Lecture #07: Conservation of Momentum

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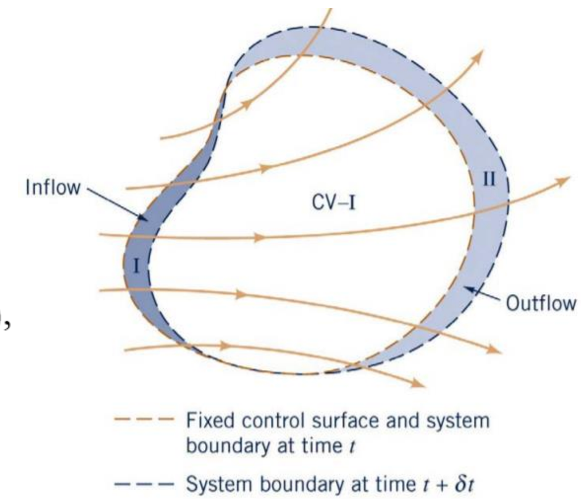
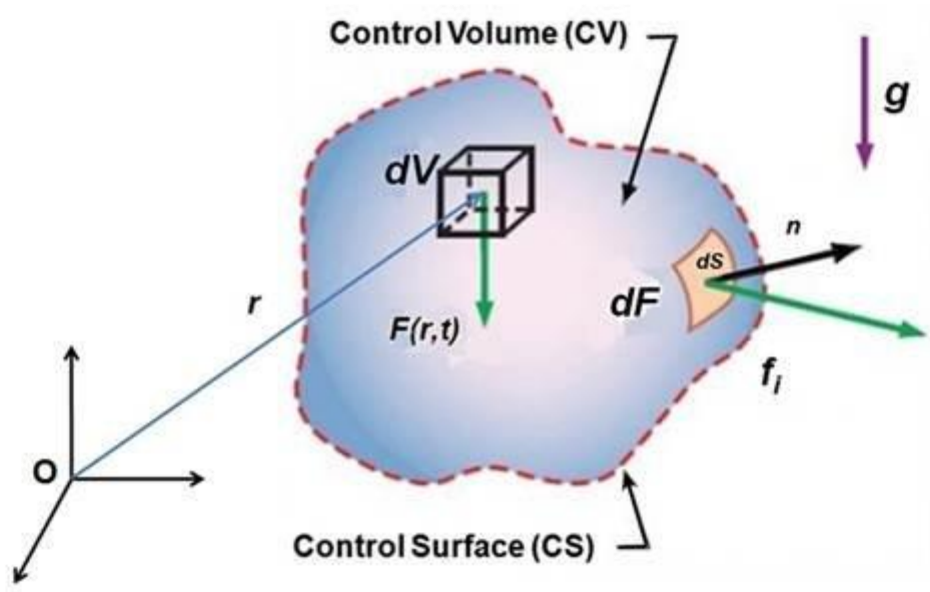
Reynolds Transport Theorem

$$\left.\frac{DN_s}{Dt}\right|_{system} = \left.\frac{D\int_{\forall} \alpha\,\rho\, d\forall}{Dt}\right|_{system} = \frac{\partial}{\partial t}\int_{C.V.} \alpha\,\rho\, d\forall + \int_{C.S.} (\alpha\,\rho \vec{V}) \bullet d\vec{A}$$



Where α is any intensive property corresponding to N. (i.e., $\alpha = N$ per unit mass), and it can be used for different quantities as follows.

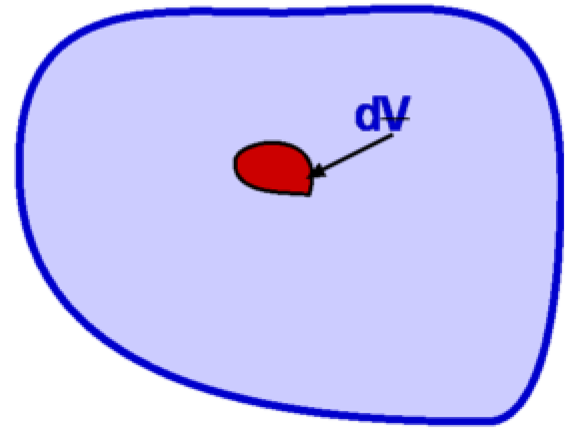
N_s	α
Mass	1
Linear momentum	$\vec{V}$
Angular momentum	$\vec{R}\times\vec{V}$
Energy	e
Entropy	s



