Lecture # 24: Airfoil Aerodynamics – Part 02: Kutta Conditions & Starting Vortices

Dr. Hui HU

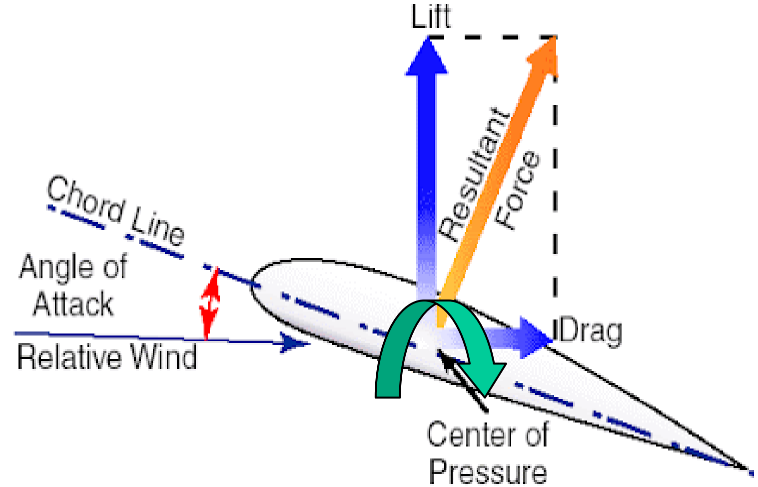
Department of Aerospace Engineering

Iowa State University, 2251 Howe Hall, Ames, IA 50011-2271

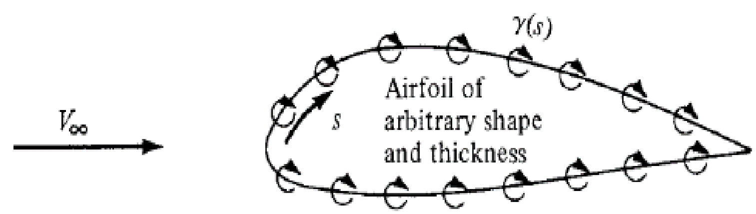
Tel: 515-294-0094 / Email: huhui@iastate.edu

❑ Incompressible Flow Around an Airfoil

- ***Strategy for finding solution of flow field around an airfoil***
  - Replace the airfoil shape with an array of vortices with variable strength.
  - Calculate the strength of this vortices such that the induced velocity field added to the free stream will make the airfoil a streamline.
  - Total circulation around the airfoil:



$$\Gamma = \sum_{i=1}^{n} \gamma_i \Delta s_i \text{ or } \Gamma = \int \gamma ds$$



Lift is calculated from Kutta-Joukowski theorem

$$L' = \rho_\infty V_\infty \Gamma$$

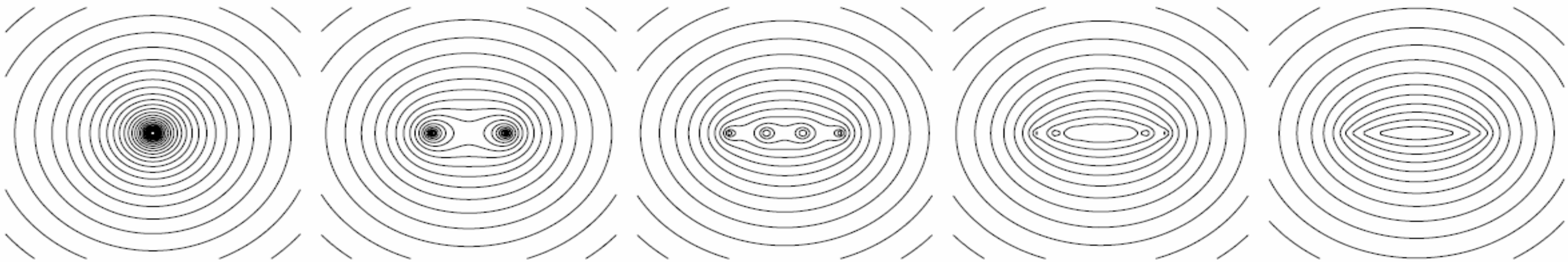
❑ Superposition of Basic Flows

Vortex Sheets

Definition

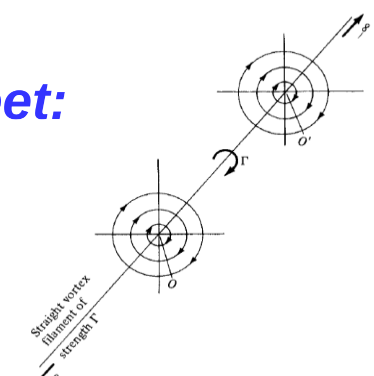
Consider a sequence of flows where a single vortex of strength Γ is repeatedly subdivided into smaller vortices which are evenly distributed along a line segment of length ℓ. The limit of this subdivision process is a *vortex sheet* of strength $\gamma = \Gamma/\ell$.

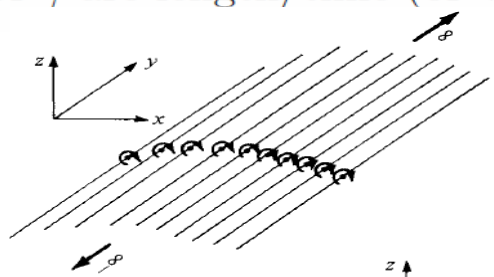
$$\Gamma \quad \rightarrow \quad 2 \times \frac{\Gamma}{2} \quad \rightarrow \quad 4 \times \frac{\Gamma}{4} \quad \rightarrow \quad 8 \times \frac{\Gamma}{8} \quad \ldots \quad \gamma$$



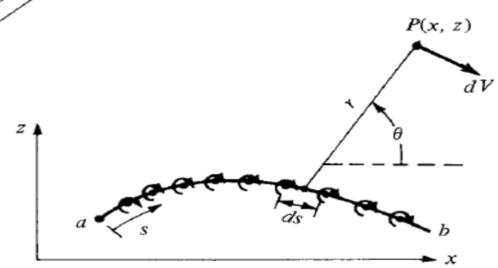
Like with the source sheet strength λ, the units of γ are length/time (or velocity).

- ***Concept of source sheet:***





Vortex sheet in perspective

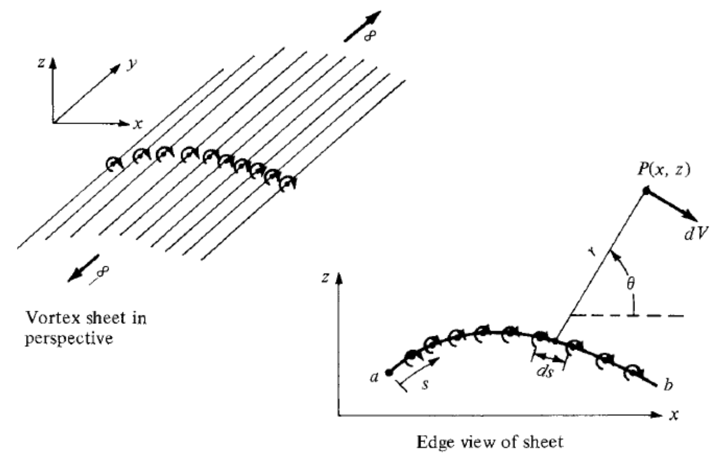


Edge view of sheet

❑ Superposition of Basic Flows

Velocity discontinuity

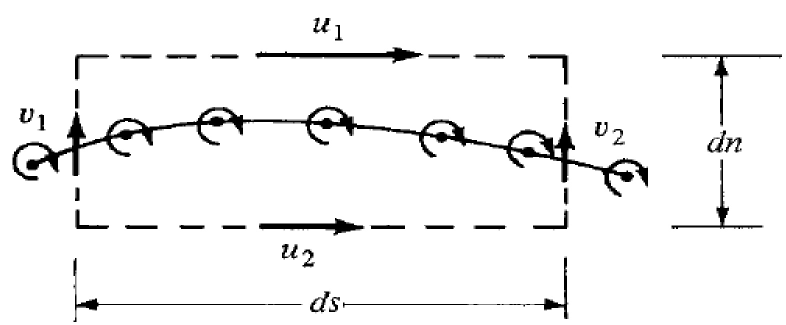
- On a source sheet the normal component of velocity is discontinuous (note 180-degree change in direction)
- On a vortex sheet the tangential component of velocity is discontinuous



