Lecture # 28: Airfoil Aerodynamics – Part 06 : Cambered Airfoil #02

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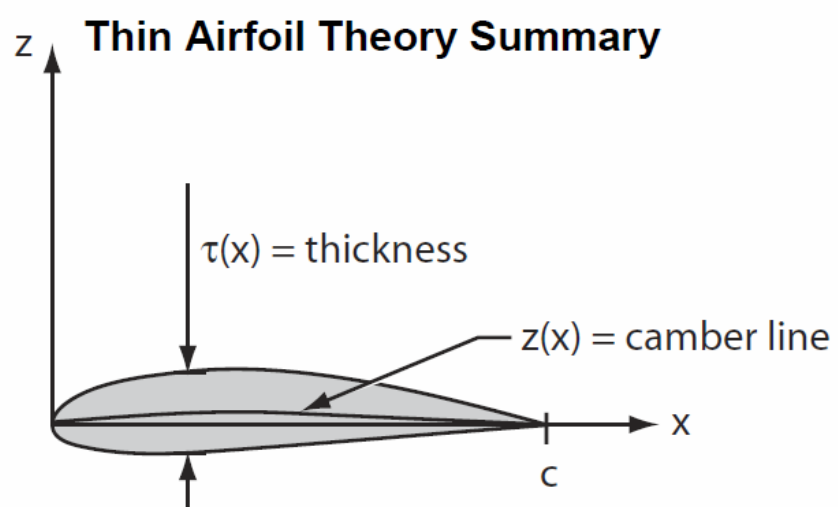
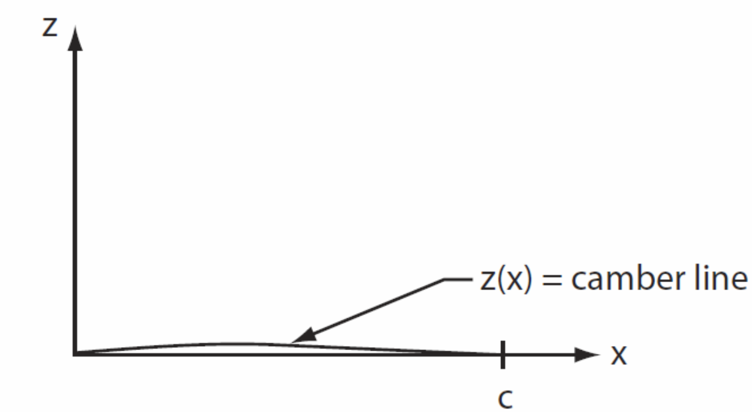
Thin Airfoil Theory

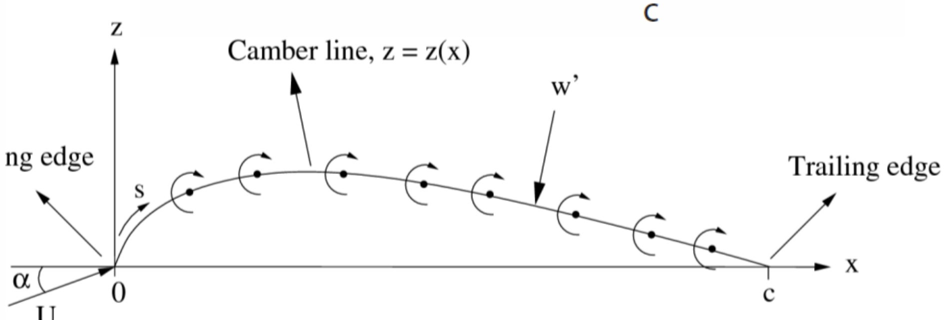
Principle:

- ***Replace thin airfoil with the mean camber line (MCL) because of the small thickness and camber of the airfoil***
- ***MCL assumed to be a streamline of the flow around the thin airfoil.***
- ***To force the MCL to be a streamline, the sum of all velocity components normal to the MCL must be equal to zero.***

$$V_{\infty,n} + V_{vortex-induced,n} = 0$$





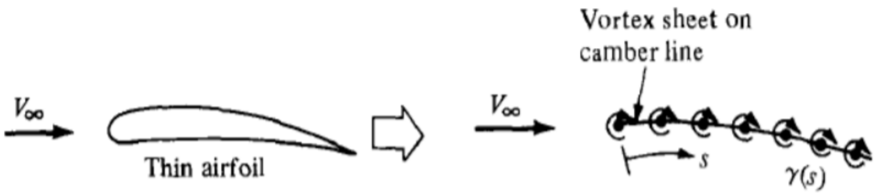


(b) Vorticity distribution on the camber line of the airfoil

❑ Thin Airfoil Theory

- *To force the mean camber line to be a streamline, the sum of all velocity components normal to the mcl must be equal to zero.*

