Lecture # 36: Introduction to Viscous Flows

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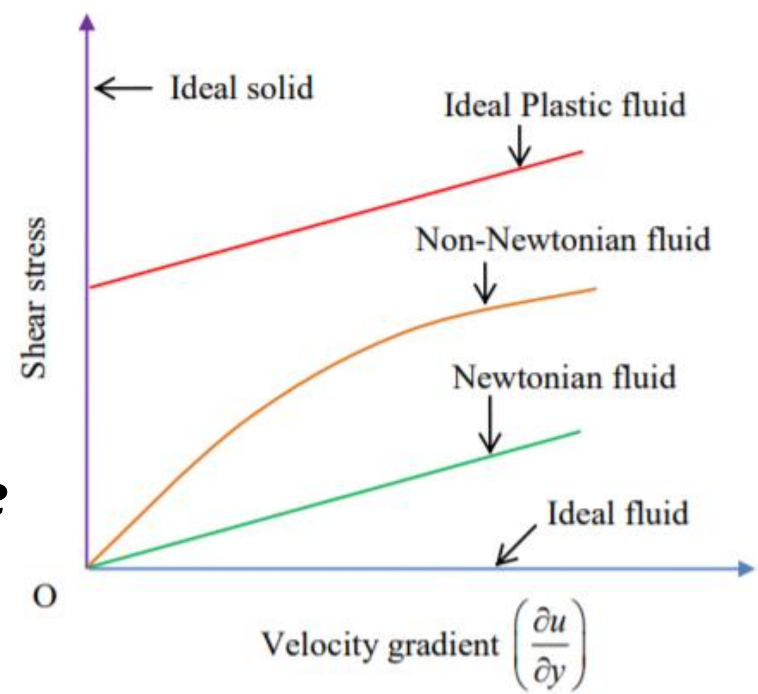
❑ IDEAL FLOW

❑ Ideal flow:

- ***Non-heat conducting, inviscid, incompressible, homogeneous fluid is defined as ideal fluid.***
- ***Assumptions used are:***
  - ***Non-heat conductive***
  - ***Homogeneous***
  - ***Incompressible***
  - ***Inviscid flow***



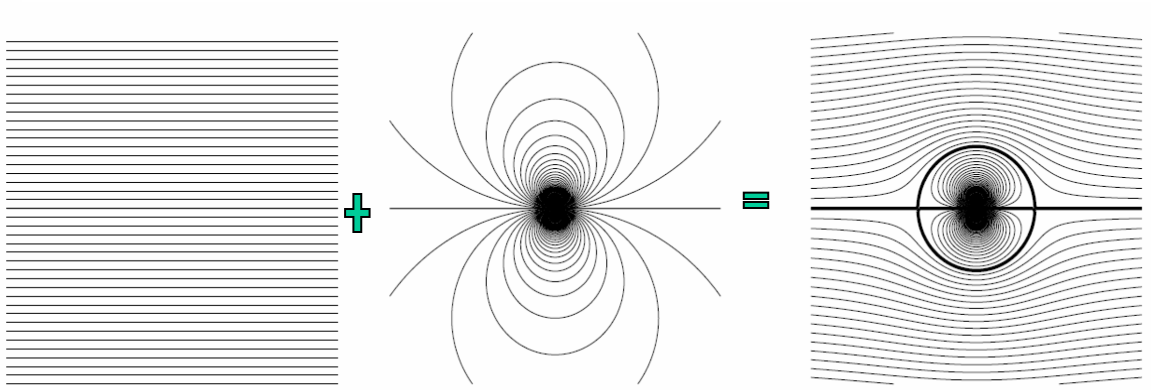
❑ Potential Flow:

- ***Definition:* A non-heat conducting, homogeneous, inviscid, incompressible (*i.e., ideal fluid*), and *irrotational* flow is defined as potential flow.**

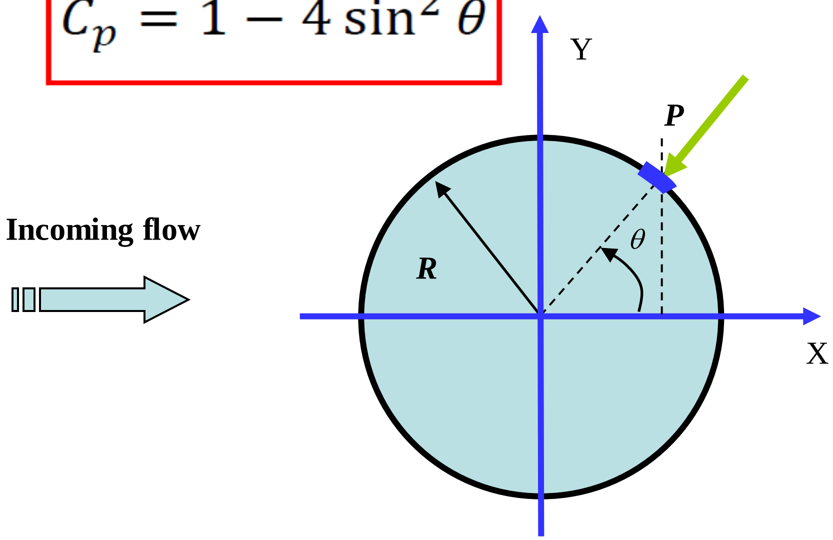
Potential & Stream Functions for Basic Flows

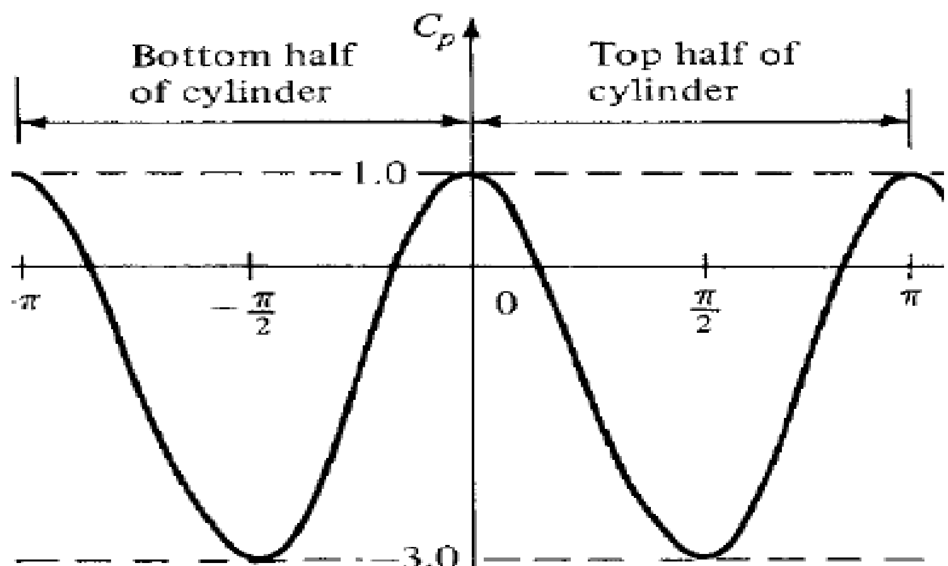
- *Uniform Flow to the Right + A 2-D Doublet*

$$
\left.
\begin{aligned}
\phi &= V_\infty r \cos\theta \left(1 + \frac{R^2}{r^2}\right) \\
\psi &= V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right) \\
V_r &= V_\infty \cos\theta \left(1 - \frac{R^2}{r^2}\right) \\
V_\theta &= -V_\infty \sin\theta \left(1 + \frac{R^2}{r^2}\right)
\end{aligned}
\right\} (r \geq R)
$$