❑ Conservation of Momentum

- ***Newton's second law states that:***
  [Time change rate of momentum of a system] = [Resultant external force acting on the system]

$$\frac{d\vec{M}_S}{dt} = \sum \vec{F}_S = \sum \vec{F}_{Surface} + \sum \vec{F}_{body}$$



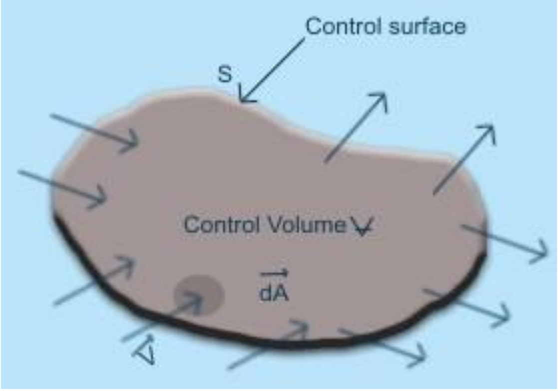
System

Reynolds Transport Theorem:

$$\left.\frac{dN_s}{dt}\right|_{system} = \frac{d\int_{\forall} \alpha\,\rho\, d\forall}{Dt} = \frac{\partial}{\partial t}\int_{C.V.} \alpha\,\rho\, d\forall + \int_{C.S.} (\alpha\,\rho\vec{V})\bullet d\vec{A}$$

$$Make: \quad \alpha = \vec{V} \qquad \Rightarrow \left.\frac{DM_s}{Dt}\right|_{system} = \frac{\partial}{\partial t}\int_{C.V.} \rho\vec{V}\, d\forall + \int_{C.S.} (\vec{V}\,\rho\vec{V})\bullet d\vec{A} = 0$$

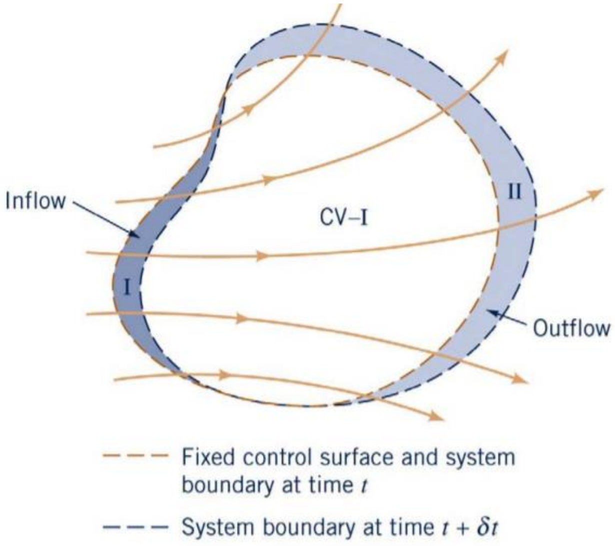
- ***Integral form of the Mass Conservation Equation:***



$$\frac{\partial}{\partial t}\int_{C.V.} \vec{V}\,\rho\, d\forall + \int_{C.S.} (\vec{V}\,\rho\vec{V})\bullet d\vec{A} = \sum \vec{F}_{surface} + \sum \vec{F}_{body}$$

❑ Conservation of Momentum – Integral form

$$\frac{\partial}{\partial t}\int_{C.V.}\vec{V}\,\rho\, d\forall + \int_{C.S.}(\vec{V}\,\rho\vec{V})\bullet d\vec{A} = \sum \vec{F}_{surface} + \sum \vec{F}_{body}$$

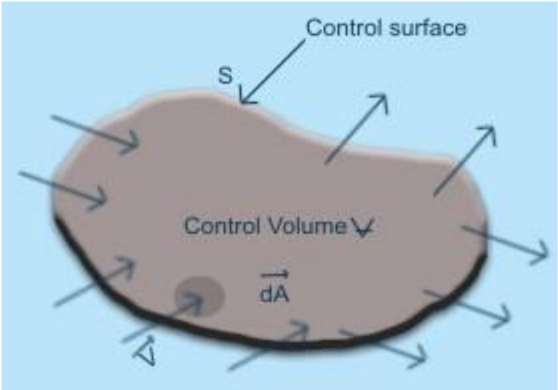


$\vec{F}_{surface}$ Surface forces such as pressure and shear stress. The surface forces usually c expressed as $\vec{F}_{surface} = \int_{C.S.} \tilde{P}\bullet d\vec{A}$, where $\tilde{P}$ is the stress tensor exerted by th surroundings on the particle surface. $\tilde{P} = -P\tilde{I} + \tilde{\tau}$

$\vec{F}_{body}$ Body forces such as electromagnetic, gravitational forces. Usually the body force can be expressed as $\vec{F}_{body} = \int_{C.V.} \rho\,\vec{f}\,d\forall$, where $\vec{f}$ is a vector which references the resultant force per unit mass.

