Lecture # 20: Stream & Potential Functions for Basic Flows – Part 2

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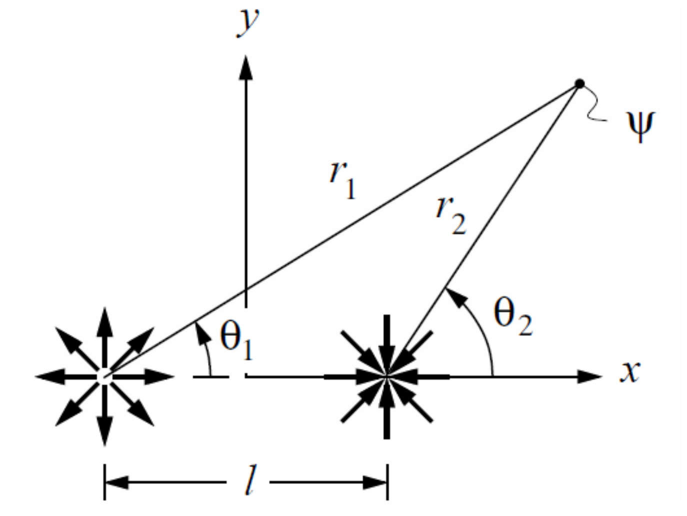
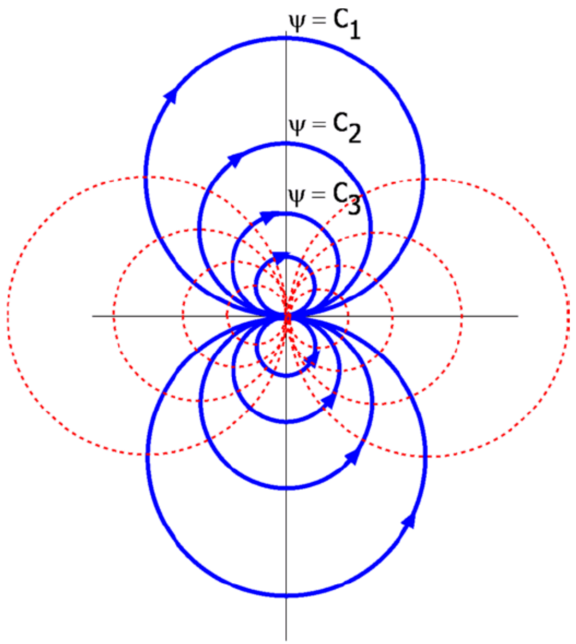
Potential & Stream Functions for Basic Flows

- *2D doublet Flow*

$$\psi = -\frac{\mu}{2\pi}\frac{\sin\theta}{r}$$

$$\phi = \frac{\mu}{2\pi}\frac{\cos\theta}{r}$$





❑ Potential & Stream Functions for Basic Flows

- ***Streamlines of a 2D doublet***

$$\psi = \lim_{\substack{\ell \to 0 \\ \kappa = \text{const.}}} -\frac{\mu}{2\pi\,\ell}\Delta\theta = -\frac{\mu}{2\pi}\frac{\sin\theta}{r}$$

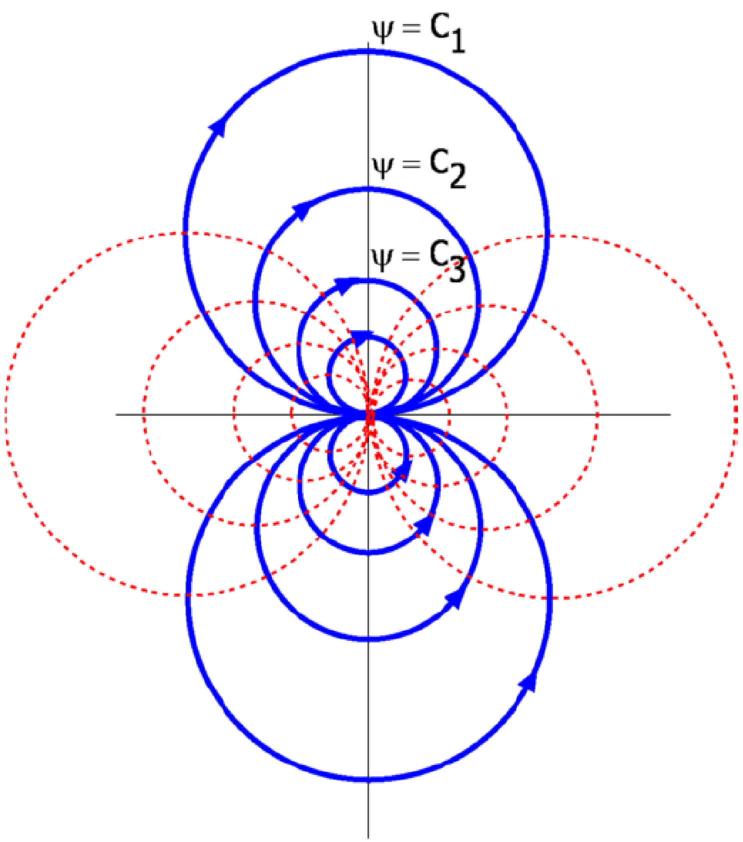
$$\psi = -\frac{\mu}{2\pi}\frac{\sin\theta}{r} = c = const.$$

$$\Rightarrow r = -\frac{\mu}{2\pi c}\sin\theta \Rightarrow r^2 = -\frac{\mu}{2\pi c} r\sin\theta$$

$$\Rightarrow x^2 + y^2 = -\frac{\mu}{2\pi c} y$$

$$\Rightarrow x^2 + y^2 + \frac{\mu}{2\pi c} y + (\frac{\mu}{4\pi c})^2 = (\frac{\mu}{4\pi c})^2$$

