Lecture # 37: Laminar and Turbulent Flows

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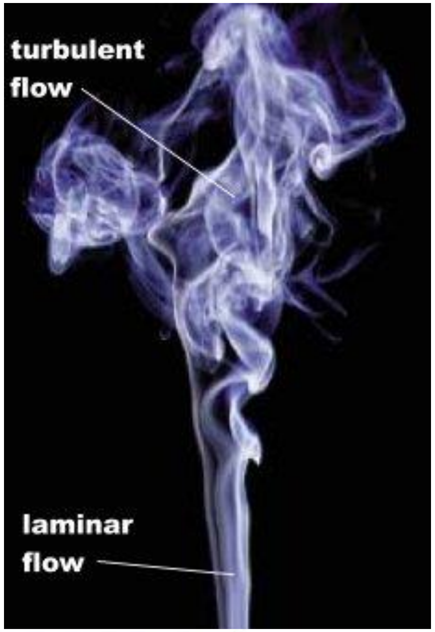
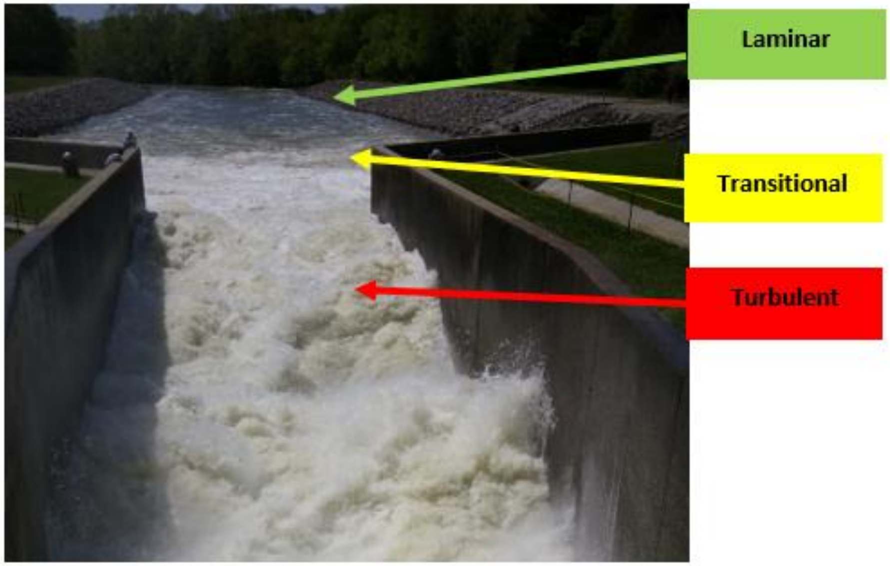
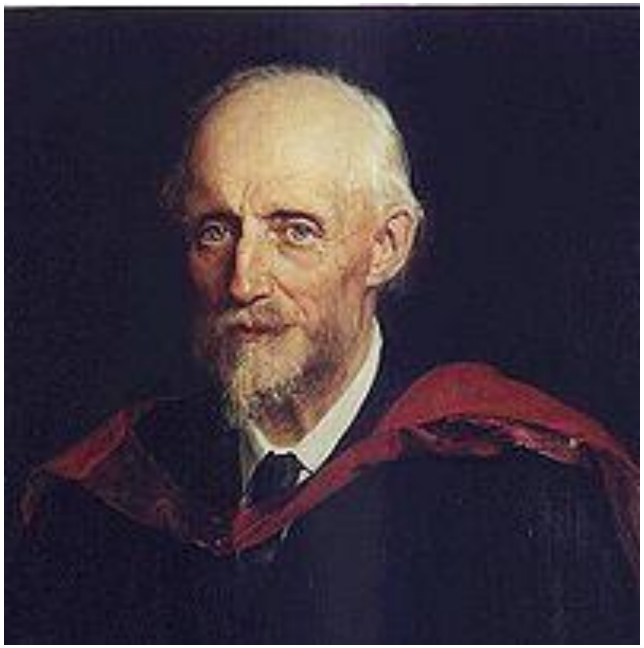
Laminar Flows and Turbulence Flows

- *Laminar flow, sometimes known as streamline flow, occurs when a fluid flows in parallel layers, with no disruption between the layers. Viscosity determines momentum diffusion.*
  - *In nonscientific terms laminar flow is "smooth," while turbulent flow is "rough."*
- *Turbulent flow is a fluid regime characterized by chaotic, stochastic property changes. Turbulent motion dominates diffusion of momentum and other scalars. The flow is characterized by rapid variation of pressure and velocity in space and time.*
  - *Flow that is not turbulent is called laminar flow*

