Lecture # 40: Turbulent Boundary Layer Flows

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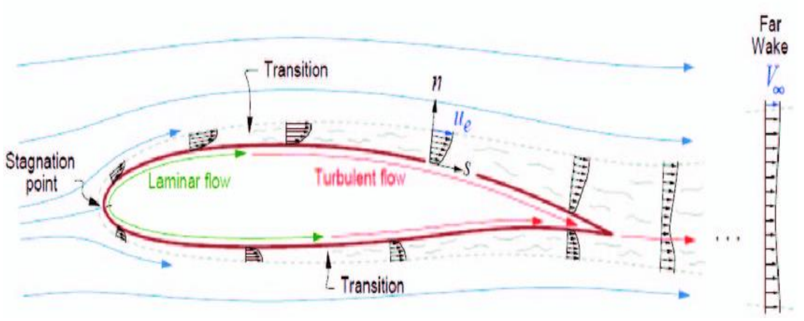
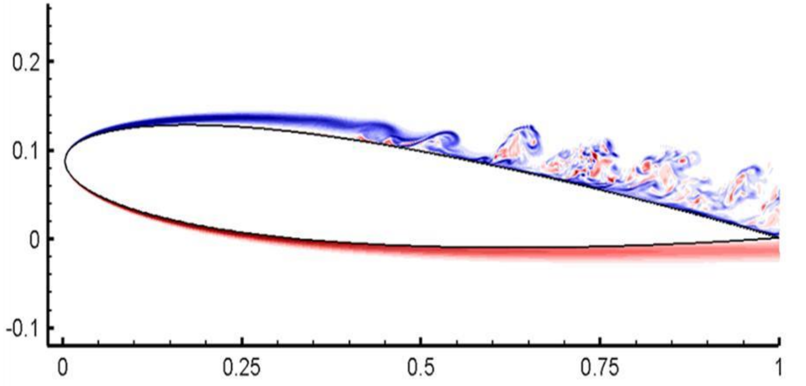
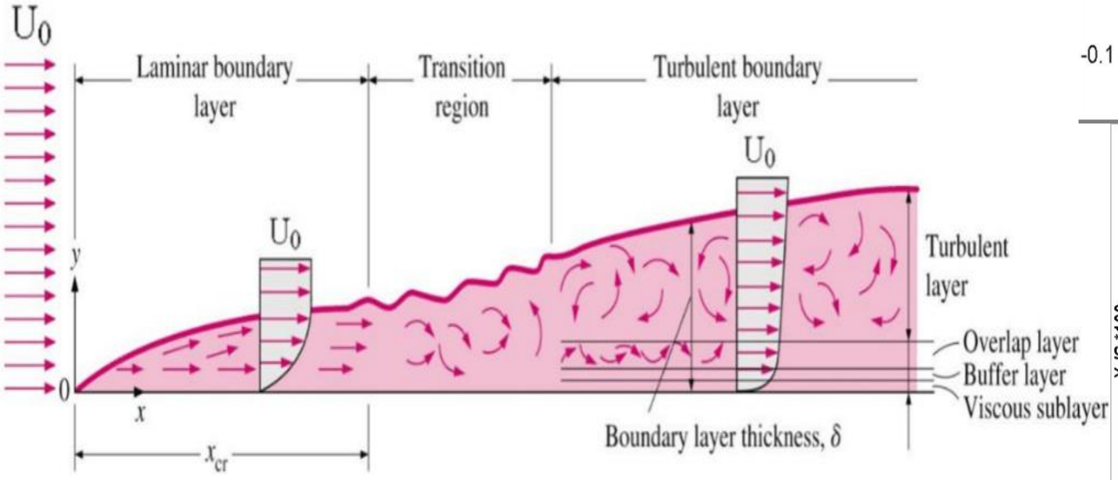
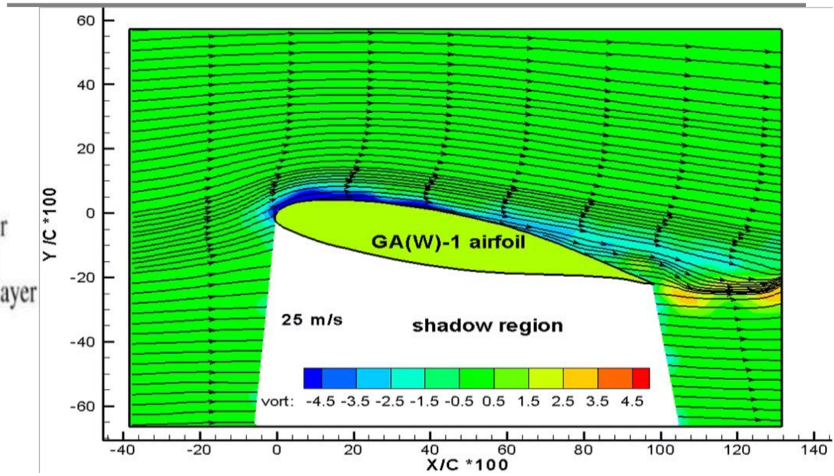
❑ Turbulence and Turbulent Boundary Layer Flows

