol{P}^t\right)\cdot\boldsymbol{F}_i\,;$$

$$J^{-1}\,\dot{\boldsymbol{Q}}\boldsymbol{:}\boldsymbol{F}_i^{-1}\cdot\boldsymbol{P}_e^t = J^{-1}\,\dot{\boldsymbol{Q}}_e\boldsymbol{:}\boldsymbol{P}_e^t = J^{-1}\,\dot{\boldsymbol{\tau}}_e^t\boldsymbol{:}\boldsymbol{\Lambda}\boldsymbol{:}\boldsymbol{P}_e^t =$$

$$= \boldsymbol{F}_i\cdot\dot{\boldsymbol{\tau}}^t\boldsymbol{:}\boldsymbol{\Lambda}\boldsymbol{:}\boldsymbol{P}_e^t = \dot{\boldsymbol{\tau}}^t\boldsymbol{:}\left(\boldsymbol{\Lambda}\boldsymbol{:}\boldsymbol{P}_e^t\right)\cdot\boldsymbol{F}_i\,.$$

Finally we obtain

$$\dot{\boldsymbol{\tau}}^t\boldsymbol{:}\dot{\boldsymbol{F}} = \dot{\boldsymbol{\tau}}^t\boldsymbol{:}\boldsymbol{\Phi} + \dot{\boldsymbol{Q}}\boldsymbol{:}\dot{\boldsymbol{P}}^t\,.$$

$$\boldsymbol{\Phi} := \boldsymbol{F}_e\cdot\dot{\boldsymbol{F}}_i + J(\boldsymbol{\Lambda}\boldsymbol{:}\dot{\boldsymbol{F}}_i\cdot\boldsymbol{P}^t)\cdot\boldsymbol{F}_i - (\boldsymbol{\Lambda}\boldsymbol{:}\boldsymbol{P}_e^t)\cdot\boldsymbol{F}_i(\dot{\boldsymbol{F}}_i\boldsymbol{:}\boldsymbol{F}_i^{-1}) = \boldsymbol{B}\boldsymbol{:}\dot{\boldsymbol{F}}_i\,, \quad (12)$$

where the fourth–rank tensor $\boldsymbol{B}$ is defined from the linear equation $\boldsymbol{\Phi} = \boldsymbol{B}\boldsymbol{:}\dot{\boldsymbol{F}}_i$. Let us calculate the right side of the equation (11). We have

$$\langle\dot{\tilde{\boldsymbol{\tau}}}^t\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}\rangle_\tau = \dot{\boldsymbol{\tau}}\boldsymbol{:}\langle\tilde{\boldsymbol{A}}^t\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}\rangle_\tau = \dot{\boldsymbol{\tau}}^t\boldsymbol{:}\langle\tilde{\boldsymbol{A}}^t\boldsymbol{:}(\dot{\tilde{\boldsymbol{F}}}_e\cdot\tilde{\boldsymbol{F}}_i + \tilde{\boldsymbol{F}}_e\cdot\dot{\tilde{\boldsymbol{F}}}_i)\rangle_\tau\,. \quad (13)$$

After similar transformations we obtain

$$\dot{\boldsymbol{\tau}}^t\boldsymbol{:}\langle\tilde{\boldsymbol{A}}^t\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_e\cdot\tilde{\boldsymbol{F}}_i\rangle_\tau = \langle\dot{\tilde{\boldsymbol{\tau}}}\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_e\cdot\tilde{\boldsymbol{F}}_i\rangle_\tau = \langle\dot{\tilde{\boldsymbol{Q}}}\boldsymbol{:}\dot{\tilde{\boldsymbol{P}}}^t\rangle_\tau +$$
$$+\langle\dot{\tilde{\boldsymbol{\tau}}}^t\boldsymbol{:}(\tilde{\boldsymbol{\Lambda}}\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_i\cdot\tilde{\boldsymbol{P}}^t)\cdot\tilde{\boldsymbol{F}}_i\,\tilde{J} - \dot{\tilde{\boldsymbol{\tau}}}^t\boldsymbol{:}\left(\tilde{\boldsymbol{\Lambda}}\boldsymbol{:}\tilde{\boldsymbol{P}}_e^t\right)\cdot\tilde{\boldsymbol{F}}_i\left(\dot{\tilde{\boldsymbol{F}}}_i\boldsymbol{:}\tilde{\boldsymbol{F}}_i^{-1}\right)\rangle_\tau\,. \quad (14)$$