$$V_{\infty,n} + V_{vortex-induced,n} = 0$$



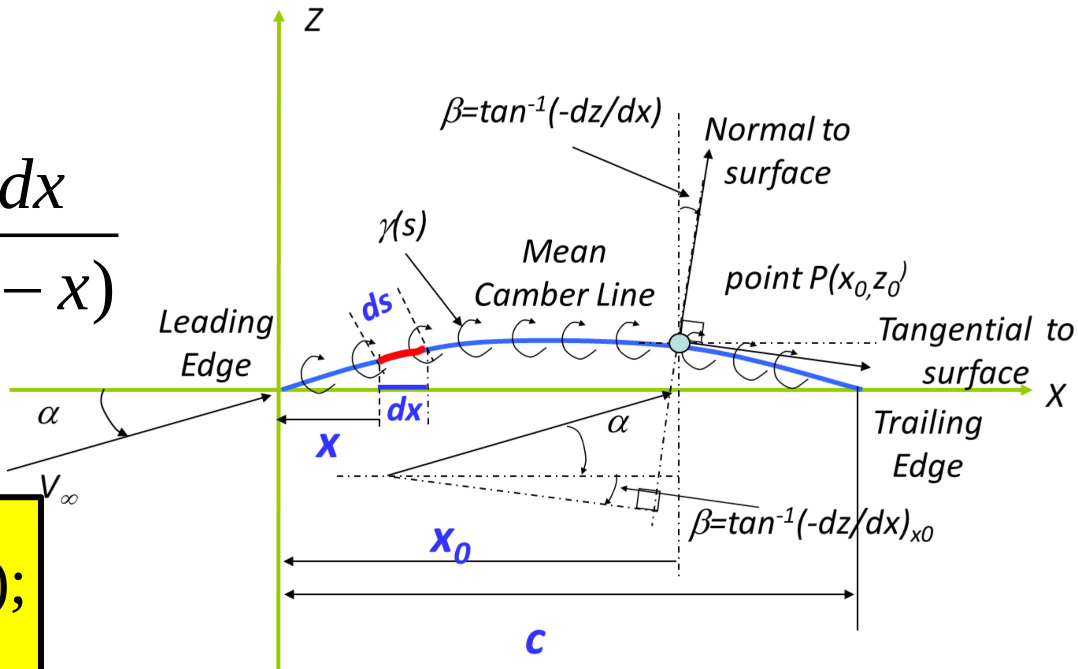
- *Since*

$$V_{\infty,n} = V_\infty(\alpha - \frac{dz}{dx});$$

$$V_{vortex-induced,n} = -\int_0^c \frac{\gamma(x)dx}{2\pi(x_0 - x)}$$

- *Therefore:*

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(x)dx}{(x_0 - x)} = V_\infty(\alpha - \frac{dz}{dx});$$



Thin Airfoil Theory for Cambered Airfoils

- ***The integral equation for a cambered airfoil:***

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(x)dx}{(x_0 - x)} = V_\infty(\alpha - \frac{dz}{dx});$$

- ***Make a transformation:***

$$x = \frac{c}{2}(1 - \cos\theta);$$

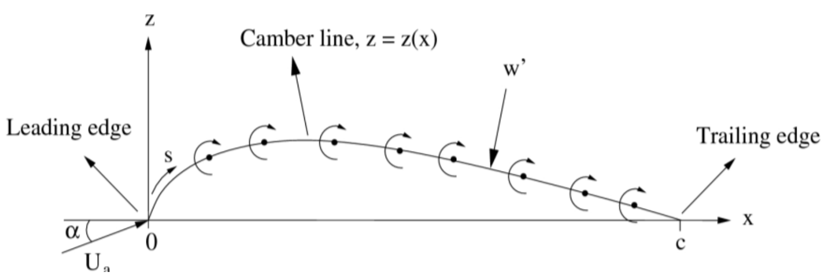
$$If\ \ \theta = 0 \quad \Rightarrow \quad x = 0, \quad at\ Leading\ \ Edge$$

$$If\ \ \theta = \pi \quad \Rightarrow \quad x = c, \quad at\ Trailing\ \ Edge$$

- ***The integral equation of cambered airfoil.***

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta - \cos\theta_0} = (V_\infty\alpha - \frac{dz}{dx})$$