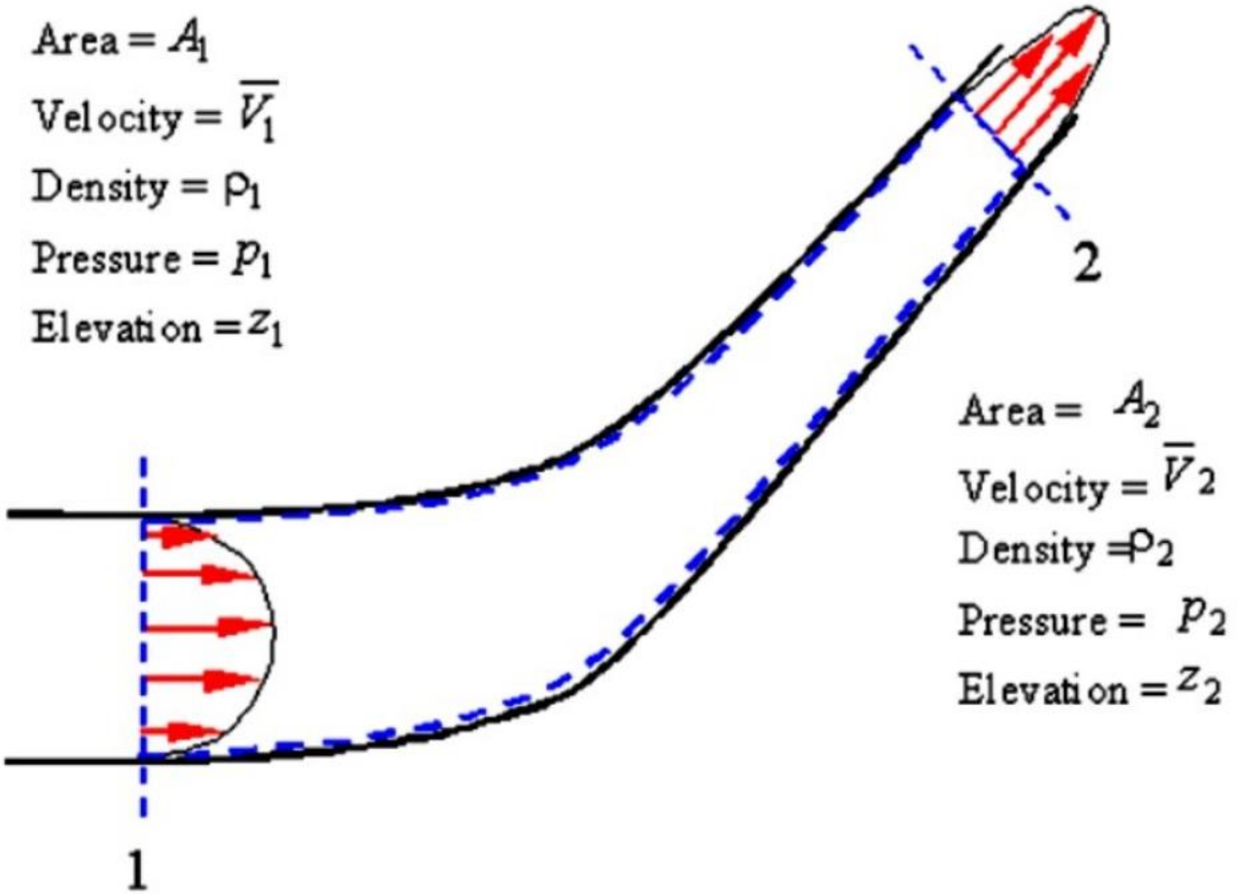
- ***Integral form of the Momentum Conservation Equation***

$$\frac{\partial}{\partial t}\int_{C.V.}\rho\vec{V}\, d\forall + \int_{C.S.}(\rho\vec{V}\vec{V})\bullet d\vec{A} = \int_{C.S.}\tilde{P}\bullet d\vec{A} + \int_{C.V.}\rho\,\vec{f}\, d\forall$$



