Lecture # 29: Airfoil Aerodynamics – Part 07 :

Airfoil with Flap & Lumped Vortex Model

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❑ Airfoil with Flap

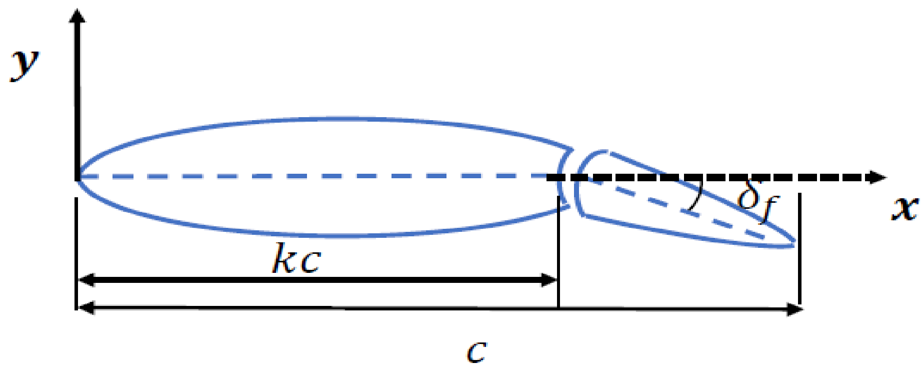
Flapped airfoil

- We can effectively incorporate a deployed flap by modifying the airfoil camber equation
- Let's consider a thin symmetric airfoil with a trailing-edge flap deflected at a small angle δ_f
  - We can write the mean camber equation as

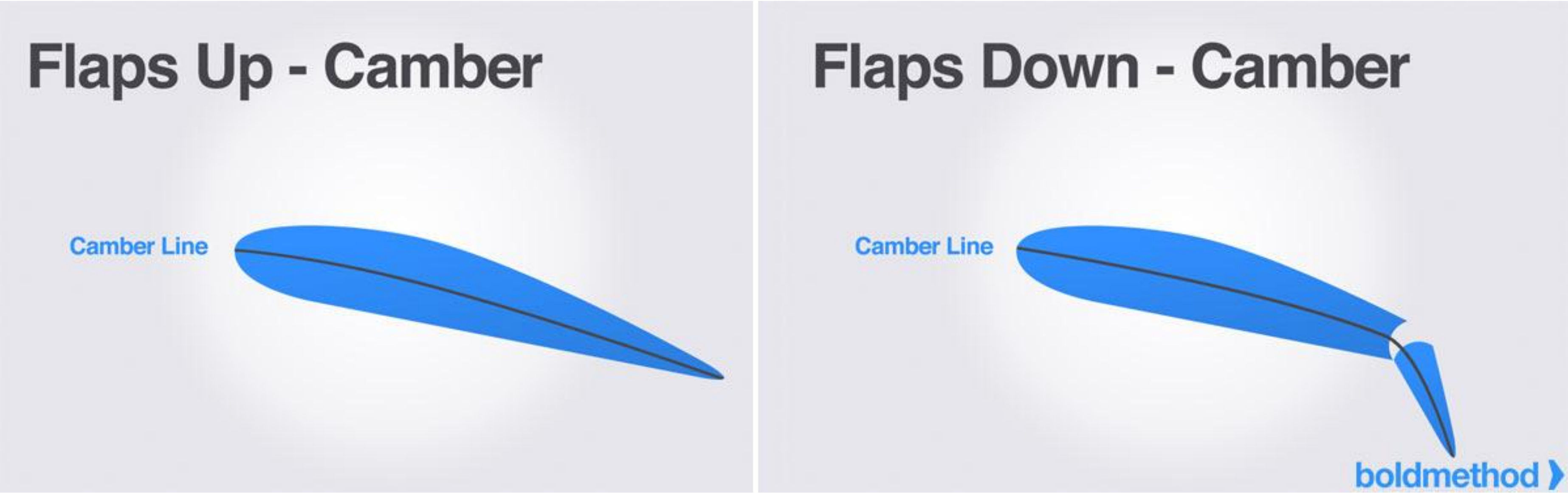
$$\eta_c(x) = \begin{cases} 0 & 0 < x \le kc \\ -\delta_f (x - kc) & kc < x < c \end{cases}$$

