Lecture # 26: Airfoil Aerodynamics - Part 04

Symmetrical Airfoils– 02

Dr. Hui HU

Department of Aerospace Engineering

Iowa State University, 2251 Howe Hall, Ames, IA 50011-2271

Tel: 515-294-0094 / Email: huhui@iastate.edu

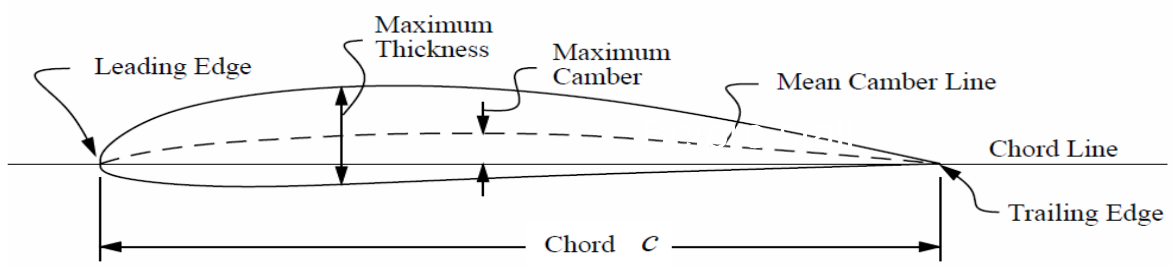
Comments and Suggestions from Quiz # 4

- *Teaching speed is fine.*
- *Less math, equations, and derivations*
- *Less congestions for PPT*
- *Prefer more writings on whiteboard*
- *More in-class practice and examples for problem solving.*
- *Enjoy in class videos.*
- *More real-life application examples.*
- *Want to know more about 2nd exam.*

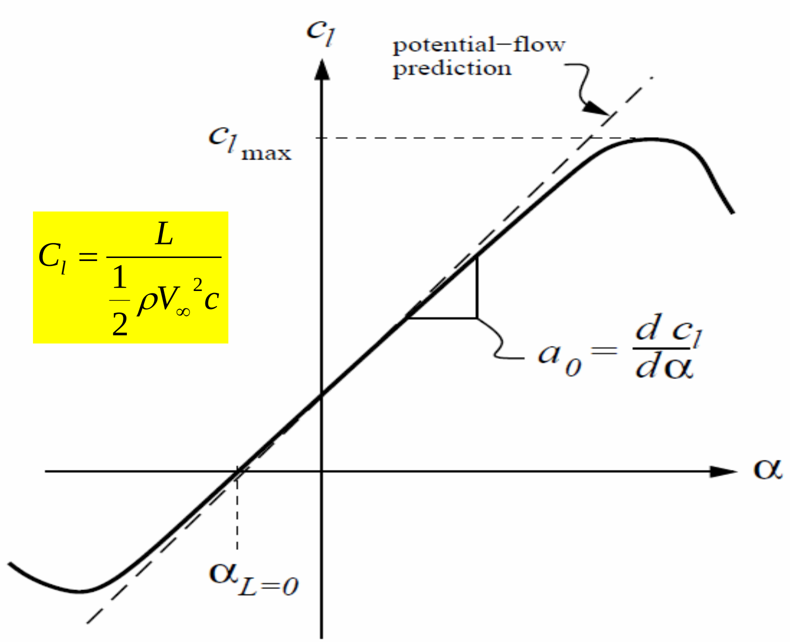
❑ Thin Airfoil Theory

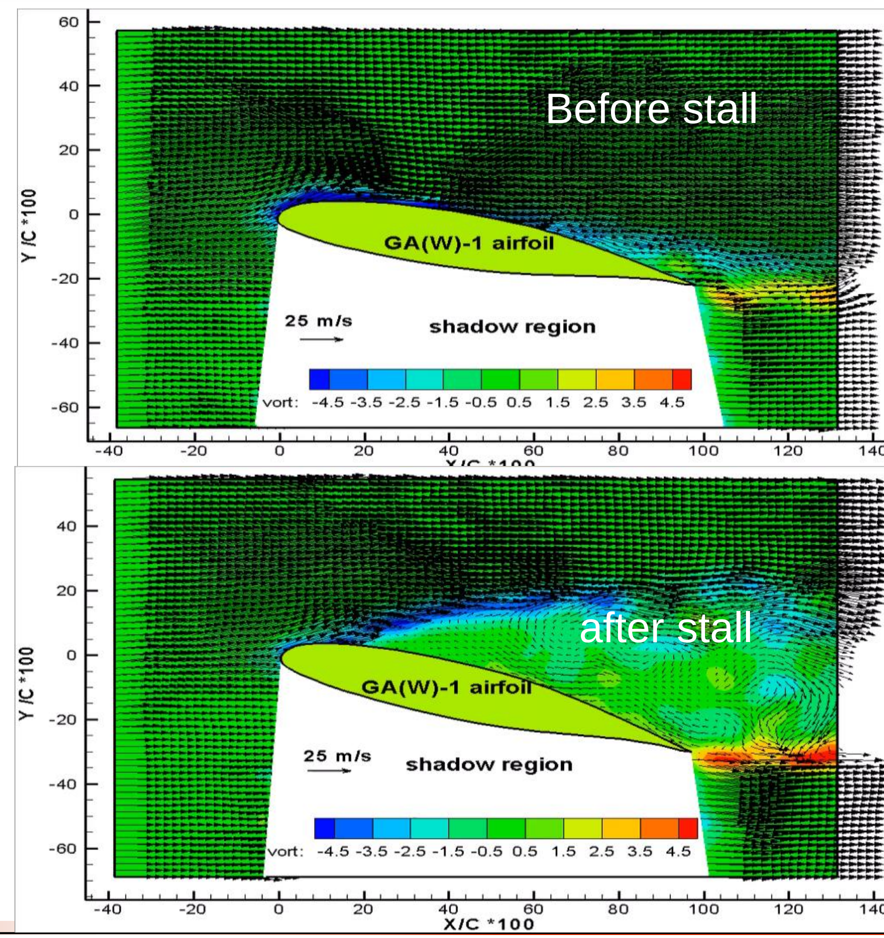
Assumptions:

- ***2-dimensions***
- ***Inviscid****
- ***Incompressible****
- ***Irrotational****
- ***Small α***
- ***Small max τ /c (i.e., airfoil thickness)***
- ***Small max z/ c ((i.e., airfoil camber)***



$$C_l = \frac{L}{\frac{1}{2}\rho V_\infty^2 c}$$



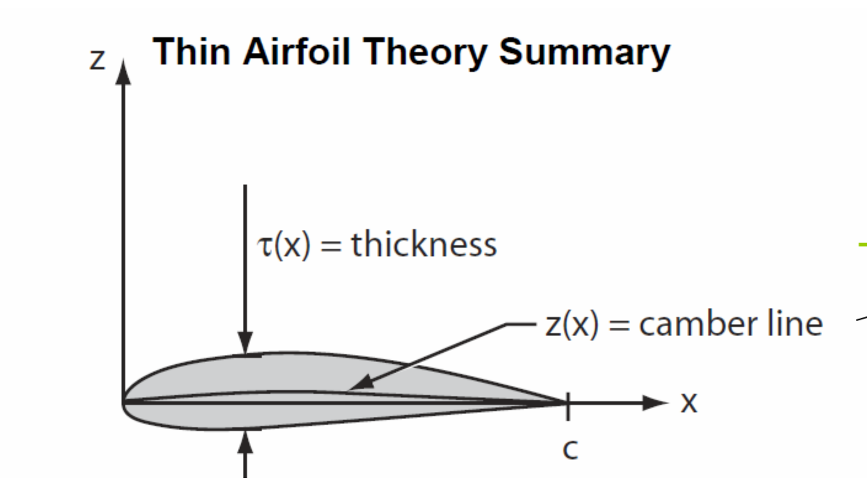


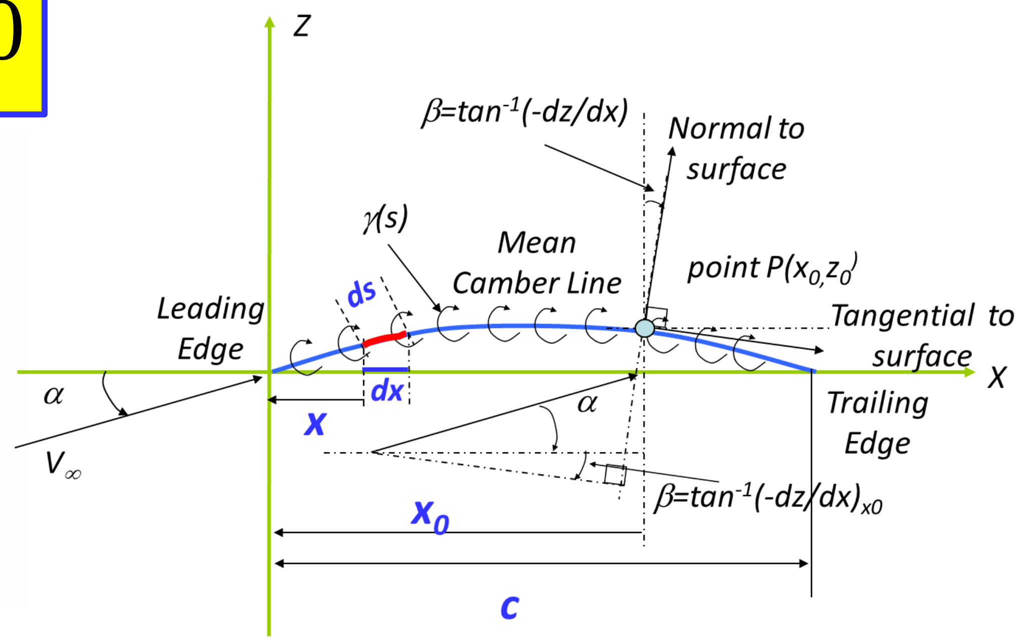
❑ Thin Airfoil Theory

Principle:

- ***Replace thin airfoil with the mean camber line (MCL) because of the small thickness and camber of the airfoil***
- ***MCL assumed to be a streamline of the flow around the thin airfoil.***
- ***To force the MCL to be a streamline, the sum of all velocity components normal to the MCL must be equal to zero.***

$$V_{\infty,n} + V_{vortex-induced,n} = 0$$

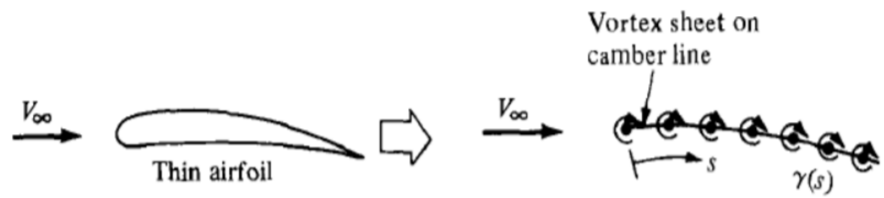




Thin Airfoil Theory

- *To force the mean camber line to be a streamline, the sum of all velocity components normal to the mcl must be equal to zero.*

$$V_{\infty,n} + V_{vortex-induced,n} = 0$$

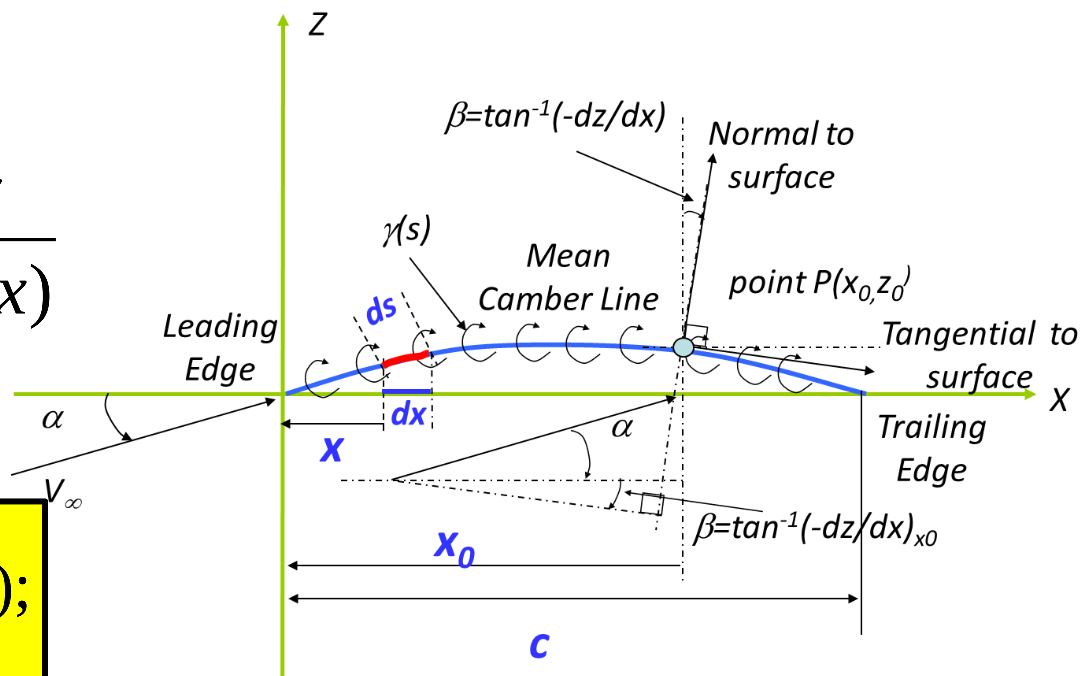


- ***Since***

$$V_{\infty,n} = V_\infty(\alpha - \frac{dz}{dx});$$

$$V_{vortex-induced,n} = -\int_0^c \frac{\gamma(x)dx}{2\pi(x_0 - x)}$$

- ***Therefore:***



$$\frac{1}{2\pi}\int_0^c \frac{\gamma(x)dx}{(x_0 - x)} = V_\infty(\alpha - \frac{dz}{dx});$$