Conservation of Momentum – Integral form

Flow past a pipe bend



- Consider the pipe bend shown above. We may first draw a free body diagram for the control volume with the forces:

❑ Conservation of Momentum – Integral form

Problem Solution:

1. *Chose a control volume:*
2. *Applying momentum conservation equation.*

$$\frac{\partial}{\partial t}\int_{C.V.}\vec{V}\,\rho\, d\forall + \int_{C.S.}(\vec{V}\,\rho\vec{V})\bullet d\vec{A} = \sum \vec{F}_{surface} + \sum \vec{F}_{body}$$

Paying due regard to the positive x and y directions, we may write the summation of forces in these two directions:

$$\sum F_x = p_1A_1 - p_2A_2\cos\theta - F_x$$
$$\sum F_y = F_y - p_2A_2\sin\theta - W$$

Relating these components to the net change of momentum flux through the inlet and exit surfaces

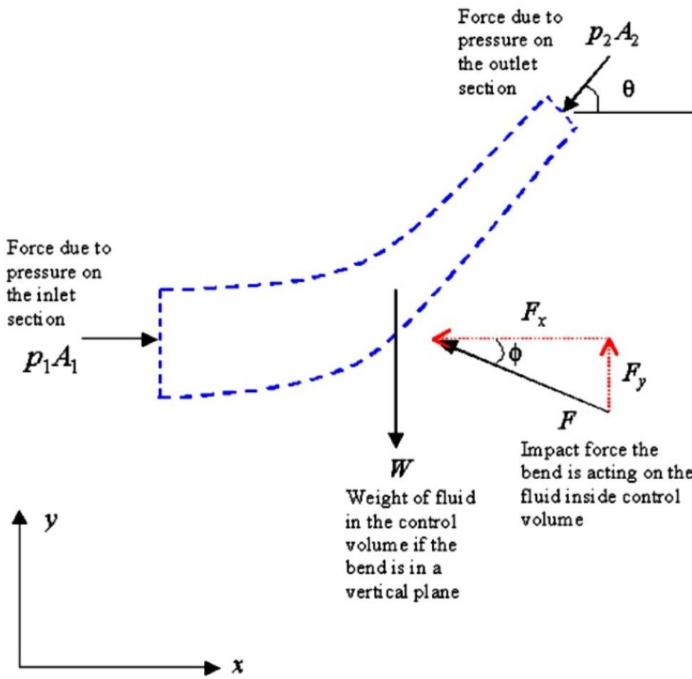
x-Direction

$$p_1A_1 - p_2A_2\cos\theta - F_x = \rho Q\left(\bar{V}_2\cos\theta - \bar{V}_1\right)$$