Consider circulation over a box with size ds and dn

$$\Gamma = v_2 dn - u_1 ds - v_1 dn + u_2 ds$$
$$\Gamma = (u_2 - u_1)ds + (v_2 - v_1)dn$$

But $\Gamma = \gamma ds$



$$\gamma ds = (u_2 - u_1)ds + (v_2 - v_1)dn$$

As $dn \rightarrow 0$ then $\gamma ds = (u_2 - u_1)ds \rightarrow$

$$\boxed{\gamma = u_2 - u_1}$$

❑ Thin Airfoil Theory – Cambered Airfoil

- ***Relationship between pressure on mcl and γ***

$$L=\int_{LE}^{TE}(p_l-p_u)\bullet ds; \quad also \quad L=-\int_{LE}^{TE}\rho V_\infty \gamma(s)ds$$

$$\Rightarrow p_l-p_u=\rho V_\infty \gamma(s)$$

Bernoulli's Equation:

$$\Rightarrow p_l+\frac{1}{2}\rho U_l^{\,2}=p_u+\frac{1}{2}\rho U_u^{\,2}$$

$$\Rightarrow p_l-p_u=\frac{1}{2}\rho(U_u^{\,2}-U_l^{\,2})=\frac{1}{2}\rho(U_u-U_l)(U_u+U_l)$$

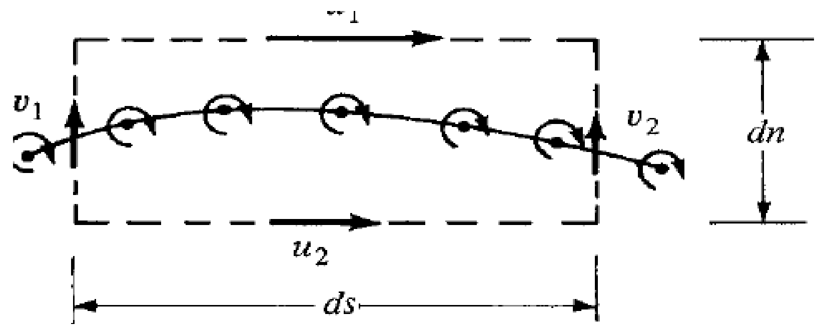
$$\because \gamma(s)=U_u-U_l$$

$$\therefore p_l-p_u=\frac{1}{2}\rho\gamma(s)(U_u+U_l)$$

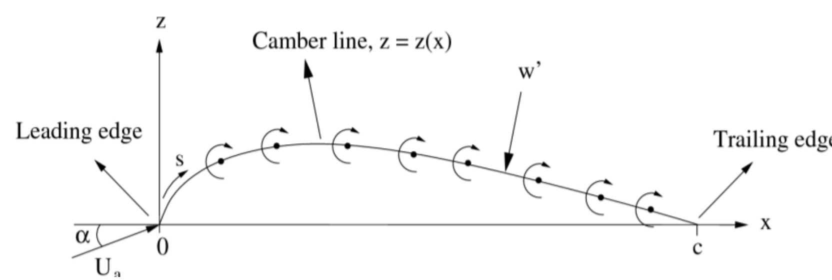
$$\Rightarrow V_\infty=\frac{(U_u+U_l)}{2}$$

$$C_{p,l}-C_{p,u}=\frac{P_l-P_\infty}{\frac{1}{2}\rho V_\infty^{\,2}}-\frac{P_u-P_\infty}{\frac{1}{2}\rho V_\infty^{\,2}}$$

$$=\frac{P_l-P_u}{\frac{1}{2}\rho V_\infty^{\,2}}=\frac{2\gamma(s)}{V_\infty^{\,2}}$$