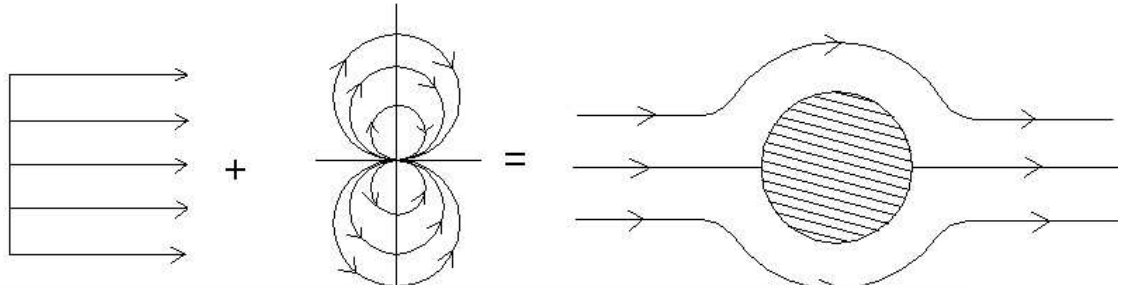
$$\Rightarrow (x-0)^2 + (y + \frac{\mu}{4\pi c})^2 = (\frac{\mu}{4\pi c})^2$$



Streamlines are circles centerd on the y-axis a distance $-\frac{\mu}{4\pi\psi}$ from the x-axis with a radius of $\left|\frac{\mu}{4\pi\psi}\right|$. All circles pass through the origin.

❑ Potential & Stream Functions for Basic Flows

- *Uniform Flow to the Right + A 2-D Doublet*



Quantity	Uniform flow	2-D doublet	Combination
$\vec{V}$	$V_\infty \hat{\imath}$		
ϕ	$V_\infty x$	$\frac{\mu}{2\pi}\frac{\cos\theta}{r}$	$V_\infty x + \frac{\mu}{2\pi}\frac{\cos\theta}{r}$
ψ	$V_\infty y$	$-\frac{\mu}{2\pi}\frac{\sin\theta}{r}$	$V_\infty y - \frac{\mu}{2\pi}\frac{\sin\theta}{r}$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left[V_\infty r\cos\theta - \frac{\mu}{2\pi}\frac{\cos\theta}{r}\right]$$

$$= V_\infty\cos\theta\left[1 - \underbrace{\frac{\mu}{2\pi V_\infty}}_{1/R^2}\frac{1}{r^2}\right] = V_\infty\cos\theta\left[1-\left(\frac{R}{r}\right)^2\right]$$

Where $R^2 = \frac{\mu}{2\pi V_\infty}$

$$V_\theta = -\frac{\partial\psi}{\partial r} = -\left[V_\infty + \frac{\mu}{2\pi r^2}\right]\sin\theta = -V_\infty\sin\theta\left[1+\left(\frac{R}{r}\right)^2\right]$$

❑ Potential & Stream Functions for Basic Flows

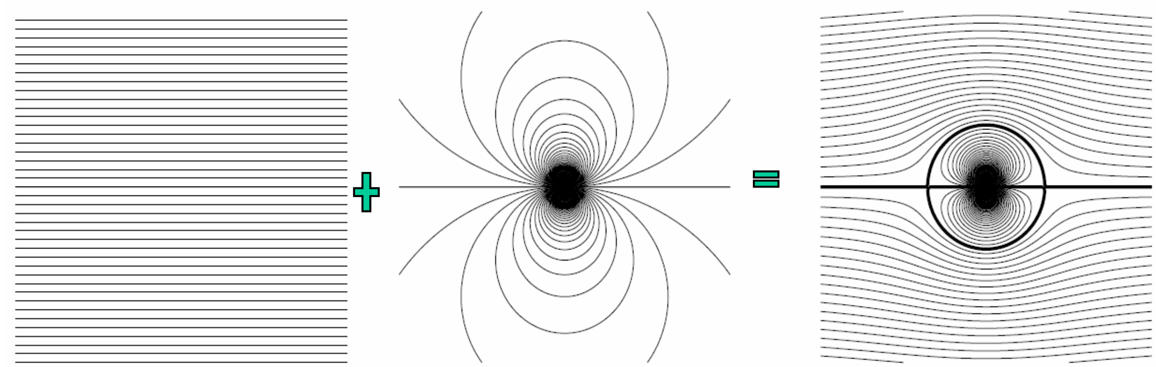
- *Uniform Flow to the Right + A 2-D Doublet*

Stagnation Points ($\vec{V} = 0$)

Set $V_\theta = 0$.

$$0 = -V_\infty \sin\theta \left[1 + \left(\frac{R}{r}\right)^2\right]$$

$\sin\theta = 0 \quad \text{or} \quad \theta_s = (0 \ \text{or} \ \pi)$



Now set $V_r = 0$.