y-Direction

$$F_y - p_2A_2\sin\theta - W = \rho Q\left(\bar{V}_2\sin\theta - 0\right)$$

$$F = \sqrt{F_x^2 + F_y^2}$$
$$\phi = \tan^{-1}\left(F_y / F_x\right)$$

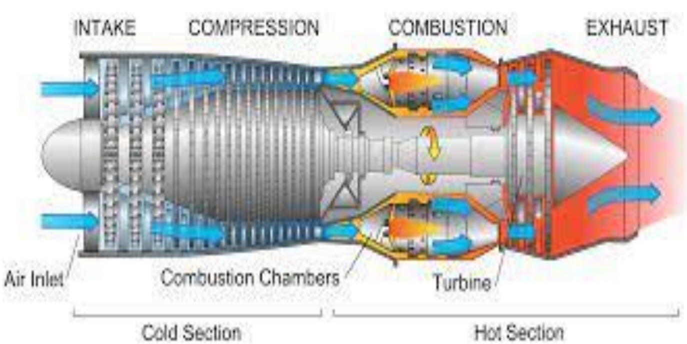


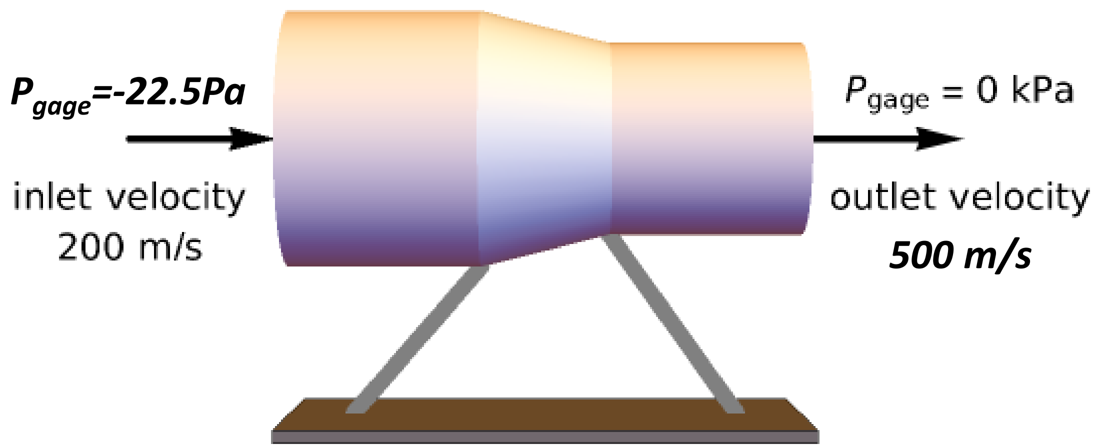
As a reaction, the impact force on the pipe bend is equal in magnitude, but opposite in direction to the one on the fluid.

Conservation of Momentum – Integral form

Example Problem:

A static thrust stand as shown in the Figure is to be designed for testing a jet engine. The flow conditions are known for a typical test: The intake flow velocity = 200m/s; exhaust gas velocity = 500 m/s; inlet cross-section area = 1.0 m^2; intake static pressure = -22.5 kPa (gauge) = 78.5 kPa(absolute); intake temperature = 268K; exhaust static pressure = 0 kPa =101 kPa(absolute). ***Estimate the thrust force generated by the jet engine on a static thrust stand.***



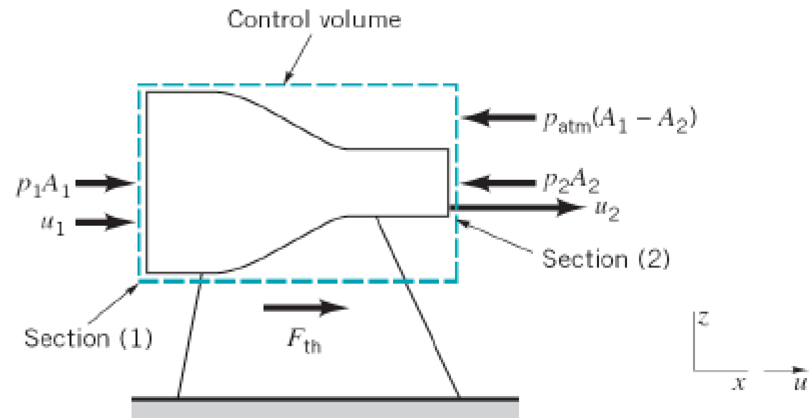


❑ Conservation of Momentum – Integral form

Problem Solution:

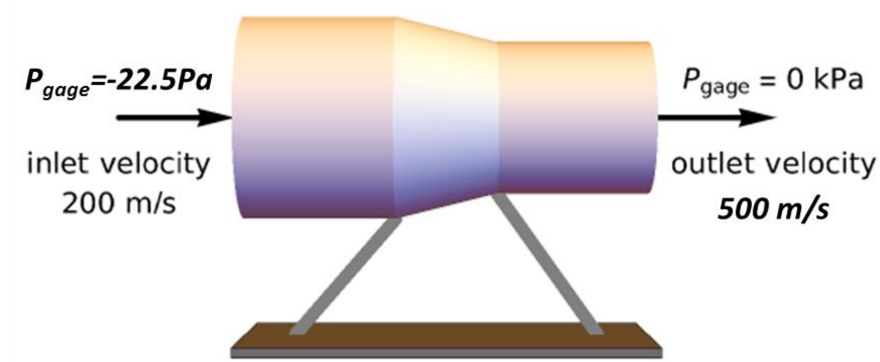
1. ***Chose a control volume:***
2. ***Applying momentum conservation equation***

$$\frac{\partial}{\partial t}\int_{C.V.} \vec{V}\rho\, d\forall + \int_{C.S.} (\vec{V}\rho\vec{V})\bullet d\vec{A} = \sum \vec{F}_{surface} + \sum \vec{F}_{body}$$