Evidently $\langle\dot{\tilde{\boldsymbol{Q}}}\boldsymbol{:}\dot{\tilde{\boldsymbol{P}}}^t\rangle_\tau = \dot{\boldsymbol{Q}}\boldsymbol{:}\dot{\boldsymbol{P}}^t$, since $\dot{\tilde{\boldsymbol{Q}}}$ and $\dot{\tilde{\boldsymbol{P}}}^t$ are kinematically and statically admissible fields. Designating

$$\tilde{\boldsymbol{\Phi}} := \tilde{\boldsymbol{F}}_e\cdot\dot{\tilde{\boldsymbol{F}}}_i + J(\tilde{\boldsymbol{\Lambda}}\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_i\cdot\tilde{\boldsymbol{P}}^t)\cdot\tilde{\boldsymbol{F}}_i - (\tilde{\boldsymbol{\Lambda}}\boldsymbol{:}\tilde{\boldsymbol{P}}_e^t)\cdot\tilde{\boldsymbol{F}}_i(\dot{\tilde{\boldsymbol{F}}}_i\boldsymbol{:}\tilde{\boldsymbol{F}}_i^{-1}) = \tilde{\boldsymbol{B}}\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_i\,,$$

where the tensor $\tilde{\boldsymbol{B}}$ is determined from the linear equation $\tilde{\boldsymbol{\Phi}} = \tilde{\boldsymbol{B}}\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_i$, we get

$$\langle\dot{\tilde{\boldsymbol{\tau}}}^t\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}\rangle_\tau = \boldsymbol{\tau}^t\boldsymbol{:}\langle\tilde{\boldsymbol{A}}^t\boldsymbol{:}\tilde{\boldsymbol{B}}\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_i\rangle_\tau + \dot{\boldsymbol{Q}}\boldsymbol{:}\dot{\boldsymbol{P}}^t\,. \quad (15)$$

Comparing formulas (12) and (15) we find

$$\dot{\boldsymbol{F}}_i = \boldsymbol{B}^{-1}\boldsymbol{:}\langle\tilde{\boldsymbol{A}}^t\boldsymbol{:}\tilde{\boldsymbol{B}}\boldsymbol{:}\dot{\tilde{\boldsymbol{F}}}_i\rangle_\tau\,. \quad (16)$$

The formula (16) is the desired expression for $\dot{\boldsymbol{F}}_i$.

Let $\tilde{\boldsymbol{F}}_e = \tilde{\boldsymbol{R}}_e \cdot \tilde{\boldsymbol{U}}_e$ and $\tilde{\boldsymbol{U}}_e = \boldsymbol{I} + \tilde{\boldsymbol{\varepsilon}}_e$, $\tilde{\boldsymbol{\varepsilon}}_e << \boldsymbol{I}$, where $\tilde{\boldsymbol{R}}_e^t = \tilde{\boldsymbol{R}}_e^{-1}$, $\tilde{\boldsymbol{U}}_e = \boldsymbol{U}_e^t$ (in a similar way, $\boldsymbol{F}_e = \boldsymbol{R}_e \cdot \boldsymbol{U}_e$, $\boldsymbol{U}_e = \boldsymbol{I} + \boldsymbol{\varepsilon}_e$ and $\boldsymbol{\varepsilon}_e << \boldsymbol{I}$), where $\boldsymbol{I}$ is a unit tensor, i.e. the elastic deformations are small. Then from the formula (16) it follows that

$$\boldsymbol{R}_e \cdot \dot{\boldsymbol{F}}_i = \langle \tilde{\boldsymbol{A}}^t : \tilde{\boldsymbol{R}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_i \rangle_\tau , \qquad \dot{\boldsymbol{F}}_i = \boldsymbol{R}_e^t \cdot \langle \tilde{\boldsymbol{A}}^t : \tilde{\boldsymbol{R}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_i \rangle_\tau , \tag{17}$$