$$0 = V_\infty \sin\theta \left[1 - \left(\frac{R}{r}\right)^2\right]$$

For $\theta = 0$ or π, $\cos\theta \neq 0$

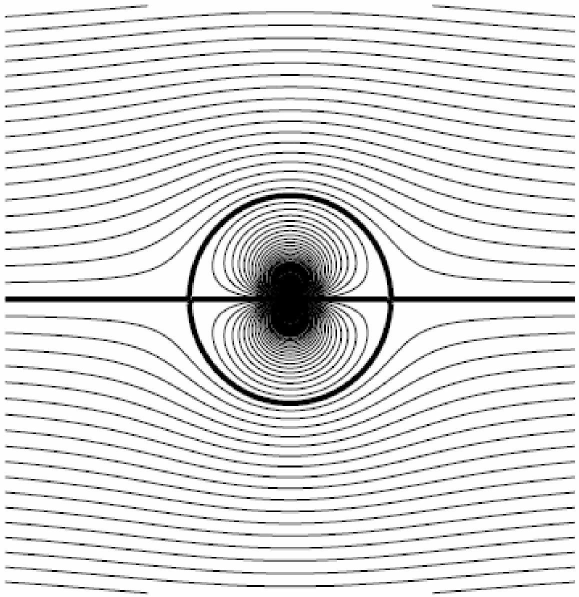
$$\therefore \left[1 - \left(\frac{R}{r}\right)^2\right] \equiv 0 \ \text{or} \ r^2 = R^2 = \frac{\mu}{2\pi V_\infty}$$

The stagnation points are located at $(r_s, \theta_s) \equiv (R, 0)$ and (R, π)

Potential & Stream Functions for Basic Flows

Uniform Flow to the Right + A 2-D Doublet

$$\left.\begin{aligned}
\phi &= V_\infty r\cos\theta\left(1+\frac{R^2}{r^2}\right)\\
\psi &= V_\infty r\sin\theta\left(1-\frac{R^2}{r^2}\right)\\
V_r &= V_\infty\cos\theta\left(1-\frac{R^2}{r^2}\right)\\
V_\theta &= -V_\infty\sin\theta\left(1+\frac{R^2}{r^2}\right)
\end{aligned}\right\}(r\geq R)$$



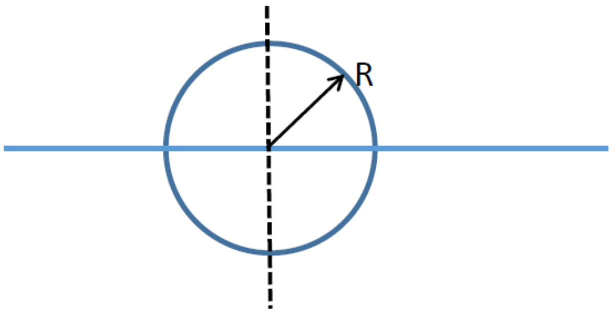
Where $R^2=\dfrac{\mu}{2\pi V_\infty}$

Substitute $(r_s,\theta_s)=(R,0)$ or (R,π) in the expression for ψ

$\psi_s=0$

at $r=R$ (surface of the cylinder)

Therefore $\psi=0$ streamline looks like



Pressure Coefficient; Lift Coefficient & Drag coefficient

Nondimensional Coefficients

The forces and moment depend on a large number of geometric and flow parameters. It is often advantageous to work with nondimensionalized forces and moment, for which most of these parameter dependencies are scaled out. For this purpose we define the following reference parameters:

Reference area: S
Reference length: ℓ
Dynamic pressure: $q_\infty = \frac{1}{2}\rho V_\infty^2$

The choices for S and ℓ are arbitrary, and depend on the type of body involved. For aircraft, traditional choices are the wing area for S, and the wing chord or wing span for ℓ. The nondimensional force and moment coefficients are then defined as follows:

Lift coefficient: $C_L \equiv \frac{L}{q_\infty S}$

Drag coefficient: $C_D \equiv \frac{D}{q_\infty S}$

Moment coefficient: $C_M \equiv \frac{M}{q_\infty S \ell}$

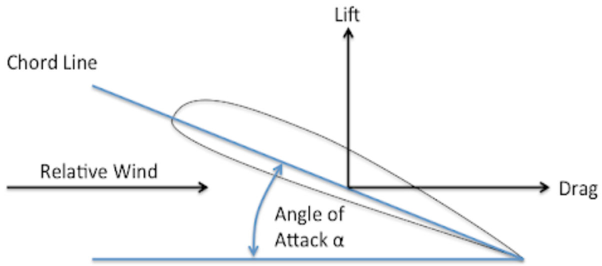
For 2-D bodies such as airfoils, the appropriate reference area/span is simply the chord c, and the reference length is the chord as well. The *local* coefficients are then defined as follows:

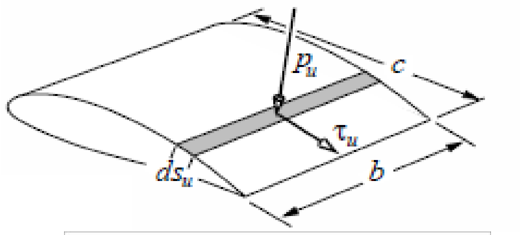
Local Lift coefficient: $c_\ell \equiv \frac{L'}{q_\infty\, c}$