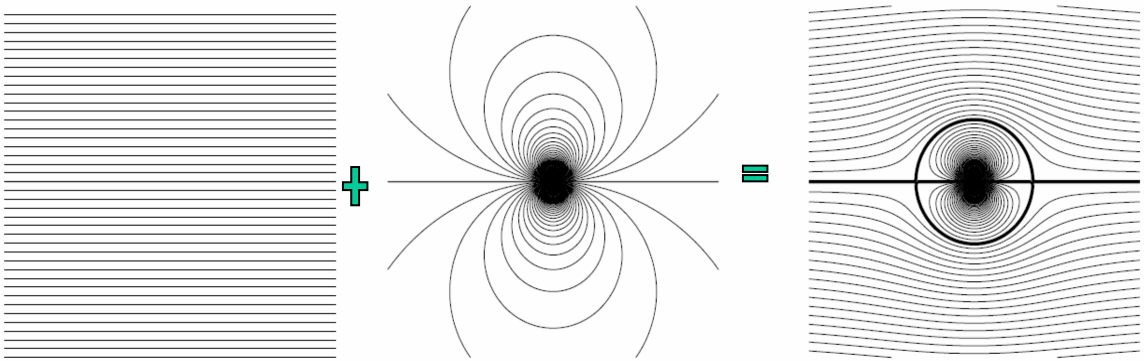


$$C_p = 1 - 4\sin^2\theta$$



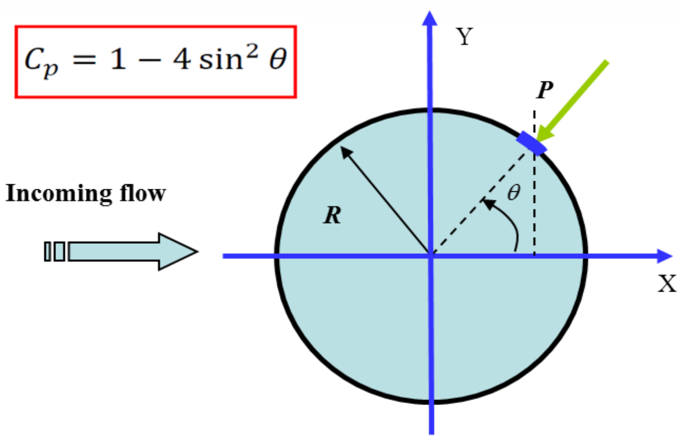


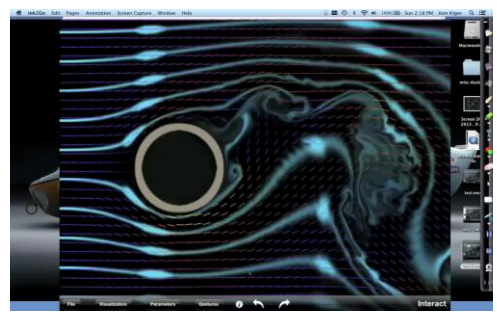
❑ Potential Flow around a Circular Cylinder

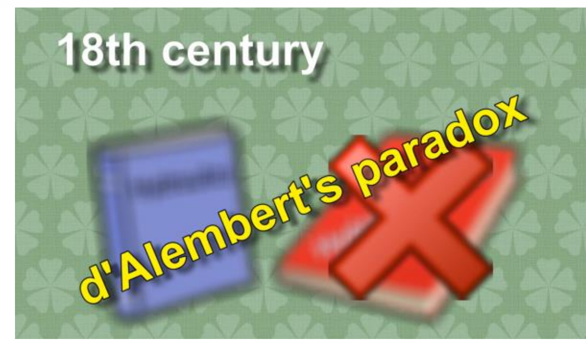


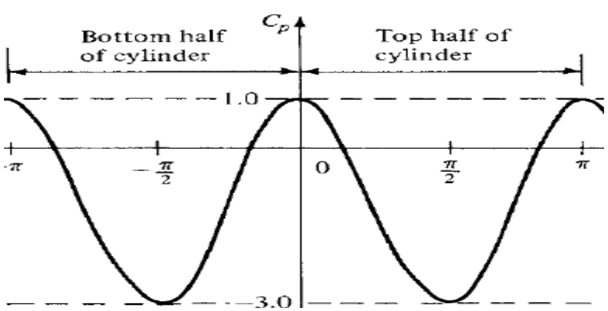
$$\left.\begin{aligned}
\phi &= V_\infty r\cos\theta\left(1+\frac{R^2}{r^2}\right)\\
\psi &= V_\infty r\sin\theta\left(1-\frac{R^2}{r^2}\right)\\
V_r &= V_\infty \cos\theta\left(1-\frac{R^2}{r^2}\right)\\
V_\theta &= -V_\infty \sin\theta\left(1+\frac{R^2}{r^2}\right)
\end{aligned}\right\}(r\ge R)$$