- *Laminar flows are restricted to finite values of a critical parameter – Reynolds number, Grashof number, …).*
- *Beyond those critical values, the laminar flow is not stable. A small perturbation is amplified and flow transitions into a different regime.*
- *This regime is dominated by fluctuating and disorderly motion called turbulence.*









- *Critical Reynolds number , ~ $1.5 \times 10^5 < Re_{critic} < 1.5 \times 10^6$*

❑ Laminar Boundary Layer Flow – Blasius Solution

- *ODE form of the BL equation :*

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \quad or \quad 2f''' + f \cdot f'' = 0$$

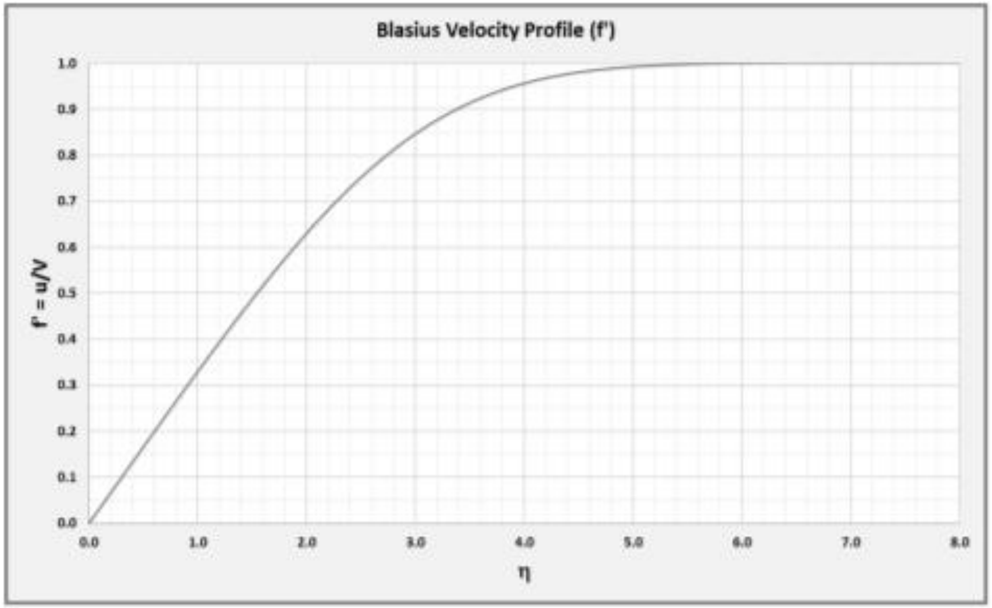
boundary conditions:

No slip ($u = 0$ @$y = 0$) ⇒ $f'(0) = 0$

No slip ($v = 0$ @$y = 0$) ⇒ $f(0) = 0$

Far field free stream ($u = V_\infty$ @$y \to \infty$) ⇒ $f'(\infty) = 1$

η	f	f'	f''
0.00	0.00000	0.00000	0.33200
0.20	0.00664	0.06641	0.33193
0.40	0.02656	0.13277	0.33142
0.60	0.05974	0.19894	0.33003
0.80	0.10611	0.26472	0.32735
1.00	0.16558	0.32980	0.32298
1.20	0.23796	0.39381	0.31657
1.40	0.32301	0.45631	0.30785
1.60	0.42037	0.51683	0.29666
1.80	0.52959	0.57486	0.28294
2.00	0.65013	0.62989	0.26676
2.20	0.78134	0.68147	0.24836
2.40	0.92249	0.72918	0.22810
2.60	1.07278	0.77269	0.20646
2.80	1.23132	0.81178	0.18400
3.00	1.39724	0.84634	0.16134
3.20	1.56963	0.87641	0.13909
3.40	1.74759	0.90211	0.11782
3.60	1.93029	0.92370	0.09802
3.80	2.11692	0.94151	0.08004
4.00	2.30676	0.95592	0.06414
4.20	2.49919	0.96736	0.05042
4.40	2.69365	0.97628	0.03887
4.60	2.88968	0.98309	0.02938
4.80	3.08689	0.98819	0.02177
5.00	3.28499	0.99194	0.01580
5.20	3.48373	0.99464	0.01124
5.40	3.68292	0.99655	0.00784
5.60	3.88244	0.99787	0.00535
5.80	4.08217	0.99876	0.00357
6.00	4.28206	0.99936	0.00233

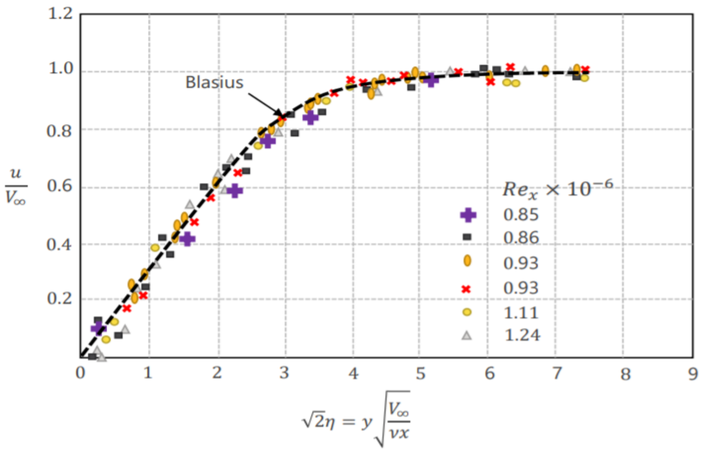


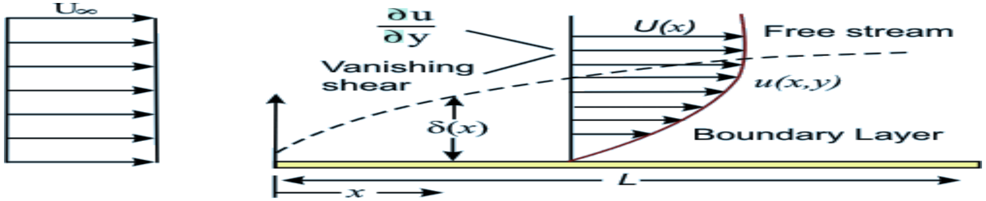
- Blasius solution calculated using a spreadsheet
- ODEs integrated numerically using Euler Predictor-Corrector method
- $f''(0) = \mathrm{h}(0)$ determined by trial and error ("shooting method") as 0.332.