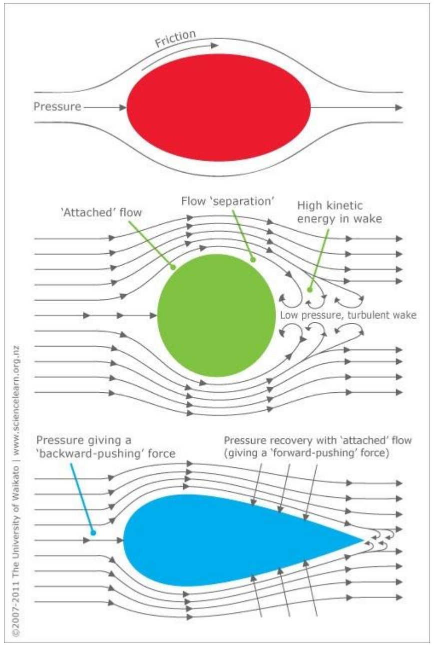
Local Drag coefficient: $c_d \equiv \frac{D'}{q_\infty\, c}$

Local Moment coefficient: $c_m \equiv \frac{M'}{q_\infty\, c^2}$

These local coefficients are defined for each spanwise location on a wing, and may vary across the span. In contrast, the C_L, C_D, C_M are single numbers which apply to the whole wing.







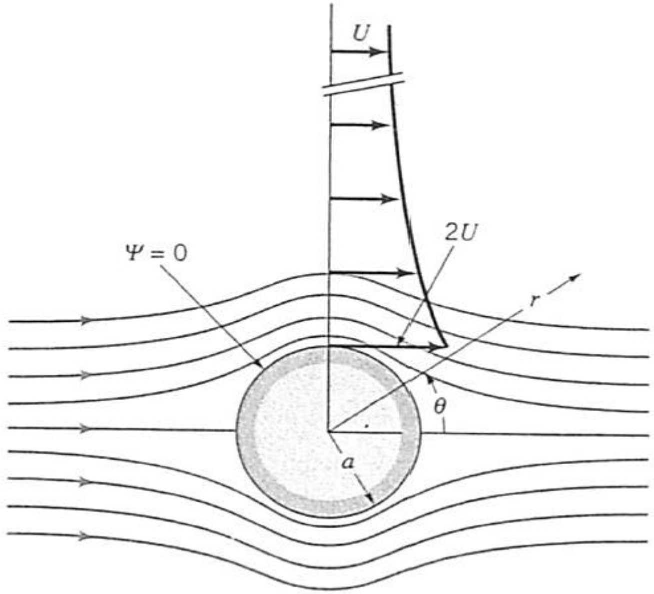
Potential & Stream Functions for Basic Flows

On the surface of the cylinder where $r = R$, we have

$$V_r = V_\infty \cos\theta \left(1 - \frac{R^2}{r^2}\right) = 0 \quad \text{(no flow out of the cylinder)}$$

$$V_\theta = -2V_\infty \sin\theta$$

The maximum surface speed of $2V_\infty$ occurs at $\theta = \pm 90°$.



The surface pressure is then obtained using the Bernoulli equation

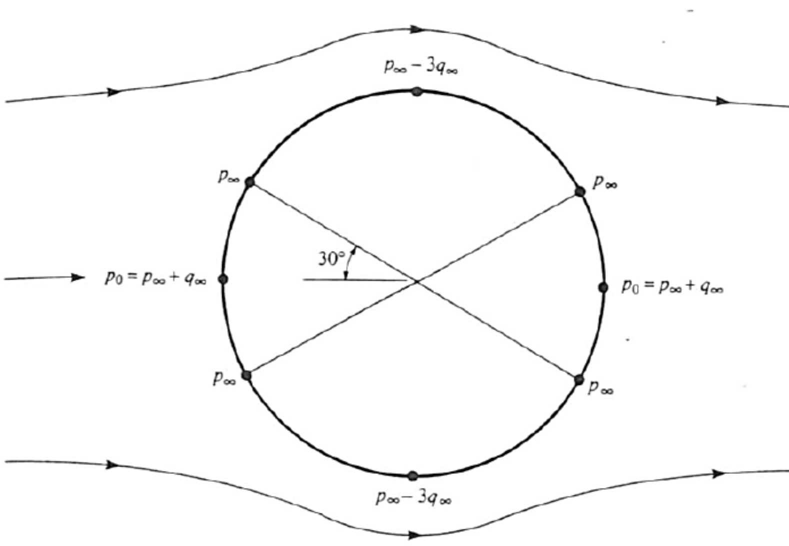
$$p(\theta) = p_o - \frac{1}{2}\rho\left(V_r^2 + V_\theta^2\right)$$

Substituting $V_r = 0$ and $V_\theta(\theta)$, and using the freestream value for the total pressur

$$p_o = p_\infty + \frac{1}{2}\rho V_\infty^2$$

gives the following surface pressure distribution.

$$p(\theta) = p_\infty + \frac{1}{2}\rho V_\infty^2\left(1 - 4\sin^2\theta\right)$$



