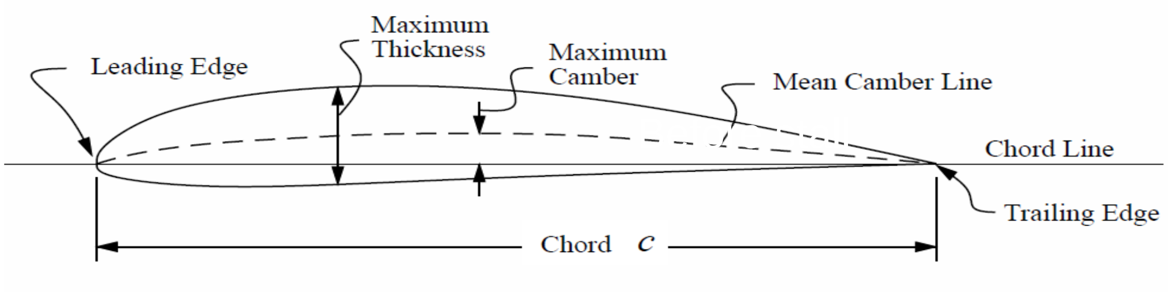
Lecture # 25: Airfoil Aerodynamics - Part 03: Symmetrical Airfoil – 01

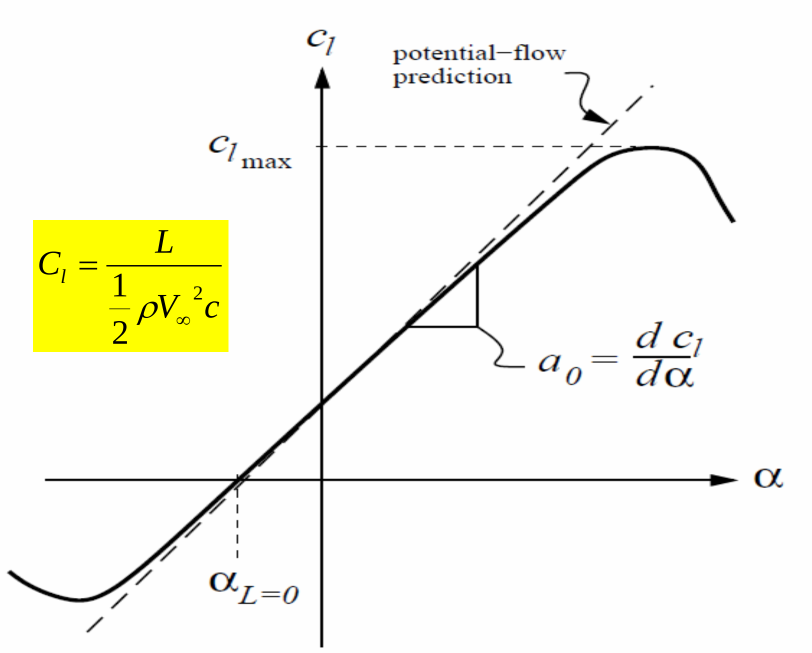
Dr. Hui HU
Department of Aerospace Engineering
Iowa State University, 2251 Howe Hall, Ames, IA 50011-2271
Tel: 515-294-0094 / Email: huhui@iastate.edu

❑ Thin Airfoil Theory

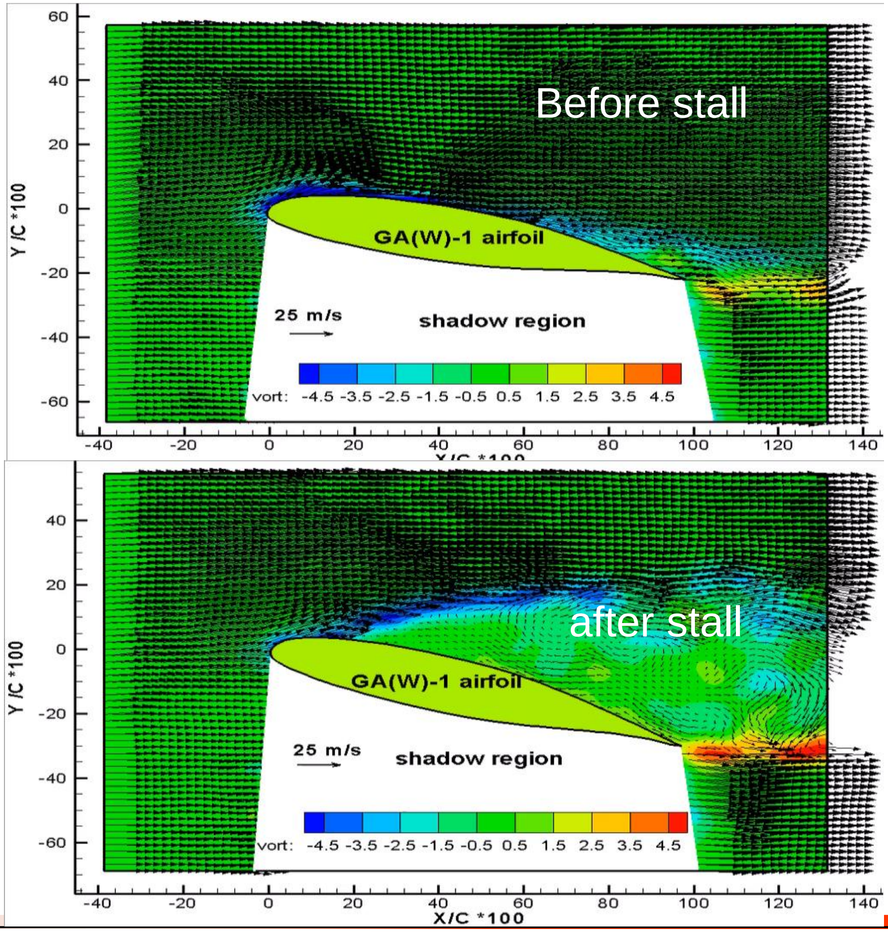
Assumptions:

- *2-dimensions*
- *Inviscid**
- *Incompressible**
- *Irrotational**
- *Small α*
- *Small max τ /c (i.e., airfoil thickness)*
- *Small max z/ c ((i.e., airfoil camber)*





$$C_l = \frac{L}{\frac{1}{2}\rho V_\infty^{\,2} c}$$



❑ Thin Airfoil Theory

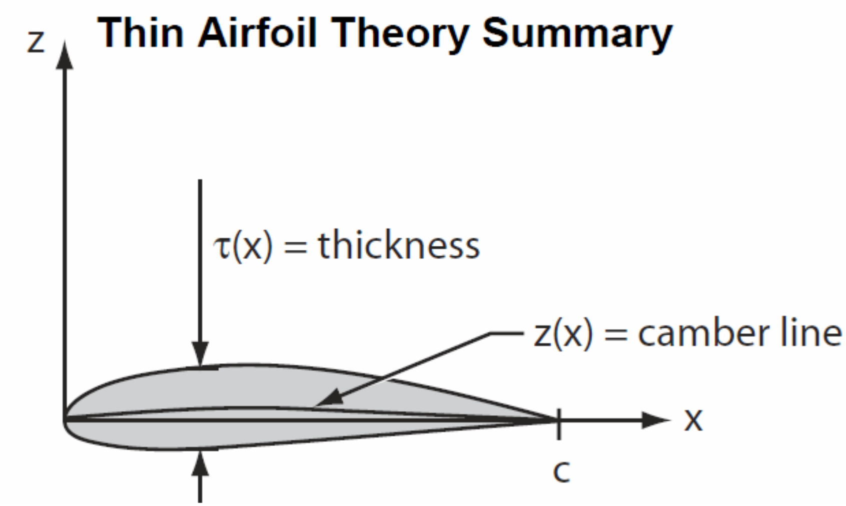
Thin Airfoil Approximation

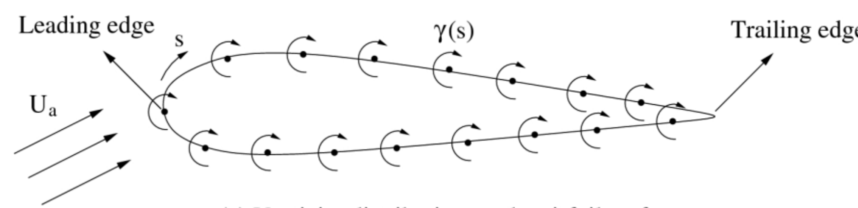
- For a sufficiently thin airfoil we can approximate the shape of th airfoil by its mean camber line.

- In the thin airfoil approximation then, we distribute vortex sheet on its mean camber line instead of top/bottom surfaces.

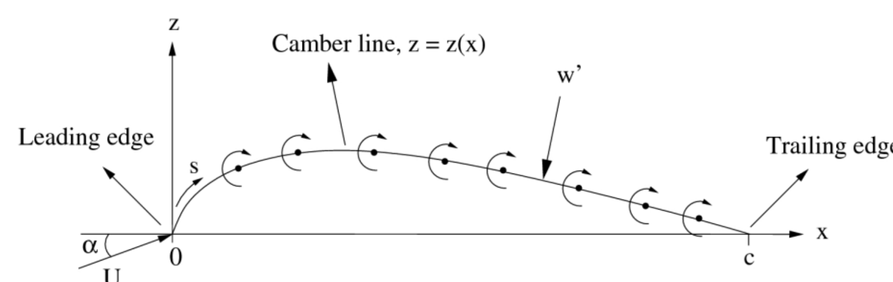
    - The strength of the vortex is adjusted such that the camber line is a streamline

- This approximation allows a closed form solution to be obtained.





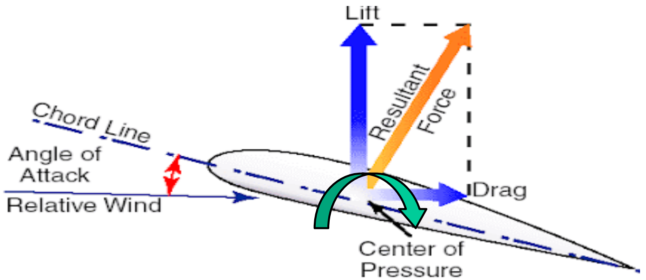
(a) Vorticity distribution on the airfoil surface

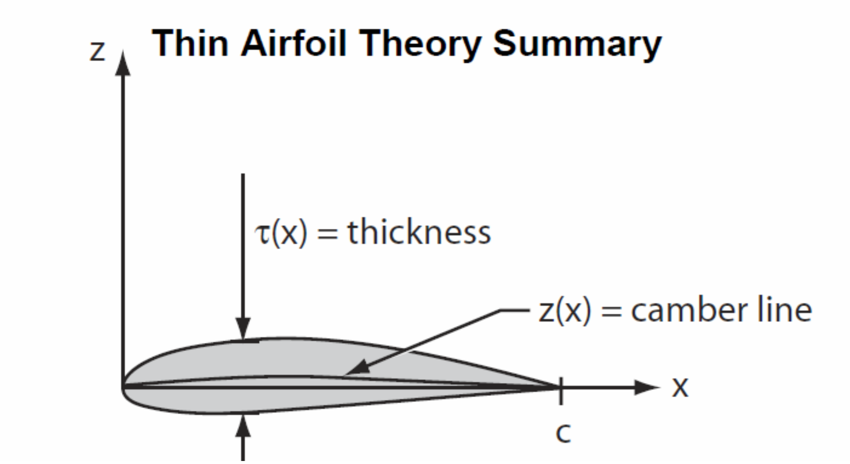


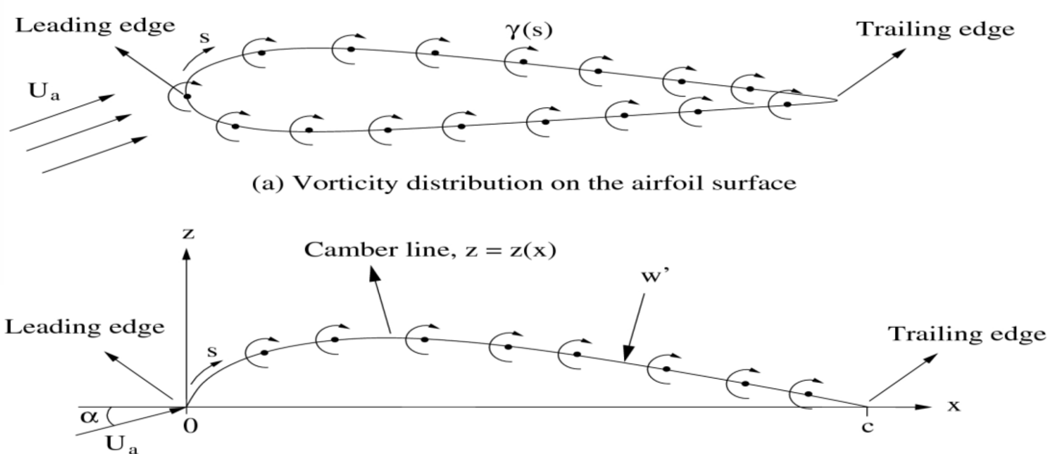
(b) Vorticity distribution on the camber line of the airfoil