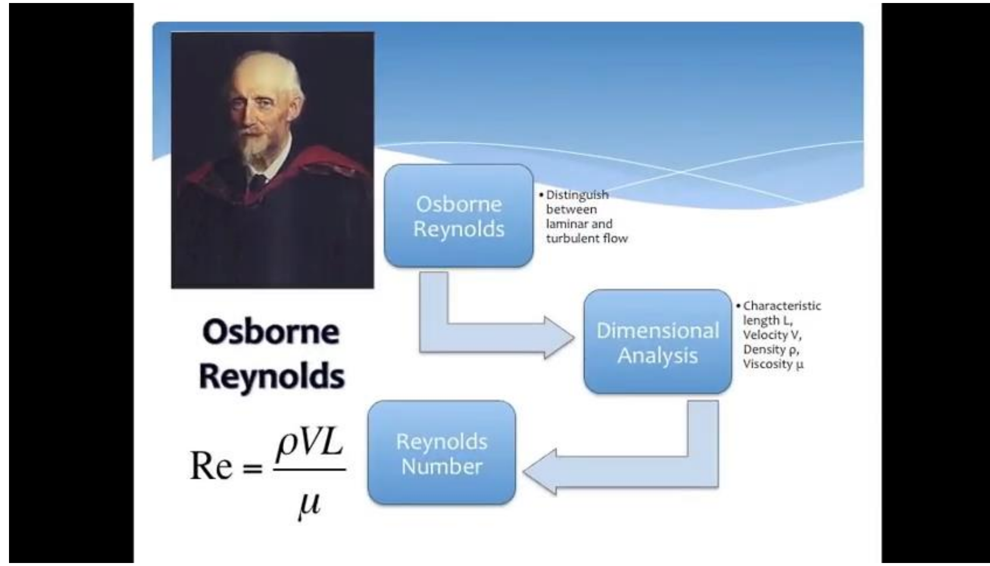


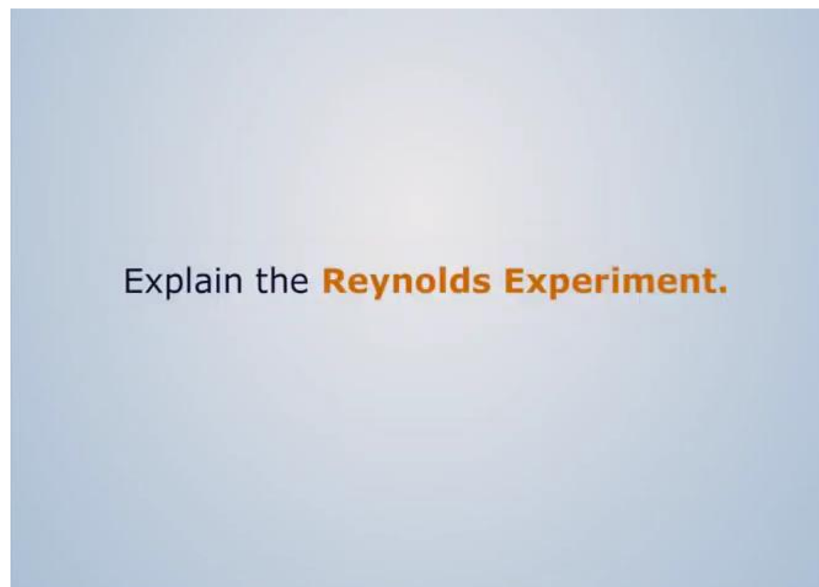


❑ Osborne Reynolds and His Famous Experiment

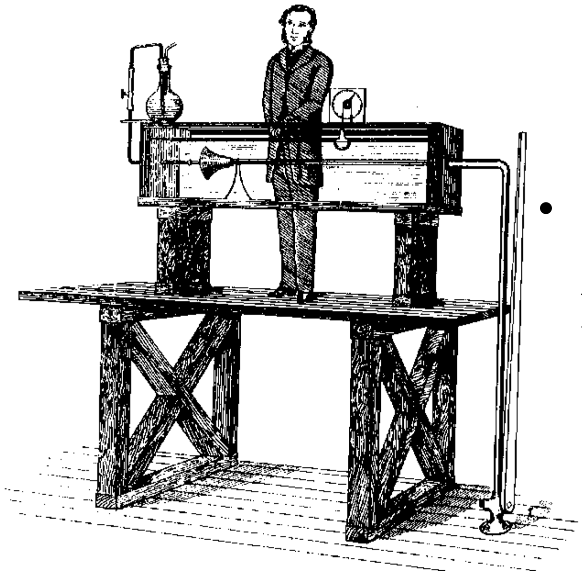


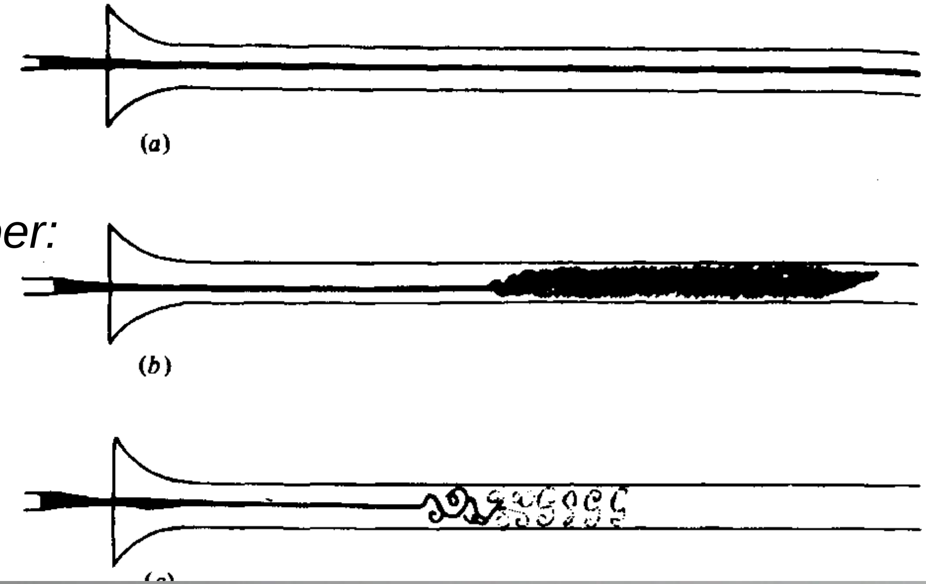
- *Osborne Reynolds (23 August 1842 – 21 February 1912) was an Irish-born innovator in the understanding of fluid dynamics.*
- *Separately, his studies of heat transfer between solids and fluids brought improvements in boiler and condenser design. He spent his entire career at what is now the University of Manchester.*