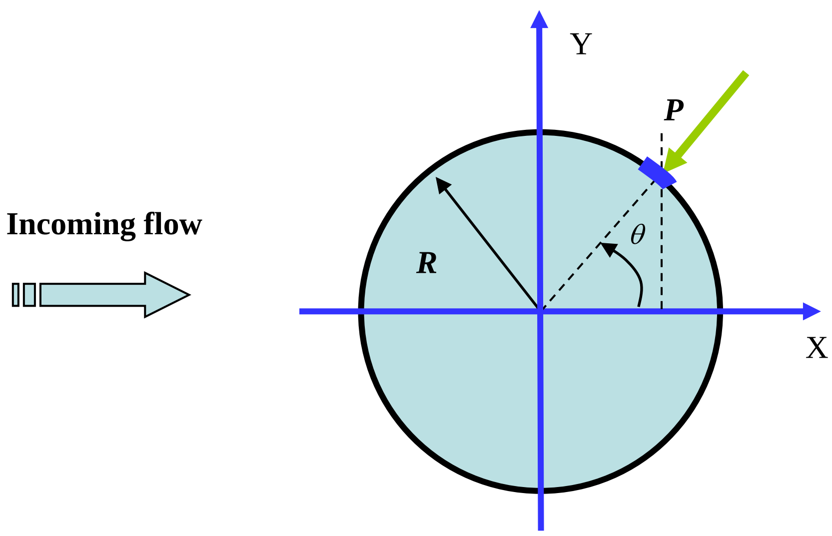
❑ Potential & Stream Functions for Basic Flows

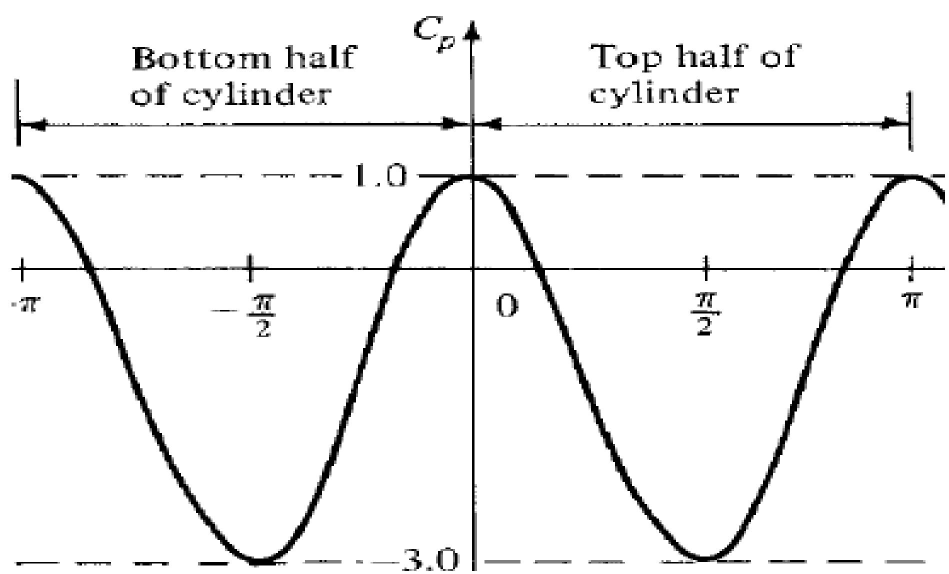
Pressure coefficient around the cylinder

$$p = p_\infty + \frac{1}{2}\rho V_\infty^2 (1 - 4\sin^2\theta)$$

$$C_p(\theta) \equiv \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - 4\sin^2\theta$$

$$\boxed{C_p = 1 - 4\sin^2\theta}$$





❑ Potential Flow Around a Circular Cylinder

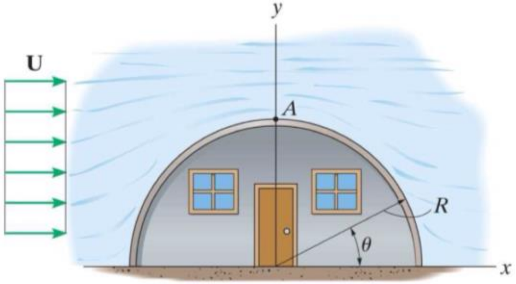
- ***Almost the same as the HW set04 -Problem#8:***

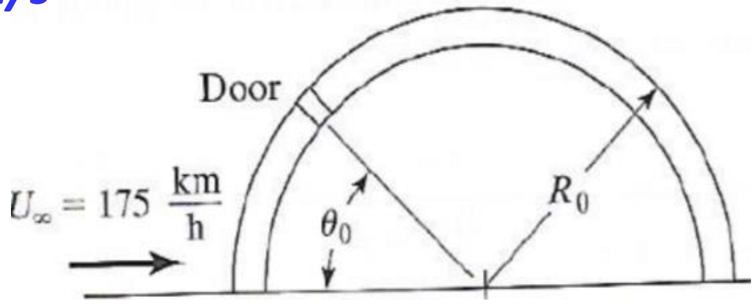
Consider the flow around the Quonset hut shown below to be represented by the superimposing a uniform flow with a doublet. Assume steady, incompressible, potential flow. The ground plane is represented by a plan of symmetry and the hut by the upper half of the cylinder. The free stream velocity is 175 km/hr; the radius R_o of the hut is 6m. The door is not well sealed, and the static pressure inside the hut is equal to that on the outer surface of the hut, where the door is located.

a. If the door to the hut is located at ground level (i.e., at the stagnation point), what is the net lift acting on the hut? What is the lift coefficient?
b. Where should the door be located (i.e., at what angle of θ_o relative to the ground) so that the net force on the hut will vanish?