❑ Thin Airfoil Theory

- ***Replace thin airfoil with a camber line (assume small thickness and camber)***
- ***Derivation of formula to predict the aerodynamic force and moment generated by a thin airfoil.***
- ***Determination of the pressure and aerodynamic centers.***
- ***Application of the thin airfoil theory to symmetric airfoils and cambered airfoils.***







(a) Vorticity distribution on the airfoil surface

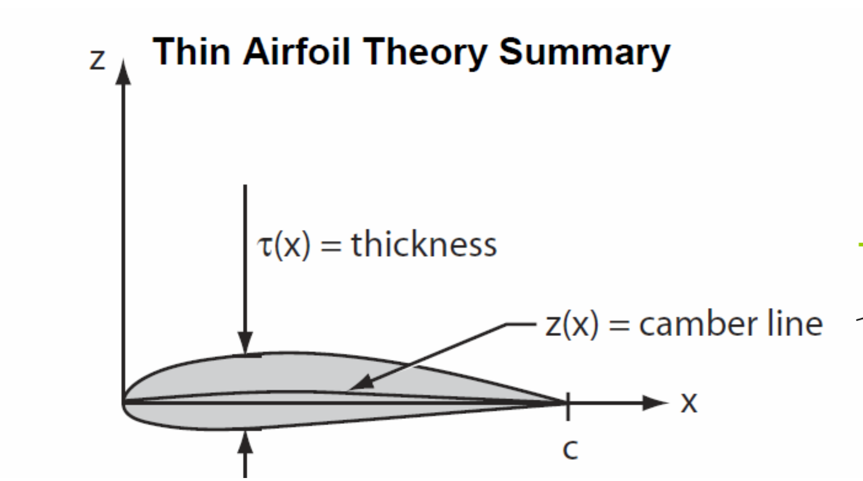
(b) Vorticity distribution on the camber line of the airfoil

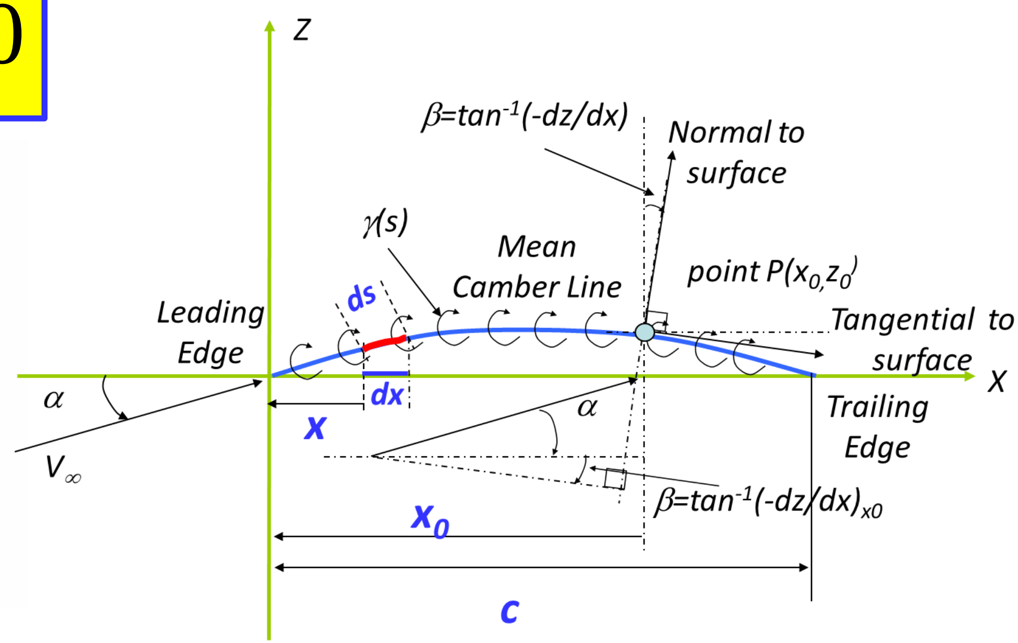
❑ Thin Airfoil Theory

Principle:

- ***Replace thin airfoil with the mean camber line (MCL) because of the small thickness and camber of the airfoil***
- ***MCL assumed to be a streamline of the flow around the thin airfoil.***
- ***To force the MCL to be a streamline, the sum of all velocity components normal to the MCL must be equal to zero.***

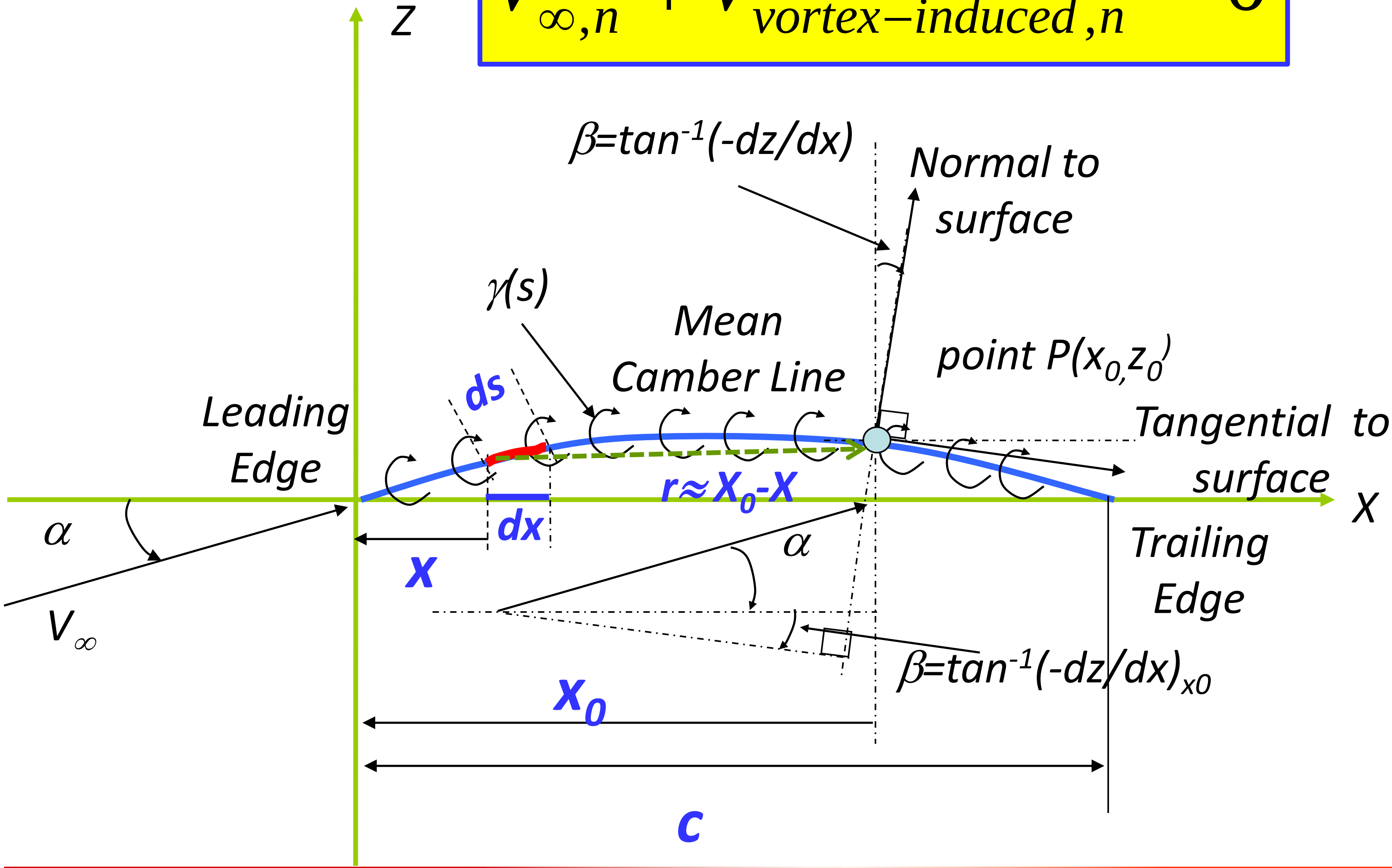
$$V_{\infty,n} + V_{vortex-induced,n} = 0$$