- Since the flow is steady, the momentum equation is simplified as:

$$\int_{C.S.} (\vec{V}\rho\vec{V})\bullet d\vec{A} = \sum \vec{F}_{surface} + \sum \vec{F}_{body}$$



- Forces along X- direction:

$$\sum F_x = F_{thrust} + P_1A_1 - P_2A_2$$

- Momentum along X- direction:

$$\int_{C.S.} (u\,\rho\vec{V})\bullet d\vec{A} = \int_{A_1} (u_1\rho\vec{V})\bullet d\vec{A} + \int_{A_2} (u\rho\vec{V})\bullet d\vec{A} = -V_1\rho_1A_1V_1 + V_2\rho_2A_2V_2$$

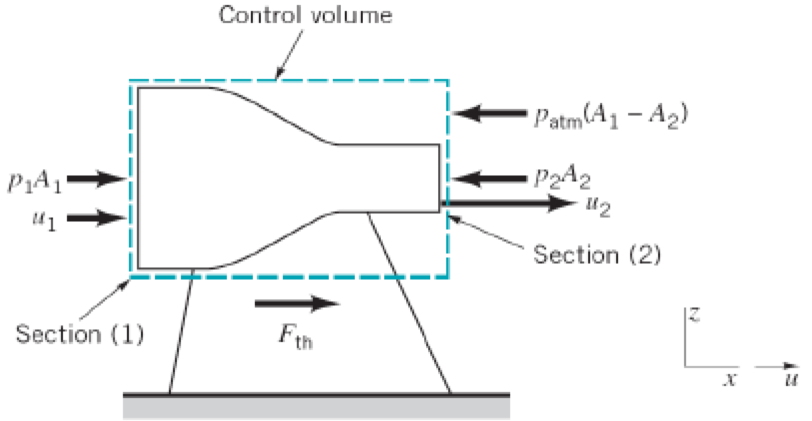
$$based\ on\ consevation\ of\ mass \Rightarrow mass\ \ flowrate\ \ \dot{m} = \rho_1A_1V_1 = \rho_2A_2V_2$$

$$Therefore: \int_{C.S.} (u\,\rho\vec{V})\bullet d\vec{A} = \dot{m}(V_2 - V_1) \quad \Rightarrow \quad F_{thrust} = \dot{m}(V_2 - V_1) - P_1A_1 + P_2A_2$$

Conservation of Momentum – Integral form

Problem Solution - continue:

1. ***Chose a control volume:***
2. ***Applying momentum conservation equation***



$$F_{thrust} = \dot{m}(V_2 - V_1) - P_1A_1 + P_2A_2$$

$$V_1 = 200m/s$$

$$P_1 = -22.5kPa(gauge) = 78.5kPa(absolute)$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{78.5\times10^3}{287.5\times268} = 1.02\ kg/m^3$$