$$C_p = 1 - 4\sin^2\theta$$



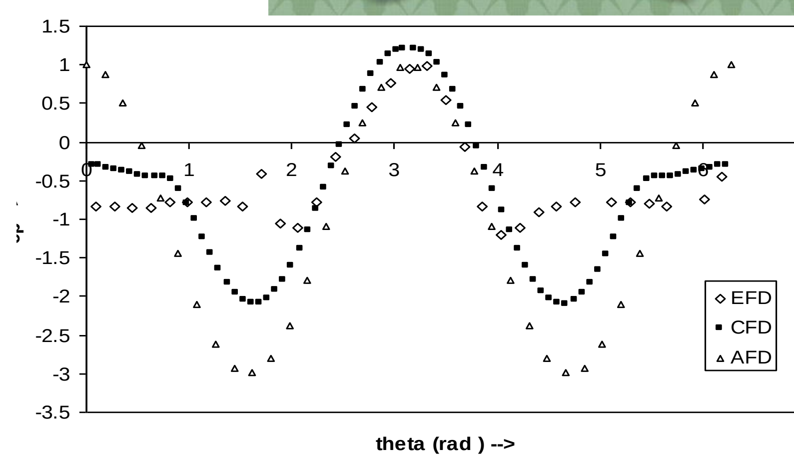




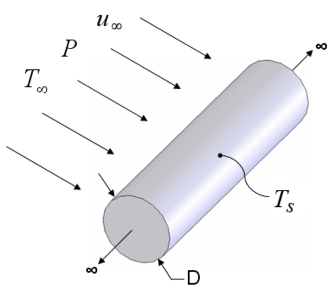


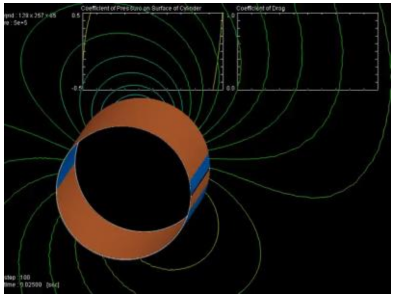
$$C_D = 0$$

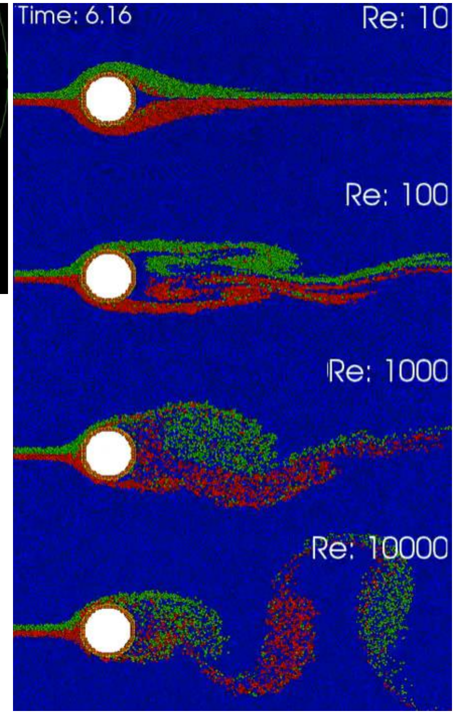
d 'Alembert paradox

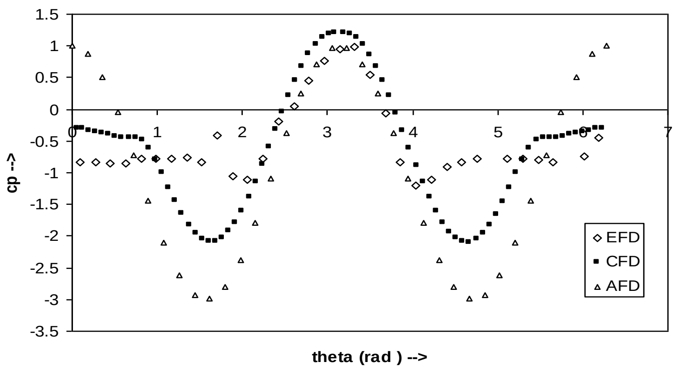


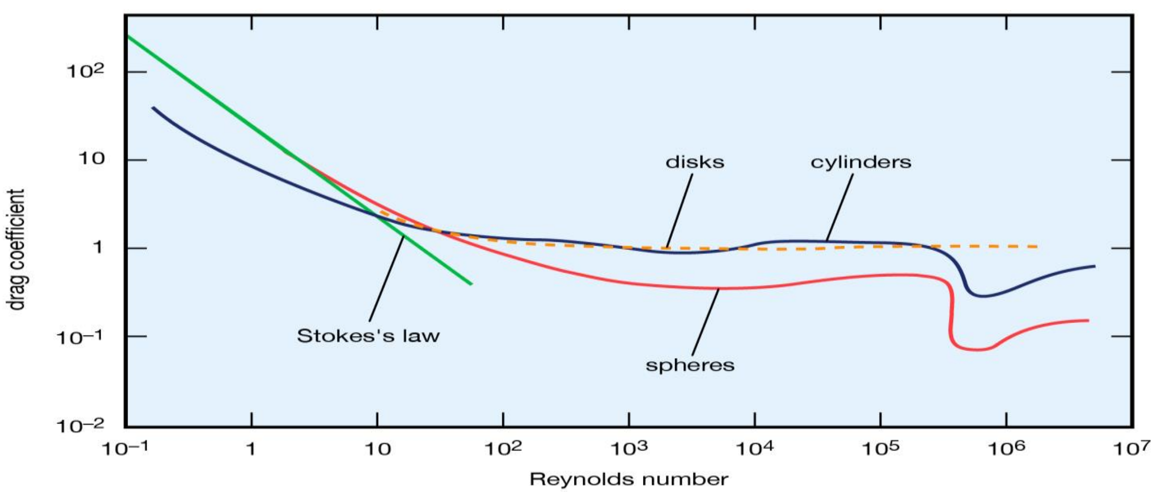
❑ Drag Coefficient of a Circular Cylinder in a Real Rlow

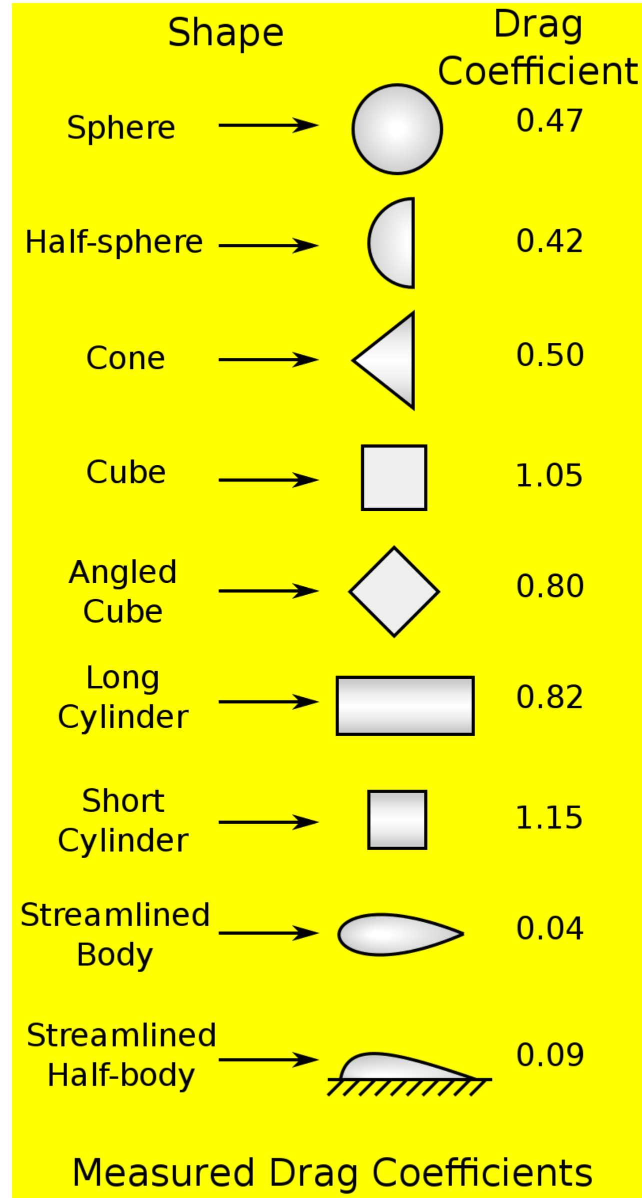






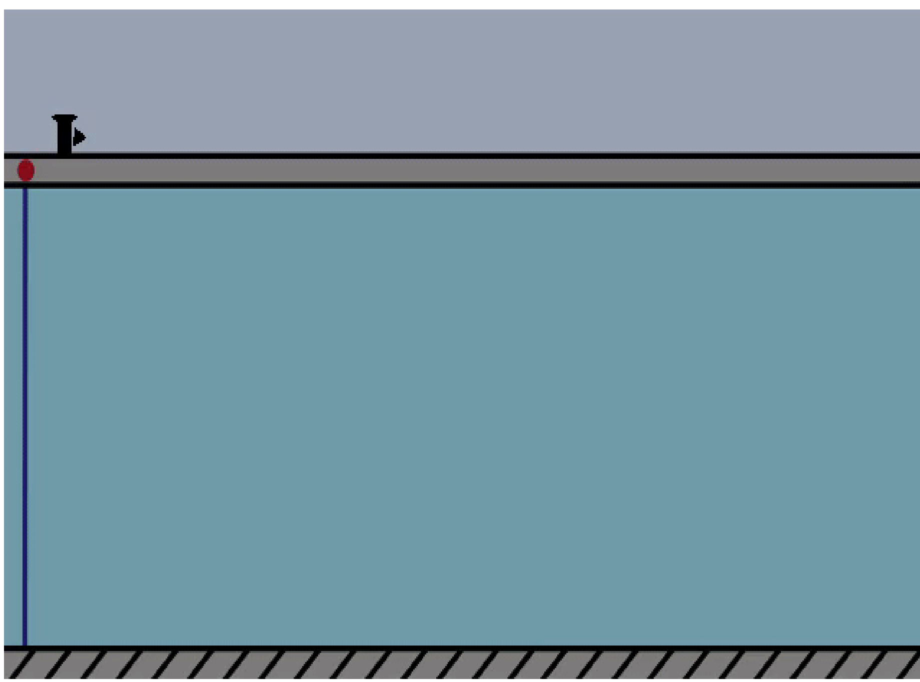






❑ Inviscid Flows vs. Viscous Flows

- ❑ *Viscosity is a measure of a fluid's resistance to flow.*
- ❑ *It describes the internal friction of a moving fluid.*
- ❑ *A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction*