❑ Thin Airfoil Theory

$$V_{\infty,n} + V_{vortex-induced,n} = 0$$

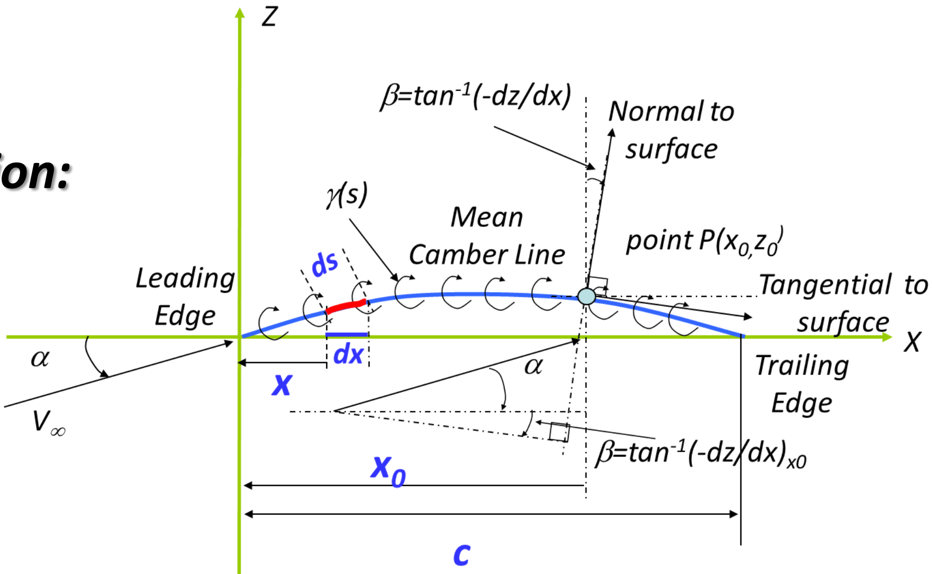


❑ Thin Airfoil Theory

- ***To force the mean camber line to be a streamline, the sum of all velocity components normal to the mcl must be equal to zero.***

- ***Now determine the component of the freestream velocity normal to the mcl.***

$$V_{\infty,n} = V_{\infty} \sin(\alpha + \varepsilon);$$



- ***Within thin airfoil theory approximation:***

Since the angle ε is small

$$\Rightarrow \beta = \tan^{-1}(-\frac{dz}{dx}) \approx -\frac{dz}{dx};$$

$$Then: \quad V_{\infty,n} = V_{\infty} \sin(\alpha + \beta) = V_{\infty} \sin(\alpha - \frac{dz}{dx});$$

Since the angle α is small $\Rightarrow \sin(\alpha + \beta) \approx \alpha + \beta \approx \alpha - \frac{dz}{dx};$

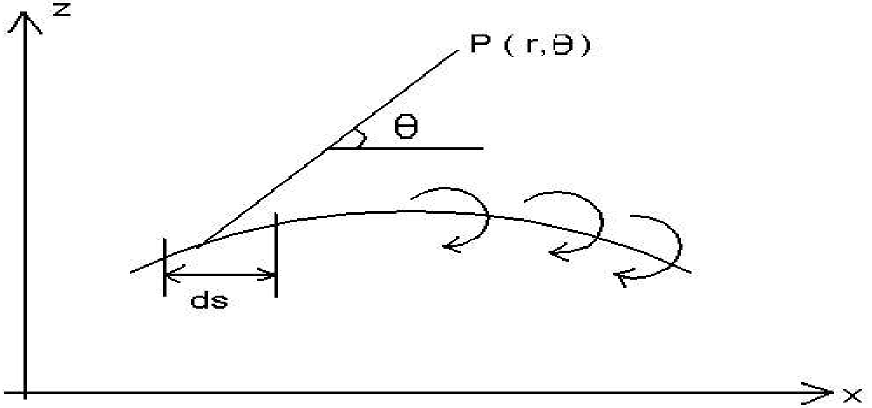
- ***Therefore:***

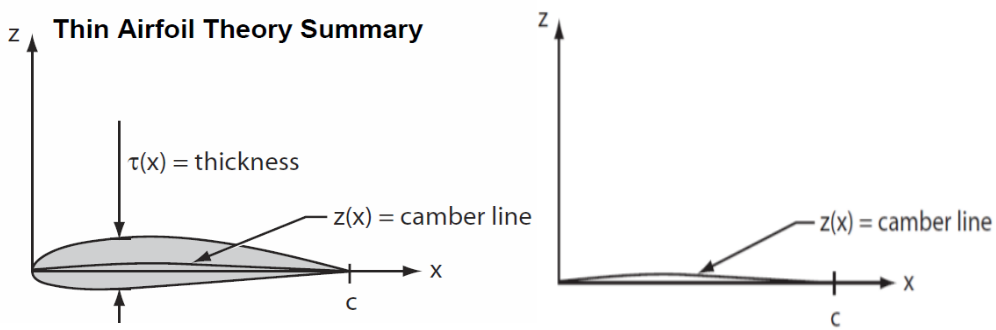
$$V_{\infty,n} = V_{\infty}(\alpha - \frac{dz}{dx});$$