since the other terms contain at $\dot{\tilde{\boldsymbol{F}}}_i$ and $\boldsymbol{F}_i$ factors of the order $\tilde{\boldsymbol{\varepsilon}}_e$ and $\boldsymbol{\varepsilon}_e$. At small elastic rotations $\tilde{\boldsymbol{R}}_e^t \approx \boldsymbol{I}$, $\boldsymbol{R}_e^t \approx \boldsymbol{I}$ we have $\dot{\boldsymbol{F}}_i = \langle \tilde{\boldsymbol{A}}^t : \dot{\tilde{\boldsymbol{F}}}_i \rangle_\tau$, at small inelastic rotations $\dot{\boldsymbol{U}}_i = \langle \tilde{\boldsymbol{A}}^t : \dot{\tilde{\boldsymbol{U}}}_i \rangle_\tau$, where $\tilde{\boldsymbol{F}}_i = \tilde{\boldsymbol{R}}_i \cdot \tilde{\boldsymbol{U}}_i$, $\boldsymbol{F}_i = \boldsymbol{R}_i \cdot \boldsymbol{U}_i$. If the inelastic deformations are also small ($\tilde{\boldsymbol{\varepsilon}}_i = \tilde{\boldsymbol{U}}_i - \boldsymbol{I} << \boldsymbol{I}$, $\boldsymbol{\varepsilon}_i = \boldsymbol{U}_i - \boldsymbol{I} << \boldsymbol{I}$), then $\dot{\boldsymbol{\varepsilon}}_i = \langle \tilde{\boldsymbol{A}}^t : \dot{\tilde{\boldsymbol{\varepsilon}}}_i \rangle$. In case of linear elasticity $\tilde{\boldsymbol{A}} = const$ and we obtain well–known Mandel–Hill relation $\boldsymbol{\varepsilon}_i = \langle \tilde{\boldsymbol{A}}^t : \tilde{\boldsymbol{\varepsilon}}_i \rangle$.

We know only one work – Hill's article [1], which tackles the problem of the relationship between the rates of effective and local plastic deformations at finite strains. The following relationship was obtained in this article

$$\dot{\boldsymbol{F}} - \boldsymbol{\Lambda} : \dot{\boldsymbol{P}}^t = \langle \tilde{\boldsymbol{A}}^t : (\dot{\tilde{\boldsymbol{F}}} - \tilde{\boldsymbol{\Lambda}} : \dot{\tilde{\boldsymbol{P}}}^t) \rangle_\tau , \tag{18}$$

but since $\dot{\boldsymbol{F}} - \boldsymbol{\Lambda} : \dot{\boldsymbol{P}}^t = \dot{\boldsymbol{F}}_e \cdot \boldsymbol{F}_i + \boldsymbol{F}_e \cdot \dot{\boldsymbol{F}}_i - J^{-1} \boldsymbol{\Lambda} : \boldsymbol{F}_i^{-1} \cdot (\dot{\boldsymbol{P}}_e^t - \dot{\boldsymbol{F}}_i \cdot \boldsymbol{F}_i^{-1} \cdot \boldsymbol{P}_e^t - \boldsymbol{P}_e^t \cdot (\dot{\boldsymbol{F}}_i : \boldsymbol{F}_i^{-1}))$, this relation contains also $\dot{\boldsymbol{F}}_e$. Therefore, $\dot{\tilde{\boldsymbol{F}}} - \tilde{\boldsymbol{\Lambda}} : \dot{\tilde{\boldsymbol{P}}}^t$ and $\dot{\boldsymbol{F}} - \boldsymbol{\Lambda} : \dot{\boldsymbol{P}}^t$ are not measures of the rate of inelastic strain.