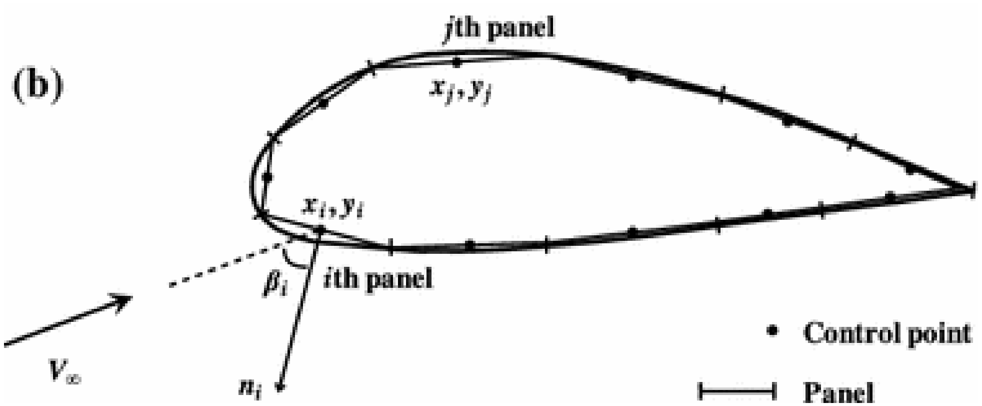


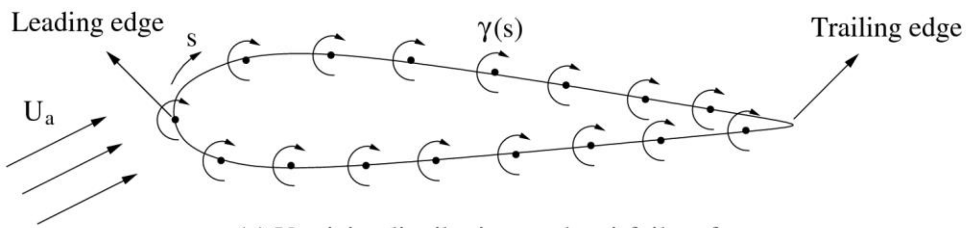
$$\gamma=u_2-u_1$$



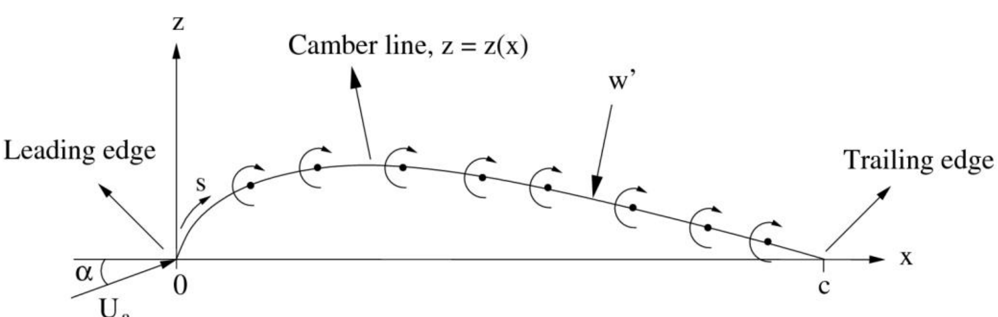
❑ Superposition of Basic Flows

- *The approach of using **panel method with vortex sheets***

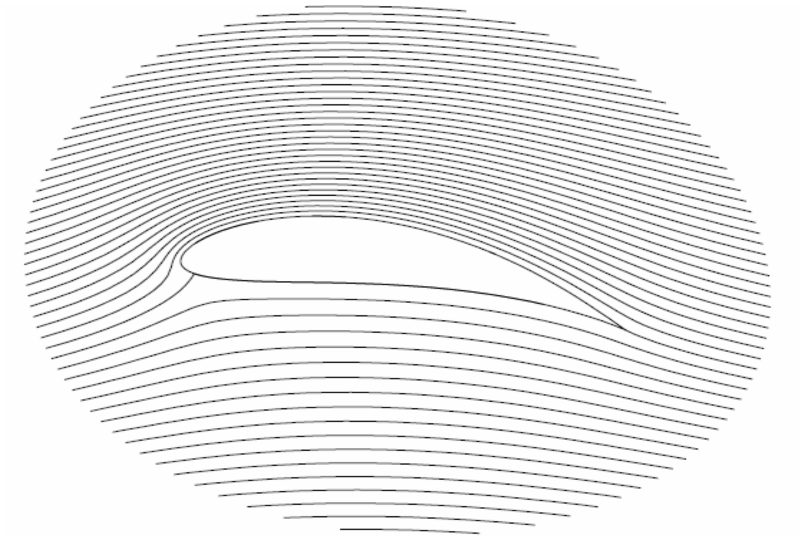




(a) Vorticity distribution on the airfoil surface



(b) Vorticity distribution on the camber line of the airfoil



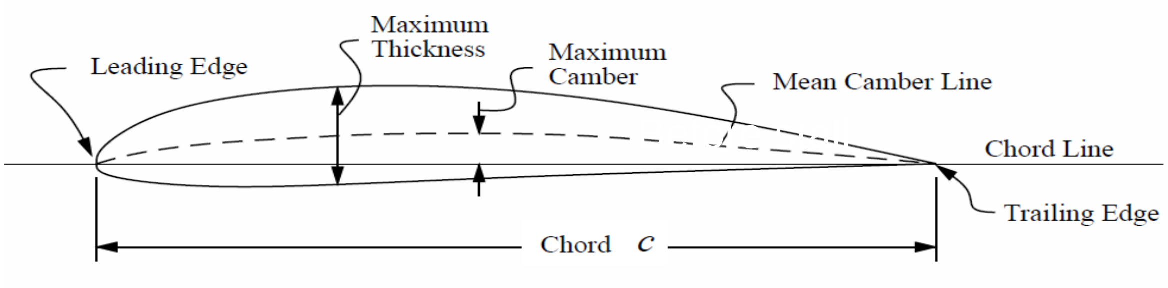
- ***The Kutta-Joukowski Lift Theorem:***

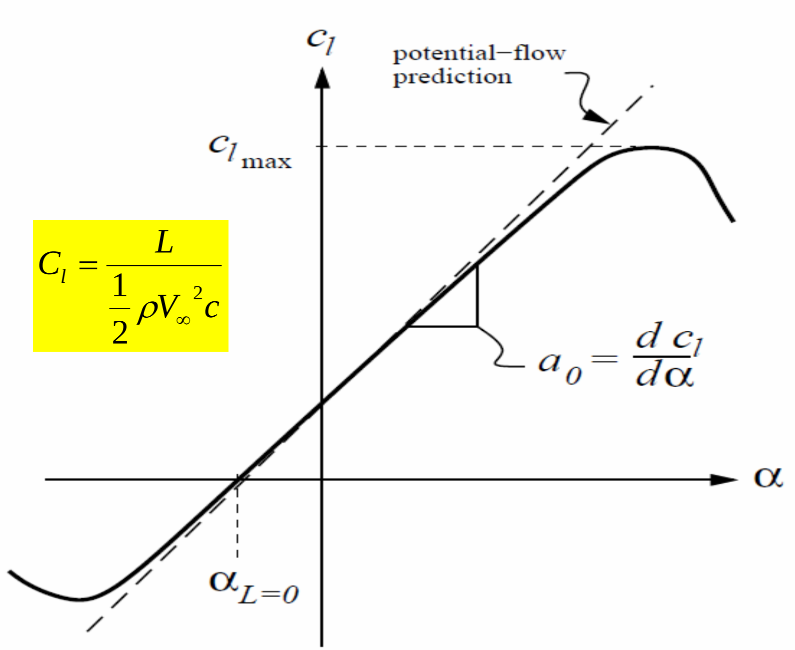
$$L' = \rho V_\infty \Gamma$$

❑ Thin Airfoil Theory

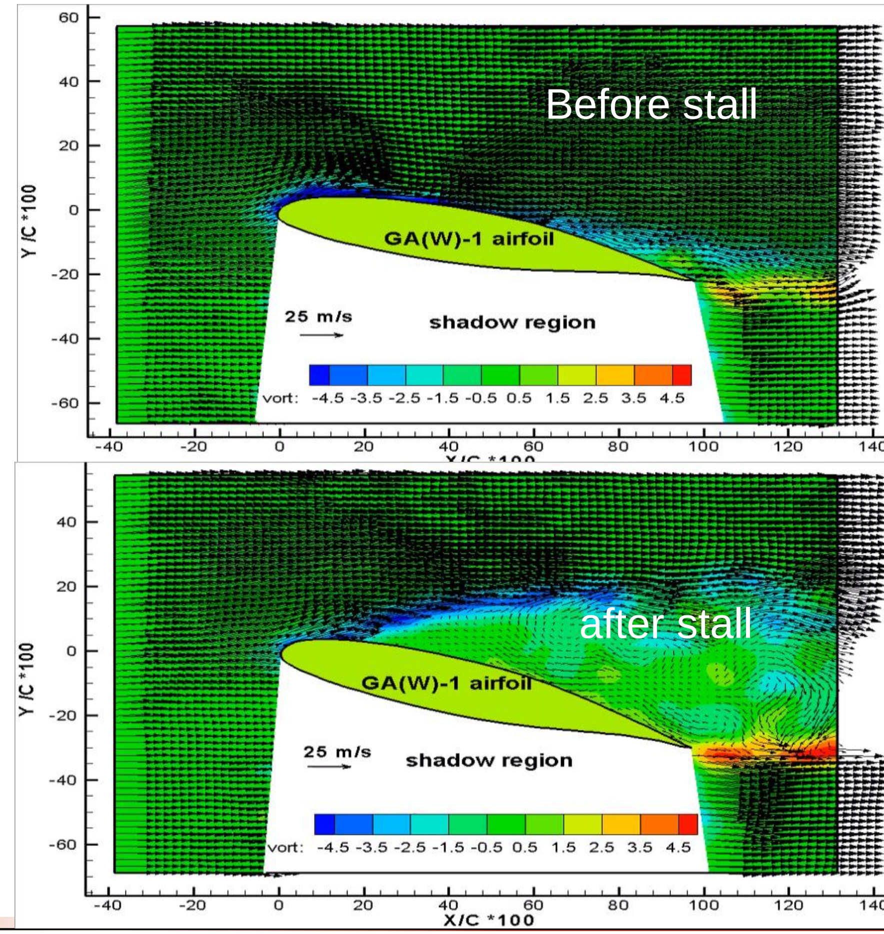
Assumptions:

- *2-dimensions*
- *Inviscid**
- *Incompressible**
- *Irrotational**
- *Small α*
- *Small max τ /c (i.e., airfoil thickness)*
- *Small max z/ c ((i.e., airfoil camber)*





$$C_l = \frac{L}{\frac{1}{2}\rho V_\infty^{\,2} c}$$



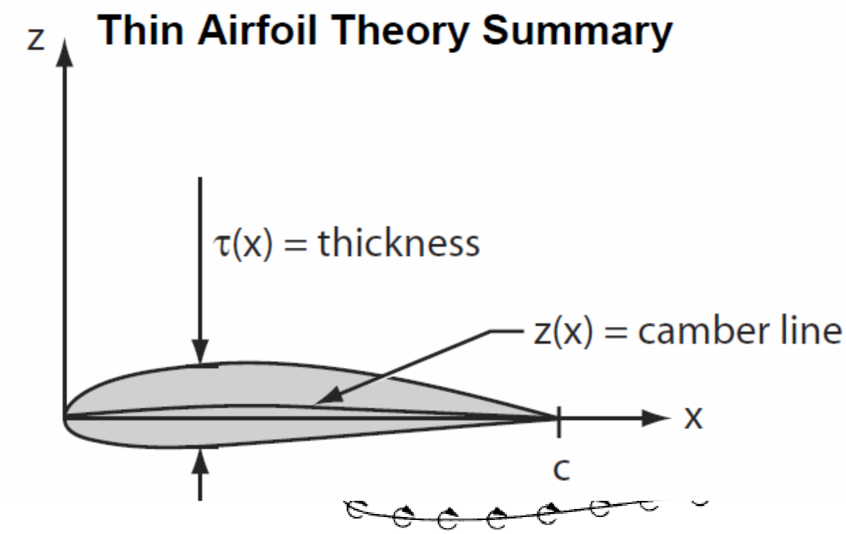
❑ Thin Airfoil Theory

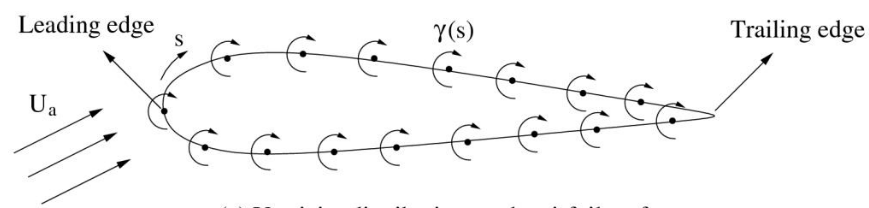
Thin Airfoil Approximation

- For a sufficiently thin airfoil we can approximate the shape of th airfoil by its mean camber line.

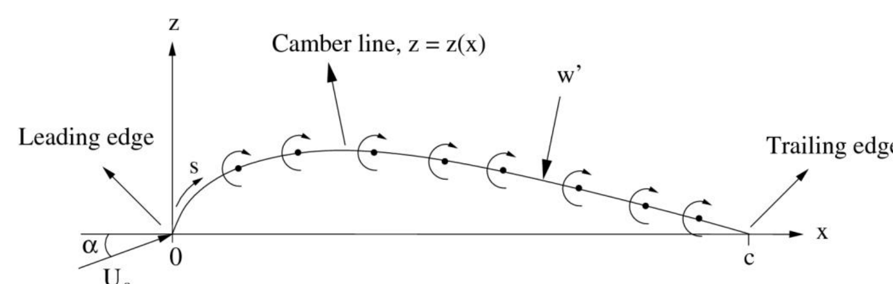
- In the thin airfoil approximation then, we distribute vortex sheet on its mean camber line instead of top/bottom surfaces.
  - The strength of the vortex is adjusted such that the camber line is a streamline

- This approximation allows a closed form solution to be obtained.





(a) Vorticity distribution on the airfoil surface



(b) Vorticity distribution on the camber line of the airfoil

• *Stream function for a thin airfoil*

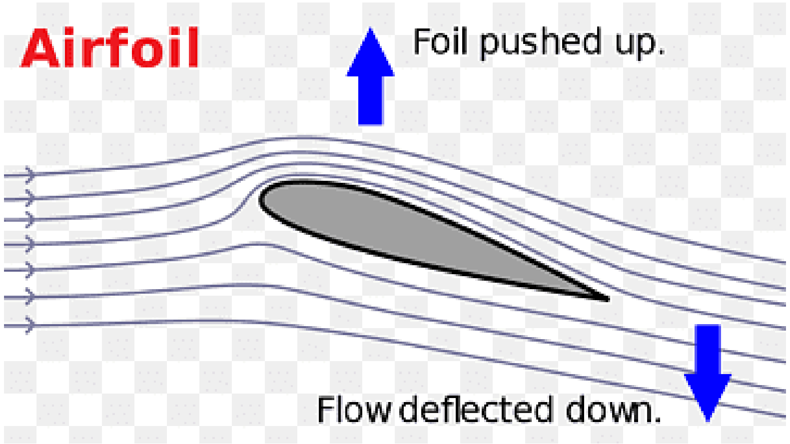
- For a thin airfoil, we can write stream function as

$$\psi = \psi_{\text{uniform}} + \psi' = V_\infty y + \psi'$$

Same as considering velocity around the airfoil is freestream plus a small disturbance

$$u = \frac{\partial \psi}{\partial y} = V_\infty + u'$$

$$v = -\frac{\partial \psi}{\partial x} = 0 + v'$$



Note that Laplace equation applies to ψ' as well

$$\nabla^2 \psi = \nabla^2 (V_\infty y + \psi') = \nabla^2 (V_\infty y) + \nabla^2 \psi' = 0$$

$$\rightarrow \nabla^2 \psi' = 0$$

Now at freestream: $\psi' = 0, u' = 0, v' = 0$

On airfoil surface $y = f(x)$: $\psi(x, y) = \text{const.}$

❑ Thin Airfoil Theory

Thin airfoil theory

The flow tangency on airfoil surface requires $v = 0$. Therefore

$$\frac{d\psi}{dx} = \frac{\partial\psi}{\partial x} + \frac{dy}{dx}\frac{\partial\psi}{\partial y} = 0$$

But $\frac{dy}{dx} = \frac{df}{dx}$ on airfoil surface. Now substitute $\psi = V_\infty y + \psi'$

$$\frac{d\psi}{dx} = \frac{\partial\psi'}{\partial x} + \frac{df}{dx}\left(V_\infty + \frac{\partial\psi'}{\partial y}\right) = 0$$

For a thin airfoil $\frac{\partial\psi'}{\partial y}$ is small and we can neglect it here, leading to

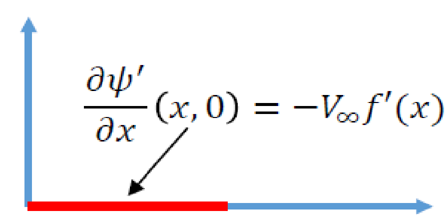
$$\frac{d\psi}{dx} = \frac{\partial\psi'}{\partial x} + \frac{df}{dx}V_\infty = 0 \rightarrow \frac{\partial\psi'}{\partial x} = -V_\infty f'(x)$$

Therefore, for a thin airfoil the problem to solve is:

$$\nabla^2\psi' = 0$$

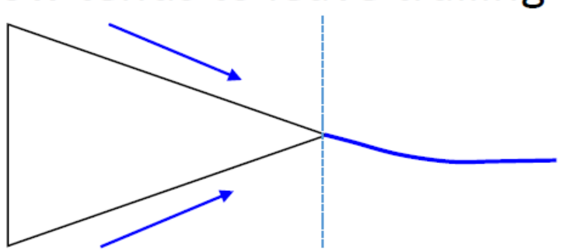
$$\frac{\partial\psi'}{\partial x}(x,0) = -V_\infty f'(x)$$