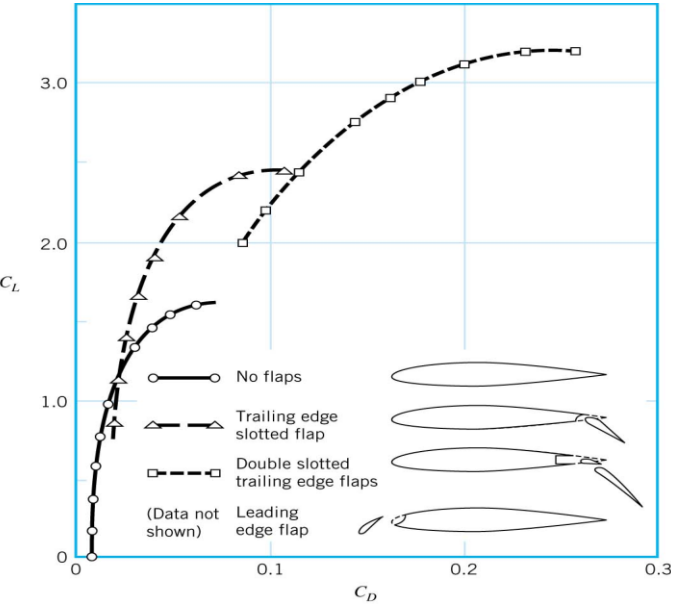
Therefore

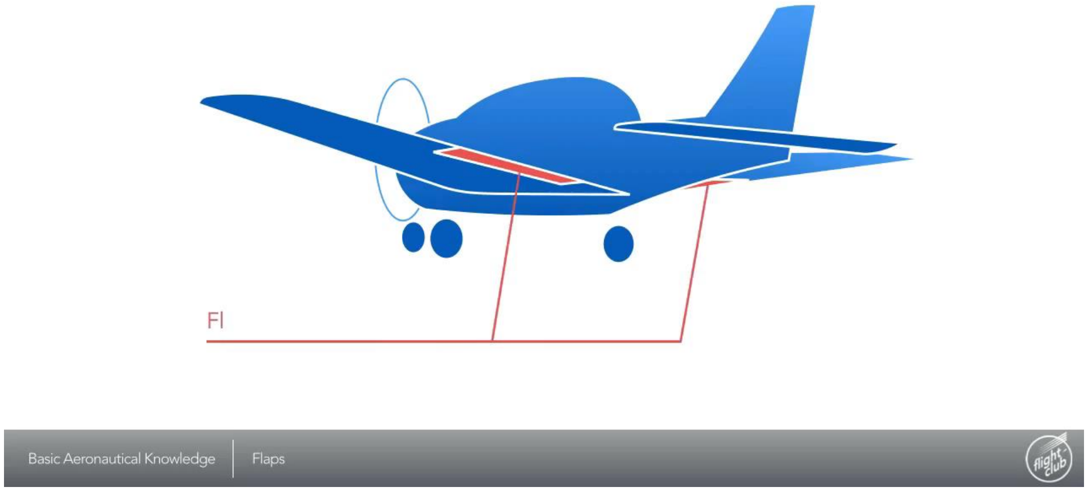
$$\frac{d\eta_c}{dx} = \begin{cases} 0 & 0 < x \le kc \\ -\delta_f & kc < x < c \end{cases}$$



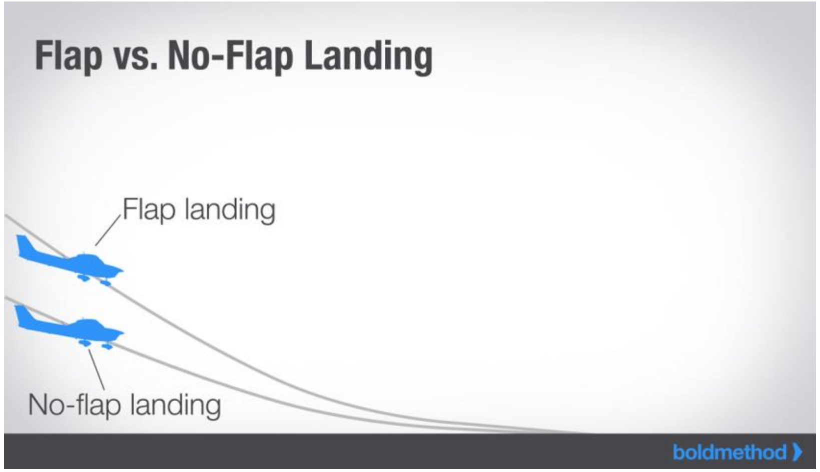
❑ Airfoil with Flap







❑ Airfoil with Flap



https://www.youtube.com/watch?v=adl1M2jzwBs

❑ Airfoil with Flap

Flapped airfoil example

In writing $d\eta_c/dx$ in terms of θ, note

$$x = \frac{c}{2}(1 - \cos\theta)$$

Location kc is a fixed point on x, that corresponds to a fixed point on θ, and we name that θ_k

$$kc = \frac{c}{2}(1 - \cos\theta_k) \rightarrow \cos\theta_k = 2k - 1$$

Now

$$\frac{d\eta_c}{dx}(\theta) = \begin{cases} 0 & 0 < \theta \leq \theta_k \\ -\delta_f & \theta_k < \theta < \pi \end{cases}$$

Then we can calculate the coefficients as

$$A_0 = \frac{1}{\pi}\int_0^{\pi} \frac{d\eta_c(\theta)}{dx} d\theta = \frac{1}{\pi}\int_0^{\theta_k} 0 d\theta - \frac{1}{\pi}\int_{\theta_k}^{\pi} \delta_f d\theta$$

$$= -\frac{\delta_f}{\pi}(\pi - \theta_k) \rightarrow A_0 = \delta_f\left(\frac{\theta_k}{\pi} - 1\right)$$

❑ Airfoil with Flap

Flapped airfoil example-continued

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{d\eta_c}{dx}\cos\theta\, d\theta = -\frac{2}{\pi}\int_{\theta_k}^{\pi} \delta_f \cos\theta\, d\theta$$
$$= -\frac{2\delta_f}{\pi}(\sin\pi - \sin\theta_k) \rightarrow A_1 = \frac{2\delta_f}{\pi}\sin\theta_k$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi} \frac{d\eta_c}{dx}\cos 2\theta\, d\theta = -\frac{2}{\pi}\int_{\theta_k}^{\pi} \delta_f \cos 2\theta\, d\theta$$
$$= -\frac{\delta_f}{\pi}(\sin 2\pi - \sin 2\theta_k) \rightarrow A_2 = \frac{\delta_f}{\pi}\sin 2\theta_k$$

Therefore

$$\alpha_{L=0} = A_0 - \frac{A_1}{2} = \delta_f\left(\frac{\theta_k}{\pi} - 1\right) - \frac{\delta_f}{\pi}\sin\theta_k$$

$$c_l = 2\pi\left[\alpha + \delta_f\left(\frac{\theta_k}{\pi} - \frac{\sin\theta_k}{\pi} - 1\right)\right]$$

❑ Airfoil with Flap

Flapped airfoil example-continued

Moment coefficient

$$c_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}\left(\frac{\delta_f}{\pi}\sin 2\theta_k - \frac{2\delta_f}{\pi}\sin\theta_k\right)$$

$$c_{m_{c/4}} = \frac{\delta_f}{4}(\sin 2\theta_k - 2\sin\theta_k)$$

Note that for $\delta_f = 0$, c_l and $c_{m_{c/4}}$ revert to clean symmetric airfoil solution.