Let the flow rule for each point is associated and the local and effective elasticity rules do not depend on plastic deformation. In this case $\tilde{\psi} = \tilde{\psi}(\tilde{\boldsymbol{E}}_e)$, $\tilde{\boldsymbol{E}}_e = 0.5(\tilde{\boldsymbol{F}}_e^t \cdot \tilde{\boldsymbol{F}}_e - \boldsymbol{I})$ and rate of dissipation is equal to $\tilde{D} = \tilde{\boldsymbol{P}}^t : \tilde{\boldsymbol{F}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_p$ [3]–[5]. Principle of maximum of dissipation results in

$$(\tilde{\boldsymbol{P}}^t - \tilde{\boldsymbol{P}}^{t*}) : \tilde{\boldsymbol{F}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_i \geq 0 \qquad \text{at} \qquad f(\boldsymbol{P}^*, \ldots) \leq 0; \qquad \tilde{\boldsymbol{F}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_i = \tilde{h} \frac{\partial \tilde{f}}{\partial \tilde{\boldsymbol{P}}},$$

where $\tilde{f}$ is a yield function. Let $\tilde{\boldsymbol{P}}^t = \tilde{\boldsymbol{q}}(\boldsymbol{P}) + \tilde{\boldsymbol{P}}_i$, where $\tilde{\boldsymbol{P}}_i$ is the stress field at $\boldsymbol{P} = 0$

$$\dot{\tilde{\boldsymbol{P}}}^t = \frac{\partial \tilde{\boldsymbol{q}}}{\partial \boldsymbol{P}} : \dot{\boldsymbol{P}}^t + \dot{\tilde{\boldsymbol{P}}}_i = \tilde{\boldsymbol{A}} : \dot{\boldsymbol{P}}^t + \dot{\tilde{\boldsymbol{P}}}_i ; \qquad \frac{\partial \tilde{\boldsymbol{P}}^t}{\partial \boldsymbol{P}} = \tilde{\boldsymbol{A}};$$

$$\tilde{\boldsymbol{F}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_i = \tilde{h} \frac{\partial \tilde{f}}{\partial \tilde{\boldsymbol{P}}} = \tilde{h} \frac{\partial \tilde{f}}{\partial \tilde{\boldsymbol{P}}} : \frac{\partial \tilde{\boldsymbol{P}}^t}{\partial \boldsymbol{P}} : \Big(\frac{\partial \tilde{\boldsymbol{P}}^t}{\partial \boldsymbol{P}} \Big)^{-1} = \tilde{h} \frac{\partial \tilde{f}}{\partial \boldsymbol{P}} : \tilde{\boldsymbol{A}}^{-1} = \tilde{h} \tilde{\boldsymbol{A}}^{t-1} : \frac{\partial \tilde{f}}{\partial \boldsymbol{P}} .$$

Substituting $\tilde{\boldsymbol{F}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_i$ in the formula (16) we get

$$\dot{\tilde{\boldsymbol{F}}}_i = \boldsymbol{B}^{-1} : \Big\langle \tilde{h}\, \tilde{\boldsymbol{A}}^t : \tilde{\boldsymbol{B}} : \tilde{\boldsymbol{F}}_e^{-1} \cdot \tilde{\boldsymbol{A}}^{t-1} : \frac{\partial \tilde{f}}{\partial \boldsymbol{P}} \Big\rangle_\tau . \tag{19}$$

The analysis of this expression shows that in the general case the flow rule for the composite as a whole is not associated. At small local and effective elastic deformations $\tilde{\boldsymbol{F}}_e \simeq \tilde{\boldsymbol{R}}_e$, $\boldsymbol{F}_e \simeq \boldsymbol{R}_e$ and using Eq. (17) we obtain

$$\boldsymbol{F}_e \cdot \dot{\boldsymbol{F}}_i = \langle \tilde{\boldsymbol{A}}^t : \tilde{\boldsymbol{F}}_e \cdot \dot{\tilde{\boldsymbol{F}}}_i \rangle_\tau = \Big\langle \tilde{h}\, \tilde{\boldsymbol{A}}^t : \tilde{\boldsymbol{A}}^{t-1} : \frac{\partial \tilde{f}}{\partial \boldsymbol{P}} \Big\rangle_\tau = \Big\langle \tilde{h} \frac{\partial \tilde{f}}{\partial \boldsymbol{P}} \Big\rangle_\tau , \tag{20}$$

i.e. flow rule is the associated.