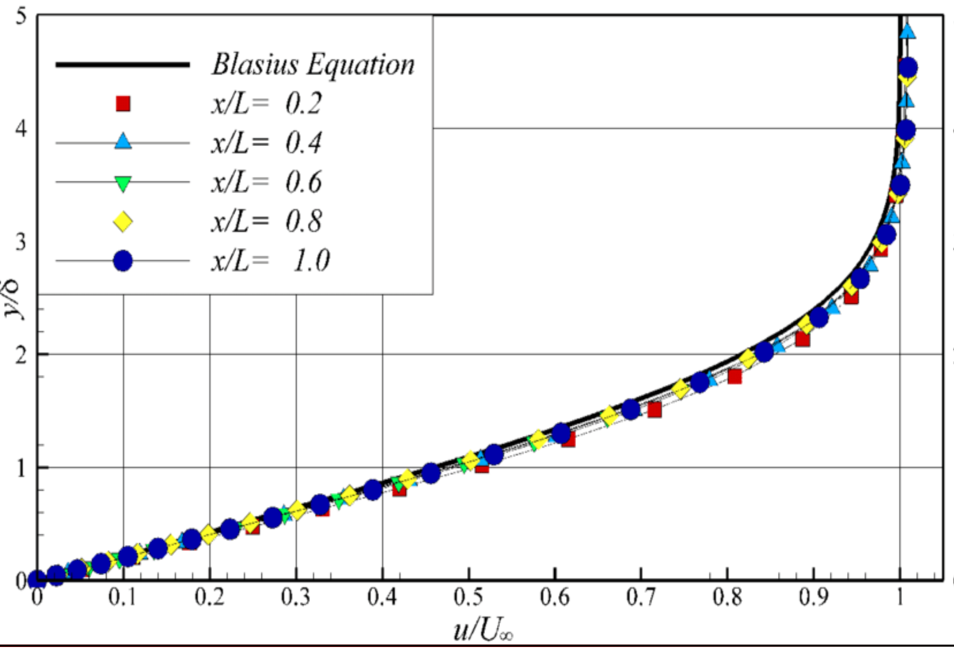
❑ Laminar Boundary Layer Flow – Blasius Solution

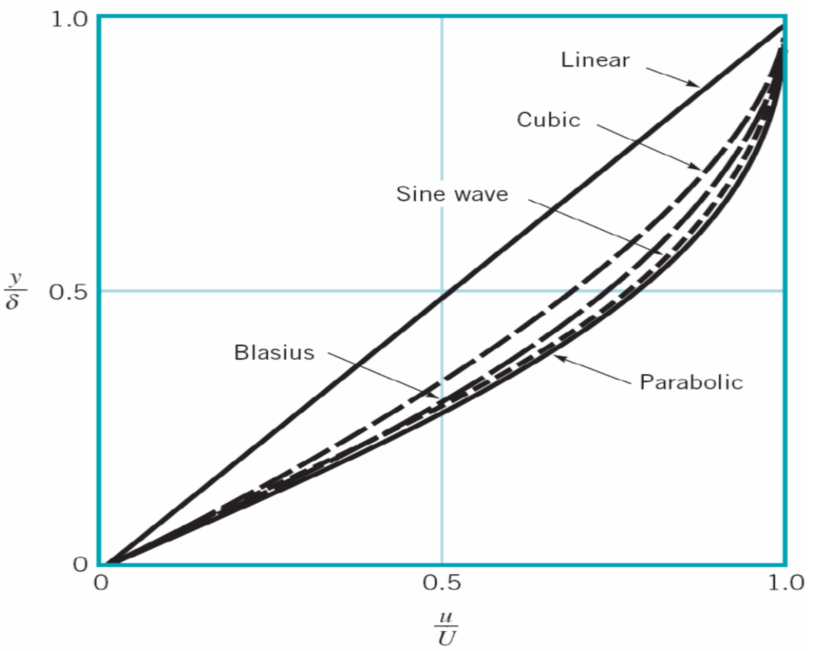
- BL approximation works very well, except very close to the leading edge where Re_x is small.