We can use the same process for a leading-edge flap or more complicated multi-element flaps.

❑ Lumped Vortex Model

Circulation around thin airfoil

- Recall from thin airfoil solution

$$\gamma(\theta) = 2V_\infty \left[(\alpha - A_0)\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]$$

For a flat plate airfoil (i.e., no camber→ $A_0 = A_1 = \cdots = A_n = 0$)

$$\gamma(\theta) = 2V_\infty \alpha \frac{1+\cos\theta}{\sin\theta}$$

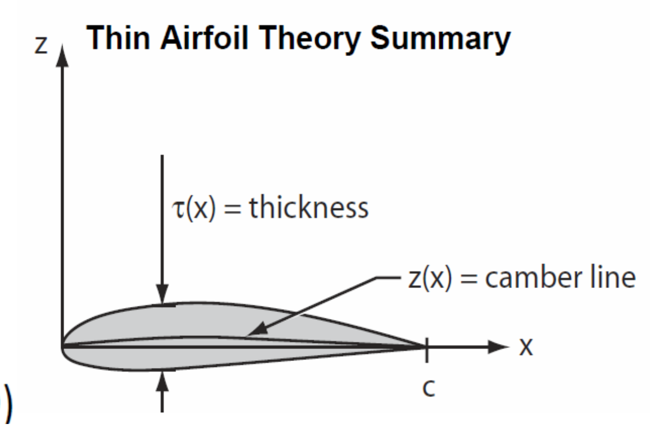
The total circulation around the airfoil is

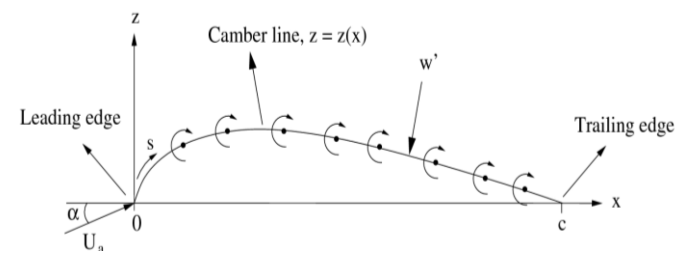
$$\Gamma = \int_0^c \gamma(x)dx = \int_0^\pi \gamma(\theta)\frac{c}{2}\sin\theta\, d\theta$$

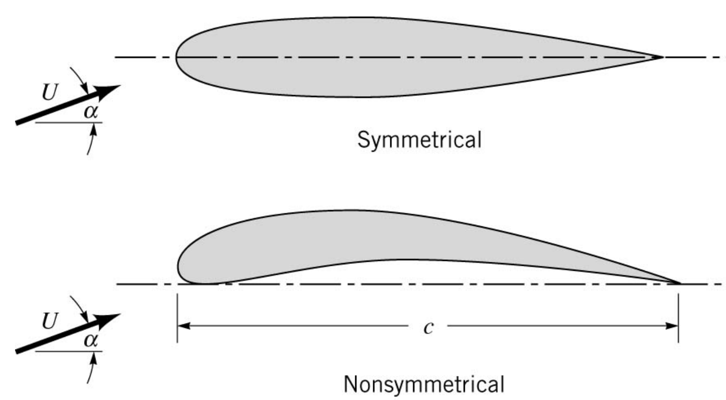
$$\Gamma = cV_\infty\alpha \int_0^\pi (1+\cos\theta)d\theta = \pi c V_\infty \alpha$$

And airfoil lift

$$L' = \rho V_\infty \Gamma$$

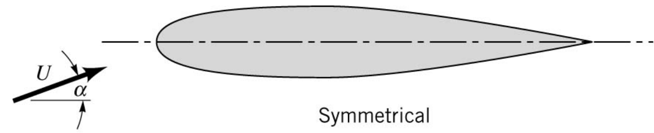






❑ Lumped Vortex Model

Lumped vortex element (model)



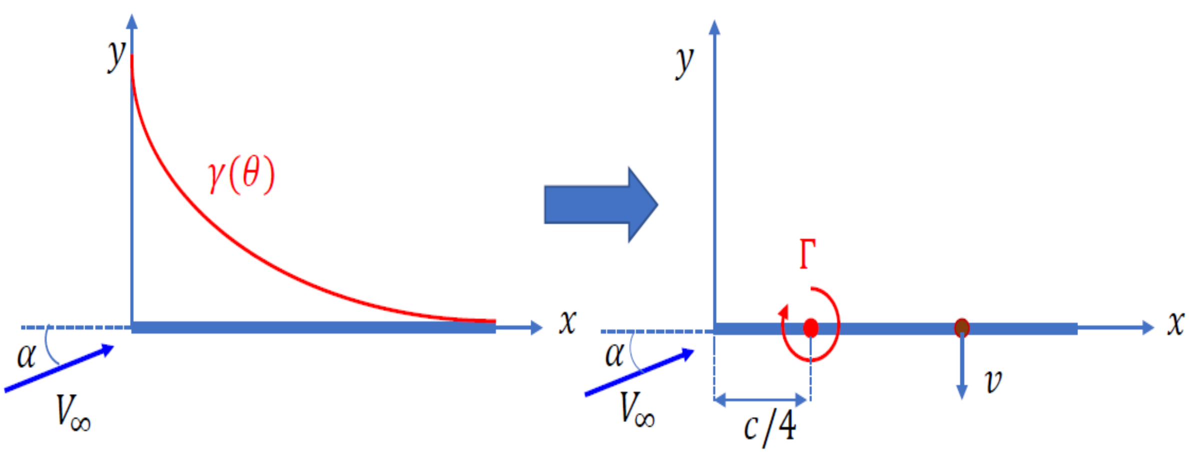
- It is sometimes useful to represent the entire circulation with a single vortex
  - Since lift acts at center of pressure, naturally the vortex should be placed there (i.e., $x = c/4$)