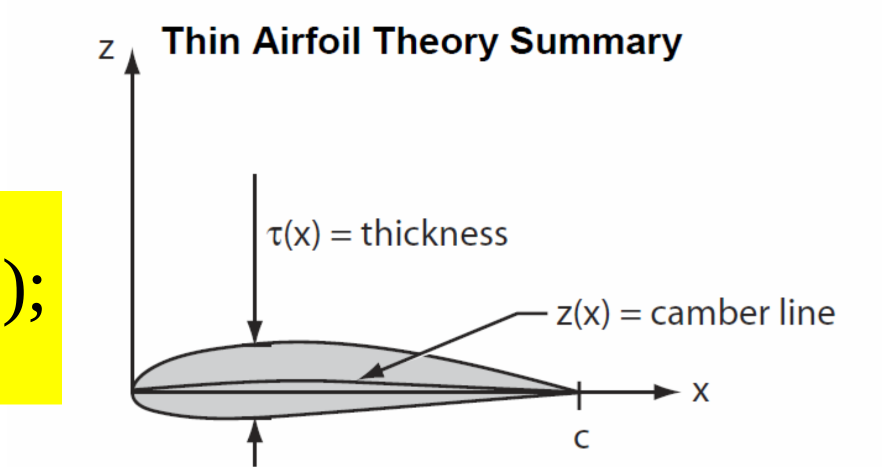
$$\frac{dz}{dx} \neq 0$$



Thin Airfoil Theory for Cambered Airfoils

- **The integral equation for a cambered airfoil:**

$$\frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty(\alpha-\frac{dz}{dx});$$

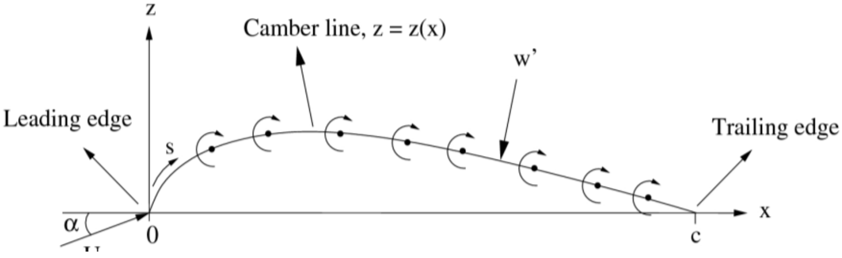


- **The solution for a cambered airfoil will be:**

$$\gamma(\theta)=2V_\infty[(\alpha-A_0)\frac{1+\cos\theta}{\sin\theta}+\sum_{n=1}^{\infty}A_n\sin(n\theta)]$$

$$A_0=\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}d\theta$$

$$A_n=\frac{2}{\pi}\int_0^{\pi}f(\theta)\cos(n\theta)d\theta=\frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos(n\theta)d\theta$$



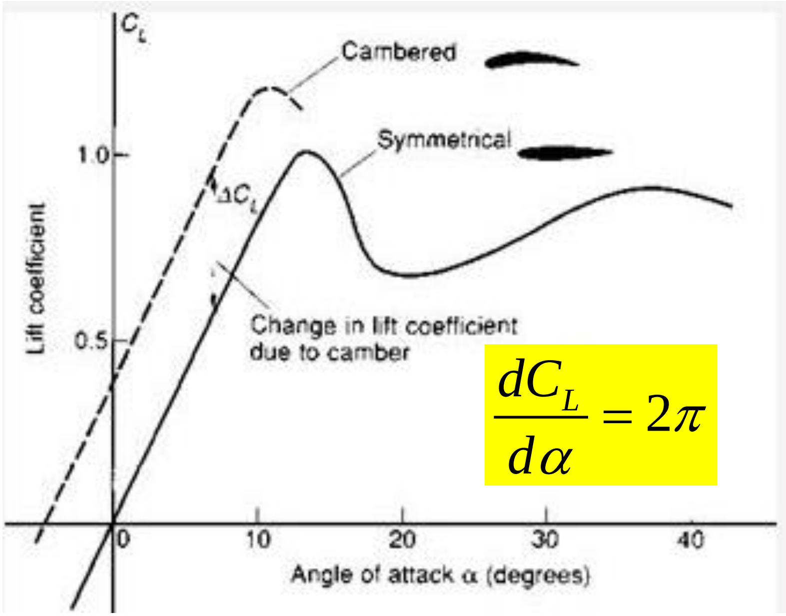
Thin Airfoil Theory – Cambered Airfoil

Lift coefficient of symmetrical airfoil

$$C_L = 2\pi[A_0 + A_1/2] = 2\pi[\alpha - \alpha_{L0}]$$

$$\alpha_{L0} = \frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(1-\cos\theta)d\theta$$

- ***The value of α_{L0} will be determined if the MCL is given for an airfoil, which is not a function of α.***
- ***The slope of the Lift coefficient profile is still 2π.***