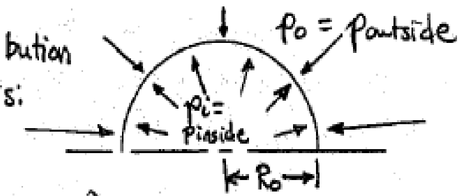
For both parts of the problem, the opening is very small compared to the radius R_o. Thus, the pressure on the door is essentially constant and equal to the value of the angle at θ_o at which the door is located. Assume that the wall is negligibly thin.

- *175 km/h = 48.6 m/s*





The pressure distribution over the outside surface is:



$$p = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 C_p$$
$$= p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 (1 - 4\sin^2\theta)$$

The lift force per unit span of the hut due to the pressure acting on the outer surface is:

$$l_o = -\int_0^\pi p_o \sin\theta R_o \, d\theta$$

$$l_o = -\int_0^\pi p_\infty \sin\theta R_o \, d\theta - \int_0^\pi \frac{1}{2}\rho_\infty U_\infty^2 \sin\theta R_o \, d\theta + \int_0^\pi 2\rho_\infty U_\infty^2 \sin\theta R_o \, d\theta$$

$$l_o = -p_\infty R_o(-\cos\theta\Big|_0^\pi - \frac{1}{2}\rho_\infty U_\infty^2 R_o(-\cos\theta\Big|_0^\pi + 2\rho_\infty U_\infty^2 R_o(-\cos\theta + \frac{1}{3}\cos^3\theta\Big|_0^\pi$$

$$l_o = -2p_\infty R_o - \frac{1}{2}\rho_\infty U_\infty^2 2R_o + 2\rho_\infty U_\infty^2 R_o\left(2 - \frac{2}{3}\right)$$

$$l_o = -2p_\infty R_o + \frac{1}{2}\rho_\infty U_\infty^2 2R_o\left(-1 + 4 - \frac{4}{3}\right)$$

$$l_o = -2p_\infty R_o + \frac{5}{3}\rho_\infty U_\infty^2 R_o$$

The lift force per unit span due to the pressure inside the hut is:

$l_i = p_i 2R_o$ (this results since the internal pressure is constant at p_i, the lift force per unit span is the pressure times the projected area)

Thus, the net lift per unit span acting on the hut is the sum of these two forces:

$$l = p_i 2R_o - p_\infty 2R_o + \frac{5}{3}\rho_\infty U_\infty^2 R_o$$

(a) If the door is located at ground level, the pressure inside the hut would be equal to the stagnation pressure

$$p_i = p_t = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2$$

Thus, $l = p_\infty 2R_o + \frac{1}{2}\rho_\infty U_\infty^2 2R_o - p_\infty 2R_o + \frac{5}{3}\rho_\infty U_\infty^2 R_o$

$$l = \left(\frac{1}{2}\rho_\infty U_\infty^2\right)(2R_o)\left(1 + \frac{5}{3}\right) = \left(\frac{1}{2}\rho_\infty U_\infty^2\right)(2R_o)\left(\frac{8}{3}\right)$$

Thus, $$C_l = \frac{l}{\frac{1}{2}\rho_\infty U_\infty^2 \, 2R_o} = \frac{8}{3}$$

Note that the characteristic dimension has been assumed to be the length across the floor of the hut, $2R_o$.

The net lift per unit span is:

$$l = \left[\frac{1}{2}(1.2250)(48.61)^2\right]\left[2(6)\right]\frac{8}{3} = 4.631 \times 10^4 \text{ N/m}$$

(b) The net force on the hut will vanish when $l = 0$. Thus,

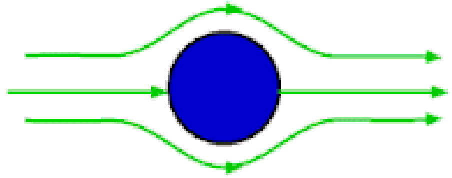
$$p_i 2R_o - p_\infty 2R_o + \frac{5}{3}\rho_\infty U_\infty^2 R_o = 0$$

Rearranging: $$\frac{p_i - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = -\frac{5}{3}$$

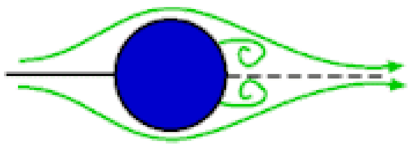
Thus, to have zero net lift, the door should be located at a point where the local pressure coefficient is $-\frac{5}{3}$. We refer to the equation defining the pressure distribution and solve for θ. Assuming that $C_p = 1 - 4\sin^2\theta = -\frac{5}{3}$; $\sin^2\theta = \frac{2}{3}$

Therefore, $\theta = 54.74°$ from the ground (on either side of the building).

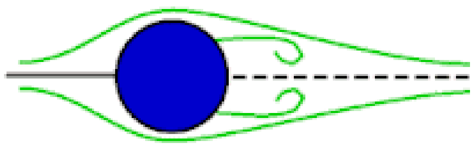
❑ REAL FLOW AROUND A CIRCULAR CYLINDER



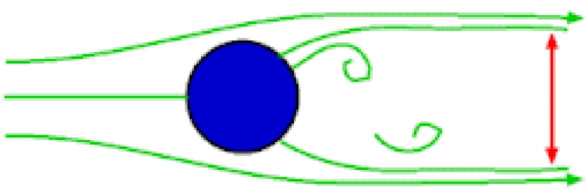
Re < 0.5, No Separation, flow resembles a potential flow.