Let us compare the obtained result with an approach by Rice [2] In [2] the macroscopic associated flow rule is proved for finite elastic strains. Let us analyze the reason of difference in results derived here and in [2]. It is assumed in [2] $\tilde{\psi} = \tilde{\psi}(\tilde{\boldsymbol{E}}, \xi_i)$, where $\tilde{\boldsymbol{E}} = 0.5\,(\tilde{\boldsymbol{F}}^t \cdot \tilde{\boldsymbol{F}} - \boldsymbol{I})$, ξ_i are scalar internal variables. As $\tilde{\boldsymbol{E}} = 0.5\,(\tilde{\boldsymbol{F}}_p^t \cdot \tilde{\boldsymbol{F}}_e^t \cdot \tilde{\boldsymbol{F}}_e \cdot \tilde{\boldsymbol{F}}_p - \boldsymbol{I}) = \tilde{\boldsymbol{F}}_p^t \cdot \tilde{\boldsymbol{E}}_e \cdot \tilde{\boldsymbol{F}}_p + \tilde{\boldsymbol{E}}_p$, $\tilde{\boldsymbol{E}}_p = 0.5\,(\tilde{\boldsymbol{F}}_p^t \cdot \tilde{\boldsymbol{F}}_p - \boldsymbol{I})$, then $\tilde{\psi} = \tilde{\psi}(\tilde{\boldsymbol{E}}_e, \tilde{\boldsymbol{F}}_p, \xi_i)$.

As for independent of plastic strain elasticity law $\tilde{\psi} = \tilde{\psi}(\tilde{\boldsymbol{E}}_e)$ [3]–[5], then in considered in [2] case $\tilde{\psi}$ and elasticity law depend in a very special and unrealistic way on plastic strain $\tilde{\boldsymbol{F}}_p$.

Let us derive the relationship between the effective and local values of tensors $\dot{\boldsymbol{F}}_i$ and $\dot{\tilde{\boldsymbol{F}}}_i$ for material with noncoherent PT. Assume that temperature θ is homogeneous in a volume v. In order to develop approximate theories of PT it is worthwhile to turn from the system of infinite number of degrees of freedom (distribution of v_n and $[\tilde{\boldsymbol{r}}]$ on Σ) to that of finite number of degrees of freedom. For this purpose Σ is divided into m–part denoted by Σ^i, motion of each parts describes a certain kind of PT and each Σ^i part is divided into k– amount of characteristic sections Σ^{ij}. Taking into consideration a volume $d\,v^{ij} = v_n\, d\,\Sigma^{ij}$, which is covered up by the section $d\,\Sigma^{ij}$ per unit time we get

$$\frac{1}{v} \int (...)\, v_n\, d\,\Sigma = \sum_{i,j} \frac{v^{ij}}{v} \frac{1}{v^{ij}} \int (...)\, d\,v^{ij} = \sum_{i,j} \langle ... \rangle_{ij}\, \dot{a}^{ij} = \langle ... \rangle_{\boldsymbol{a}} : \dot{\boldsymbol{a}},$$

$$\text{where} \quad \langle ... \rangle_{ij} = \frac{1}{v^{ij}} \int (...)\, d\,v^{ij}, \quad \dot{\boldsymbol{a}} \equiv \{\dot{a}_{ij}\}, \quad \langle ... \rangle_{\boldsymbol{a}} \equiv \{\langle ... \rangle_{ij}\}.$$

Parameters a^{ij} at a fixed i can characterize the shape of a region occupied by phases, e.g. they can be the principal axes of ellipsoidal regions. When Σ^i surfaces are not divided into sections, $\dot{a}^i = \frac{v^i}{v}$ are the rate of change of volume fraction at the i–th PT.