❑ Thin Airfoil Theory

❑ ***The integral equation of thin airfoil theory:***

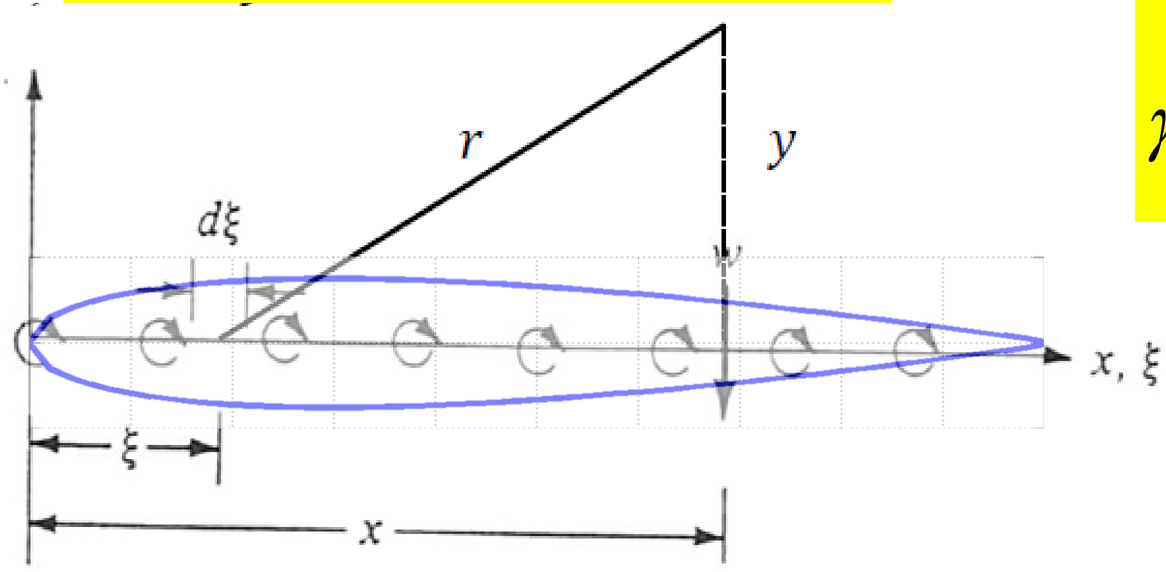
$$\frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty(\alpha-\frac{dz}{dx});$$

For a Symmetric Airfoil with : $\frac{dz}{dx}=0$

$$\frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty\alpha;$$

The solution is :

$$\gamma(\theta)=2\alpha V_\infty\frac{1+\cos\theta}{\sin\theta}$$



Thin Airfoil Theory

- ***The Kutta-Joukowski Lift Theorem*** ***states the lift per unit length of an airfoil is equal to the density (ρ) of the air times the strength of the rotation (Γ) times the velocity (V) of the air.***

$$L' = \rho V_\infty \Gamma$$

Lift generated by the airfoil

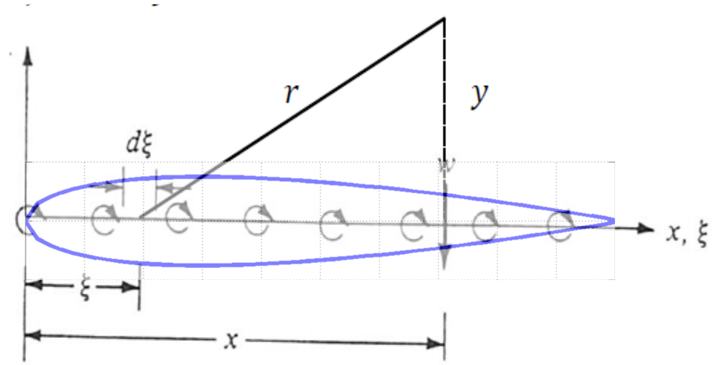
$$\Gamma = \int_{LE}^{TE} \gamma(x)dx$$

$$\because x = \frac{c}{2}(1-\cos\theta) \Rightarrow dx = \frac{c}{2}\sin\theta d\theta$$

$$\therefore \Gamma = \int_{LE}^{TE} \gamma(x)dx = \int_0^\pi \gamma(\theta)\cdot\frac{c}{2}\cdot\sin\theta\cdot d\theta = \frac{c}{2}\cdot\int_0^\pi 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta}\cdot\sin\theta d\theta$$

$$= \alpha V_\infty c\int_0^\pi (1+\cos\theta)d\theta = \alpha V_\infty c[\theta+\sin\theta]\Big|_0^\pi = \alpha V_\infty c\pi$$

$$\Rightarrow L = \rho V_\infty \Gamma = \rho V_\infty^{\ 2} c\pi\alpha$$

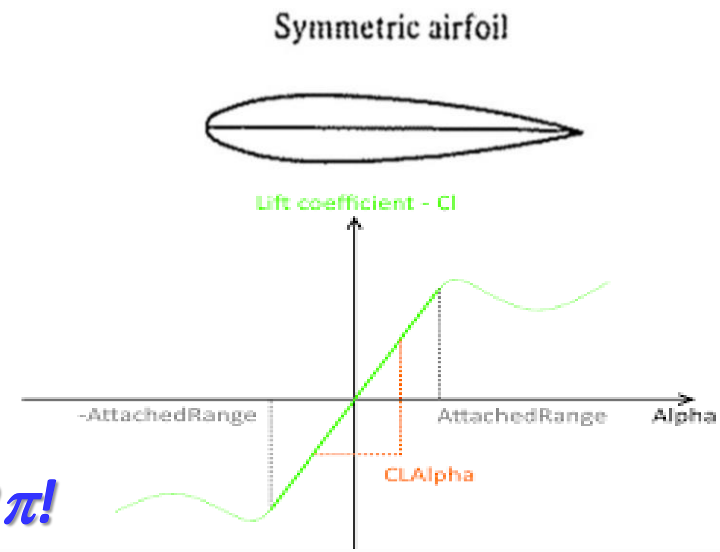


Thin Airfoil Theory

- **_Lift coefficient of symmetrical airfoil_**

$$L = \rho V_\infty \Gamma = \rho V_\infty{}^2 c \pi \alpha$$

$$C_L = \frac{L}{\frac{1}{2}\rho V_\infty{}^2 c} = \frac{\rho \alpha V_\infty{}^2 c \pi}{\frac{1}{2}\rho V_\infty{}^2 c} = 2\pi\alpha$$