- ***What is Viscosity?***
- https://www.youtube.com/watch?v=9NYs3Y-IjGw

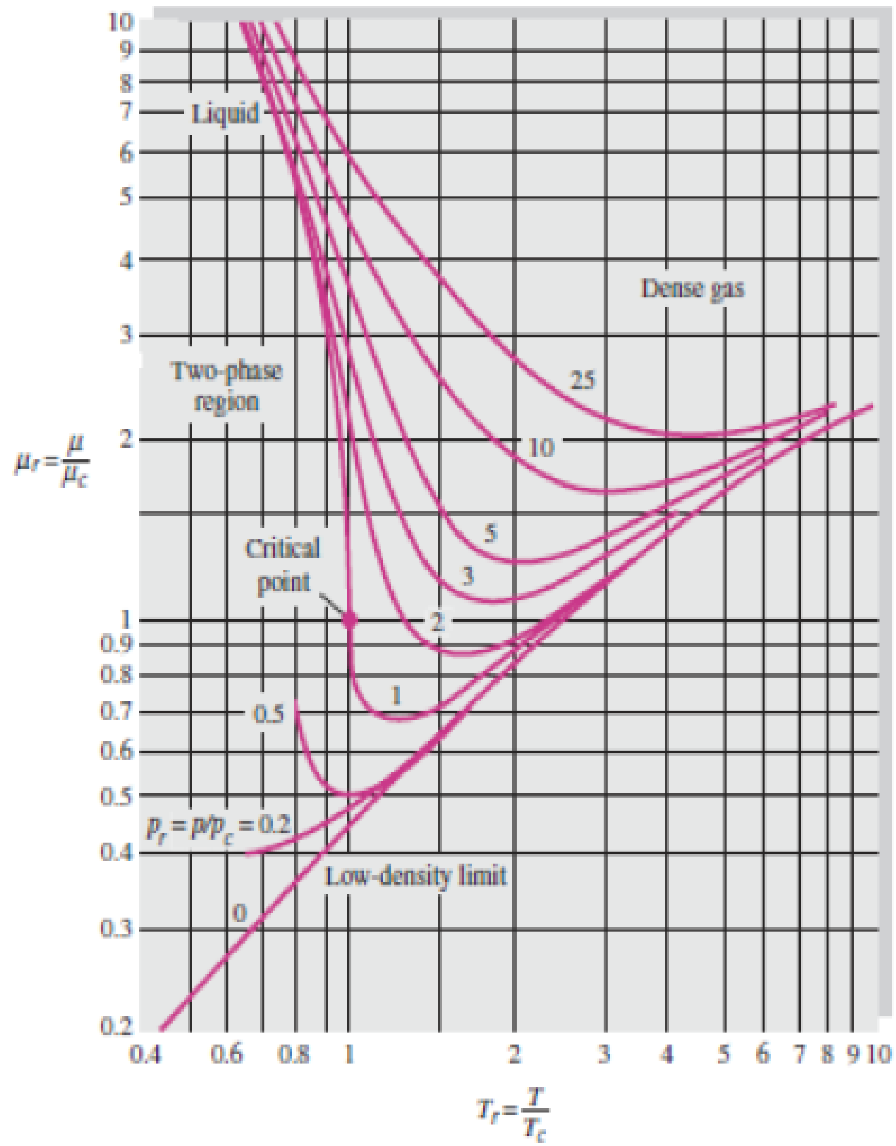
How to Measure Viscosity

- A viscometer (also called viscosimeter) is an instrument used to measure the viscosity of a fluid.
- For liquids with viscosities which vary with flow conditions, an instrument called a rheometer is used. Thus, a rheometer can be considered as a special type of viscometer.

Inviscid Flows vs. Viscous Flows

Viscosity

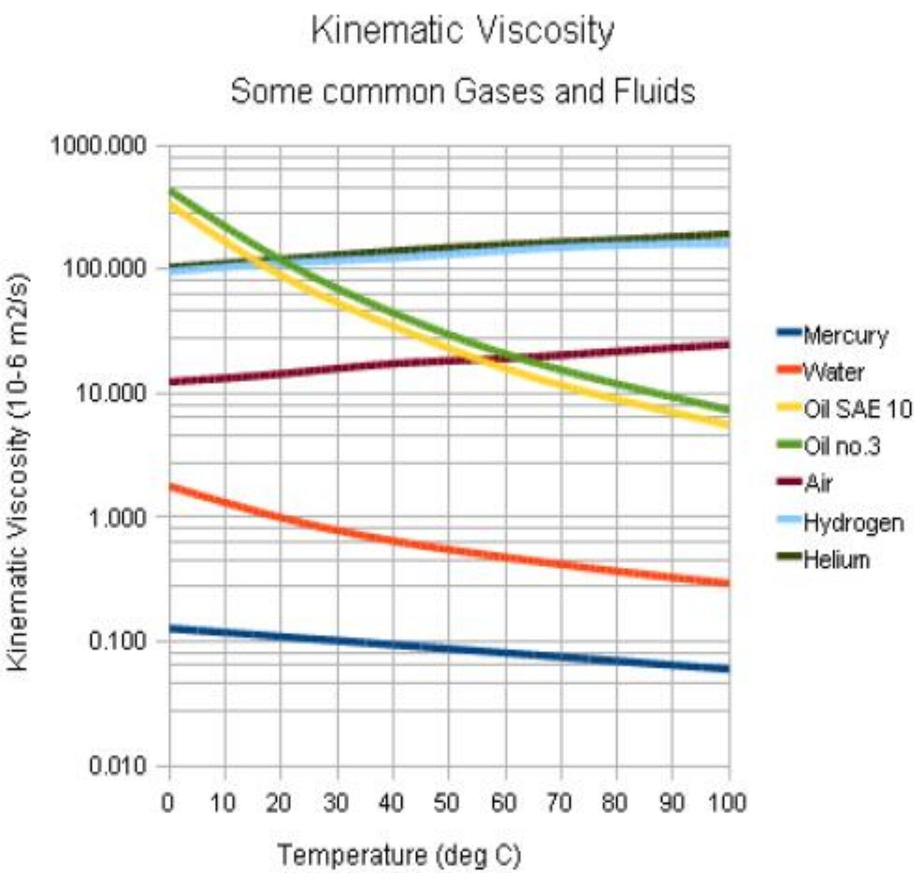
- Dynamic viscosity μ is a property of the fluid
- Units : $\frac{N.s}{m^2} = Pa.s$ also 1 poise=0.1 $Pa.s$
- Kinematic viscosity $v = \frac{\mu}{\rho}$ with unit of $St = cm^2/s$
- Similar to other fluid properties, viscosity is in general a function of temperature (and pressure)
- Typically, viscosity of gases increases with temperature while viscosity of liquids decreases.