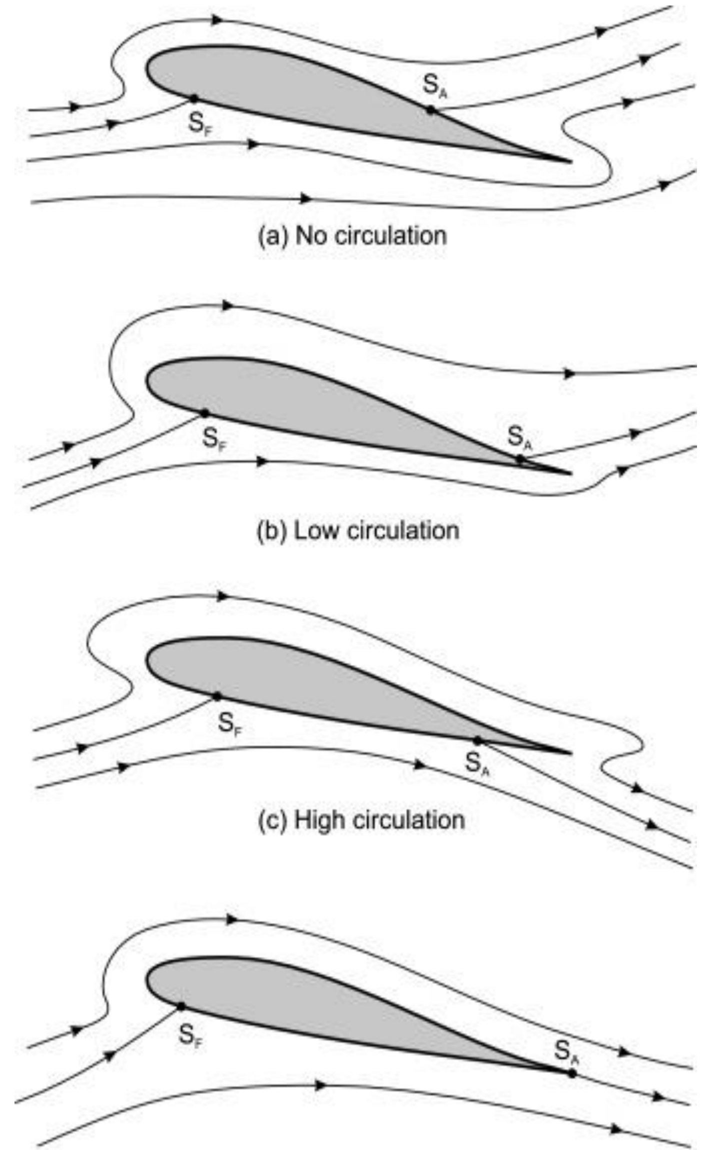
$$\frac{\partial\psi'}{\partial x} = \frac{\partial\psi'}{\partial y} \rightarrow 0 \text{ at } \infty$$



❑ Airfoil Aerodynamics

- ## *Kutta condition at Airfoil TE*
  - ***It is named after German mathematician and aerodynamicist - Martin Kutta.***
  - ***The Kutta condition is a principle in steady-flow fluid dynamics, especially aerodynamics, that is applicable to solid bodies with sharp corners, such as the trailing edges of airfoils.***
  - ***For a given airfoil at a given angle of attack, the value of Γ around the airfoil is such that the flow would leaves the trailing edge smoothly.***
  - Flow tends to leave trailing edge smoothly





(a) No circulation

(b) Low circulation

(c) High circulation

(d) Circulation satisfying Kutta condition

❑ Kutta Condition at Airfoil Trailing Edge (TE)

- For a finite angle trailing edge, can't have two different velocity at the same point, hence $V_1 = V_2 = 0$
- For a cusped trailing edge, V_1 and V_2 can be non-zero since they are in the same direction.

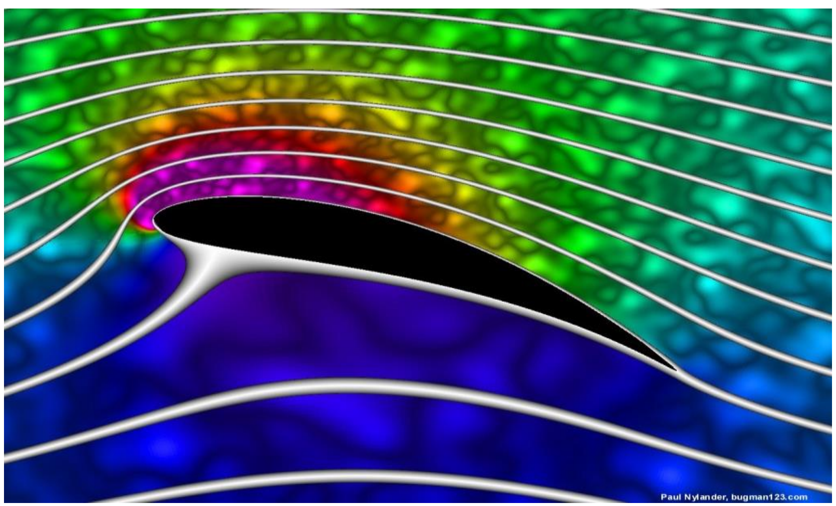
from Bernoulli's equation:

$$p_a + \frac{1}{2}\rho V_1^2 = p_a + \frac{1}{2}\rho V_2^2 \rightarrow V_1 = V_2$$

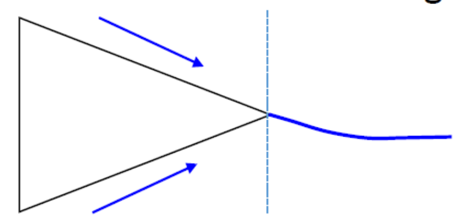
- In relation to the vortex sheet discontinuity

$$\gamma(\text{TE}) = V_2 - V_1$$

$$\gamma(\text{TE}) = 0$$



- Flow tends to leave trailing edge smoothly



Pressure can be different across solid airfoil

Pressure must be the same on upper and lower surface of stagnation streamline at TE

Finite angle

At point a: $V_1 = V_2 = 0$

- ***Case #1***

Cusp

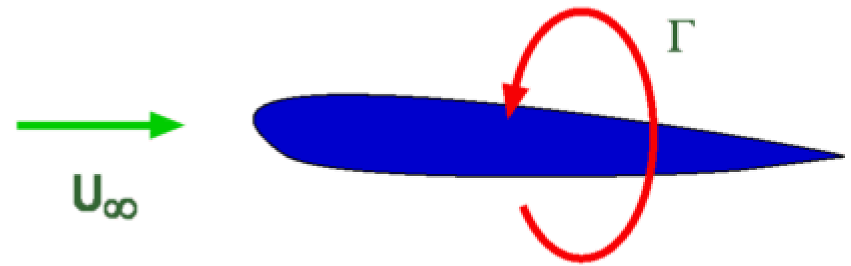
At point a: $V_1 = V_2 \neq 0$

- ***Case #2***

Thin Airfoil Theory

The Kutta-Joukowski Lift Theorem:

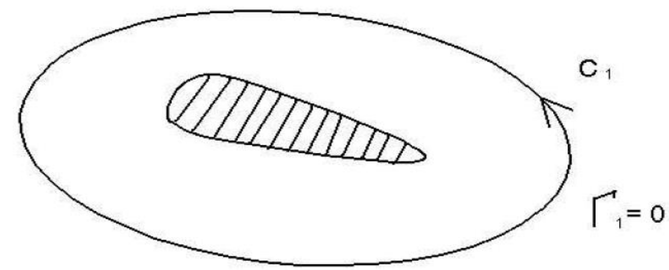
$$L' = \rho V_\infty \Gamma$$



$L = -\rho U_\infty \Gamma$

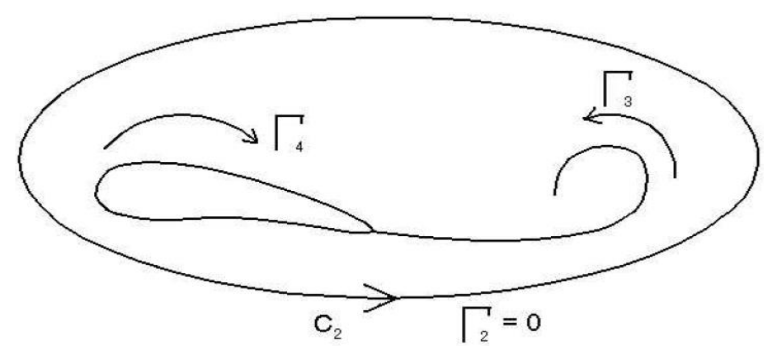
Helmholtz's theorem:

- If $\Gamma = 0$ originally in a flow, it remains zero.



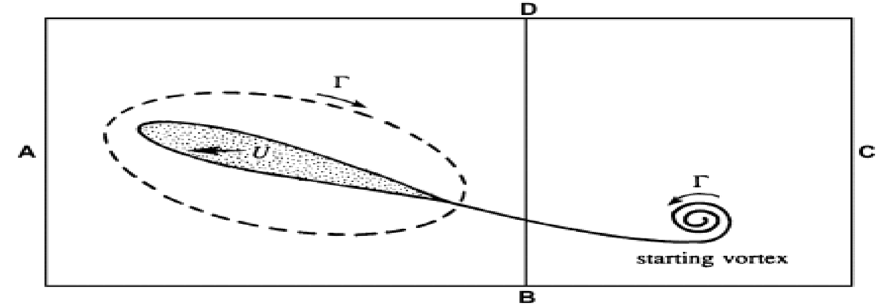
Kelvin's theorem:

- Circulation around a closed curve formed by a set of continuous fluid elements remains constant as the fluid elements move through the flow:
  - $D\Gamma/Dt = 0$.
- Substantial derivative gives the time rate of change following a given fluid element.

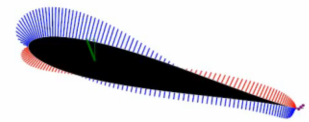


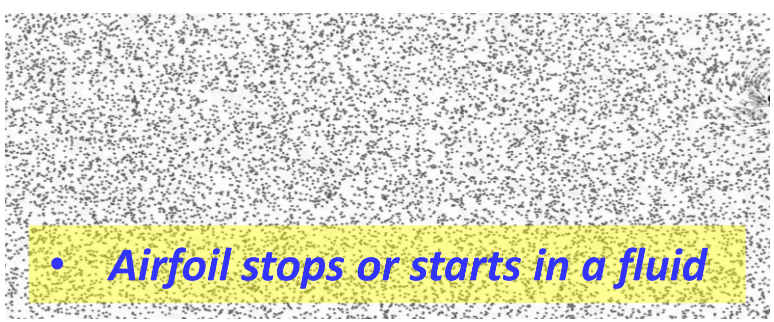
❑ Airfoil Aerodynamics

- Initially circulation around airfoil is zero
- At start of the motion a vortex forms at the trailing edge and rolls up downstream (due to a high velocity gradient at trailing edge)
- Circulation around the airfoil is equal and opposite to that of vortex moving downstream
- Once airfoil reaches steady state, vorticity no longer sheds from the trailing edge and flow leaves smoothly.
- When airfoil stops, another vortex sheds from trailing edge, with equal and opposite strength, such that the net circulation is again zero.