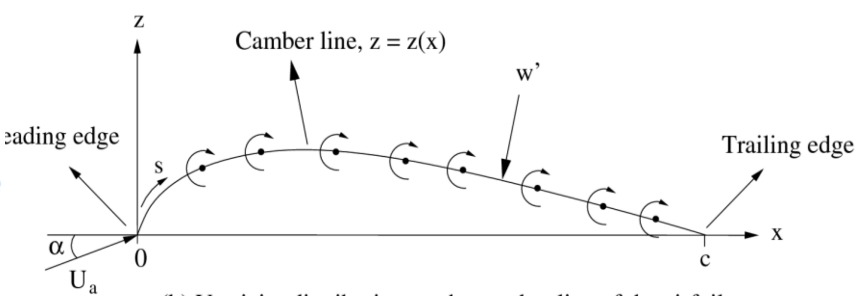
Thin Airfoil Theory

- *Consider the flow induced by an elemental vortex sheet ds at a point P on the vortex sheet.*
- *Velocity induced by a 2-D vortex is:* $$\vec{V} = V_\theta \hat{e}_\theta = -\frac{\Gamma}{2\pi r}\hat{e}_\theta; \quad .$$
- *Similarly, the velocity at the point P induced by the vortex sheet of infinitesimal length ds is given by:*

$$d\vec{V}_P = -\frac{\gamma(s)ds}{2\pi r}\hat{e}_\theta; \quad .$$







(b) Vorticity distribution on the camber line of the airfoil

❑ Thin Airfoil Theory

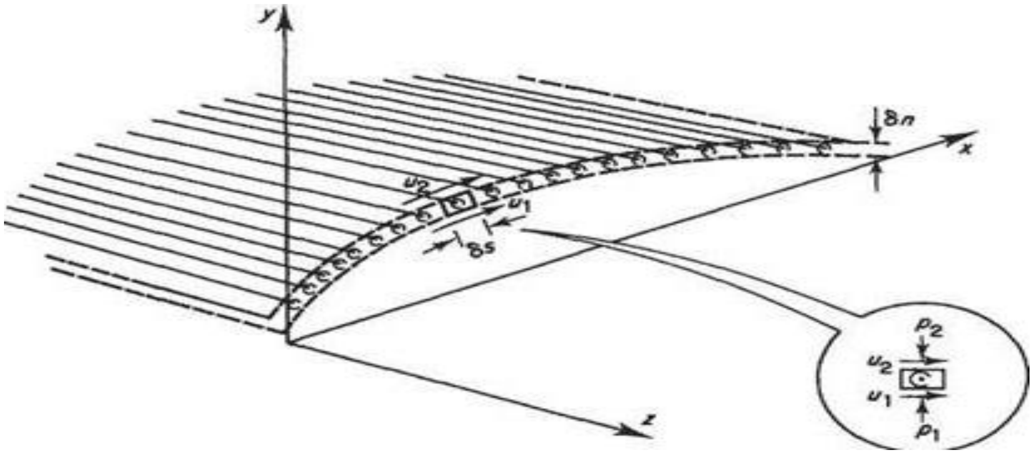
- ***Within thin airfoil theory approximation:***

$$\text{Thin airfoil} \Rightarrow ds \approx dx$$

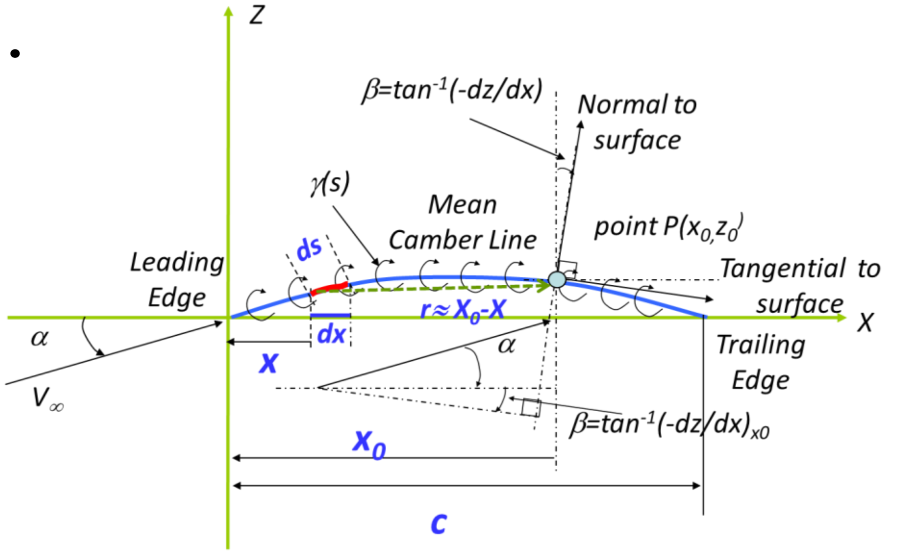
$$\text{therefore:}$$

$$d\vec{V}_P = -\frac{\gamma(s)ds}{2\pi r}\hat{e}_\theta;$$

$$\Rightarrow dV_{induced,n} = -\frac{\gamma(x)dx}{2\pi(x_0 - x)}\quad .$$



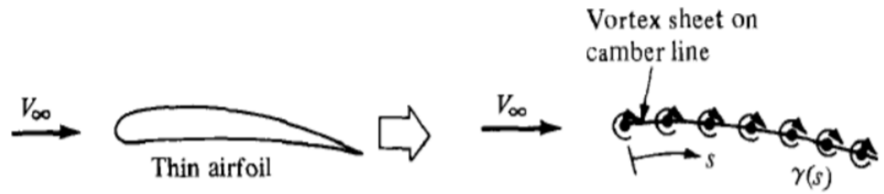
$$V_{vortex-induced,n} = -\int_0^c \frac{\gamma(x)dx}{2\pi(x_0 - x)}$$



❑ Thin Airfoil Theory

- ***To force the mean camber line to be a streamline, the sum of all velocity components normal to the mcl must be equal to zero.***