- **_Slope of the Lift coefficient profile is 2π!_**

$$L = \rho V_\infty \Gamma = \rho V_\infty{}^2 c \pi \alpha$$

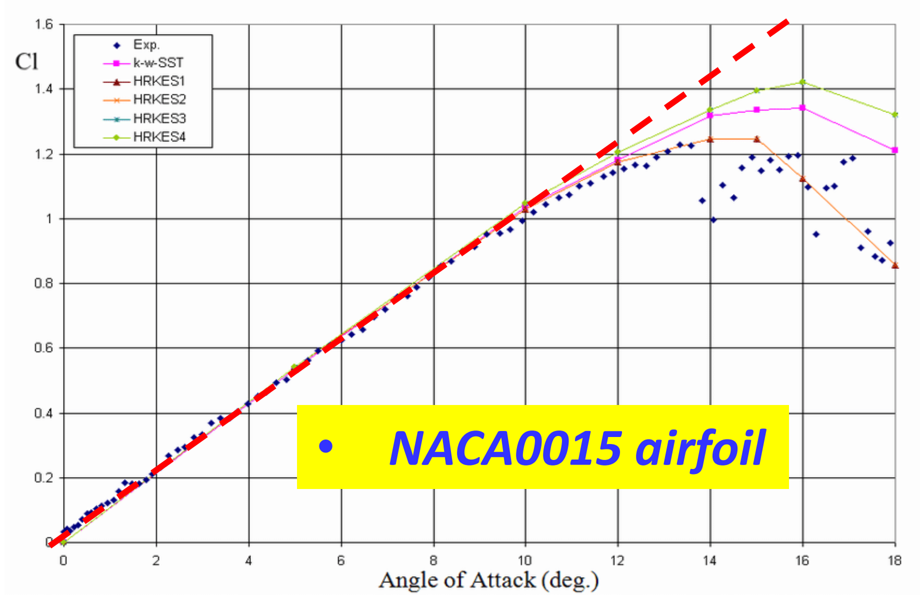
$$\because C_L = 2\pi\alpha$$

$$\therefore$$

$$\alpha = 0 \quad \Rightarrow \quad C_L = 0$$

$$\alpha = 5^o \quad \Rightarrow \quad C_L = 0.55$$

$$\alpha = 10^o \quad \Rightarrow \quad C_L = 1.10$$



❑ Thin Airfoil Theory

❑ Momentum about the airfoil leading edge

$$M_{LE} = -\int_{LE}^{TE} x dL; \quad dL = \rho V_\infty \gamma(x) dx$$

$$\Rightarrow M_{LE} = -\int_{LE}^{TE} x \rho V_\infty \gamma(x) dx$$

$$\because x = \frac{c}{2}(1-\cos\theta); \quad \gamma(\theta) = 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta}$$

$$\therefore dx = \frac{c}{2}\sin\theta d\theta$$

$$\Rightarrow M_{LE} = -\int_{LE}^{TE} x \rho V_\infty \gamma(x) dx$$

$$= -\int_{LE}^{TE} \frac{c}{2}(1-\cos\theta)\rho V_\infty 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta}\frac{c}{2}\sin\theta d\theta$$

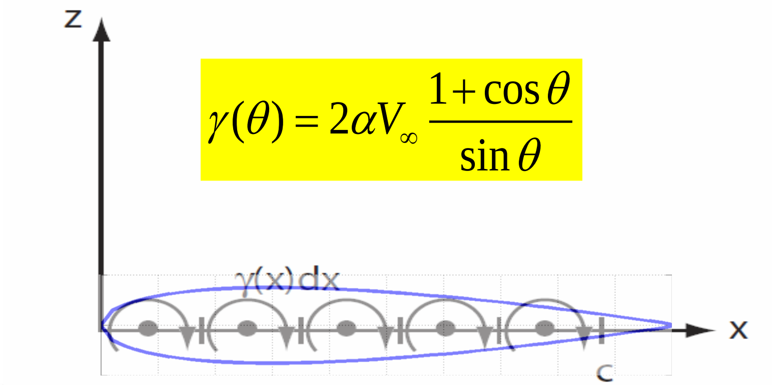
$$= -\frac{\rho V_\infty{}^2 \alpha c^2}{2}\int_0^\pi (1-\cos\theta)(1+\cos\theta) d\theta$$

$$= -\frac{\rho V_\infty{}^2 \alpha c^2}{2}\int_0^\pi \sin^2\theta d\theta = -\frac{\rho V_\infty{}^2 \alpha c^2}{2}\int_0^\pi \frac{1-\cos 2\theta}{2} d\theta$$