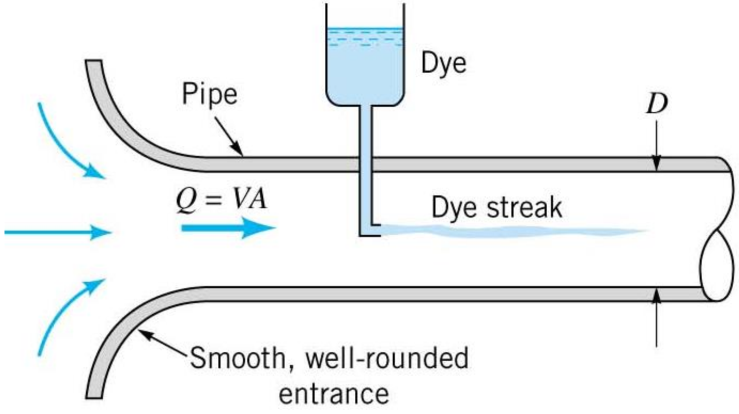
❑ Reynolds' Experiment

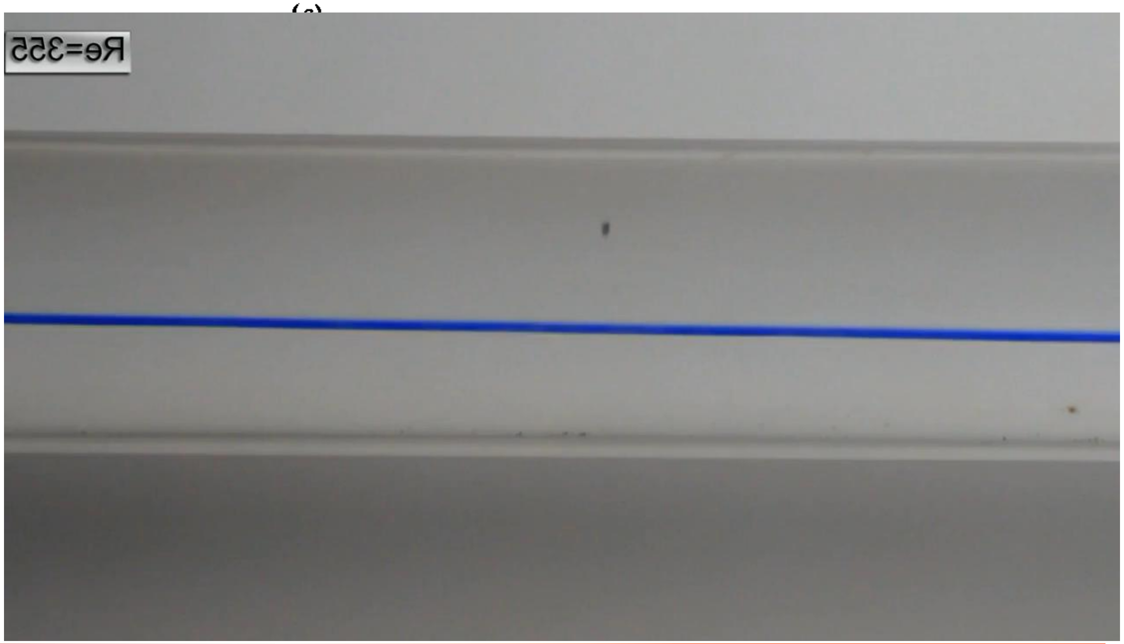




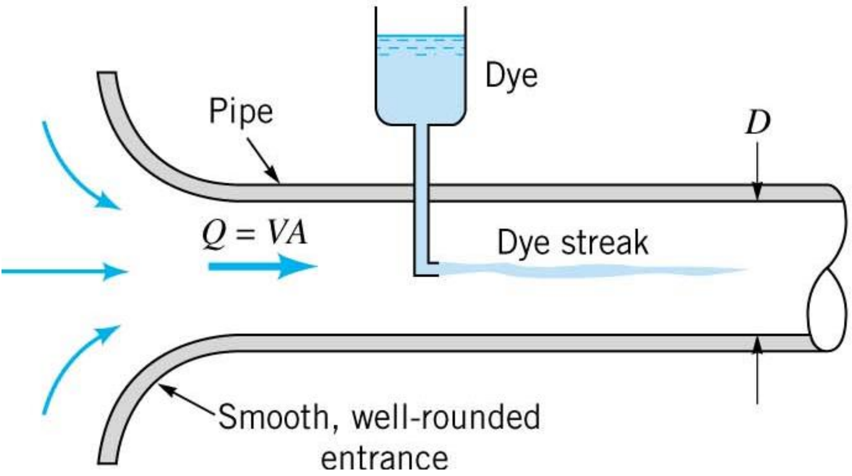
- *Reynolds number:*

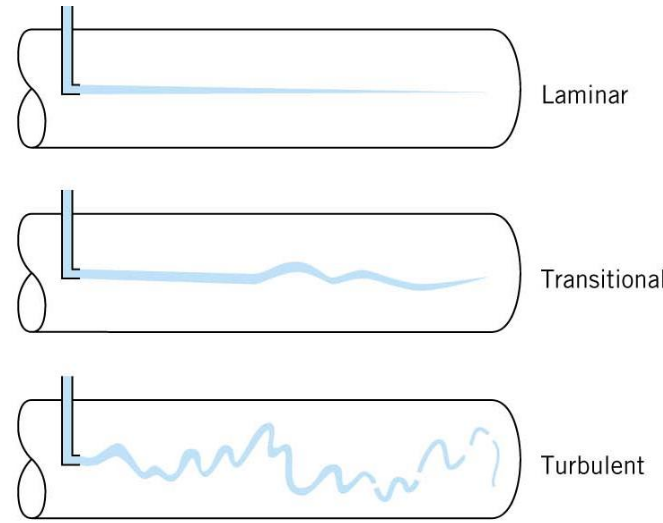
$$\mathrm{Re} = \frac{\rho D U}{\mu}$$

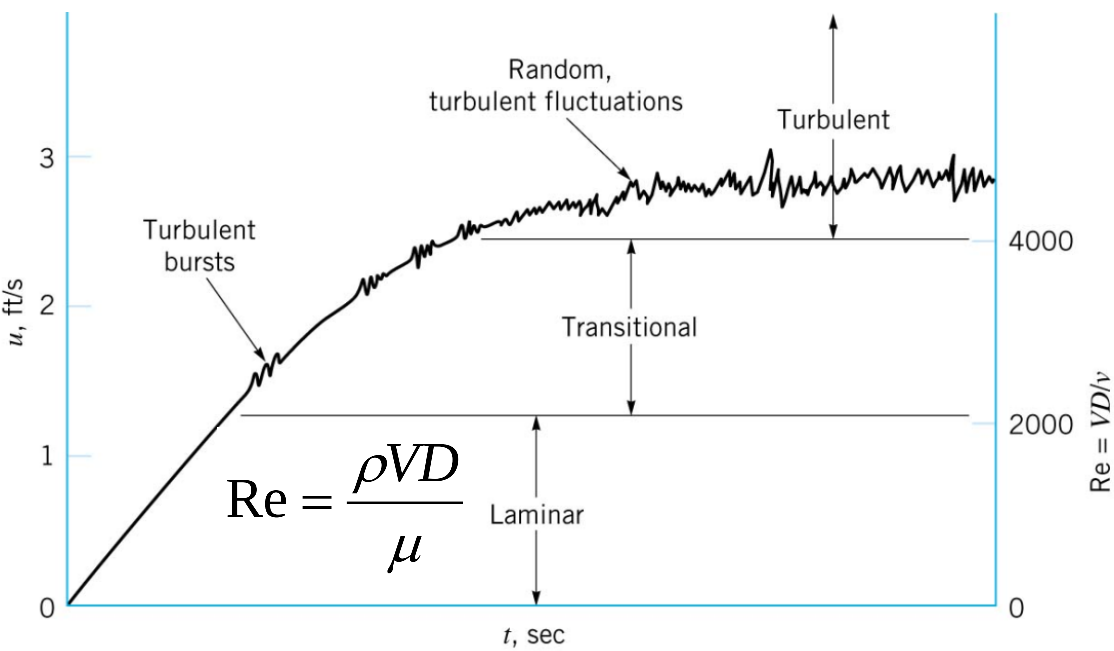




❑ Turbulent flows in a Pipe