$$V_{\infty,n} + V_{vortex-induced,n} = 0$$



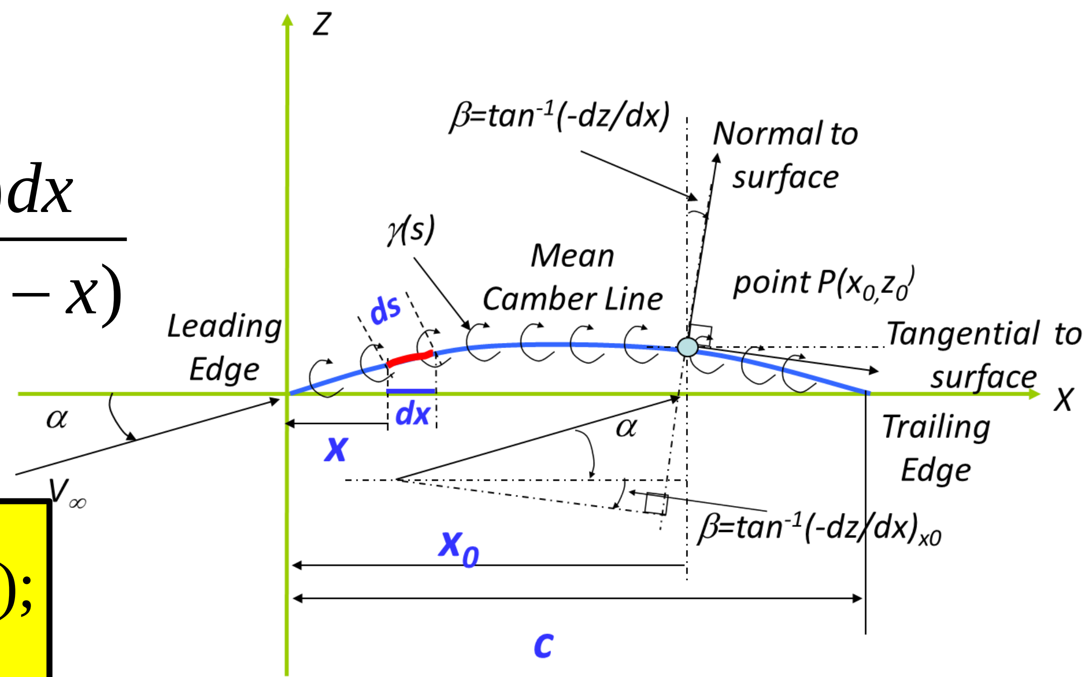
- ***Since***

$$V_{\infty,n} = V_\infty(\alpha - \frac{dz}{dx});$$

$$V_{vortex-induced,n} = -\int_0^c \frac{\gamma(x)dx}{2\pi(x_0 - x)}$$

- ***Therefore:***

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(x)dx}{(x_0 - x)} = V_\infty(\alpha - \frac{dz}{dx});$$

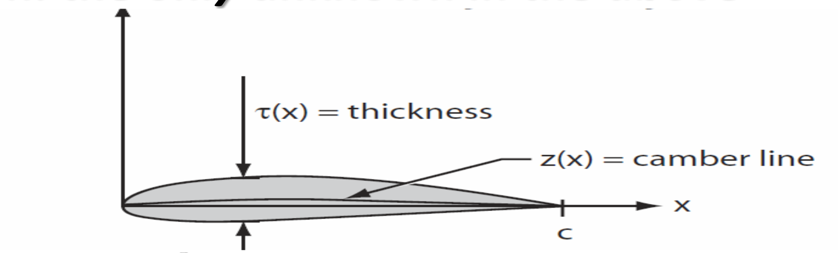


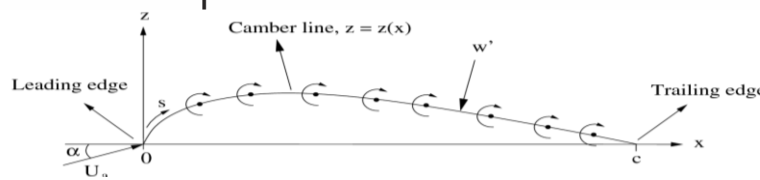
Thin Airfoil Theory

- ***The integral equation of thin airfoil theory:***

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(x)dx}{(x_0 - x)} = V_\infty(\alpha - \frac{dz}{dx});$$

- ***For a given airfoil geometry, dz/dx is known. the only unknown in the above equation is γ(x).***



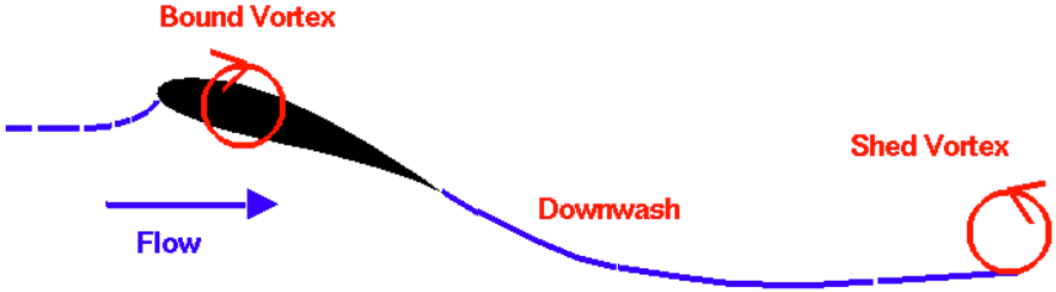


- ***If γ(x) can be determined, then***

$$\Gamma = \int_0^c \gamma(x)dx;$$

- ***The Kutta-Joukowski Lift Theorem states the lift per unit length of an airfoil is equal to the density (ρ) of the air times the strength of the rotation (Γ) times the velocity (V) of the air.***

$$L' = \rho V_\infty \Gamma$$



Thin Airfoil Theory

- ***The integral equation of thin airfoil theory:***

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(x)dx}{(x_0-x)} = V_\infty(\alpha - \frac{dz}{dx});$$

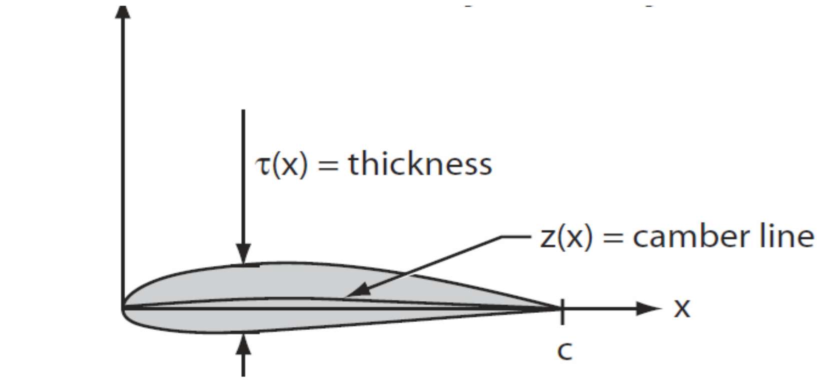
- ***To solve the integral equation, we first make a transformation:***

