STRESS DISTRIBUTION IN DEFORMABLE GASKETS OF TOROIDAL HIGH PRESSURE EQUIPMENT

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The stress distribution in deformable gaskets and containers of toroidal high pressure equipment is studied with the aid of a set of "Ductility" programs by constructing slip line field. It is shown that the main role of toroidal recesses consists in increasing the level of contact friction when burrs are relatively thick, i.e., at different stages of compression. Possible mechanisms for increasing the contact friction to suite the geometry of the deformation center are studied.

A study aimed at exploring the stress distribution in deformable gaskets and containers used in toroidal high pressure equipment [1] and at analyzing the reasons for the increase in pressure compared with the pressure in equipment having a die with a flat end face is described. It is assumed that the container material is entirely in the ductile state and that the die is absolutely rigid. In a cylindrical system of coordinates $r\theta z$, four equations are available for determining the four components $(\sigma_z, \sigma_r, \sigma_\theta, \tau=\tau_{rz})$ of the stress tensor $\bar{\sigma}$, i.e., two equilibrium equations, the Coulomb ductility condition $\tau_n=k+\sigma_n \operatorname{tg}\rho$, and the complete ductility condition $\sigma_3=\sigma_2=\sigma_\theta$, where τ_n and σ_n are the tangential and normal stresses on the slip area, ρ is the internal friction angle, and k is the shear strength at $\sigma_n = 0$ and σ_i represents the main stresses, and i = 1,2,3 [2].

A set of "Ductility" programs [2] has been developed for solving the given system of equations by the slip line method. In this paper calculations are given for a container made of pressed lithographic stone, for k = 0.051 gPa and ρ = 0.158. The diagram of the high pressure equipment is shown in Fig. 1, and the slip line field for the case under consideration is shown in Fig. 2, the field being formed by two isogonal families (α and β) of slip lines intersecting at angles $\frac{\pi}{2} \pm \rho$.

The boundary conditions will be considered. The surface AB (see Fig. 2) is assumed free from stress and $\sigma_r = \tau = 0$. On the symmetry axis $z = 0$, $\tau = 0$. The friction angle φ, which is equal to the angle between the β-slip lines and the contact surface of the deformable gasket is considered to be constant. The angle φ depends on the roughness of the contact surface, the stresses, the properties of the materials, the temperature, etc., and is bound to be varied.

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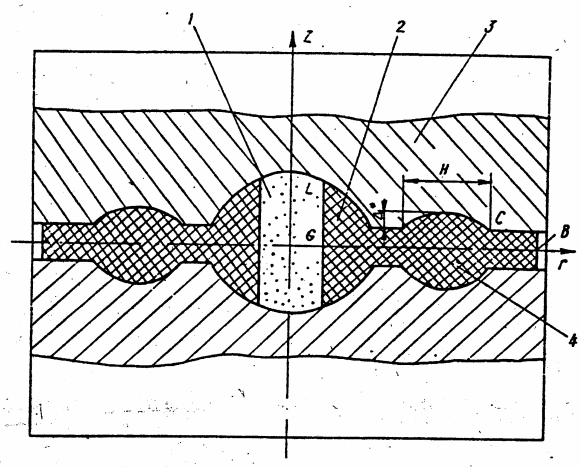


Fig. 1. Diagram of toroidal high pressure equipment: 1) reactive charge; 2) container; 3) die; 4) deformable gasket.

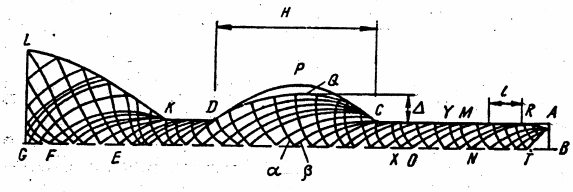


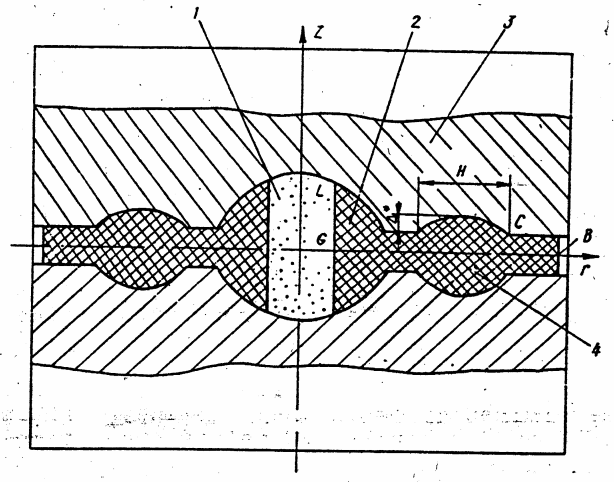
Fig. 2. Slip line field in a high pressure container of toroidal type.