$$\text{Re} = \frac{\rho V D}{\mu}$$

Empirically,

- *Re < 1,000, laminar flow*
- *Re ≈ 1,000 ~ 3,000, transition*
- *Re > 3000, turbulent flow.*

Re_C ~ critical Reynolds number, above which flow exhibits turbulent characteristics

❑ Characterization of Turbulent Flows

$$u = \overline{u} + u'; \quad v = \overline{v} + v' \quad w = \overline{w} + w'$$

$$\overline{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x,y,z,t)dt; \quad \overline{v} = \frac{1}{T}\int_{t_0}^{t_0+T} v(x,y,z,t)dt; \quad \overline{w} = \frac{1}{T}\int_{t_0}^{t_0+T} w(x,y,z,t)dt$$

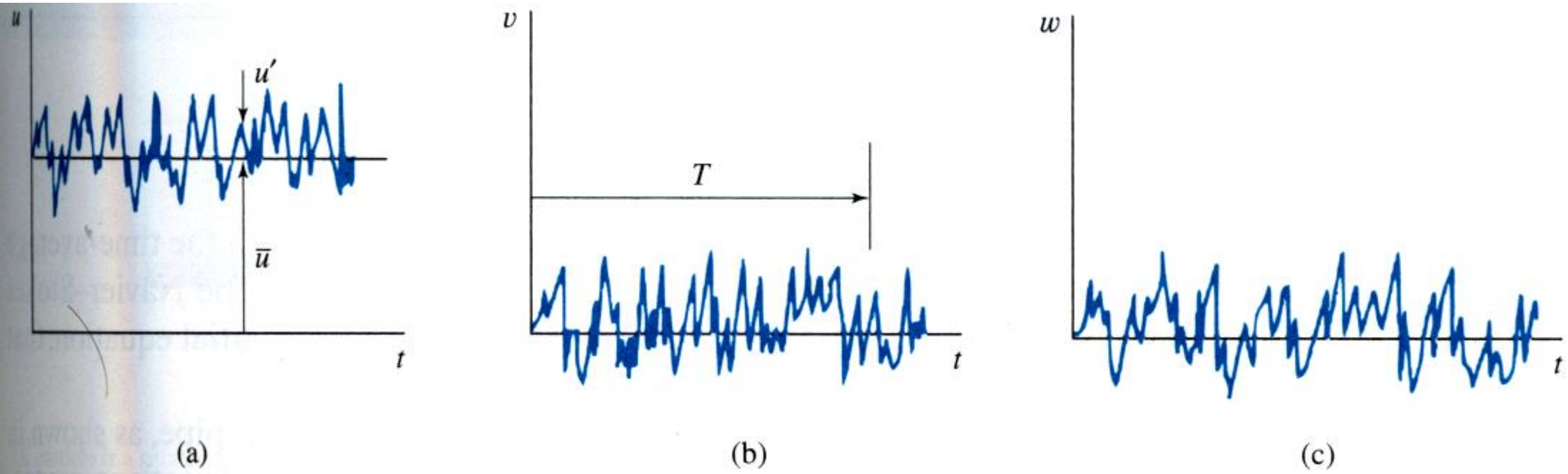
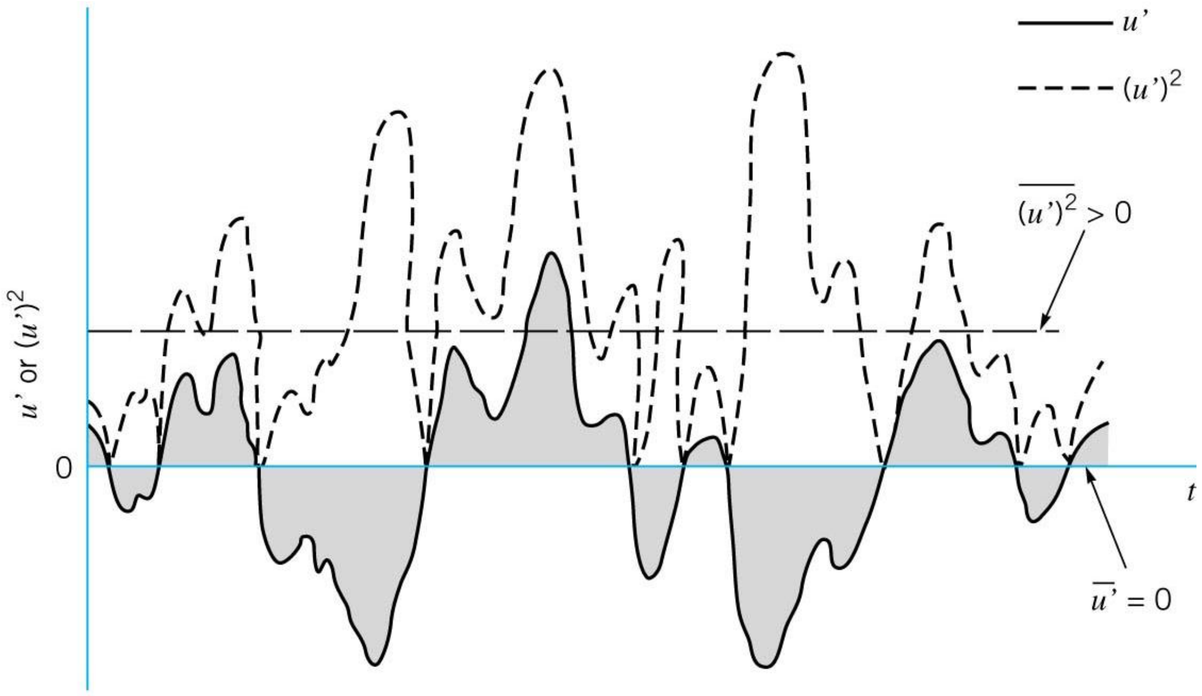