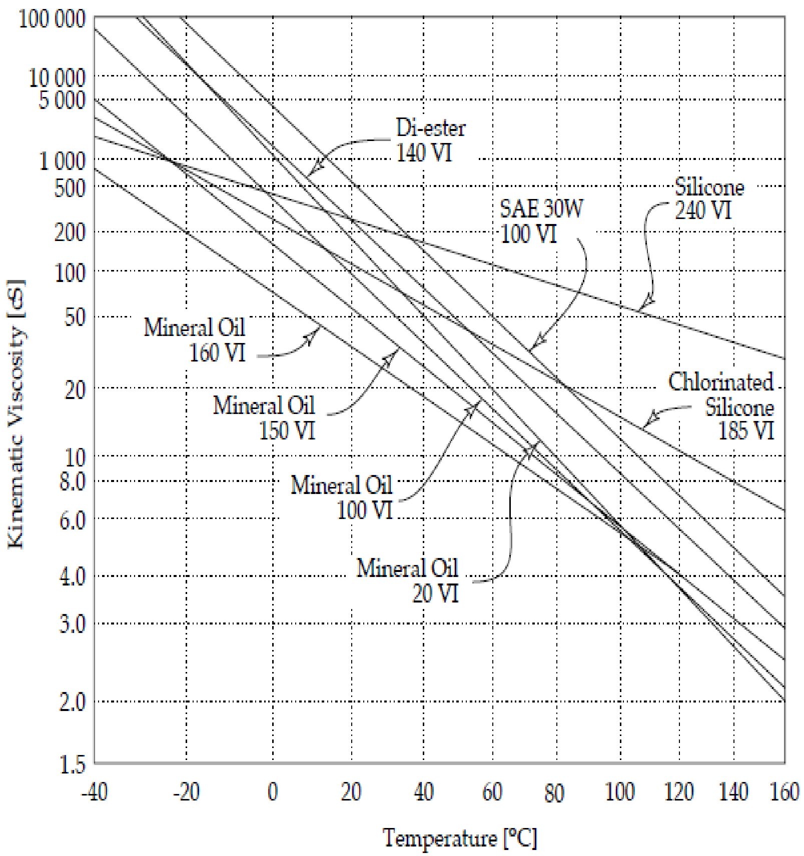


❑ Inviscid Flows vs. Viscous Flows

❑ *Effects of temperature*

- *The viscosity of liquids decreases with increase the temperature.*
- *The viscosity of gases increases with the increase the temperature.*



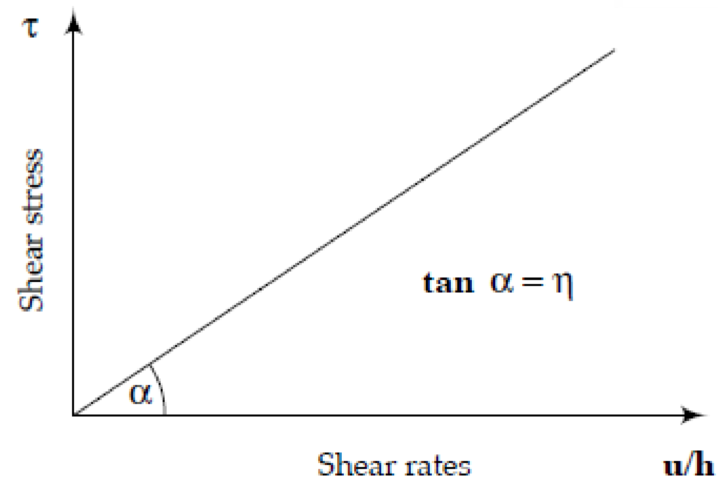


❑ Inviscid Flows vs. Viscous Flows

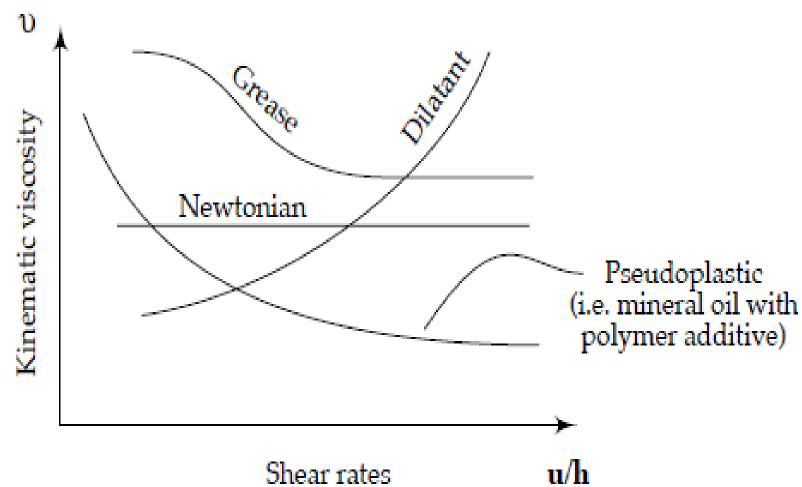
- Viscosity - shear relationship

- *For Newtonian fluids, shear stress linearly vary with the shear rate as shown in Figure. Viscosity is constant for this kind of fluid.*

$$\tau = \eta\ (u/h)$$



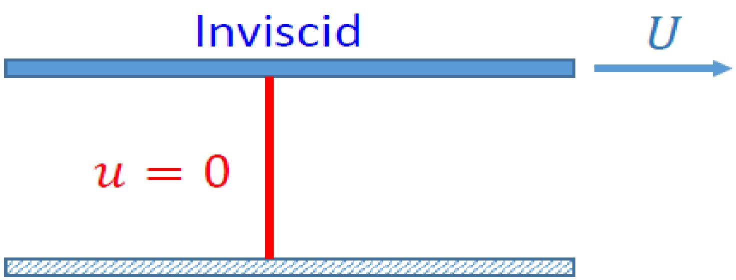
- *Non-Newtonian fluid doesn't follow the linear relation between viscosity and shear rate.*



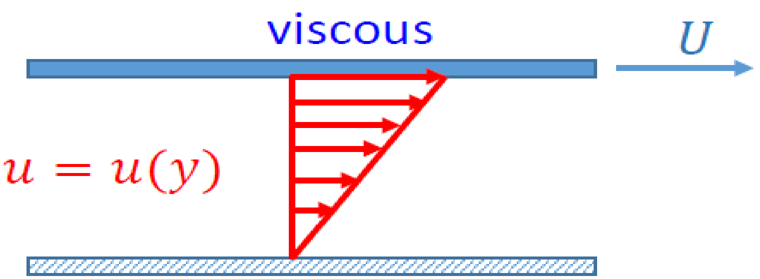
❑ Inviscid Flows vs. Viscous Flows

- ***There is no friction in inviscid flow. Fluid element next to a surface can freely move along the surface.***
- ***In viscous flow there is friction between fluid elements and between fluid and surface.***
- ***In viscous flow fluid element next to the surface moves with the surface (doesn't move if the surface is stationary).***

- **Understanding Viscosity and Viscous Force**



Fluid initially at rest
Plate moves with U
Fluid remains at $u = 0$



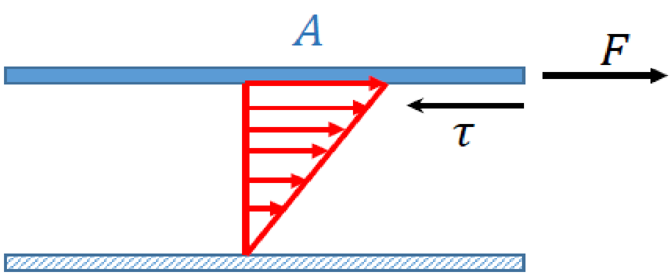
Fluid initially at rest
Plate moves with U
Fluid element next to plate moves with U