$$u(x) = V_\infty f'$$





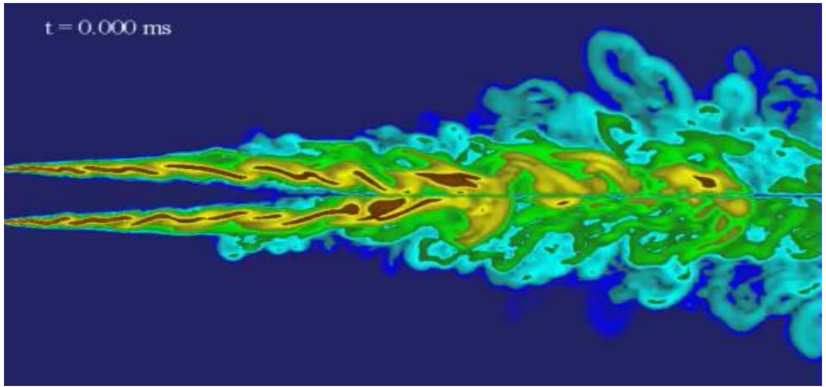




❑ Turbulence and Turbulent Boundary Layer Flows

Turbulent flow

- The turbulent motion may be described by:
  - Fluctuations : velocity, pressure,…
    - velocity fluctuates in all three dimensions and in time
  - Eddies
    - Fluid packets of many sizes, larger ones break down into smaller ones. At the smallest size, eddies dissipate due to viscosity.
  - Random
    - Fluid properties have random variations but have a particular form. Each property has a specific continuous energy spectrum which drops off to zero at small eddy size region
  - Self-sustaining
    - Turbulent flow once triggered, can sustain itself and create new eddies replacing the one lost due to dissipation.
  - Mixing
    - Much stronger than laminar case (molecular diffusion). Eddies actively move in all directions and cause rapid mixing.



❑ Turbulent Boundary Layer Flows

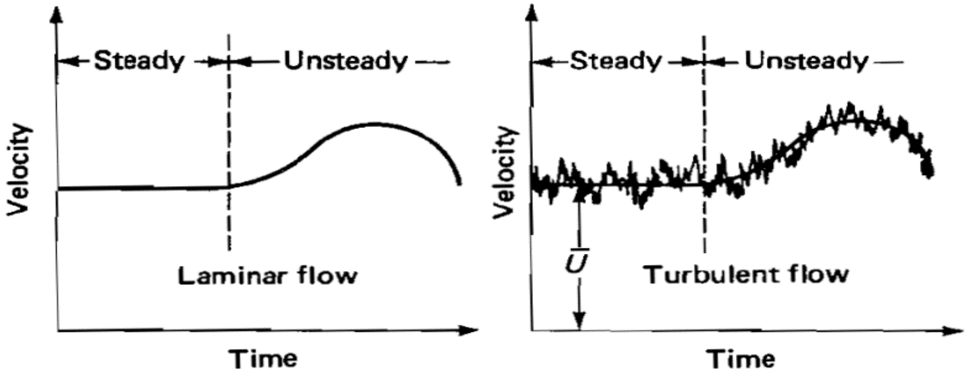
Mathematical model and solutions

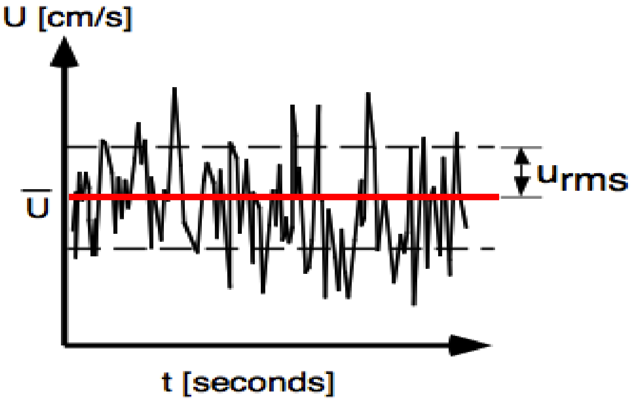
- Navier-Stokes equations apply to turbulent flows as well.
- In general NS equations can be solved numerically to obtain the flow field.
- Due to wide range of ‘scales’ however, a very high-resolution grid is required. This makes solution impractical for majority of the flows.