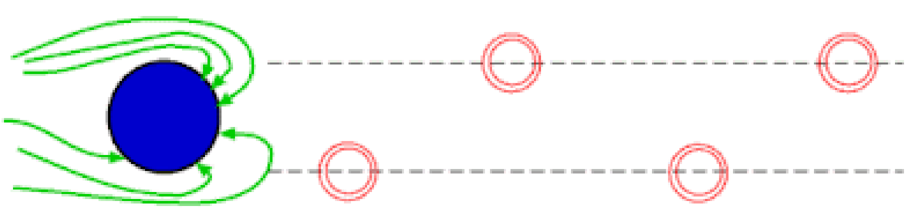
2<Re<30, Separation observed with eddies holding on to their position.



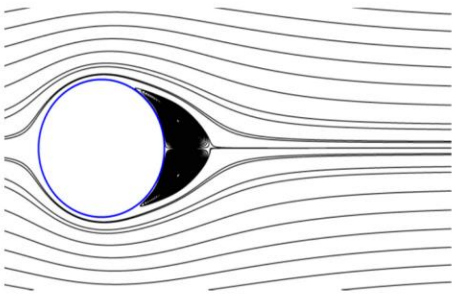
At sligthly higher Re, eddies get elongated



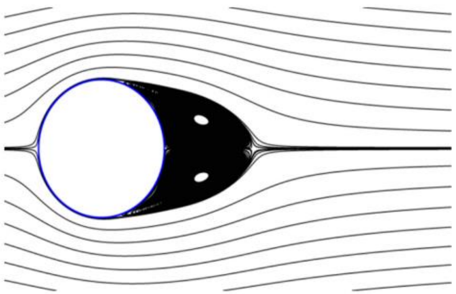
40<Re<70, Vortex arrangement unstable, Wake oscilates periodically as shown.



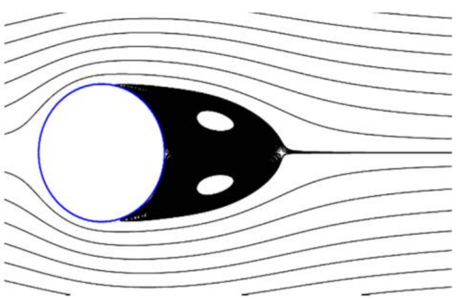
Re around 90, Eddies break off and are washed downstream giving what is called Karman Vortex Street