$$\dot{m} = \rho_1V_1A_1 = 1.02\times200\times1.0 = 204\ kg/s$$

$$V_2 = 500m/s$$

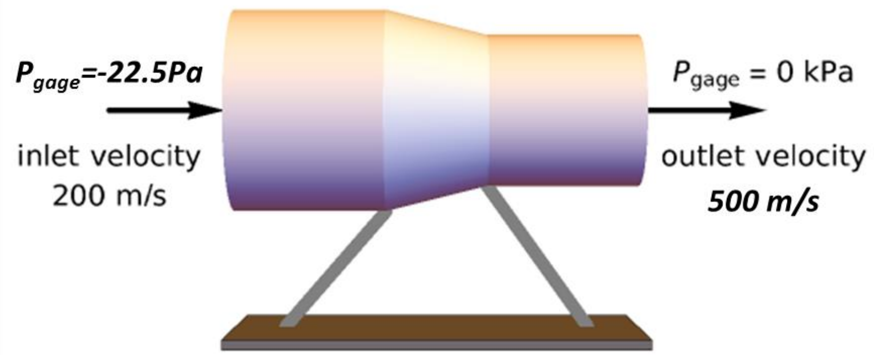
$$Therefore:$$

$$F_{thrust} = \dot{m}(V_2 - V_1) - P_1A_1 + P_2A_2$$

$$= 204\times(500-200) - (-22.5*10^3\times1.0) + 0\times A_2$$

$$= 61200 + 22500$$

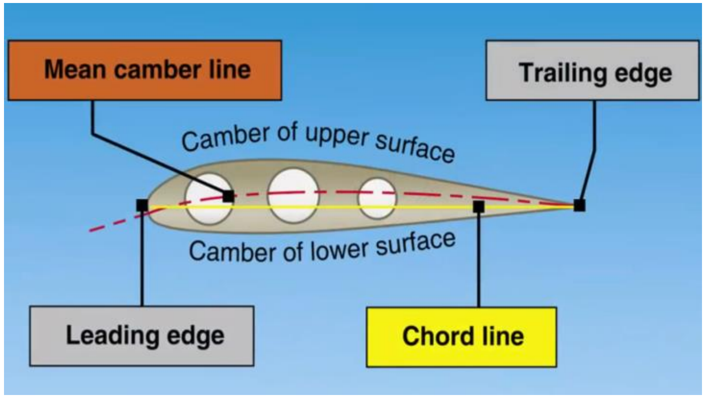
$$= 83700N$$



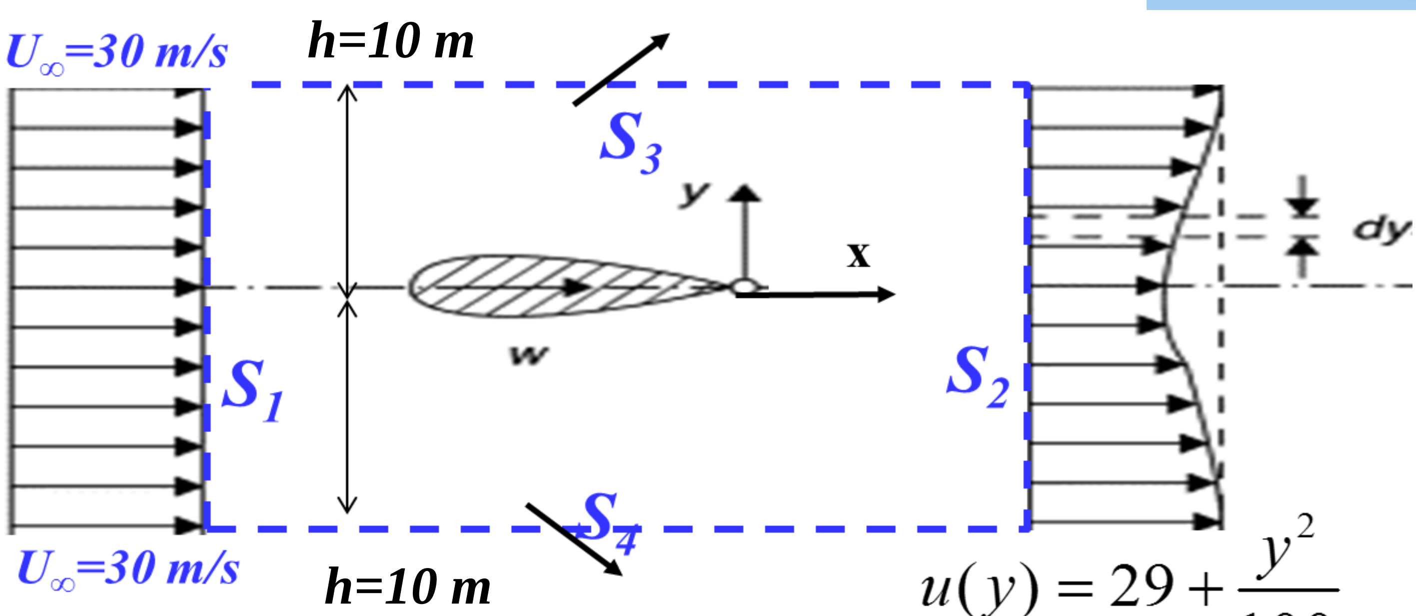
Conservation of Momentum – Integral form

Example-Drag on an airfoil

- ***Airfoil aerodynamics***
- https://www.youtube.com/watch?v=8fk2J5LtdSg



- Problem : Two-dimensional steady flow around the airfoil where velocity profile is given (measured) at upstream and a downstream location. Find an expression for the drag force on the airfoil.



$$u(y) = 29 + \frac{y^2}{100}$$