- Alternatively, fluid velocity may be considered as the sum of a mean and a fluctuating part:

$$u(x, y, z, t) = \bar{u}(x, y, z, t) + u'(x, y, z, t)$$

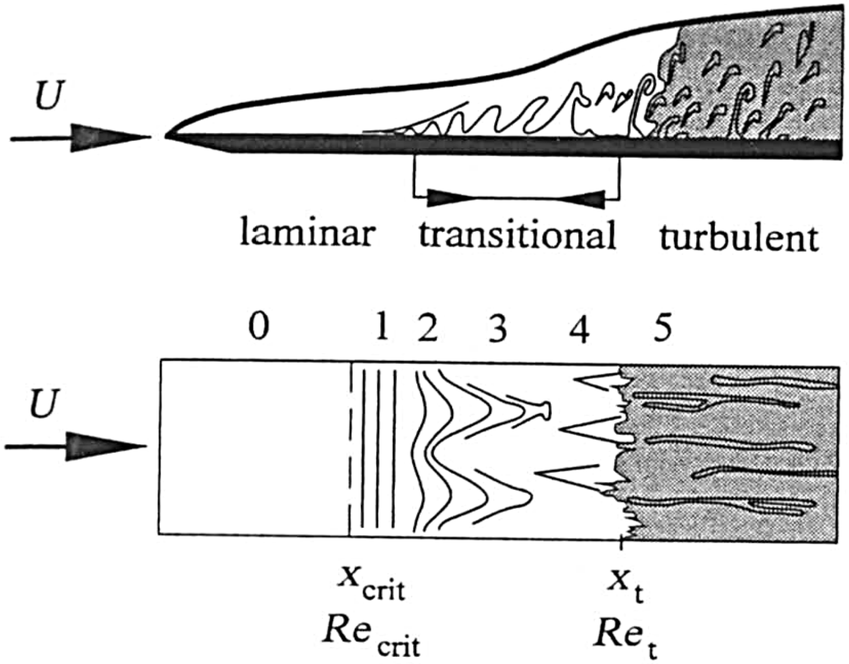
- The equations then are solved for the average field $(\bar{u}, \bar{v}, \bar{w})$, where the terms involving the fluctuating variables are modeled (turbulence modeling).