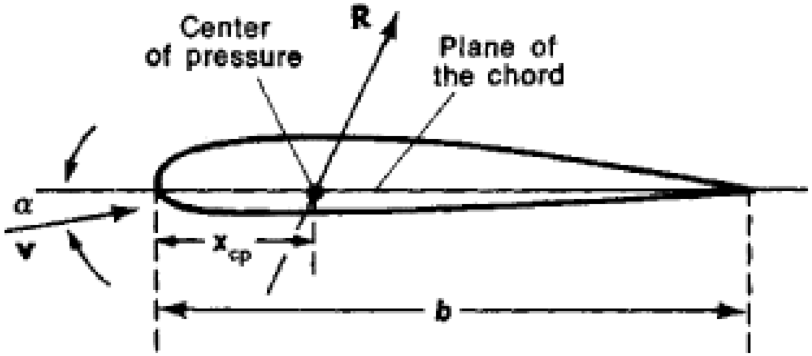
$$\frac{dC_L}{d\alpha} = 2\pi$$

Moment about the airfoil leading edge

$$C_{M,LE} = -[\frac{C_L}{4} + \frac{\pi}{4}(A_1 - A_2)]$$

$$C_{M,c/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{CP} = \frac{1}{4}[c + \frac{\pi c}{C_L}(A_1 - A_2)]$$



- ***No dependency on α for aerodynamic center***

Thin Airfoil Theory – Cambered Airfoil

- For a thin airfoil with chord length of $c = 1.5\ m$ flying at altitude 1000 m ($\rho_\infty = 1.112\ kg/m^3$) with $V_\infty = 100\ m/s$ and $\alpha = 2°$
  - Find L' if $\alpha_{L=0} = -\frac{\pi}{60}$
  - Find M'_{LE} if $c_{m_{c/4}} = -0.3$

❑ Thin Airfoil Theory – Cambered Airfoil

- For a thin airfoil with chord length of $c = 1.5\ m$ flying at altitude 1000 m ($\rho_\infty = 1.112\ kg/m^3$) with $V_\infty = 100\ m/s$ and $\alpha = 2°$
  - Find L' if $\alpha_{L=0} = -\frac{\pi}{60}$
  - Find M'_{LE} if $c_{m_{c/4}} = -0.3$

$$c_l = 2\pi(\alpha - \alpha_{L=0}) = 2\pi\left(2 \times \frac{\pi}{180} + \frac{\pi}{60}\right) = \frac{\pi^2}{18} = 0.5483$$

$$L' = c_l\left(\frac{1}{2}\rho_\infty V_\infty^2 c\right) = 0.5483 \times \frac{1}{2} \times 1.112 \times 100^2 \times 1.5$$

$$L' = 4573\ N/m$$

$$c_{m_{LE}} = c_{m_{c/4}} - \frac{1}{4}c_l = -0.3 - \frac{1}{4} \times 0.5483 \rightarrow c_{m_{LE}} = -0.4371$$

$$M'_{LE} = \frac{1}{2}\rho_\infty V_\infty^2 c^2 c_{m_{LE}} = \frac{1}{2} \times 1.112 \times 100^2 \times 1.5^2 \times -0.4371$$

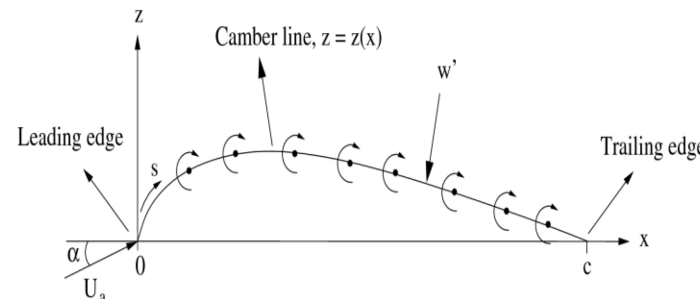
$$M'_{LE} = -5468\ N$$

❑ Thin Airfoil Theory – Cambered Airfoil

Cambered airfoil example

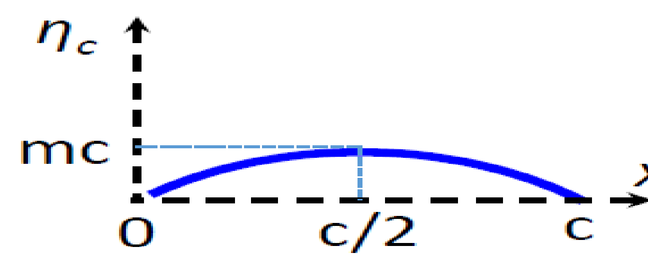
Let's consider a thin airfoil with the mean camber line equation given as:

$$\eta_c(x) = 4mc\left(\frac{x}{c}\right)\left(1 - \frac{x}{c}\right), \qquad m \geq 0$$