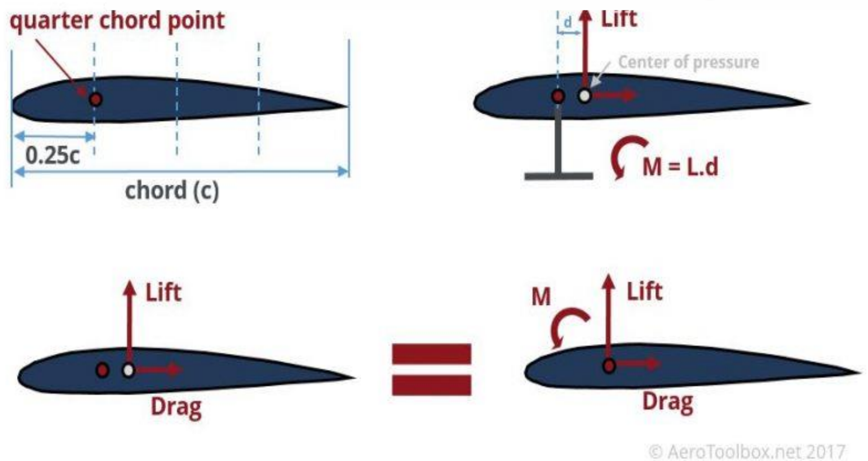
$$= -\frac{\rho V_\infty{}^2 \alpha c^2}{2}[\int_0^\pi [\frac{1}{2} d\theta - \int_0^\pi \frac{\cos 2\theta}{4} d2\theta] = -\frac{\rho V_\infty{}^2 \alpha c^2}{2}[\frac{\pi}{2} - 0]$$

$$= -\frac{\rho V_\infty{}^2 \alpha c^2 \pi}{4}$$

$$C_{M,LE} = \frac{M_{LE}}{\frac{1}{2}\rho V_\infty{}^2 c^2} = -\frac{\rho \alpha V_\infty{}^2 c^2 \pi}{4 \cdot \frac{1}{2}\rho V_\infty{}^2 c^2} = -\frac{\pi\alpha}{2}$$



$$\gamma(\theta) = 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta}$$



$$\alpha = 0 \quad \Rightarrow \quad C_{M,LE} = 0$$

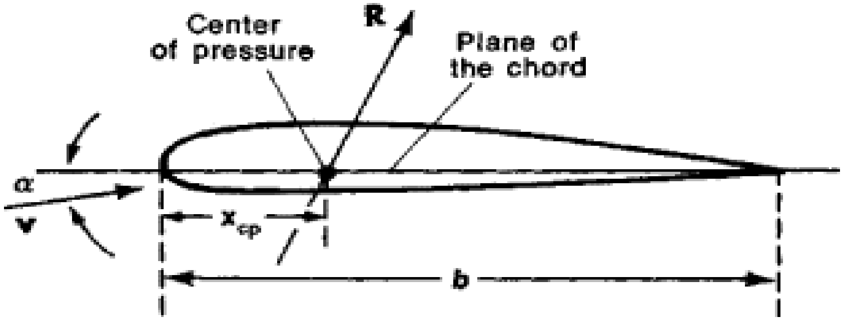
$$\alpha = 5^o \quad \Rightarrow \quad C_{M,LE} = 0.11$$

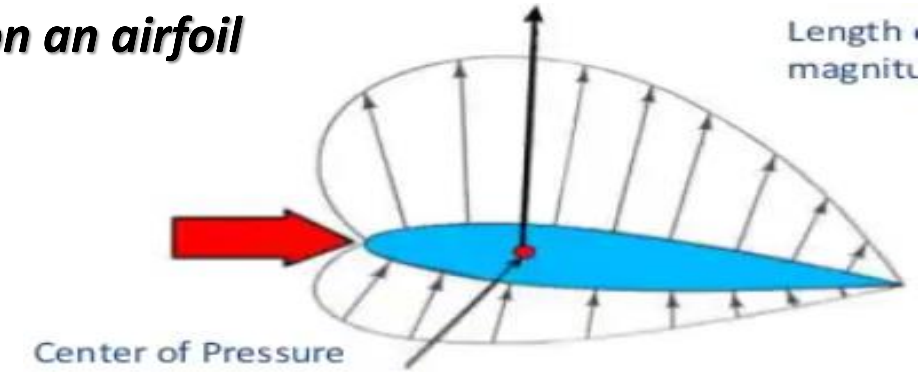
$$\alpha = 10^o \quad \Rightarrow \quad C_{M,LE} = 0.22$$