❑ Turbulent Boundary Layer Flows

- **Boundary layer flow over flat plate**



0 stable, laminar flow

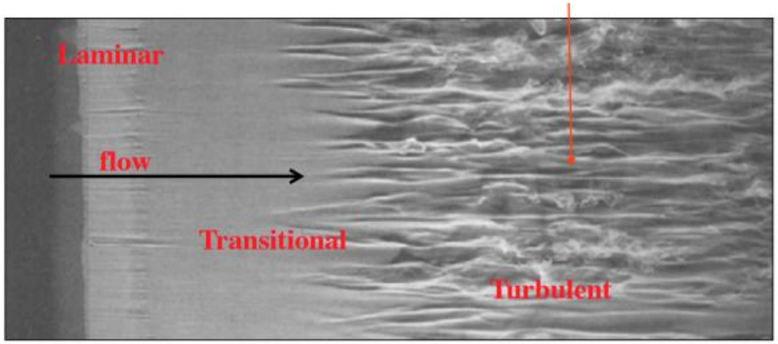
1 unstable Tollmien–Schlichting waves

2 three-dimensional waves, Λ-vortices

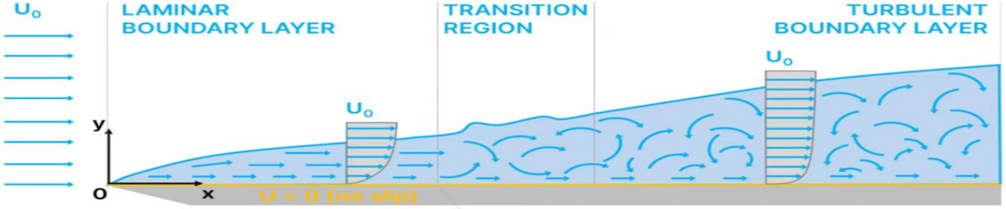
3 vortex decay

4 formation of turbulent spots

5 turbulent flow



- Laminar --> *orderly*
- Turbulent --> *chaotic*



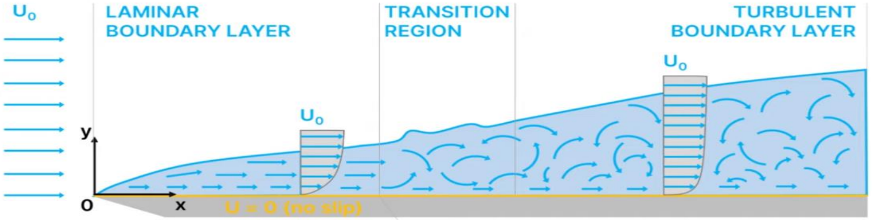
$$Re_x = \frac{\rho U x}{\mu}$$

$$Re_{x,critical} \approx 5 * 10^5$$
for a flat plate

❑ Turbulent Boundary Layer Flows

- ***Some characteristics of turbulent boundary layer***

  - Flow outside the boundary layer is irrotational
  - Flow inside the boundary layer is rotational ($\omega \neq 0$)
  - Fluid particles deform and rotate in the laminar boundary layer. They become highly distorted in the turbulent region due to irregular nature of turbulence.
  - The transition to turbulence occurs at Re_{cr} ~ 2×10^5 to 3×10^6 depending on the roughness of the surface and upstream turbulence.
  - The velocity gradient at the surface is higher in turbulent flow
  - Wall shear stress is then higher in the turbulent flow.



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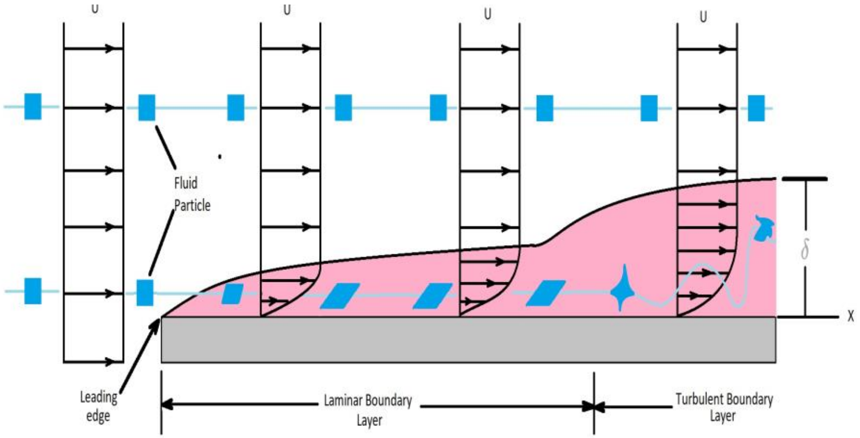
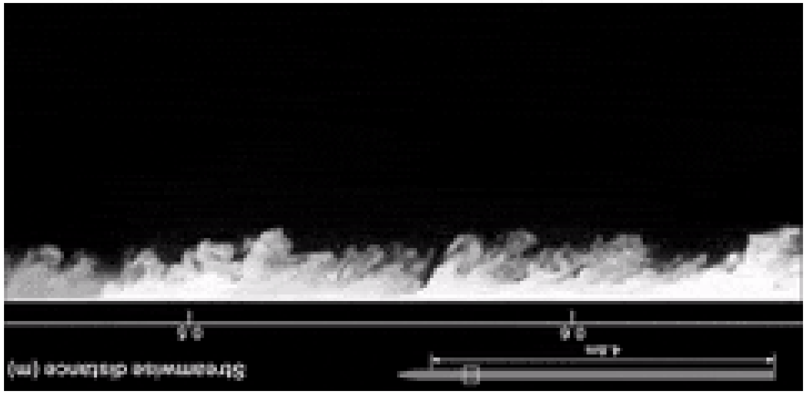
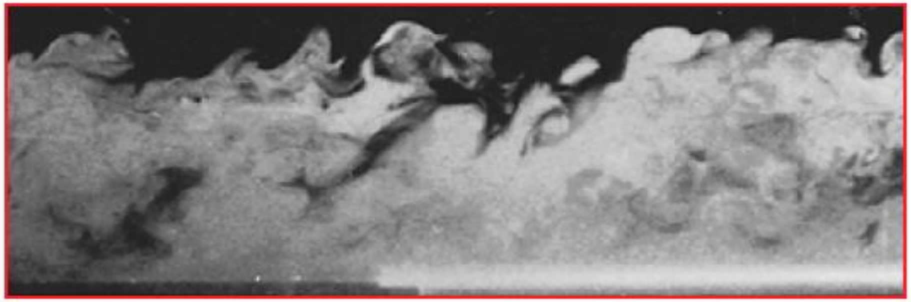
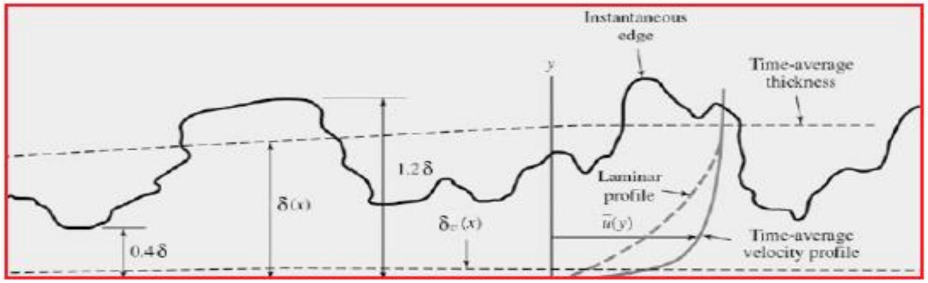