First calculate $\frac{d\eta_c(x)}{dx}$ (with η_c still as a function of x)

$$\frac{d\eta_c(x)}{dx} = 4m\left(1 - \frac{2x}{c}\right)$$



Then make the change of variable

$$x = \frac{c}{2}(1 - \cos\theta)$$

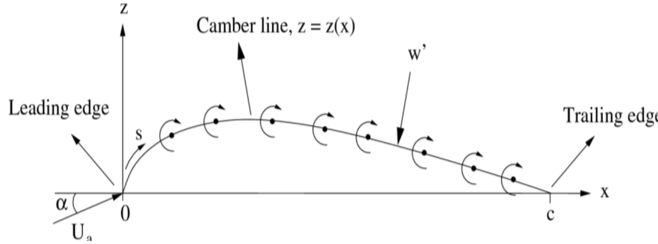
$$\frac{d\eta_C(\theta)}{dx} = 4m[1 - (1 - \cos\theta)]$$

$$\frac{d\eta_c(\theta)}{dx} = 4m\cos\theta$$

$$\eta_c(0) = 0$$
$$\eta_c(c) = 0$$
$$\eta_c\left(\frac{c}{2}\right) = mc$$

Thin Airfoil Theory – Cambered Airfoil

Finding coefficients



Recall

$$\frac{d\eta_c}{dx} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

There are two methods for finding A_n coefficients for a given $d\eta_c/dx$:

Method A: Read the coefficients

Only works when $d\eta_c/dx$ can be written as a single function of $\cos n\theta$ over the entire airfoil

Method B: Use the integrals

This is the general method and always works

❑ Thin Airfoil Theory – Cambered Airfoil

Lift coefficient

$$\eta_c(x) = 4mc\left(\frac{x}{c}\right)\left(1-\frac{x}{c}\right), \qquad m \geq 0$$

Method A

$$\frac{d\eta_c}{dx} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta = A_0 + A_1 \cos\theta + A_2 \cos 2\theta + \cdots$$

here

$$\frac{d\eta_c}{dx} = 4m\cos\theta$$

Compare the two

$$A_0 = 0\,, A_1 = 4m\,, A_2 = 0$$

Then

$$\alpha_{L=0} = A_0 - \frac{A_1}{2} = -\frac{4m}{2} = -2m < 0$$

$$c_l = 2\pi(\alpha + 2m)$$

❑ Thin Airfoil Theory – Cambered Airfoil

Method B

$$\frac{d\eta_c}{dx} = 4m\cos\theta$$

$$A_0 = \frac{1}{\pi}\int_0^{\pi}\frac{d\eta_c(\theta)}{dx}d\theta = \frac{1}{\pi}\int_0^{\pi}4m\cos\theta\, d\theta$$

$$A_0 = \frac{4m}{\pi}(\sin\pi - \sin 0) = 0$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi}\frac{d\eta_c}{dx}\cos\theta\, d\theta = \frac{2}{\pi}\int_0^{\pi}4m\cos^2\theta\, d\theta$$

$$A_1 = \frac{4m}{\pi}\int_0^{\pi}(1+\cos 2\theta)\, d\theta = \frac{4m}{\pi}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi} = 4m$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi}\frac{d\eta_c}{dx}\cos 2\theta\, d\theta = \frac{2}{\pi}\int_0^{\pi}4m\cos\theta\cos 2\theta\, d\theta$$

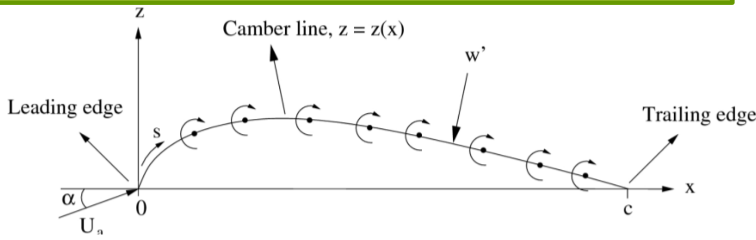
$$A_2 = \frac{8m}{\pi}\int_0^{\pi}\cos\theta\cos 2\theta\, d\theta = 0$$

Same for A_n , $n \geq 2$

$$\gamma(\theta) = 2V_\infty[(\alpha - A_0)\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty}A_n\sin(n\theta)]$$

$$A_0 = \frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}d\theta$$

$$A_n = \frac{2}{\pi}\int_0^{\pi}f(\theta)\cos(n\theta)d\theta = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos(n\theta)d\theta$$



$$\alpha_{L=0} = A_0 - \frac{A_1}{2} = -\frac{4m}{2} = -2m < 0$$

$$c_l = 2\pi(\alpha + 2m)$$

❑ Thin Airfoil Theory – Cambered Airfoil

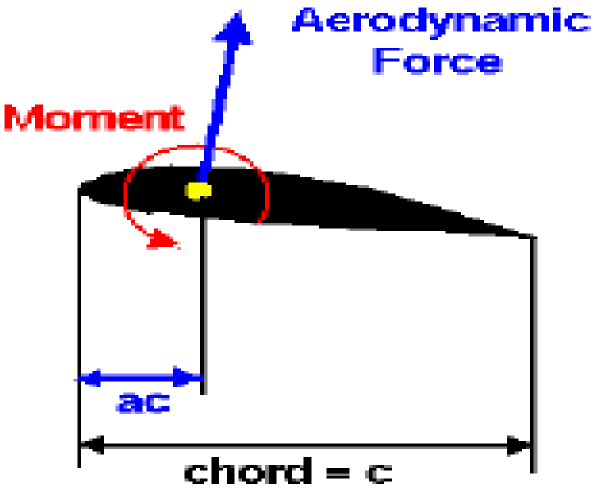
Moment coefficient and center of pressure