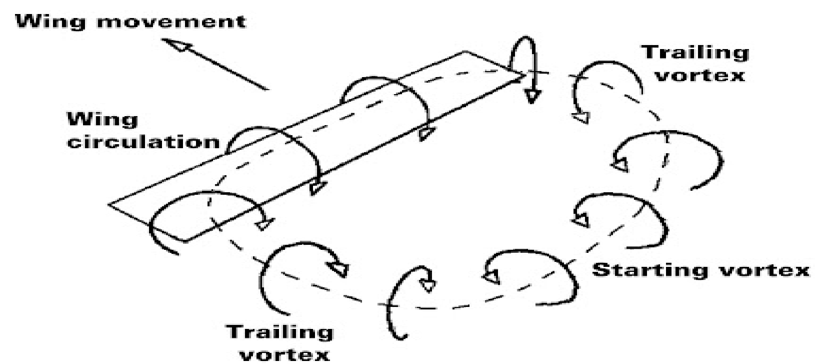


- *Starting vortex*

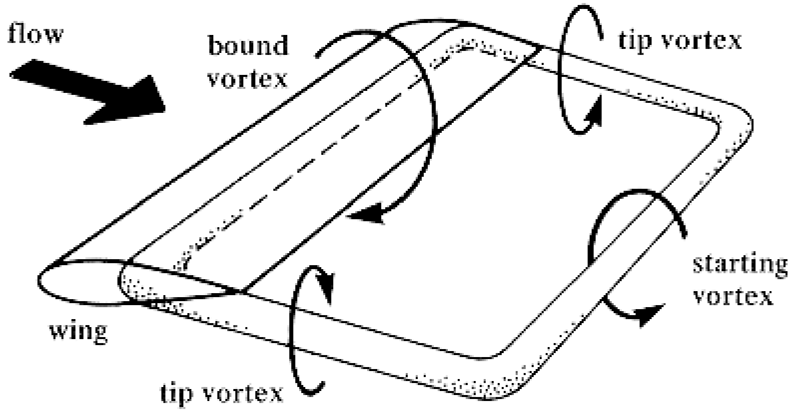


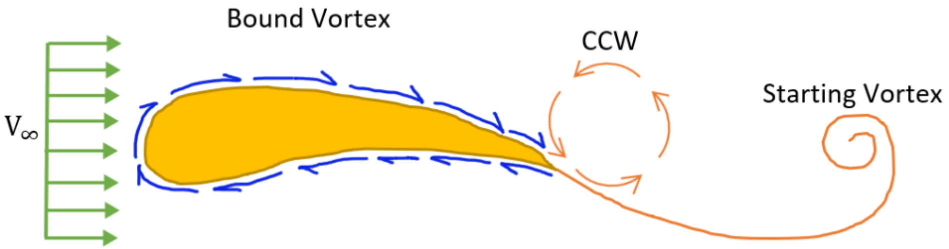


- *Airfoil stops or starts in a fluid*



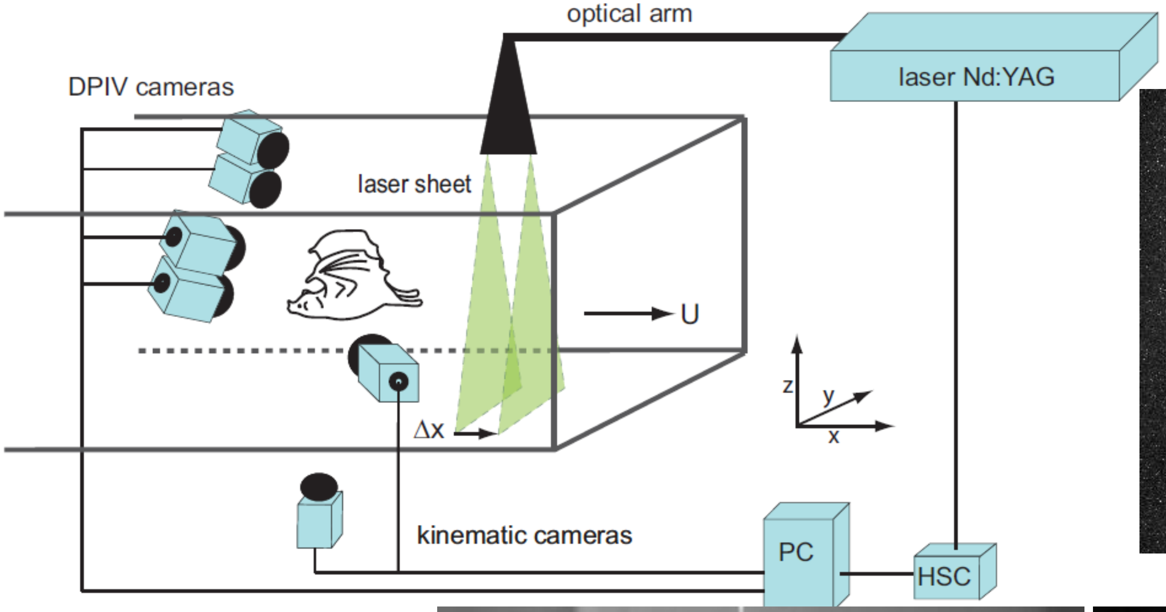
❑ Starting Vortex from Shedding from a Wing/Airfoil