❑ Thin Airfoil Theory

❑ Thin Airfoil Theory

❑ *Center of Lift or center of pressure on an airfoil*

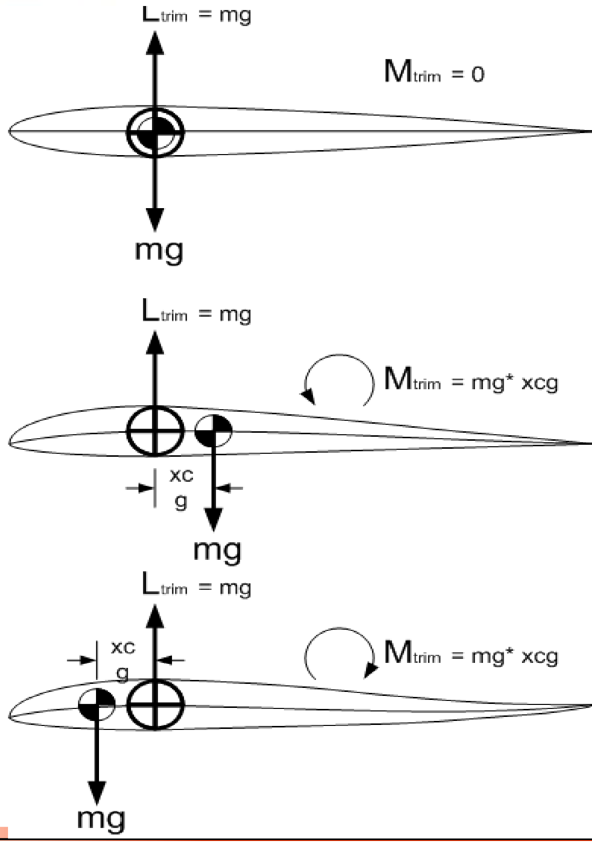




$$M_{LE} = -LX_{CP}$$

$$\because L = \pi\alpha c\rho V_\infty^2 \ and \quad M_{LE} = -\frac{\rho V_\infty^2 \alpha c^2 \pi}{4}$$

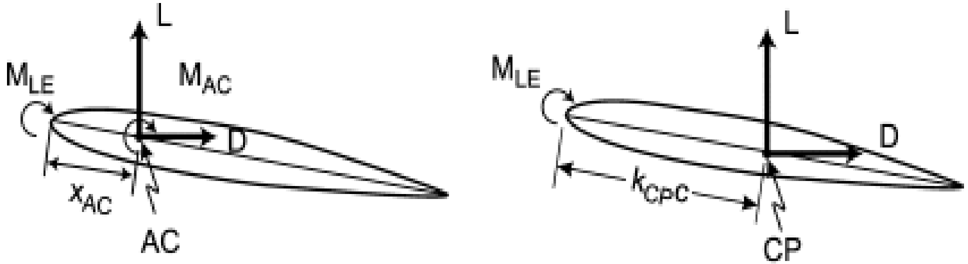
$$\Rightarrow X_{CP} = \frac{C_L}{4}$$



- ***Center of Lift* is located at quarter chord point (i.e., C/4) of an airfoil.**