❑ Inviscid Flows vs. Viscous Flows

Shear stress

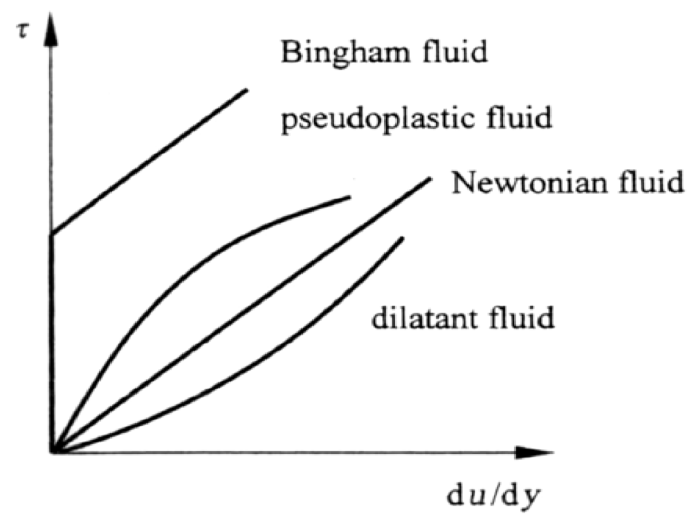
- If the force required to move the plate is F

Shear stress $\tau = \frac{F}{A}$



- The faster you move the plate, the more force you need

- Shear stress is related to the velocity gradient



- For a Newtonian fluid, the relationship is linear

$$\tau = \mu \frac{\partial u}{\partial y}$$

Inviscid Flows vs. Viscous Flows

Navier-Stokes Equation:

$$\rho \frac{D\vec{V}}{Dt} + \nabla P - \nabla \bullet \tilde{\tau} - \rho \vec{f} = 0$$

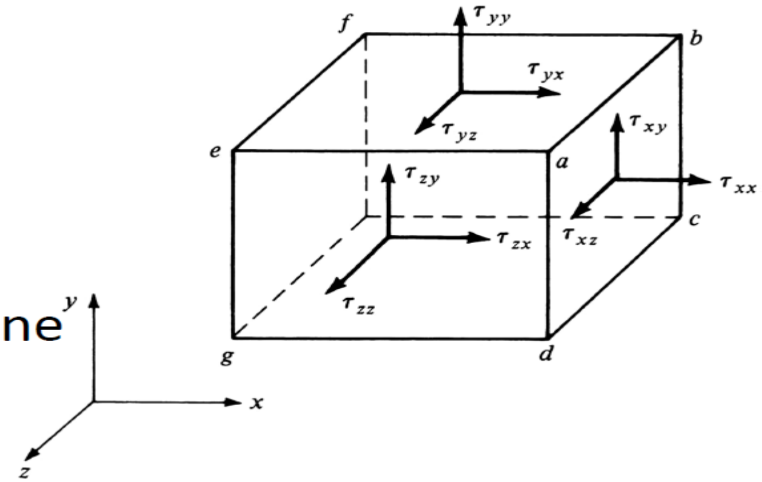
Shear stress on a fluid element

- Shear stress is a tensor variable

There is three-component of shear stress on each face of the fluid element.

τ_{ii} is normal stress and τ_{ij} is the tangential stress.

τ_{xy}: stress in y direction on a plane normal to x.



The Navier-Stokes Equations

$$\rho \frac{D\vec{V}}{Dt} + \nabla P - \nabla \bullet \tilde{\tau} - \rho \vec{f} = 0$$

Stress Tensor

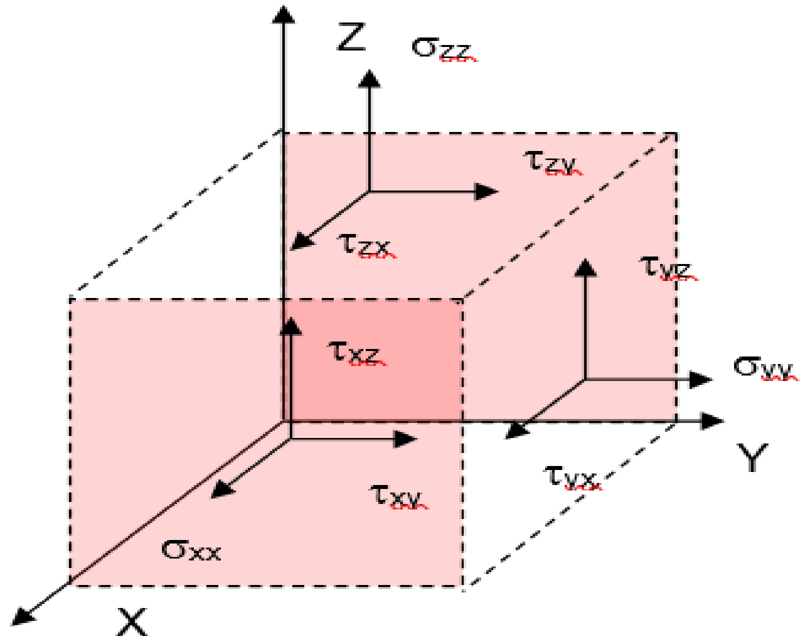
The stress tensor has nine components:

$$\tilde{\tau} = \begin{vmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$

Newtonian fluid,

$$\tilde{\tau} = \mu[\nabla\vec{V} + (\nabla\vec{V})^T - \frac{2}{3}(\nabla \bullet \vec{V})\tilde{I}]$$

For incompressible flow, in Cartesian coordinate system

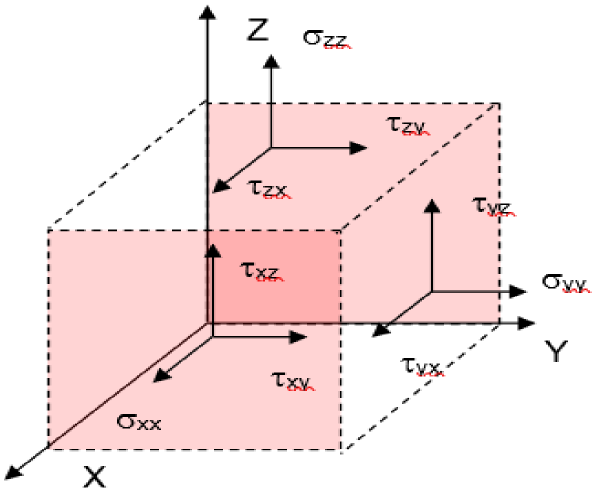


$$\tau_{ij} = \mu(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$$

$$\tau_{xy} = \tau_{yx}; \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{zy} = \tau_{yz}$$

❑ Inviscid Flows vs. Viscous Flows

Conservation of momentum



For Newtonian fluid

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{xx} = -p + 2\mu\frac{\partial u}{\partial x}$$

Therefore, the x-momentum becomes

$$\rho\frac{Du}{Dt} = \frac{\partial}{\partial x}\left(-p + 2\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right)$$

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right)$$

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\underbrace{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)}_{\nabla^2 u} + \mu\frac{\partial}{\partial x}\underbrace{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}_{\vec{\nabla}.\vec{V} = 0}$$