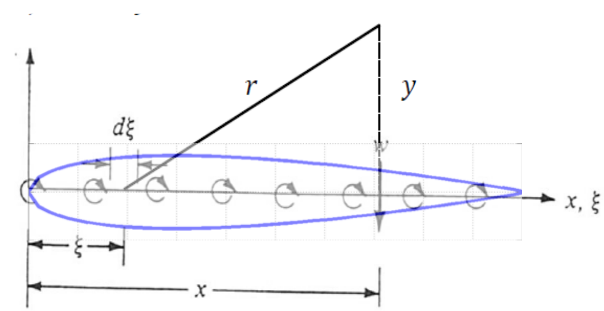
$$\frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty\alpha;$$

  - However, with distributed γ, the zero normal velocity is enforced on the surface of airfoil. Where should we enforce this boundary condition for single vortex case?

$$\gamma(\theta)=2\alpha V_\infty\frac{1+\cos\theta}{\sin\theta}$$

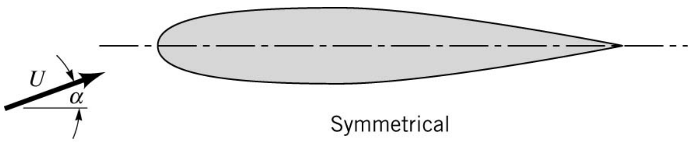
$$\Gamma=\alpha V_\infty c\pi$$





Lumped Vortex Model

Lumped vortex element (model)



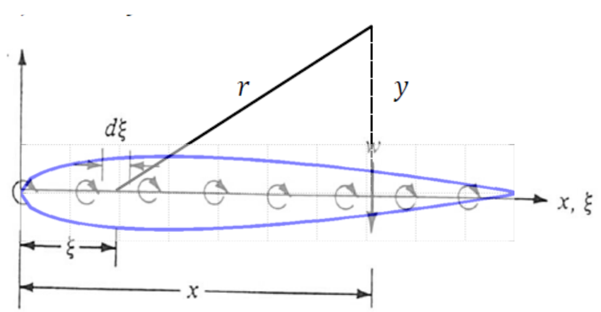
- Zero normal velocity at $x = x_c$

$$-\frac{\Gamma}{2\pi\left(x_c - \frac{c}{4}\right)} + V_\infty \alpha = 0$$

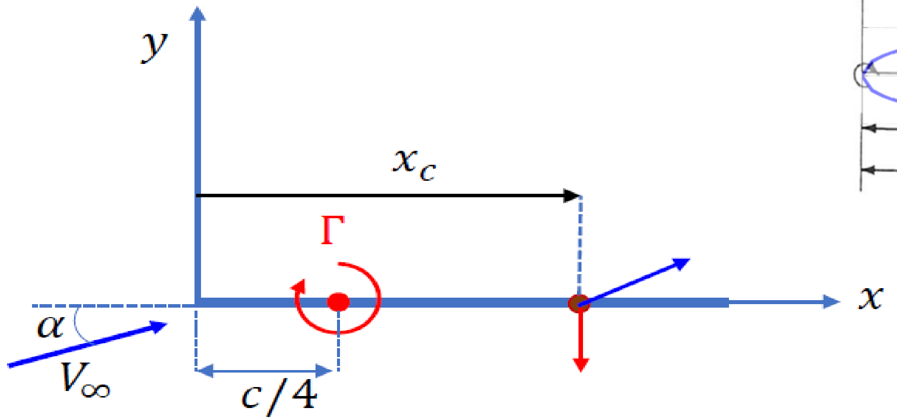
And $\Gamma = \pi c V_\infty \alpha$

$$-\frac{\pi c V_\infty \alpha}{2\pi\left(x_c - \frac{c}{4}\right)} + V_\infty \alpha = 0$$

$$2\left(x_c - \frac{c}{4}\right) = c \rightarrow \boxed{x_c = \frac{3}{4}c}$$



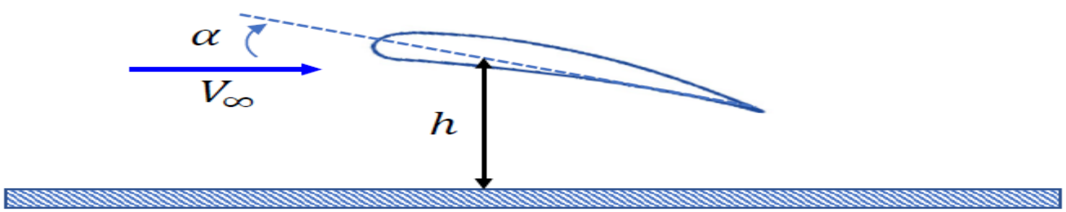
This point is called *collocation point.*