❑ Thin Airfoil Theory for Symmetrical Airfoils

❑ *Center of Lift or center of pressure on an airfoil*



$$M'_{LE} = M'_{c/4} - L'_{c/4} \quad \text{(b)}$$

$$C_{m,LE} = C_{m,c/4} - C_l/4$$

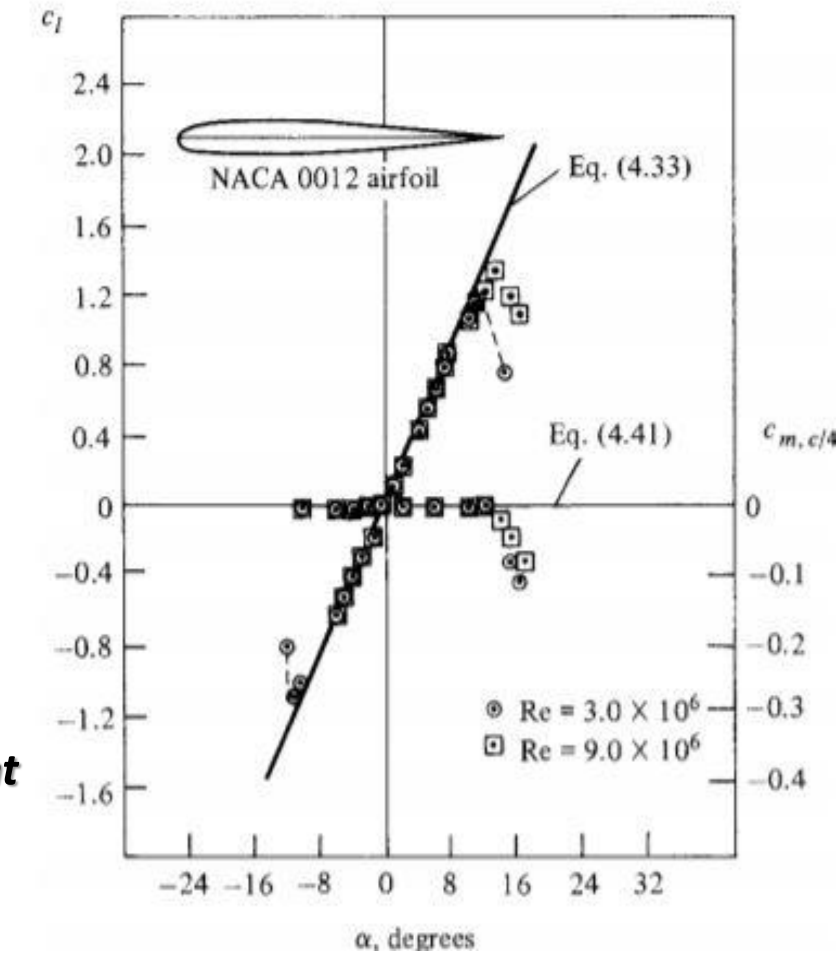
$$C_{m,LE} = -C_l/4$$

$$C_{m,c/4} = 0$$

$C_{m,c/4}$ is equal to zero for all values of α.

- ***C/4 is the Aerodynamic center,* which is the point on an airfoil about which the aerodynamically generated moment is independent of angel of attack.**

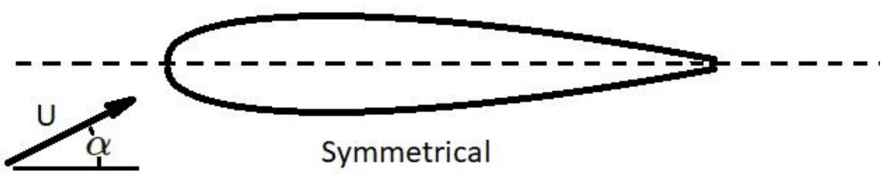
- ***NACA0012 airfoil***

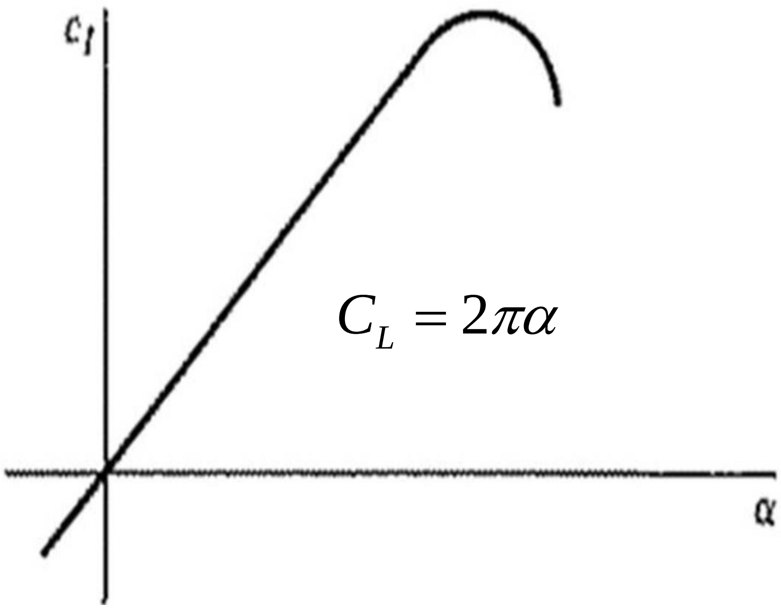


❑ Thin Airfoil Theory for Symmetrical Airfoils

Example problem:

- *For a symmetrical airfoil with a chord length of 2 meters placed in an air stream of 50 m/s at standard sea level conditions (ρ=12.3 kg/m3), if the lift per unit span is 1353 N/m, what is the angle of attack (in degree) of the airfoil model?*





$$C_L = 2\pi\alpha$$

❑ Thin Airfoil Theory for Symmetrical Airfoils

Solution:

The lift can be generated by per unit span of a thin airfoil at the AOA of α can be:

$$L' = \rho V_\infty^{\ 2} \cdot c \cdot \alpha \ \pi \quad \Rightarrow \quad \alpha = \frac{L'}{\rho V_\infty^{\ 2} \cdot c \cdot \pi}$$

For the present conditions:

$$L' = 1353N; \quad \rho = 1.23; \quad V_\infty = 50\ m/s^2; \quad c = 2; \quad \pi = 3.14$$

Therefore, $$\alpha = \frac{L'}{\rho V_\infty^{\ 2} \cdot c \cdot \pi} = \frac{1353}{1.23 * 50 * 50 * 2 * 3.14} = 0.07\ rad = 4.0\deg$$

The angle of attack of the airfoil is 0.07 rad or 4.0 degree

Or it can be solved by:

$$C_l = \frac{L'}{\frac{1}{2}\rho V_\infty^{\ 2} \cdot c} = \frac{1353}{\frac{1}{2} * 1.23 * 50 * 50 * 2} = 0.44$$

$$C_l = 2\alpha\ \pi = 0.44 \quad \Rightarrow \quad \alpha = \frac{0.44}{2 * 3.14} = 0.07\ rad = 4.0\deg$$