For the statically admissible field $\dot{\boldsymbol{\tau}}$ and kinematically admissible field $\dot{\tilde{\boldsymbol{F}}}$ at homogeneous boundary conditions it follows from Eq. (7)

$$\dot{\boldsymbol{\tau}}:\dot{\boldsymbol{F}} = \langle\dot{\tilde{\boldsymbol{\tau}}}:\dot{\tilde{\boldsymbol{F}}}\rangle_\tau + \langle\dot{\tilde{\boldsymbol{\tau}}}^t:[\tilde{\boldsymbol{F}}]\rangle_{\boldsymbol{a}}:\dot{\boldsymbol{a}} - \langle\dot{\boldsymbol{\tau}}^t:[\dot{\tilde{\boldsymbol{r}}}]_2\,\boldsymbol{n}\rangle_\Sigma\,, \tag{21}$$

where $\langle ... \rangle_\Sigma = \frac{1}{v}\int(...)\,d\Sigma$. Let us take into account that $\dot{\tilde{\boldsymbol{Q}}}_e = \tilde{\boldsymbol{\Lambda}}:\dot{\tilde{\boldsymbol{\tau}}}^t_e$, $\dot{\tilde{\boldsymbol{F}}}_e = \tilde{\boldsymbol{\Lambda}}:\dot{\boldsymbol{P}}^t_e + \tilde{\boldsymbol{m}}\,\dot{\theta}$, $\dot{\boldsymbol{Q}}_e = \boldsymbol{\Lambda}:\dot{\boldsymbol{\tau}}^t_e$, $\dot{\boldsymbol{F}}_e = \boldsymbol{\Lambda}:\dot{\boldsymbol{P}}^t_e + \boldsymbol{m}\,\dot{\theta} + \boldsymbol{g}:\dot{\boldsymbol{a}}$, $\boldsymbol{g} = \frac{\partial\boldsymbol{F}_e}{\partial\boldsymbol{a}}$, $\tilde{\boldsymbol{m}} = \frac{\partial\tilde{\boldsymbol{F}}_e}{\partial\theta}$, $\boldsymbol{m} = \frac{\partial\boldsymbol{F}_e}{\partial\theta}$ and making calculations, we obtain

$$\dot{\boldsymbol{\tau}}^t:\dot{\boldsymbol{F}} = \dot{\boldsymbol{\tau}}^t:[\boldsymbol{B}:\dot{\boldsymbol{F}}_i + \boldsymbol{m}\cdot\tilde{\boldsymbol{F}}_i\,\dot{\theta} + (\boldsymbol{g}:\dot{\boldsymbol{a}})\cdot\boldsymbol{F}_i] + \dot{\boldsymbol{Q}}:\dot{\boldsymbol{P}}^t\,; \tag{22}$$

$$\langle\dot{\tilde{\boldsymbol{\tau}}}:\dot{\tilde{\boldsymbol{F}}}\rangle_\tau = \dot{\boldsymbol{\tau}}^t:\langle\tilde{\boldsymbol{A}}^t:(\tilde{\boldsymbol{B}}:\dot{\tilde{\boldsymbol{F}}}_i + \tilde{\boldsymbol{m}}\cdot\tilde{\boldsymbol{F}}_i\,\dot{\theta})\rangle_\tau + \langle\dot{\boldsymbol{Q}}:\dot{\tilde{\boldsymbol{P}}}^t\rangle_\tau\,. \tag{23}$$

Taking into account that $\dot{\boldsymbol{Q}}:\dot{\boldsymbol{P}}^t = \langle\dot{\tilde{\boldsymbol{Q}}}:\dot{\tilde{\boldsymbol{P}}}^t\rangle_\tau$, we finally obtain from Eq. (21)

$$\begin{aligned}\dot{\boldsymbol{F}}_i = \boldsymbol{B}^{-1}:[\,&\langle\tilde{\boldsymbol{A}}^t:(\tilde{\boldsymbol{B}}:\dot{\tilde{\boldsymbol{F}}}_i + \tilde{\boldsymbol{m}}\cdot\tilde{\boldsymbol{F}}_i\,\dot{\theta})\,\rangle_\tau - \boldsymbol{m}\cdot\boldsymbol{F}_i\,\dot{\theta} - \\ &-(\boldsymbol{g}:\dot{\boldsymbol{a}})\cdot\boldsymbol{F}_i + \langle\tilde{\boldsymbol{A}}^t:[\tilde{\boldsymbol{F}}]\,\rangle_{\boldsymbol{a}}:\dot{\boldsymbol{a}} - \langle\tilde{\boldsymbol{A}}^t:[\tilde{\boldsymbol{r}}]_2\,\boldsymbol{n}\rangle_\Sigma\,]\,.\end{aligned} \tag{24}$$

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