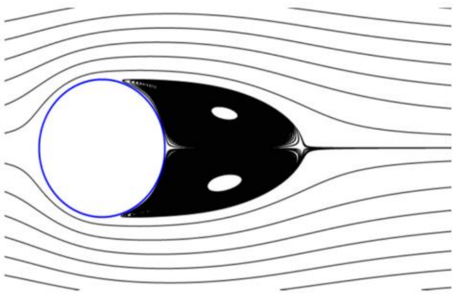
(a) $\hat{t} = 2$ $(Re = 2035)$



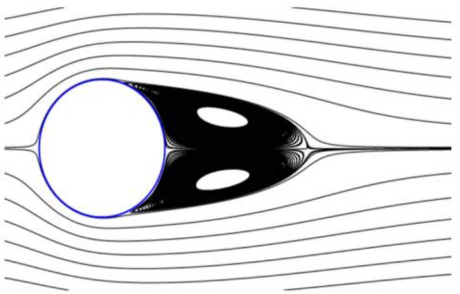
(b) $\hat{t} = 4$ $(Re = 2861)$



(c) $\hat{t} = 6$ $(Re = 3496)$

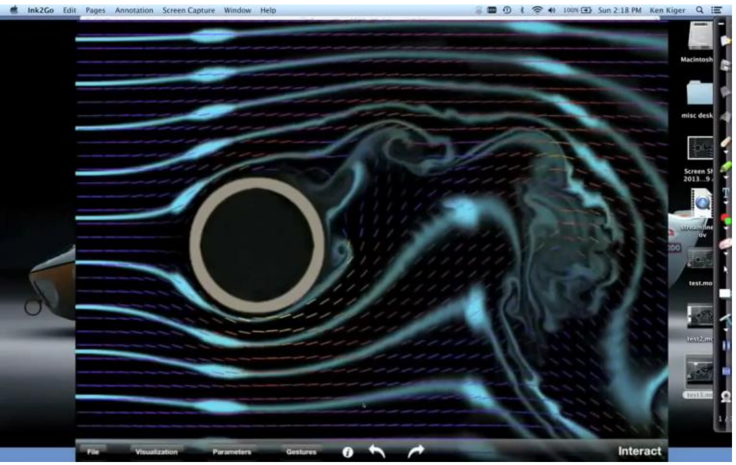


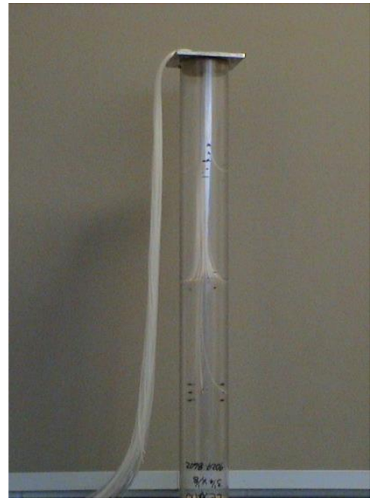
(d) $\hat{t} = 8$ $(Re = 4031)$

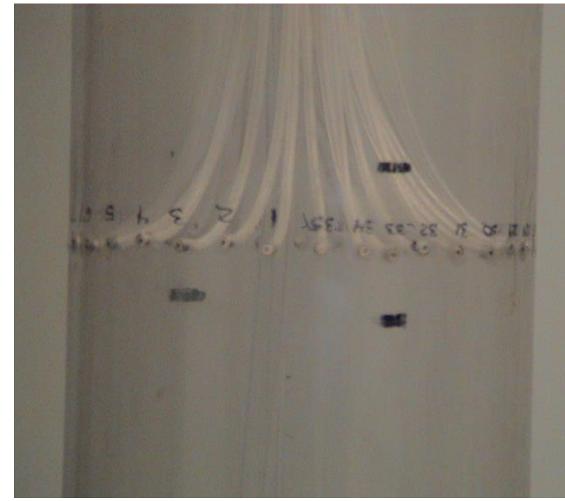


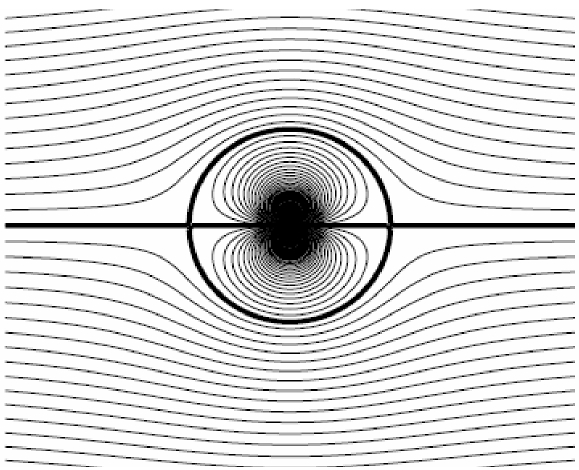
(e) $\hat{t} = 10$ $(Re = 4501)$

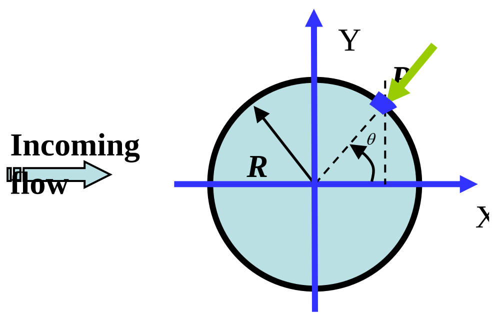
AerE344 Lab#04 : Pressure Distributions around a Circular Cylinder

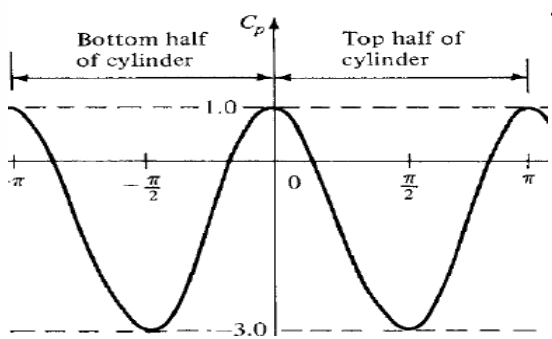


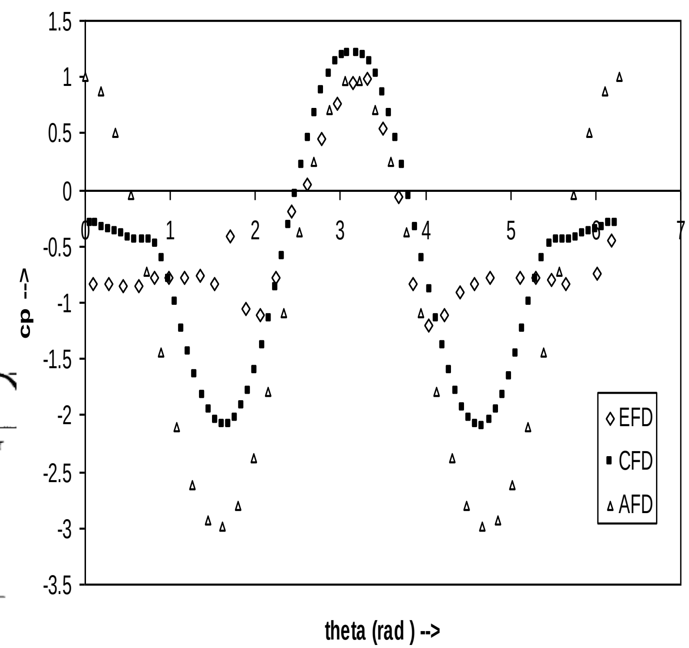












$$C_p = 1 - 4\sin^2\theta$$

❑ Drag Coefficient of a Circular Cylinder in a Real Rlow