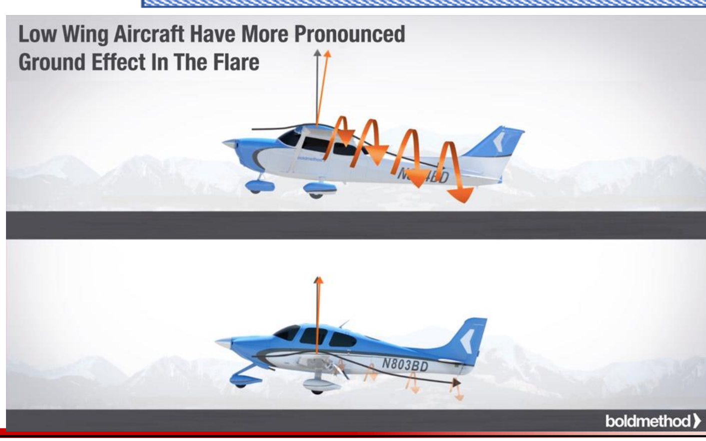


❑ Lumped Vortex Model

Example – Ground effect

- A thin airfoil next to a solid wall (near ground)
  - We can model the airfoil with a lumped (bound) vortex
  - Then use image method to consider the wall effect
  - Determine the circulation such that the boundary condition at collocation point is satisfied





❑ Lumped Vortex Model

Example – continued

- Add velocities in the normal to the plate (airfoil) direction

Velocity contribution from the bound vortex

$$v_1 = -\frac{\Gamma}{2\pi\left(\frac{c}{2}\right)}$$

Contribution from the image vortex

$$v_2 = -\frac{\Gamma}{2\pi r}\sin(\alpha - \beta)$$

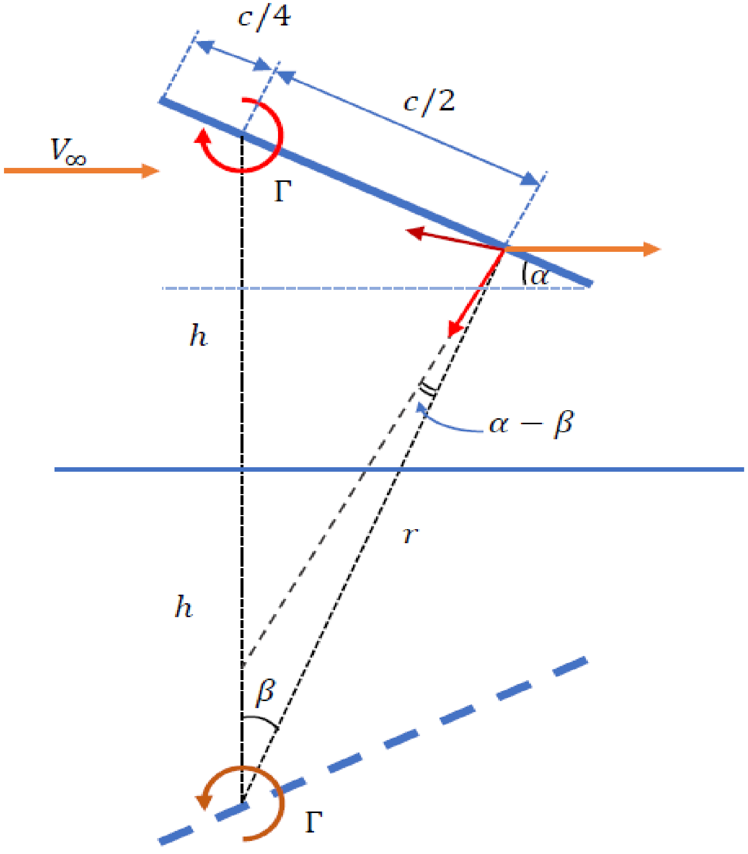
With small α

$$v_2 = \frac{\Gamma}{2\pi r}\sin\beta$$

And

$$\sin\beta \approx \frac{c/2}{r}$$

$$r^2 = 4h^2 + \frac{c^2}{4}$$



Lumped Vortex Model

Example – continued

Contribution from free stream

$$v_3 = V_\infty \sin\alpha \approx V_\infty \alpha$$

Boundary condition at $x_c = \frac{3}{4}c$

$$-\frac{\Gamma}{\pi c} + \frac{\Gamma}{4\pi}\frac{c}{r^2} + V_\infty \alpha = 0$$

$$\Gamma = \frac{\pi V_\infty \alpha}{\frac{1}{c} - \frac{c}{4r^2}} = \pi c V_\infty \alpha \frac{4r^2}{4r^2 - c^2}$$

$$\Gamma = \pi c V_\infty \alpha \frac{16h^2 + c^2}{16h^2} = \pi c V_\infty \alpha \left[1 + \left(\frac{c}{4h}\right)^2\right]$$

$$\Gamma = \Gamma_\infty \left[1 + \left(\frac{c}{4h}\right)^2\right]$$

Circulation(and lift) increases by a factor of $\left[1 + \left(\frac{c}{4h}\right)^2\right]$ due to the ground effect.