Eliminating σ from the expression for the stress [2] and allowing for the fact that $\delta = \frac{\pi}{2} - \rho + \varphi$ (δ is the angle between the α-slip lines and the axis r), the following relation is obtained between the tangential and normal stresses on the contact surface of the deformable gasket:

$$\tau_n = (k + \sigma_n \operatorname{tg} \rho) \frac{\cos \rho \cdot \cos (2\varphi - \rho)}{1 + \sin \rho \sin (2\varphi - \rho)}.$$

The angle φ changes within the limits $0 \leqslant \varphi \leqslant \frac{\pi}{4} + \frac{\rho}{2}$. If $\varphi = 0$, the friction is a maximum and is equal to the internal friction, and $\tau_n = k + \sigma_n \operatorname{tg} \rho$. When $\varphi = \frac{\pi}{4} + \frac{\rho}{2}$ friction is absent and $\tau_n = 0$.

These data are used to construct a slip line field up to the β-line passing through the point K. In the region ABT, which is a region of uniform stress distribution, the slip lines are straight lines. In the region ART the degenerate Riemann problem is solved, and the slip lines form a centered fan. Point A is a singular point, and at this point the stresses undergo abrupt changes (discontinuities). The problem is then solved in the following way. A point on the contact surface is first obtained, a β-slip line is then constructed, and a



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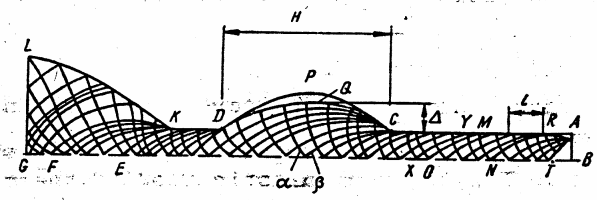

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point is obtained on the symmetry axis, the construction being taken to point C. The points of intersection of the torus surface with the flat surface of the deformable gasket C and D are singular points, and at these points there are abrupt changes in the angle δ and all the components of the stress tensor. In the construction the last slip line leaving the point C should fall on point D. From the edge of the matrix K the slip line field, both under compression conditions and on heating, is constructed just as in the case of an anvil with a depression [3]. Thus, under compression conditions it is necessary to determine two constants from the experiments, i.e., a constant characterizing the level of contact friction in the depression, i.e., the angle Δ between the slip lines KF and the contact surface of the depression at the point K (or the position of the point of flow separation F), and the difference S of any stresses at points F and G, with integral allowance for the elastoplastic properties of the reaction mixture and container.

The density of the network of slip lines and consequently the accuracy of the solution, are determined by the number of lines starting from the singular points A, C, D, and K, and the number of points on section GF. The abrupt changes in the angles between the slip lines, starting from points A, C, and K, are assumed to be 3°, as against 2° from the point D. At the section GF eight points are assigned. The abrupt change in angle at the point A was corrected by a special subprogram for obtaining contact between the slip lines ARN and the line AC.

The given slip line field is used to construct isolines of the tensors $\bar{\sigma}$ and σ_0 for compression conditions (Fig. 3). The stresses σ_z in the region of $r > r_E$ are weakly dependent on z and increase monotonically with decrease in r. Regions with negative values of τ are absent. The stresses σ_0 and σ_r increase monotonically with decrease in r and with increase in z. In the region of the depression the maximum stresses are σ_z.