Also

$$c_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}(0 - 4m)$$

$$c_{m_{c/4}} = -\pi m < 0$$

This is a nose-down pitching moment



Center of pressure

$$x_{cp} = \frac{c}{4}\left(1 + \frac{\pi(A_1 - A_2)}{c_l}\right) = \frac{c}{4}\left(1 + \frac{\pi(4m - 0)}{2\pi(\alpha + 2m)}\right)$$

$$x_{cp} = \frac{c}{4}\left(1 + \frac{2m}{\alpha + 2m}\right)$$

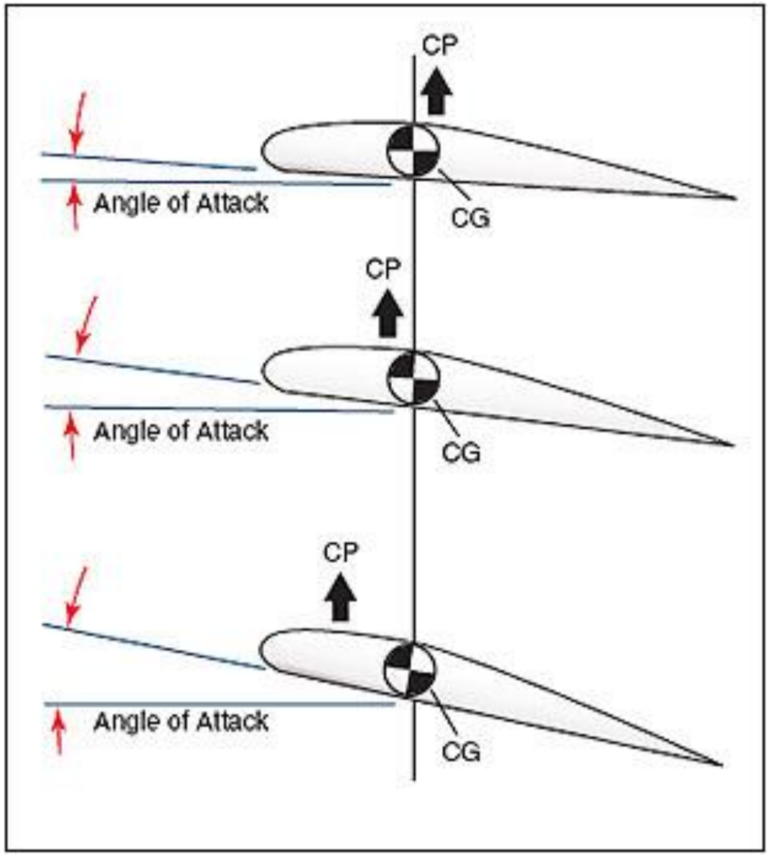
❑ Thin Airfoil Theory – Cambered Airfoil

Center of pressure

$$x_{cp} = \frac{c}{4}\left(1 + \frac{2m}{\alpha + 2m}\right)$$

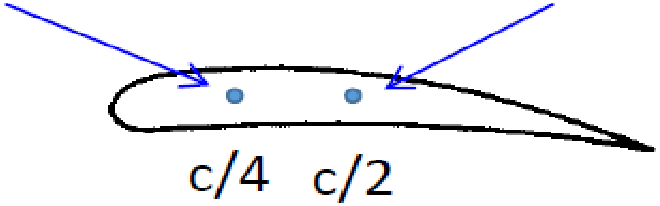
$$\alpha = 0 \Longrightarrow x_{cp} = \frac{c}{2}$$

$$\alpha \rightarrow \infty \Longrightarrow x_{cp} = \frac{c}{4}$$



x_{cp} moves here as α increases

x_{cp} starts here at $\alpha = 0$



❑ Thin Airfoil Theory – Cambered Airfoil

The NACA 4412 airfoil has a mean camber line given by

$$\frac{\eta_c}{c} = \begin{cases} 0.25\left[0.8\dfrac{x}{c} - \left(\dfrac{x}{c}\right)^2\right] & \text{for } 0 \le \dfrac{x}{c} \le 0.4 \\ 0.111\left[0.2 + 0.8\dfrac{x}{c} - \left(\dfrac{x}{c}\right)^2\right] & \text{for } 0.4 \le \dfrac{x}{c} \le 1 \end{cases}$$