For an aifoil with $c = 4.0m$

1). *when* $h = c = 4.0mm \Rightarrow \Gamma = 1.0625\Gamma_\infty$

$\Rightarrow$ ~ 6.25% *more lift with ground effect.*

2). *when* $h = c/2 = 2.0m \Rightarrow \Gamma = 1.25\Gamma_\infty$

$\Rightarrow$ ~ 25% *more lift with ground effect.*

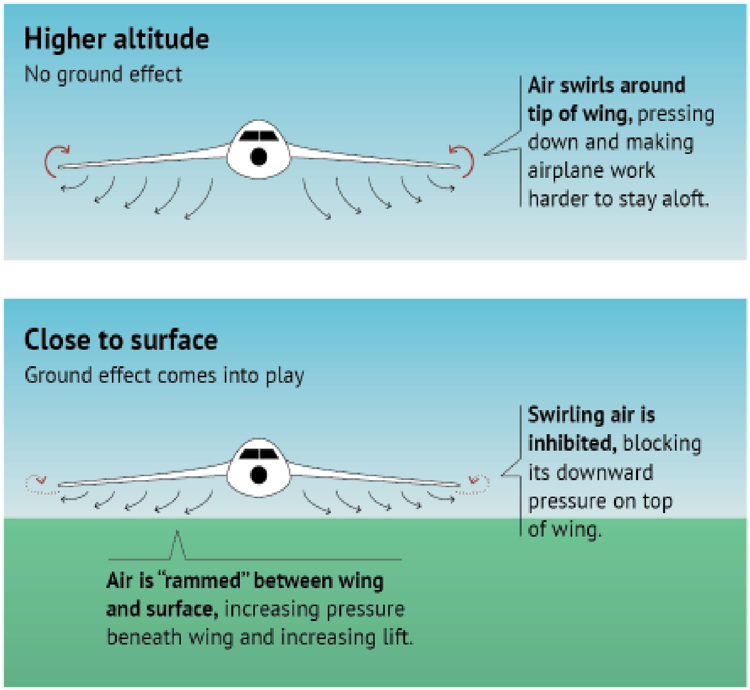
3). *when* $h = c/4 = 1.0m \Rightarrow \Gamma = 2.0\Gamma_\infty$

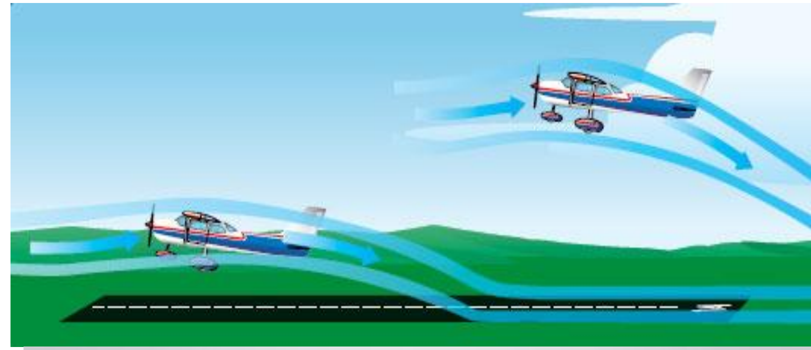
$\Rightarrow$ ~ 100% *more lift with ground effect.*

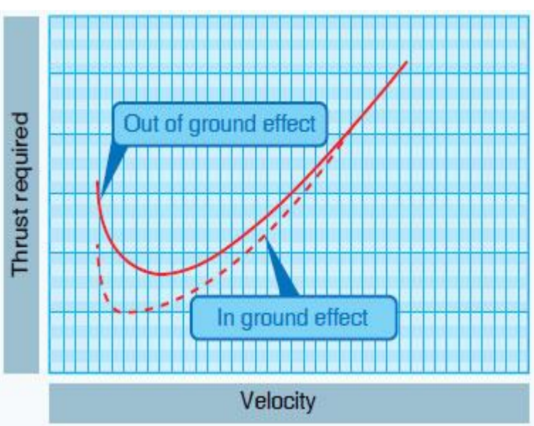
❑ Ground Effect Air Vehicles

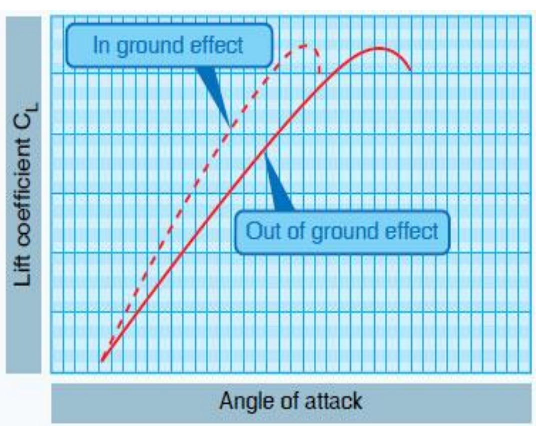
HOW GROUND EFFECT VEHICLES WORK

These vehicles take advantage of the "ground effect" to fly more efficiently. They have the potential to be faster than trains and hydrofoil ships and are more efficient at lower speeds than commercial aircraft. Also, since the effect works over water and dry ground, at cruise speed the vehicle could be as amphibious as a hovercraft.