In the case shown in Fig. 2, when plastic flow of the material in the region of the recess takes place below the slip line DQC, the material above this line remains rigid (undeformed). Consequently, the contact friction in the region of the recess is equal to the shear resistance at points of the DQC slip line, i.e., $\varphi=0$ and the friction is the maximum possible. Therefore, if in the region of the deformable gasket $\varphi>0$, the presence of a toroidal recess leads an increase in the contact friction and consequently to an increase in the level of attainable pressures, which is the more significant the greater the angle φ in the region of the deformable gasket, the angle ρ, and the width of H of the recess. Thus, when $\rho = 0.158$, $k = 0.051$ gPa, $\varphi=30°$, $H = 2z_1$ (where z_1 is the thickness of the deformable gasket at the edge K), and the stresses σ_r at the point G in the presence of the recess are 22% higher than without it. On the other hand, at the maximum possible contact friction in the region of the deformable gasket ($\varphi=0$), the presence of a toroidal recess, with an accuracy up to the calculation error, does not affect the calibration curve (plot of σ_r at the point G versus the force p), irrespective of the value of H. It is thus clear that when the maximum possible friction is attained by any method (e.g., by application of frictional coatings) the production of recesses is inadvisable.

If the contact friction for the given container material obeys the general laws of friction under plastic compression [4], then, with decrease in thickness of the deformable gasket the friction level increases, and beginning from a certain thickness (more accurately from a certain ratio of the thickness of the deformable gasket to its length) becomes the maximum possible. Since in practice the ratio of the thickness of the deformable gasket to its length is fairly

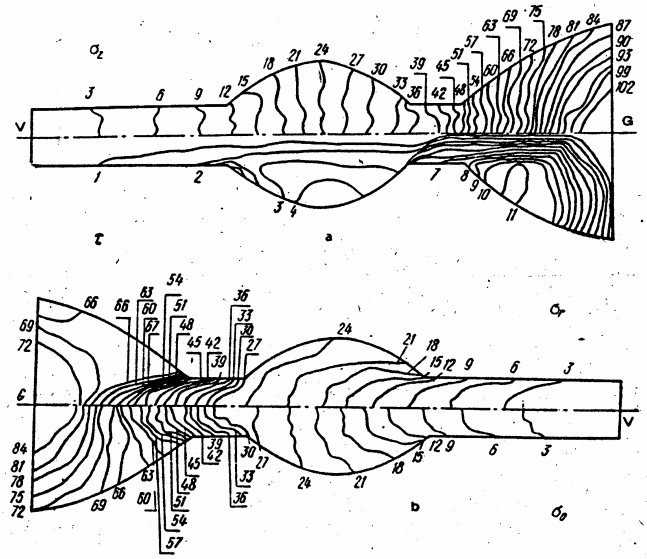

Fig. 3. Isolines of the components of the stress tensors σ_z, τ (a), and σ_r and the hydrostatic pressure in the container σ_0 (b). All values in 10^2 MPa.

small in the compressed state, it is to be expected that the friction will attain the maximum possible value, although during the compression process it can be much less than the limiting value. The level attained in the working pressure zone depends on the whole prehistory of the compression process, and the greater the friction the less the flow of container material from the cavity of the depression into the region of the deformable gasket and the greater the flow in the direction of the working zone, thus sealing it and increasing the pressure in it. Accordingly, the main role of toroidal recesses consists in increasing the level of contact friction when the deformable gasket is relatively thick, i.e., in the early stages of compression.

If slip occurs along the DQC line relative displacement at the interface between the deformable gasket and the matrix DPC is not observed, which decreases the probability of the formation of surface cracks in the matrix and leads to an extension of its life. When the DPC line lies below the DQC line slip in the recess takes place along the matrix surface, shortening its service life, and the contact friction (at $\varphi > 0$ in the region of the deformable gasket) fails to attain the maximum possible value, i.e., the presence of the recess is ineffective. On the other hand, in the present of a rigid DPCQ zone a significant increase in this is unsuitable since this does not increase the contact friction and the attainable pressure and leads to an increase in stress concentrations in the matrix and shortens its life. Accordingly, there is an optimum ratio between the width and depth of the recess, i.e., the depth of the recess should be 20-30% more than the depth Δ of penetration of the slip line into it for a given width H.

As a result of these calculations it is established that the relation $\Delta(H)$ is parabolic, $\Delta(0) = 0$, and for a fixed value of H the quantity Δ increases with decrease in thickness z_1 of the deformable gasket. When the values of z_1 and H

are fixed, the penetration depth Δ depends on the angle ρ and is independent of k. Increase in thickness of the deformable gasket because of the recess also establishes more favorable conditions for operation of the matrix under compressive loads [5].