FIGURE 7.7 Velocity components in a turbulent pipe flow: (a) x-component velocity; (b) r-component velocity; (c) θ-component velocity.

❑ Turbulence intensities

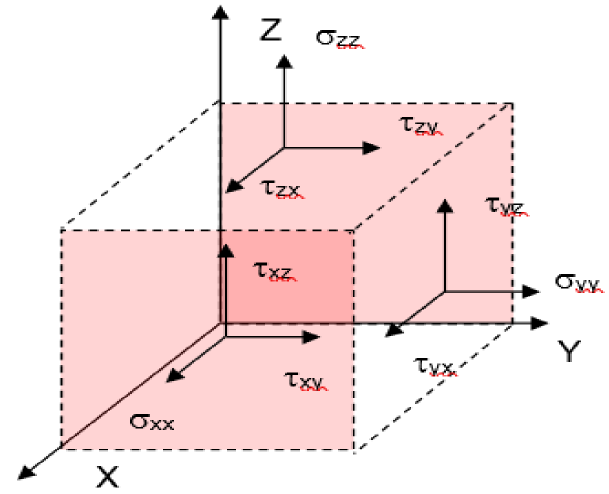
$$\overline{u'} = 0; \quad \overline{v'} = 0 \quad \overline{w'} = 0$$

$$\overline{(u')^2} = \frac{1}{T}\int_{t_0}^{t_o+T}(u')^2\,dt > 0; \quad \overline{(v')^2} > 0 \quad \overline{(w')^2} > 0$$



❑ EQUATION VISCOUS FLOWS

Conservation of momentum



For Newtonian fluid

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{xx} = -p + 2\mu\frac{\partial u}{\partial x}$$

Therefore, the x-momentum becomes

$$\rho\frac{Du}{Dt} = \frac{\partial}{\partial x}\left(-p + 2\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right)$$

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right)$$

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\underbrace{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)}_{\nabla^2 u} + \mu\frac{\partial}{\partial x}\underbrace{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}_{\vec{\nabla}.\vec{V} = 0}$$

Reynolds Stress

$$u_i = \bar{u}_i + u_i'$$

where by definition

$$\bar{u}_i = \frac{1}{\tau}\int_0^\tau u_i dt$$

It immediately follows that

$$\bar{u_i'} = \overline{u_i - \bar{u}_i} = \bar{u}_i - \bar{u}_i = 0, \text{ also } \frac{\partial}{\partial x}\bar{u}_i = \overline{\frac{\partial u_i}{\partial x}} \text{ etc.}$$