❑ Inviscid Flows vs. Viscous Flows

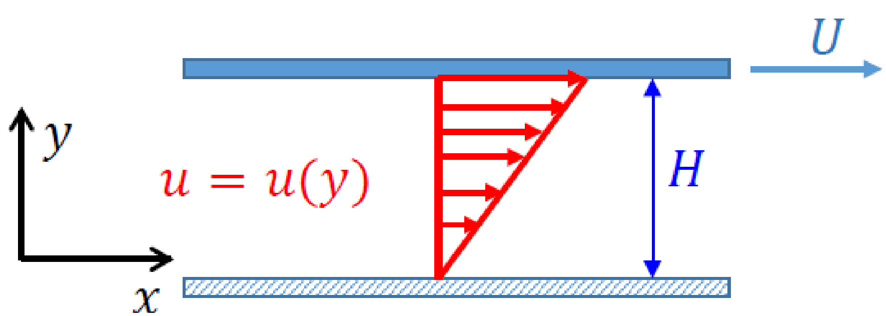
Couette flow – a simple example

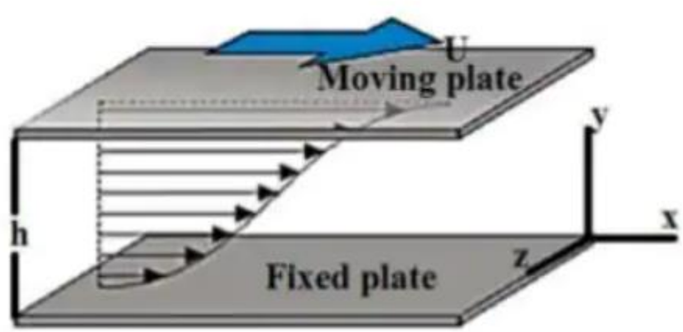
- Flow between two (infinitely large) parallel plates. Bottom plate is stationary, and the top plate moves with a constant steady speed of U.

- Flow is steady with parallel streamlines.

- With infinitely large plates there is no gradient in x direction, therefore

$$\frac{\partial u}{\partial x} = \frac{\partial p}{\partial x} = 0$$





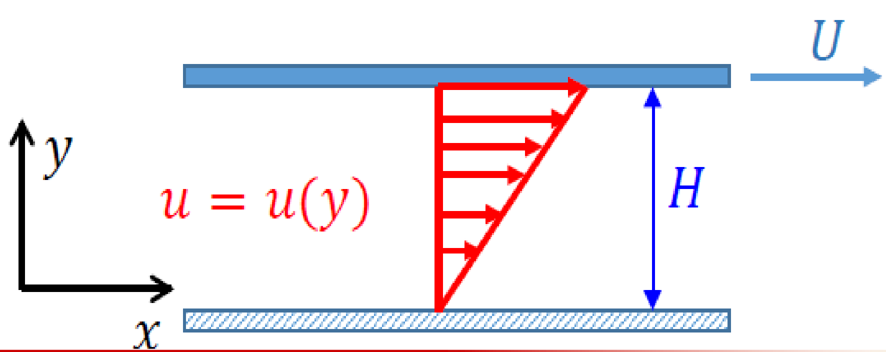
❑ Inviscid Flows vs. Viscous Flows

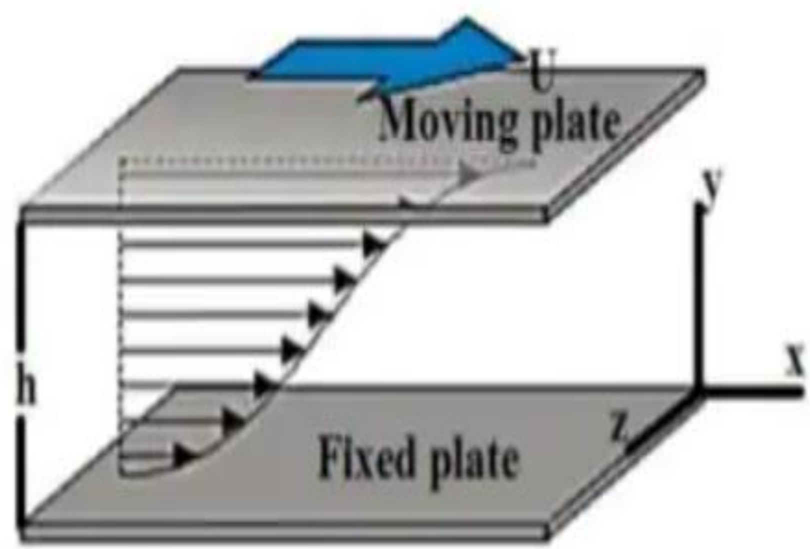
Momentum x-component: $$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\rho g_x-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

Momentum y-component: $$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=\rho g_y-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

Momentum z-component: $$\rho\left[\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]=\rho g_z-\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

Conservation of Mass: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$



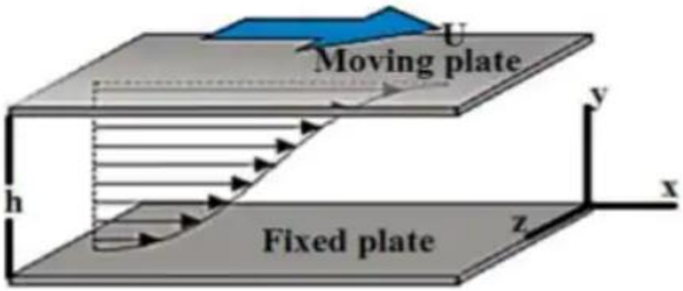


❑ Inviscid Flows vs. Viscous Flows

Velocity distribution in Couette flow

x-momentum

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$



$$0 = 0 + \mu\left(0 + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow \frac{\partial u}{\partial y} = c_1 \Rightarrow u(y) = c_1 y + c_2$$

Boundary conditions

$$u(y = 0) = 0 \Rightarrow c_2 = 0$$

$$u(y = H) = U \Rightarrow c_1 H = U \Rightarrow c_1 = U/H$$

$$u(y) = U\frac{y}{H}$$

❑ Inviscid Flows vs. Viscous Flows

Shear stress and pressure

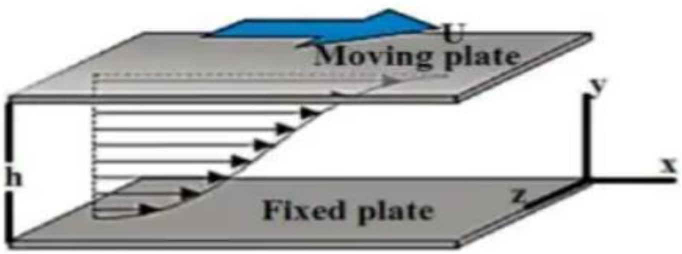
- Velocity profile

$$u(y) = U\frac{y}{H}$$

Shear stress

$$\tau = \mu\frac{du}{dy} = \mu\frac{U}{H}$$

Shear stress is constant throughout the flow.