Another phenomenon associated with contact friction should be noted. The slip line ARN has to touch the contact surface at the point R (evidently there is no other variant for constructing the slip line field), so that, irrespective of the friction level (angle $\varphi_0>0$) attained at the contact surface in the remaining region of the deformable gasket, at the point R angle $\varphi=0$. At this point the angle φ_0, cannot change abruptly from 0 to φ, since in this case the slip lines starting from singular point A cover the region below the line ARN, already covered by the slip lines. It is assumed that the angle φ_0 changes linearly when r changes from 0 to r at a certain section l.

It is established that when calculations are carried out for l less than a certain value l_0, the slip lines of one family begin to intersect, e.g., the lines MO and YX. Mathematically this means that at such boundary conditions the solution cannot be extended in the direction of decreasing r beyond a certain slip line envelope, beyond which there is no solution and plastic flow cannot arise. Physically this means that if flow occurs, this friction law is not realized, and l must be increased. Accordingly, to determine the minimum value of l_0 the following natural criterion is assumed: the value of l must be decreased until a correct slip line field can be constructed in the whole volume of material in the plastic state (i.e., a slip line field in which the slip lines of one family do not intersect). The value of l_0 is directly proportional to the values of φ and z_1 and to the distance between the points R and K (since the lines of one family intersect far away from the point R). For example, when $\varphi=20°$, $l_0 = 1.5 z_1$.

A similar phenomenon is observed at relatively small values of the ratio H/z_1 in the region of point D of a toroidal recess: the angle φ_D, formed at the point D of the slip line DQC and the contact surface, can be less than the angle φ, of the contact surface being provided. In this case the angle of friction will also increase from φ_D to φ at a certain section l_0.

Analysis of slip line fields thus shows that the contact friction depends essentially on the geometry of the plastic deformation center and not just on the roughness of the contact surface. In particular, the theoretically calculated phenomenon of an increase in friction for certain types of deformation center geometry can be used in practice for increasing the level of the established pressure.

The case where the matrix has several recesses was studied in a similar manner, but no qualitative differences were observed. In this connection, from the point of view of the maximum increase in friction level, the optimum situation is the presence of a multiplicity of narrow recesses with a distance between them less than l_0, which can be attained, for example, by establishing a corresponding microrelief on the contact surface.

In fact, a small recess produces little weakening of the matrix. Furthermore, a small angle φ_D is established at the exit from the recess, and when $l<l_0$ the friction in the region between the recesses is higher than in the absence of friction. However, the width of the recess should exceed a specific value H_0 for which the material of the deformable gasket can flow into it.

The effect of recesses during heating has been analyzed qualitatively. During the heating process the shear strength of the material of the deformable gasket is decreased and the contact friction in the region of the gasket undergoing deformation, which is equal to the shear strength in its thin contact layer, is also decreased. In the presence of a toroidal recess, and when the dimensions of the DPCQ region are significant, the temperature on the DQC slip line is less than in the contact layer in the absence of a recess by an amount ΔT, so that the contact friction increases.

It should be noted in the first place that the effect of increase in friction is the greater the more marked is the fall in shear strength of the material of the deformable gasket with rise in temperature. Hence, in the absence of a relation between τ_n and the temperature within the given range the toroidal recesses do not increase the load carrying capacity of the deformable gasket on heating, but do not decrease the load carrying capacity of the matrix.

In the second place, it should be noted that the increase in friction is the more significant the larger the value of ΔT, i.e., the larger the size of the DPCQ region and of the recess close to the edge of the matrix. Accordingly, small recesses and corresponding microrelief of the contacting surface do not increase contact friction during the heating process. However, the larger the size of the DPCQ region and of the recess close to the edge of the matrix, the greater the matrix weakening.

It follows from the above that the optimum dimensions and position of a recess must be determined after calculating the temperature distribution, the distribution of thermoelastic stresses in the matrix, and the distribution of thermoplastic stresses in the container and the deformable gasket, with due allowance for the relation between τ_n and the temperature. Moreover, in varying the positions of the points K, D, P, and C of the matrix it is necessary to obtain the maximum possible increase in the maintained pressure for a given deformation force. In addition, a further condition should be fulfilled, requiring that the maximum equivalent stresses which are in accordance with the selected strength criterion in the region between the recess and the depression do not exceed the maximum equivalent stresses in the most heavily loaded region of the matrix.

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