Substitute Eq. (1) into continuity and average over τ, i.e., take $\overline{(\quad)}$

$$\overline{\frac{\partial u_i}{\partial x_i}} = \overline{\frac{\partial \bar{u}_i}{\partial x_i}} + \underbrace{\overline{\frac{\partial u_i'}{\partial x_i}}}_{\downarrow \atop 0} = 0, \qquad \Longrightarrow \boxed{\frac{\partial \bar{u}_i}{\partial x_i} = 0}$$

but $$\frac{\partial u_i}{\partial x_i} = 0 = \underbrace{\frac{\partial \bar{u}_i}{\partial x_i}}_{\downarrow \atop 0\text{ , just shown}} + \frac{\partial u_i'}{\partial x_i}, \qquad \Longrightarrow \boxed{\frac{\partial u_i'}{\partial x_i} = 0}$$

❑ Reynolds Stress

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i$$

$$\overline{\frac{\partial u_i}{\partial t}} = \frac{\partial \bar{u}_i}{\partial t} + \underbrace{\overline{\frac{\partial u_i'}{\partial t}}}_{0}; \text{ similarly } \begin{cases} \overline{\nu \nabla^2 u_i} = \nu \nabla^2 \overline{u_i} \\ \overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\overline{(\bar{p} + p')} = \frac{\partial \bar{p}}{\partial x_i} \text{ etc.} \end{cases}$$

$$\overline{u_j \frac{\partial u_i}{\partial x_j}} = \overline{(\bar{u}_j + u_j')\frac{\partial}{\partial x_j}(\bar{u}_i + u_i')} = \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \underbrace{\overline{u_j' \frac{\partial \bar{u}_i}{\partial x_j}}}_{0} + \underbrace{\overline{\bar{u}_j \frac{\partial u_i'}{\partial x_j}}}_{0} + \overline{u_j' \frac{\partial}{\partial x_j} u_i'}$$

- ***From continuity equation, we have***

$$\overline{u_j' \frac{\partial}{\partial x_j} u_i'} = \frac{\partial}{\partial x_j}\overline{u_j' u_i'} - \overline{u_i' \underbrace{\frac{\partial u_j'}{\partial x_j}}_{0 \to \text{by continuity}}}$$

❑ Reynolds Stress

$$\overline{u'_j \frac{\partial}{\partial x_j} u'_i} = \frac{\partial}{\partial x_j} \overline{u'_j u'_i} - \overline{u'_i \underbrace{\frac{\partial u'_j}{\partial x_j}}_{0 \to \text{by continuity}}}$$

and thus we finally obtain

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \underbrace{-\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 \bar{u}_i}_{\frac{1}{\rho}\frac{\partial}{\partial x_j}\overline{\tau_{ij}}} - \frac{\partial}{\partial x_j}\overline{u'_i u'_j}$$

Reynolds averaged N-S equation: $\boxed{\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\overline{\tau_{ij}} - \rho \overline{u'_i u'_j}\right]}$

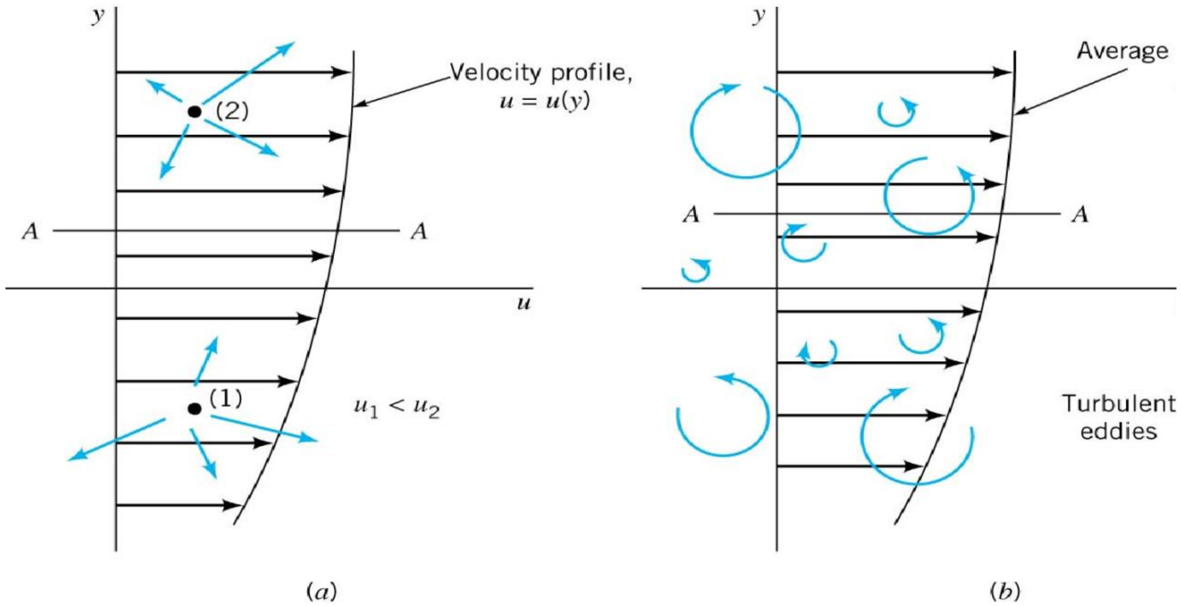
Reynolds stress: $\boxed{\tau_{R_{ij}} \equiv -\rho \overline{u'_i u'_j}}$