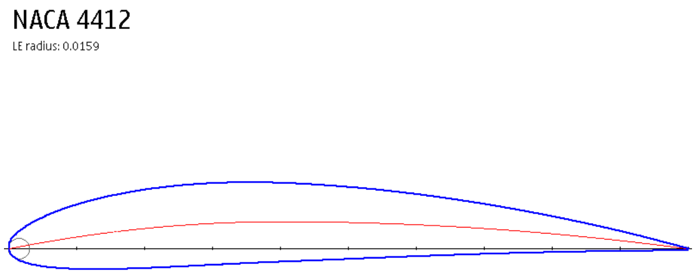
Using thin airfoil theory, calculate

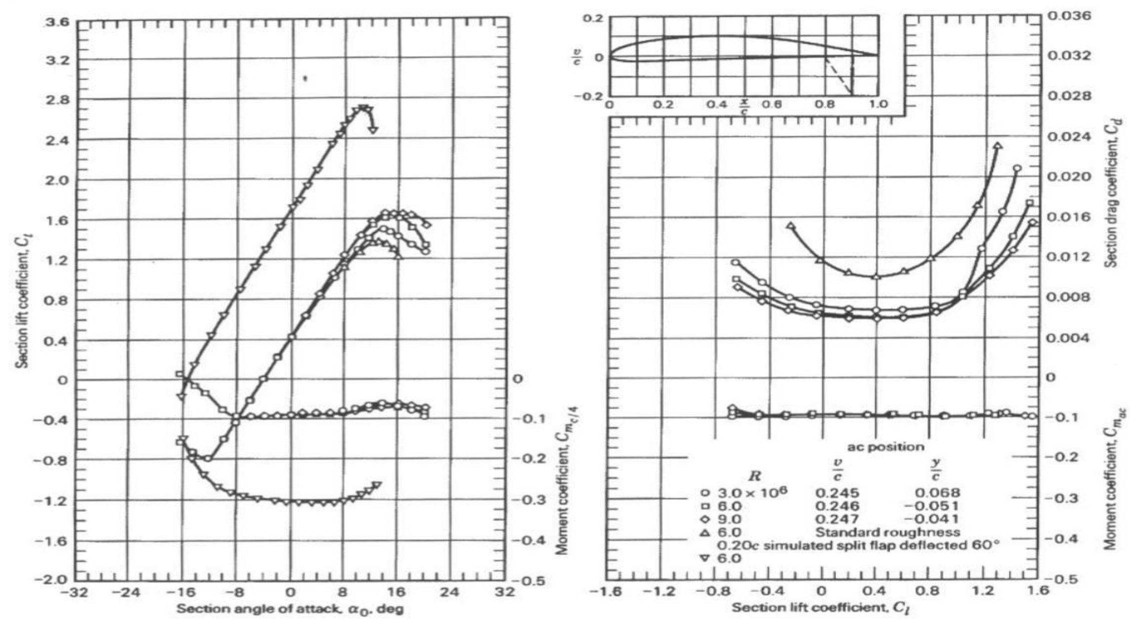
a. $\alpha_{L=0}$ and c_l when $\alpha = 3°$.

b. $c_{m,c/4}$ and x_{cp}/c for $\alpha = 3°$.

c. Compare the results of part (a) and (b) with experimental data of NACA 4412 airfoil (see plots below and quiz 7).

d. Lift per unit length of span and circulation for an airfoil with chord length of 2 m flying at a standard altitude of 3 km and velocity of 60 m/s (same angle of attack of 3°).





Aerodynamic characteristics of the NACA 4412 airfoil.

Thin Airfoil Theory – Cambered Airfoil

$$\frac{d\eta_c}{dx} = \frac{d(\eta_c/c)}{d(x/c)} = \begin{cases} 0.25\left[0.8 - 2\left(\frac{x}{c}\right)\right] = 0.2 - 0.5\frac{x}{c} & \text{for } 0 \le \frac{x}{c} \le 0.4 \\ 0.111\left[0.8 - 2\left(\frac{x}{c}\right)\right] = 0.089 - 0.222\frac{x}{c} & \text{for } 0.4 \le \frac{x}{c} \le 1 \end{cases}$$

For $\frac{x}{c} = 0.4$ & $\frac{x}{c} = \frac{1}{2}(1 - \cos\theta) \rightarrow \cos\theta = 0.2 \rightarrow \theta = 1.369\ rad$

$$\frac{d\eta_c}{dx} = \begin{cases} 0.2 - 0.25(1 - \cos\theta) = -0.05 + 0.25\cos\theta & \text{for } 0 \le \theta \le 1.369 \\ 0.089 - 0.111(1 - \cos\theta) = -0.0223 + 0.111\cos\theta & \text{for } 1.369 \le \theta \le \pi \end{cases}$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \frac{d\eta_c(\theta)}{dx} d\theta$$
$$= \frac{1}{\pi}\int_0^{1.369} (-0.05 + 0.25\cos\theta) d\theta + \frac{1}{\pi}\int_{1.369}^{\pi} (-0.0223 + 0.111\cos\theta) d\theta = 0.0089$$