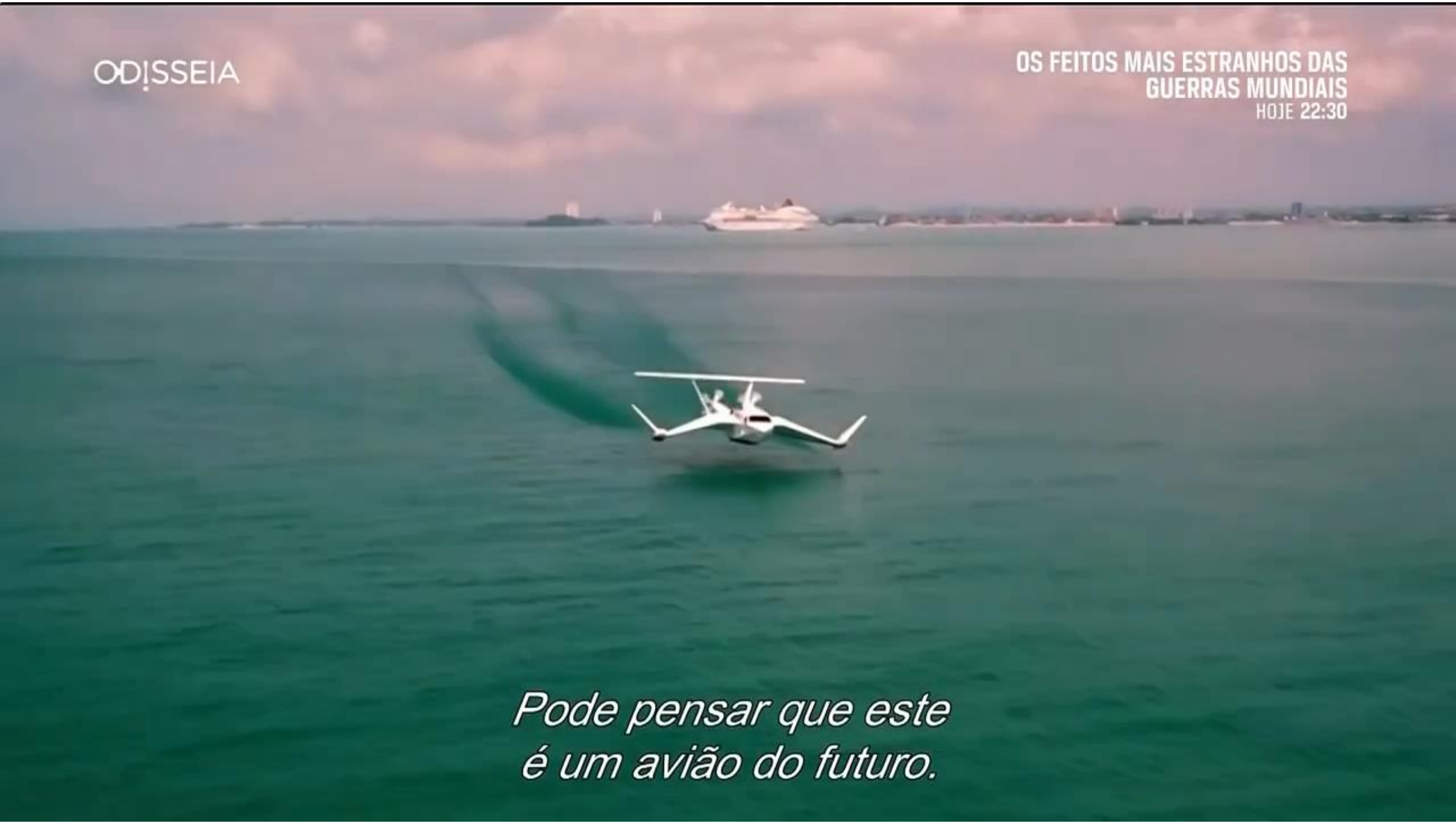






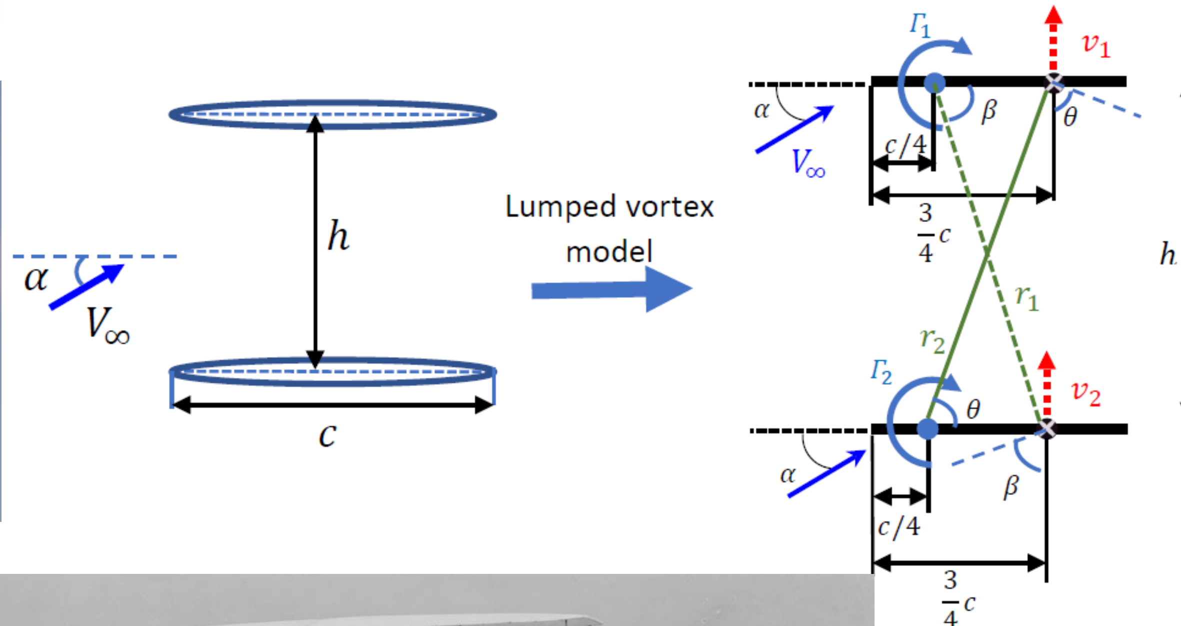
- ***Ground Effect***

❑ Ground Effect Air Vehicles



❑ Airfoil with Flap

Consider a uniform flow with speed V_∞ at a small angle of attack of α past a biplane consisting of two thin symmetric airfoils of chord length c separated by distance h. Use the lumped vortex model to find the lift coefficient of each airfoil. Check your solution for the limit case of $h \to \infty$ and see if that make sense.



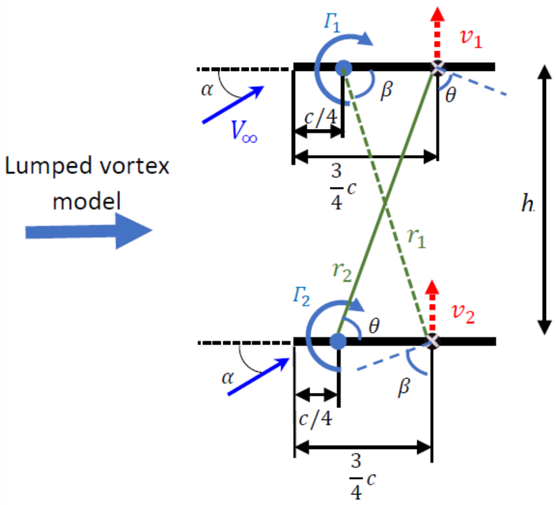

Wright Brothers' Biplane Design

❑ Lumped Vortex Model

Solution

In the lumped vortex theory, the airfoil circulation is replaced with a single vortex at the center of pressure ($\frac{c}{4}$) while the zero normal velocity at the surface is enforced at the collocation point ($\frac{3}{4}c$).