$$x = \frac{c}{2}(1-\cos\theta); \quad \theta = (0,\pi)$$

$$If \ \theta = 0 \quad \Rightarrow \quad x = 0, \quad at\ Leading\ \ Edge$$

$$If \ \theta = \pi \quad \Rightarrow \quad x = c, \quad at\ Trailing\ \ Edge$$

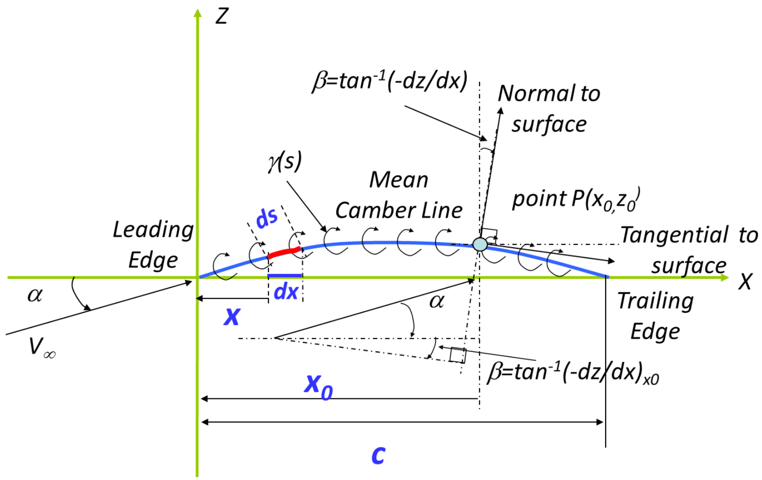
$Then$, x is a point on x_0 , corresponding to θ_0 :



$$Then: \quad x = \frac{c}{2}(1-\cos\theta_0);$$

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\bullet c\bullet\sin\theta d\theta}{\frac{c}{2}(1-\cos\theta_0)-\frac{c}{2}(1-\cos\theta)} = V_\infty(\alpha - \frac{dz}{dx});$$

$$\Rightarrow \frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta - \cos\theta_0} = V_\infty(\alpha - \frac{dz}{dx});$$



Thin Airfoil Theory

- ***The integral equation of thin airfoil theory:***

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty(\alpha-\frac{dz}{dx});$$

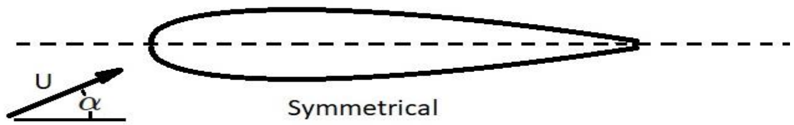
- ***Solution of the integral equation for a symmetrical airfoil:***

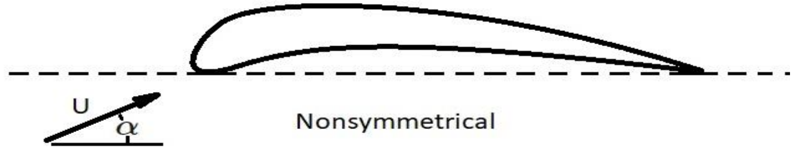
Since it is symmetrical airfoil, therefore: $\frac{dz}{dx}=0$

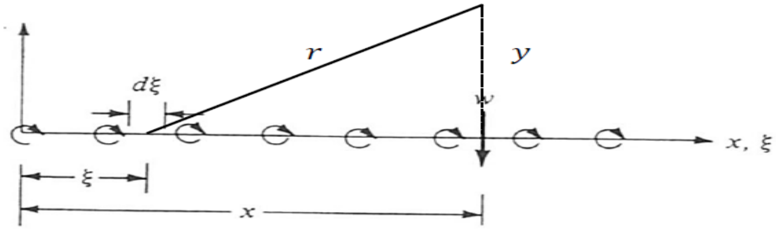
$$\frac{dz}{dx}=0\Rightarrow\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty\alpha;$$

The solution will be:

$$\gamma(\theta)=2\alpha V_\infty\frac{1+\cos\theta}{\sin\theta}$$







Thin Airfoil Theory

- ***The integral equation of thin symmetrical airfoil.***

$$\frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty\alpha;$$

- ***Solution of the equations:***

$$\gamma(\theta)=2\alpha V_\infty\frac{1+\cos\theta}{\sin\theta}$$

- ***Verification of the solution:***

$$\frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=\frac{1}{2\pi}\int_0^{\pi}\frac{2\alpha V_\infty\frac{1+\cos\theta}{\sin\theta}\sin\theta d\theta}{\cos\theta-\cos\theta_0}=\frac{\alpha V_\infty}{\pi}\int_0^{\pi}\frac{(1+\cos\theta)d\theta}{\cos\theta-\cos\theta_0}$$

$$\because\quad\int_0^{\pi}\frac{\cos(n\theta)d\theta}{\cos\theta-\cos\theta_0}=\frac{\pi\sin(n\theta_0)}{\sin\theta_0}$$

$$\therefore\frac{\alpha V_\infty}{\pi}\int_0^{\pi}\frac{(1+\cos\theta)d\theta}{\cos\theta-\cos\theta_0}=\frac{\alpha V_\infty}{\pi}[\int_0^{\pi}\frac{d\theta}{\cos\theta-\cos\theta_0}+\int_0^{\pi}\frac{\cos\theta d\theta}{\cos\theta-\cos\theta_0}]$$

$$=\frac{\alpha V_\infty}{\pi}[\frac{\pi\sin(0\bullet\theta_0)}{\sin\theta_0}+\frac{\pi\sin(1\bullet\theta_0)}{\sin\theta_0}]=\frac{\alpha V_\infty}{\pi}[0+\pi]=\alpha V_\infty$$