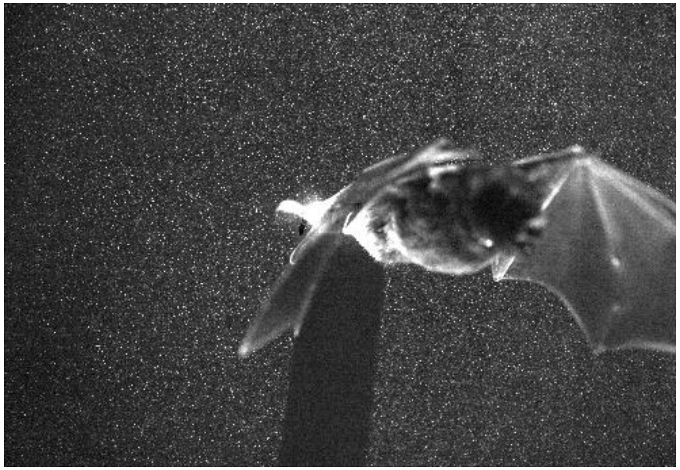


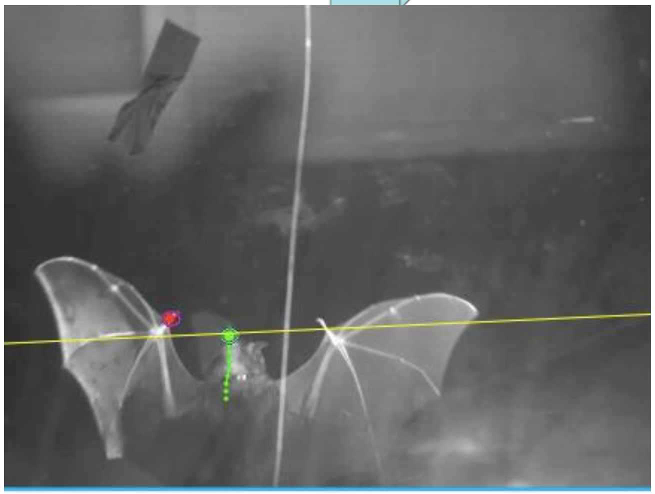


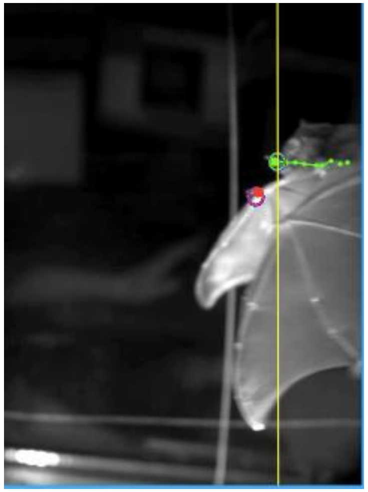
❑ Aerodynamics of Bat Flight

PIV Experiments: Bat flight

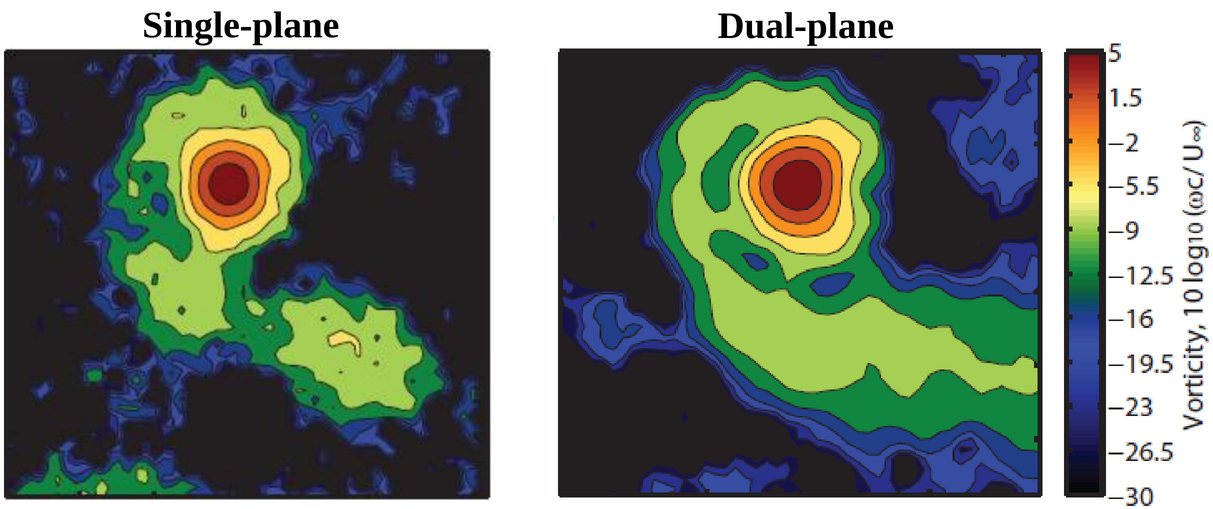


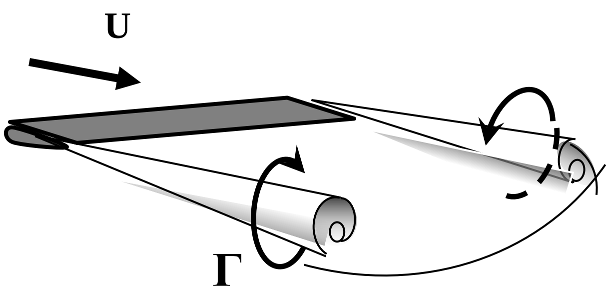




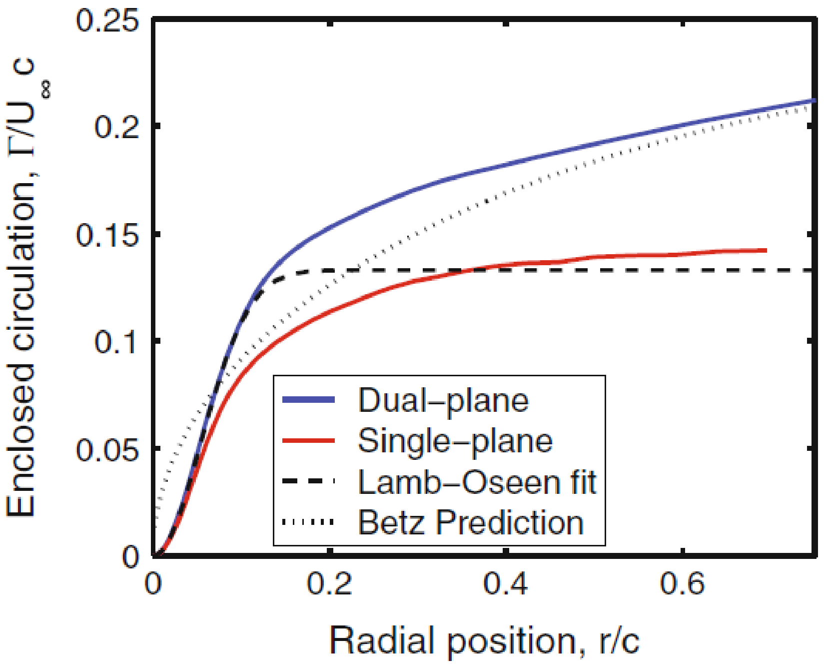


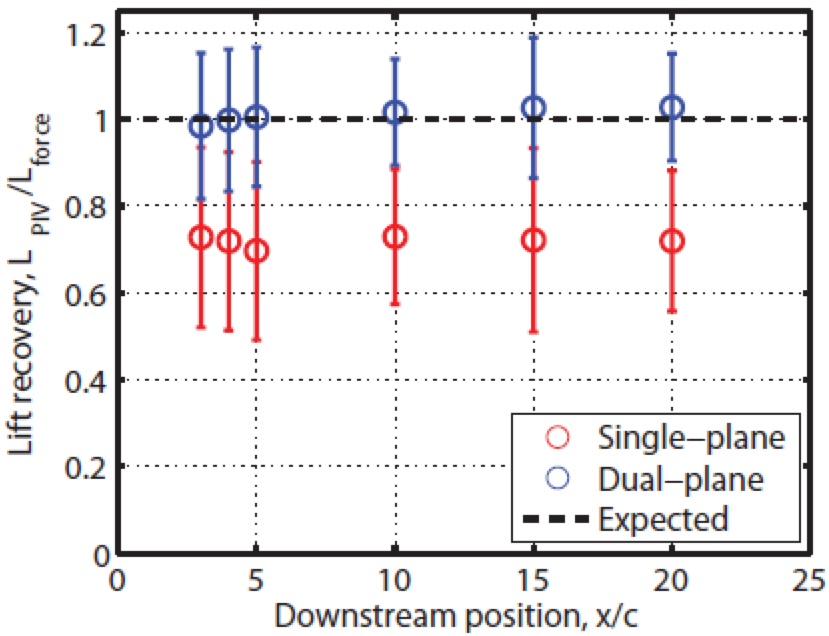
Example PIV experiments: Dual-plane PIV



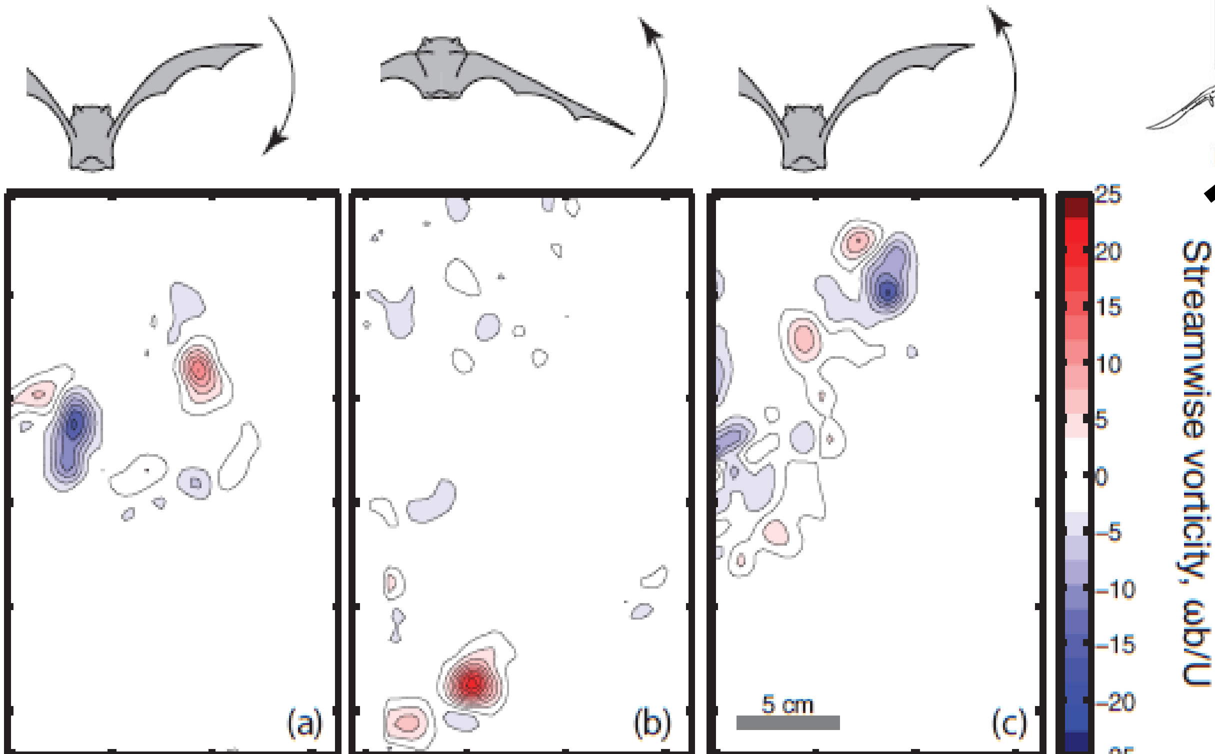


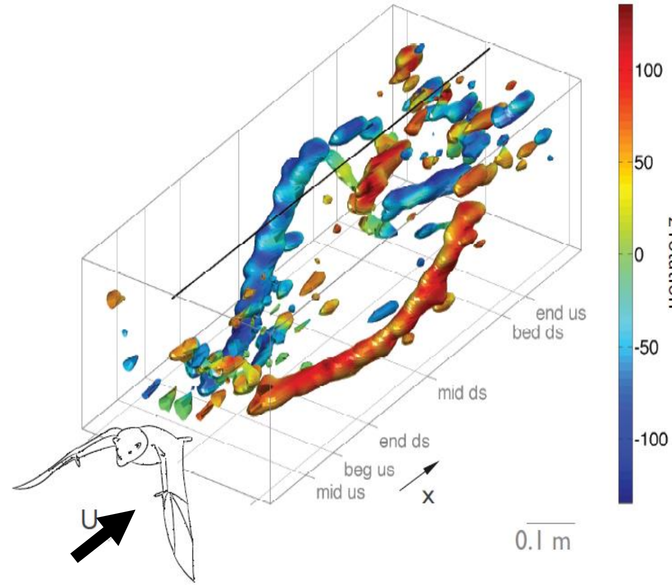
$$L = \int_{span} \rho U_\infty \Gamma(s) ds$$





Example PIV experiments: Bat flight





Example PIV experiments: Bat flight

$$L = \frac{\rho U_\infty}{T} \int_{\text{wb}} \int_{\text{Trefftz}} y \cdot \omega \; dA \; dt$$