For the biplane configuration, there are two airfoils with circulations of Γ_1 and Γ_2. Boundary condition is enforced at the collocation point of each airfoil.



For the top airfoil

$$v_1 = V_\infty \sin\alpha - \frac{\Gamma_1}{2\pi(c/2)} - \frac{\Gamma_2}{2\pi r_2}\cos\theta = 0$$

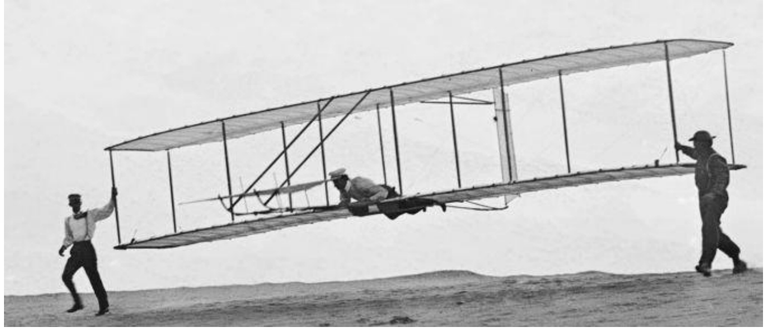
From geometry

$$r_2^2 = \left(\frac{c}{2}\right)^2 + h^2 \rightarrow r_2^2 = \frac{1}{4}(c^2 + 4h^2)$$

$$\cos\theta = \frac{c/2}{r_2}$$

And for small angle of attack $\sin\alpha \approx \alpha$

$$\rightarrow V_\infty \alpha - \frac{\Gamma_1}{\pi c} - \frac{\Gamma_2}{2\pi r_2}\frac{c/2}{r_2} = 0$$



❑ Lumped Vortex Model

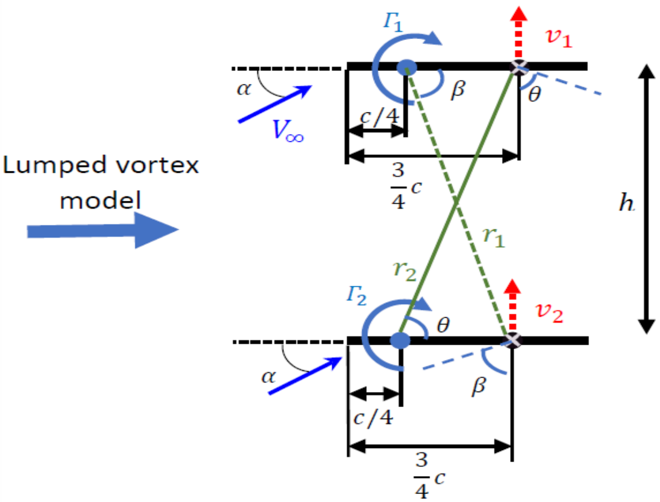
Similarly for the bottom airfoil

$$v_2 = V_\infty \alpha - \frac{\Gamma_1}{2\pi r_1}\cos\beta - \frac{\Gamma_2}{2\pi(c/2)} = 0$$

$$r_1^2 = \frac{1}{4}(c^2 + 4h^2)$$

$$\cos\beta = \frac{c/2}{r_1}$$

$$\rightarrow V_\infty \alpha - \frac{\Gamma_1}{2\pi r_1}\frac{\frac{c}{2}}{r_1} - \frac{\Gamma_2}{\pi c} = 0 \rightarrow V_\infty \alpha - \frac{\Gamma_1 c}{\pi(c^2 + 4h^2)} - \frac{\Gamma_2}{\pi c} = 0$$



Solve the two equations for Γ_1 and Γ_2 to arrive at:

$$\Gamma_1 = \Gamma_2 = \pi c V_\infty \alpha \frac{c^2 + 4h^2}{2c^2 + 4h^2} = \Gamma_\infty \frac{c^2 + 4h^2}{2c^2 + 4h^2}$$

The equal value of circulation is to be expected since the airfoils are aligned vertically, and it does not matter which airfoil is at the top.

To write the result in terms of the lift coefficient, note that

$$L' = \rho V_\infty \Gamma \rightarrow \frac{1}{2}\rho V_\infty^2 c_l c = \rho V_\infty \Gamma \rightarrow c_l = \frac{2\Gamma}{V_\infty c}$$

$$c_{l_1} = c_{l_2} = c_{l_\infty} \frac{c^2 + 4h^2}{2c^2 + 4h^2}$$

Where $c_{l_\infty} = 2\pi\alpha$ is the airfoil lift coefficient in infinitely open medium.

Wright Brothers' Biplane Design

1). when $h = c \Rightarrow c_{l_1} = c_{l_2} = \frac{5}{6} c_{l_\infty}$

$\Rightarrow$ ~ 67% more lift with biplane design

2). when $h = c/2 \Rightarrow c_{l_1} = c_{l_2} = \frac{2}{3} c_{l_\infty}$

$\Rightarrow$ 33% more lift with biplane design

3). when $h = c/4 \Rightarrow c_{l_1} = c_{l_2} = \frac{5}{9} c_{l_\infty}$

$\Rightarrow$ ~ 11% more lift with biplane design

❑ Lumped Vortex Model