Turbulent Shear Stress

Laminar flows: $$\tau_{lam} = \mu \frac{\partial u}{\partial y}$$

Turbulent flows: $$\overline{\tau_{turb}} = -\rho \overline{u'v'}$$

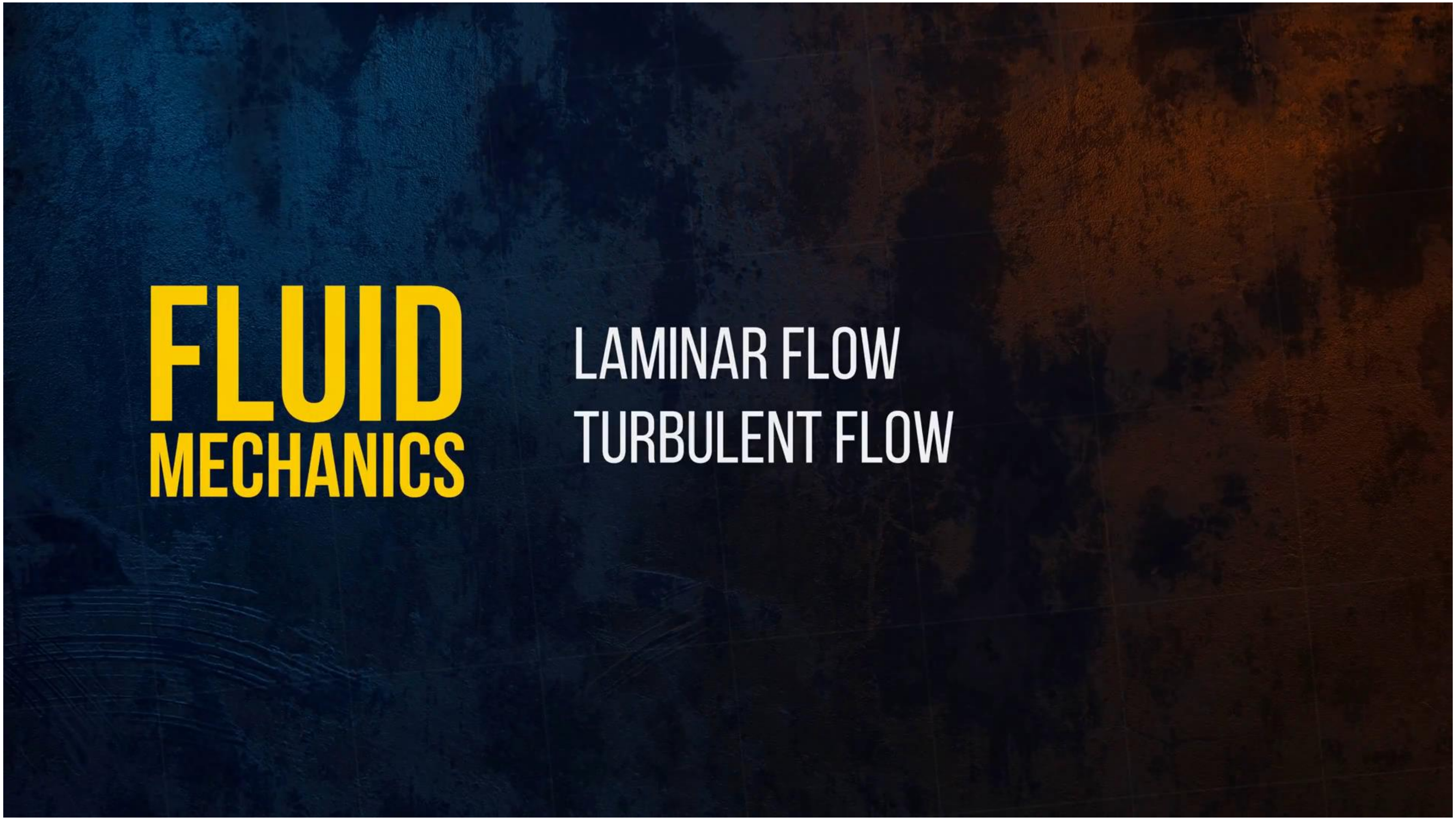
$$\bar{\tau} = \overline{\tau_{lam}} + \overline{\tau_{turb}} = \mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'}$$



(a) laminar flow (b) turbulent flow

❑ Laminar and Turbulent Flows

- *Understanding Laminar and Turbulent Flow*



https://www.youtube.com/watch?v=9A-uUG0WR0w&t=308s