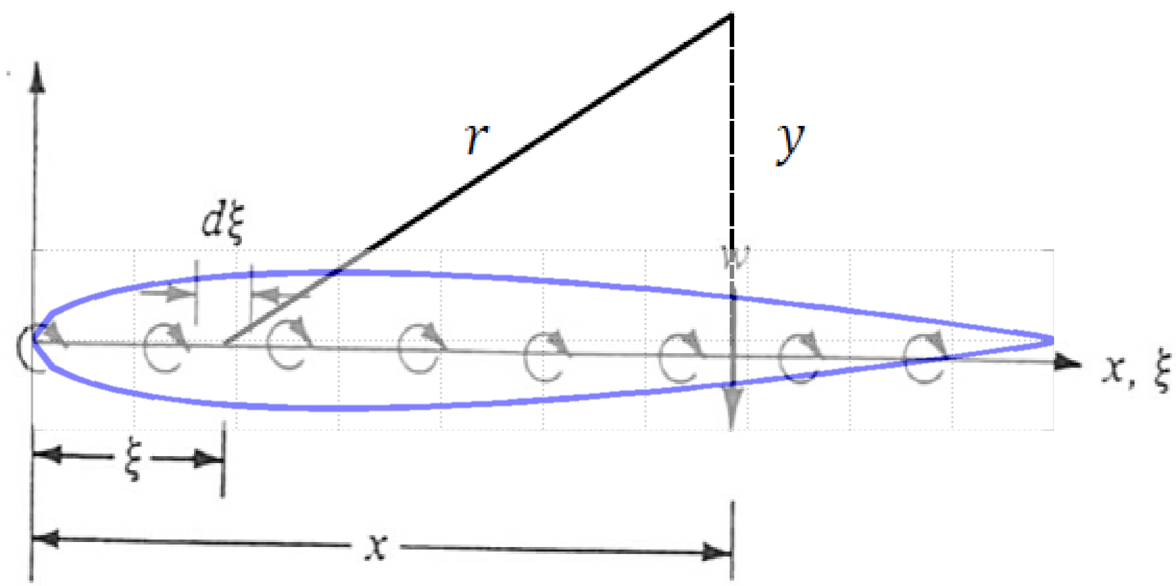
Thin Airfoil Theory

- ***The integral equation of thin airfoil theory for symmetric airfoils:***

$$\frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty\alpha;$$

- ***Solution of the equation:***

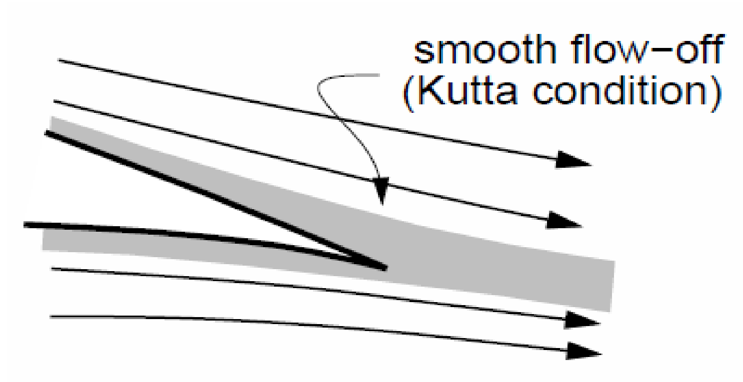
$$\gamma(\theta)=2\alpha V_\infty\frac{1+\cos\theta}{\sin\theta}$$

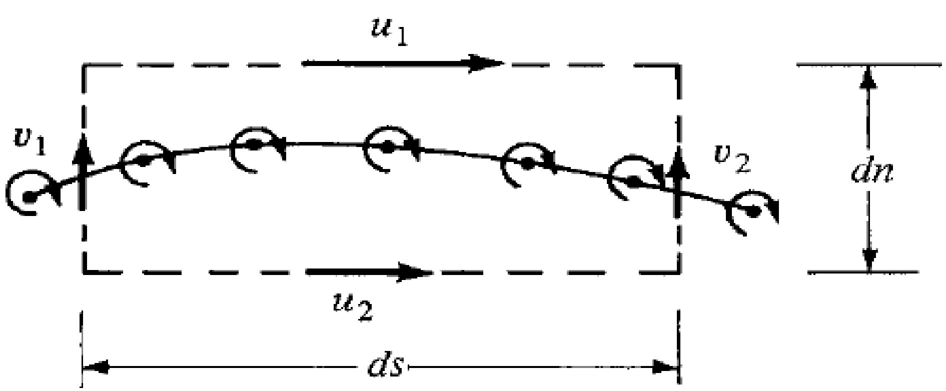


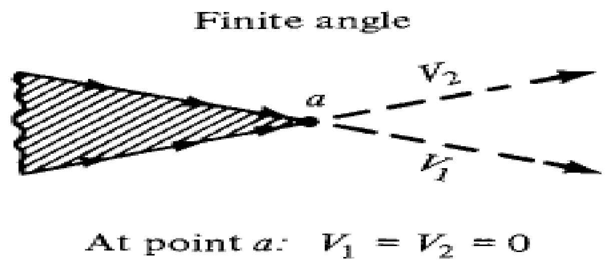
Thin Airfoil Theory

Kutta condition:

- ***For a given airfoil at a given angle of attack, the value of Γ around the airfoil is such that the flow would leaves the trailing edge smoothly.***







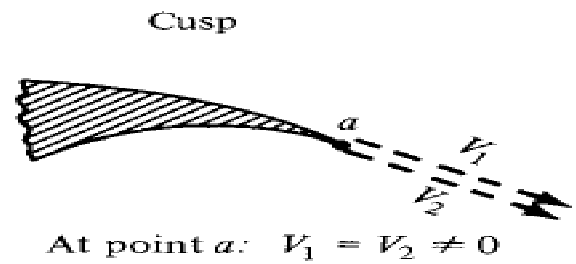
- **Case #1**

e vortex sheet

$$\gamma = u_2 - u_1$$

$$\gamma(\mathrm{TE}) = V_2 - V_1$$

$$\gamma(\mathrm{TE}) = 0$$



- **Case #2**

Thin Airfoil Theory

Is Kutta condition satisfied at TE?

$$\gamma(\theta) = 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta}$$

$$at\ airfoil\ LE: \quad \theta = \pi$$

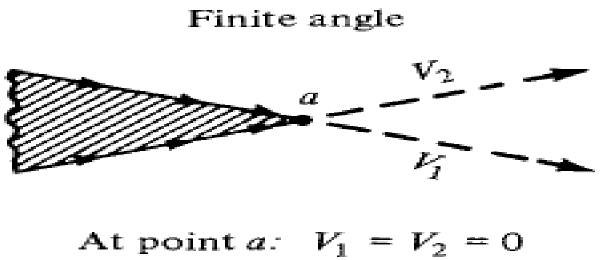
$$\Rightarrow \gamma(\pi) = 2\alpha V_\infty \frac{1+\cos\pi}{\sin\pi} = \frac{0}{0}$$

$$Therefore:$$

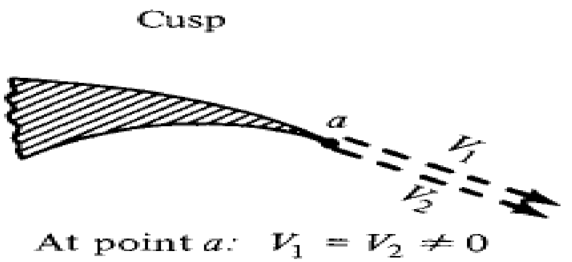
$$\gamma(\theta)\Big|_{\theta\to\pi} = \left.\frac{\dfrac{d(1+\cos\theta)}{d\theta}}{\dfrac{d(\sin\theta)}{d\theta}}\right|_{\theta\to\pi}$$

$$= \left.\frac{-\sin\theta}{\cos\theta}\right|_{\theta\to\pi} = \frac{0}{-1} = 0$$

$$\Rightarrow Kutta\ condition\ is\ satisfied\ at\ TE!$$



- **Case #1**



- **Case #2**

- In relation to the vortex sheet discontinuity

$$\gamma(\mathrm{TE}) = V_2 - V_1$$

$$\gamma(\mathrm{TE}) = 0$$