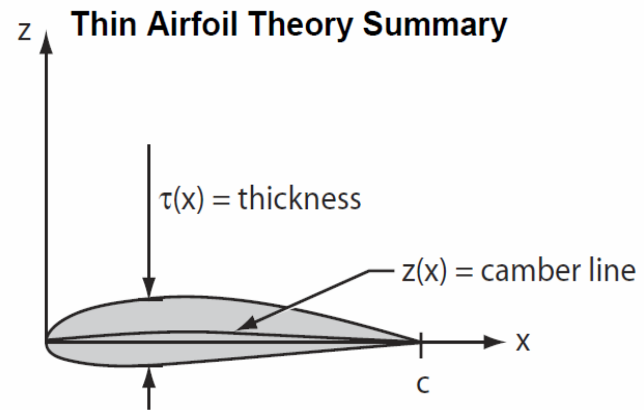
$$A_1 = \frac{2}{\pi}\int_0^\pi \frac{d\eta_c(\theta)}{dx} \cos\theta\, d\theta$$
$$= \frac{1}{\pi}\int_0^{1.369} (-0.05 + 0.25\cos\theta)\cos\theta\, d\theta + \frac{1}{\pi}\int_{1.369}^{\pi} (-0.0223 + 0.111\cos\theta)\cos\theta\, d\theta = 0.163$$

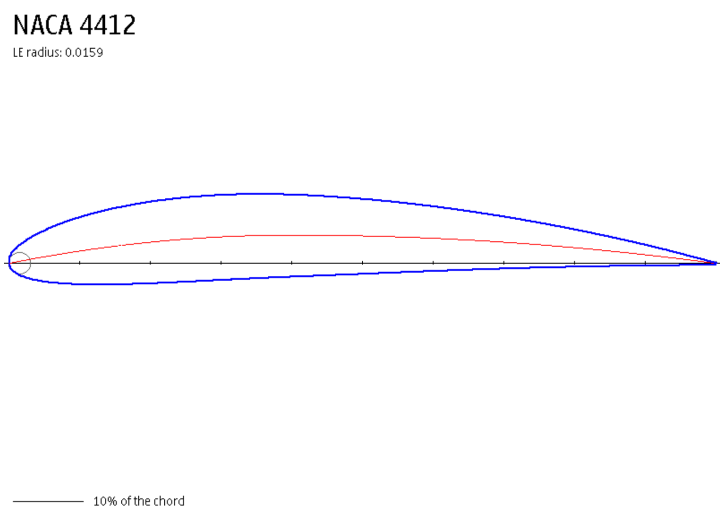
$$A_2 = \frac{2}{\pi}\int_0^\pi \frac{d\eta_c(\theta)}{dx} \cos 2\theta\, d\theta$$
$$= \frac{1}{\pi}\int_0^{1.369} (-0.05 + 0.25\cos\theta)\cos 2\theta\, d\theta + \frac{1}{\pi}\int_{1.369}^{\pi} (-0.0223 + 0.111\cos\theta)\cos 2\theta\, d\theta = 0.0277$$

$$\alpha_{L=0} = A_0 - \frac{A_1}{2} = 0.0089 - \frac{0.163}{2} = -0.0726\ rad$$
$$\alpha_{L=0} = -4.16°$$

$$c_l = 2\pi(\alpha + 0.0726)$$

For $\alpha = 3° = 0.0524\ rad \rightarrow c_l = 0.7854$





❑ Thin Airfoil Theory – Cambered Airfoil

b.

$$c_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}(0.0277 - 0.163) = -0.1063$$

$$\frac{x_{cp}}{c} = \frac{1}{4}\left[1 + \frac{\pi}{c_l}(A_1 - A_2)\right] = \frac{1}{4}\left[1 + \frac{\pi}{0.7854}(0.163 - 0.0277)\right]$$

$$\frac{x_{cp}}{c} = 0.3853$$

c.

Comparison with experimental data

	Inviscid theory	**Experiment**	**Error (%)**
c_l	0.7854	0.76	3.3
$c_{m_{c/4}}$	-0.1063	-0.095	11.9

d.

$$L' = \frac{1}{2}\rho_\infty V_\infty^2 c_l c, \qquad \rho_\infty = 0.9093\frac{kg}{m^3} \quad \text{(standard atmosphere h=3 km)}$$

$$L' = \frac{1}{2}(0.9093)(60^2)(0.7854)(2) = 2571\frac{N}{m}$$

$$L' = \rho_\infty V_\infty \Gamma \rightarrow \Gamma = \frac{L'}{\rho_\infty V_\infty} = \frac{2571}{0.9090 \times 60} = 47.12\frac{m^2}{s}$$