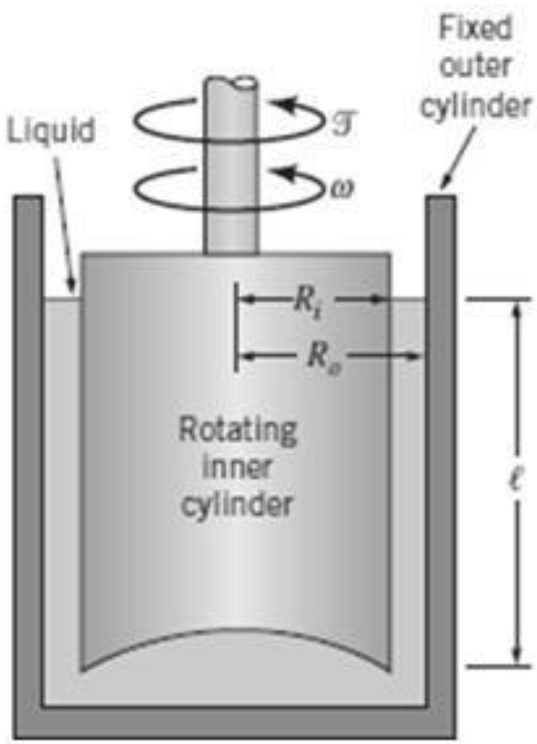
y-momentum

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \mu\nabla^2 v$$

Y-momentum simplifies to:

$$\frac{\partial p}{\partial y} = 0 \Rightarrow p = const.$$

Pressure is also constant throughout the flow.



- *The viscosity of liquids can be measured.*

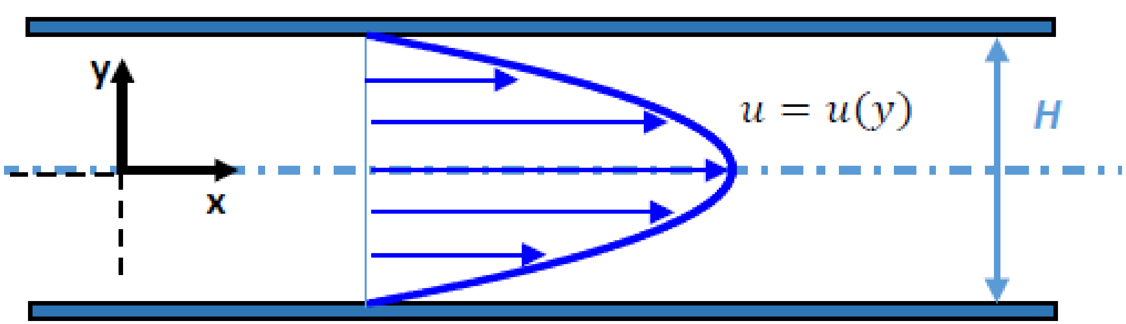
❑ Inviscid Flows vs. Viscous Flows

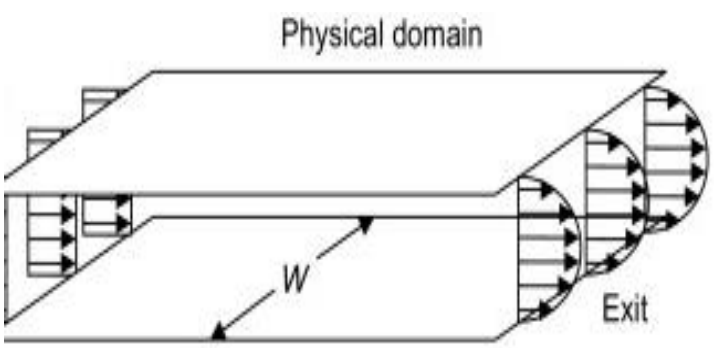
Second example-pressure driven flow

- Now consider both top and bottom plates are stationary but there is known pressure gradient dp/dx applied along the x axis. The streamlines are still parallel and

$$\frac{\partial u}{\partial x} = 0$$

Find the velocity distribution $u = u(y)$





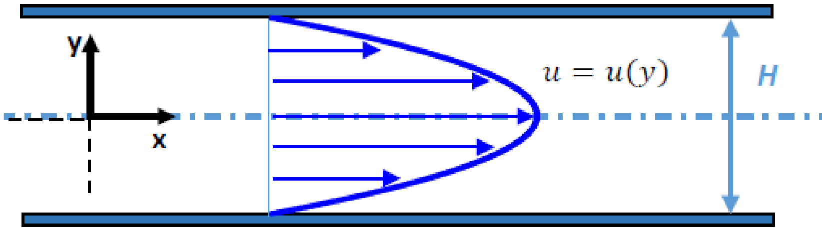
❑ Inviscid Flows vs. Viscous Flows

Solution

- X momentum

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$0 = -\frac{dp}{dx} + \mu \left(0 + \frac{\partial^2 u}{\partial y^2} \right)$$



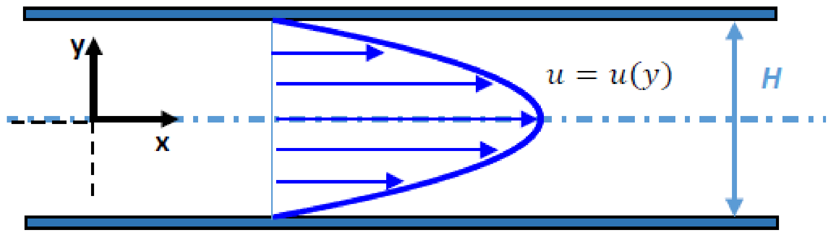
Since $\frac{\partial u}{\partial x} = 0$ you can write $\partial u / \partial y$ as du/dy

$$\frac{d^2 u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx} \Rightarrow \frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx} y + c_1$$

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx} y^2 + c_1 y + c_2$$

❑ Inviscid Flows vs. Viscous Flows

Solution-continued



$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}y^2 + c_1 y + c_2$$

Boundary conditions

$$u\left(y = \frac{H}{2}\right) = 0 \Rightarrow \frac{1}{2\mu}\frac{dp}{dx}\left(\frac{H}{2}\right)^2 + c_1\frac{H}{2} + c_2 = 0$$

$$u\left(y = -\frac{H}{2}\right) = 0 \Rightarrow \frac{1}{2\mu}\frac{dp}{dx}\left(-\frac{H}{2}\right)^2 - c_1\frac{H}{2} + c_2 = 0$$

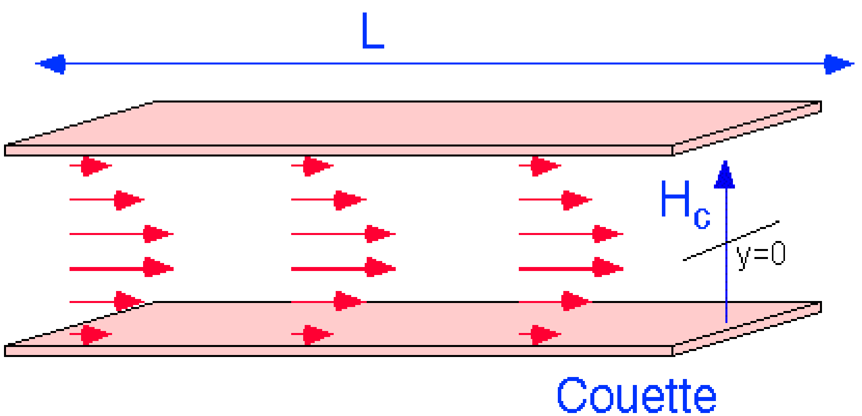
Subtract the two equation from each other

$$2c_1\frac{H}{2} = 0 \Rightarrow c_1 = 0$$

Then use one to get

$$c_2 = -\frac{1}{2\mu}\frac{dp}{dx}\left(\frac{H}{2}\right)^2$$

$$u(y) = -\frac{H^2}{8\mu}\frac{dp}{dx}\left(1 - \frac{y^2}{(H/2)^2}\right)$$



❑ Inviscid Flows vs. Viscous Flows

Understanding Viscosity – 10 minutes lecture

- https://www.youtube.com/watch?v=VvDJyhYSJv8&t=244s