❑ Turbulent Boundary Layer Flows

Turbulent Boundary Layer Model

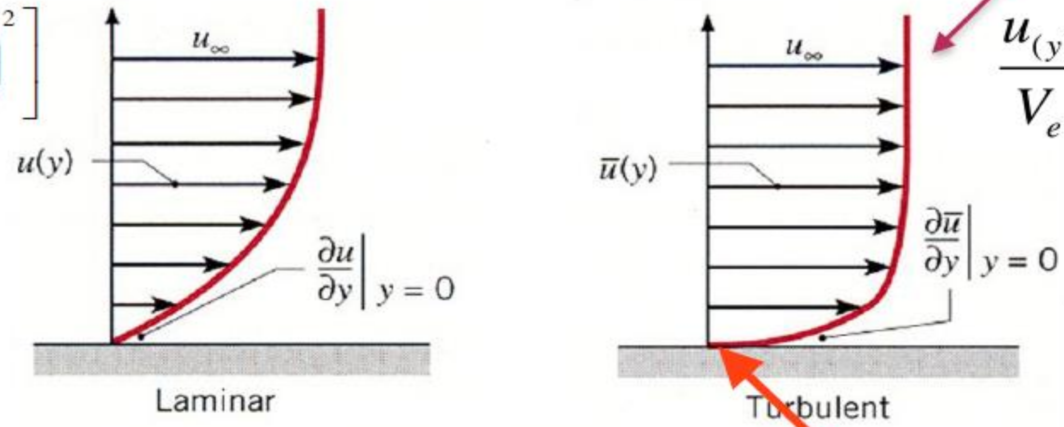


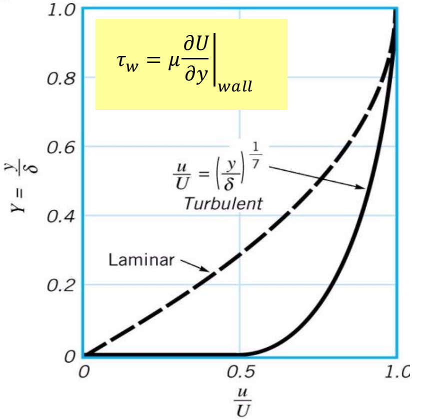


Comparison of laminar and turbulent velocity profile in the boundary layer

$$\frac{u_{(y)}}{V_e} = \left[\frac{2\cdot y}{\delta} - a\cdot\left(\frac{y}{\delta}\right)^2\right]$$

$$\frac{u_{(y)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$





$$\left.\frac{\partial u}{\partial y}\right|_{y=0,\ \text{lam}} < \left.\frac{\partial \bar{u}}{\partial y}\right|_{y=0,\ \text{turb}}$$

❑ Turbulent Boundary Layer Flows

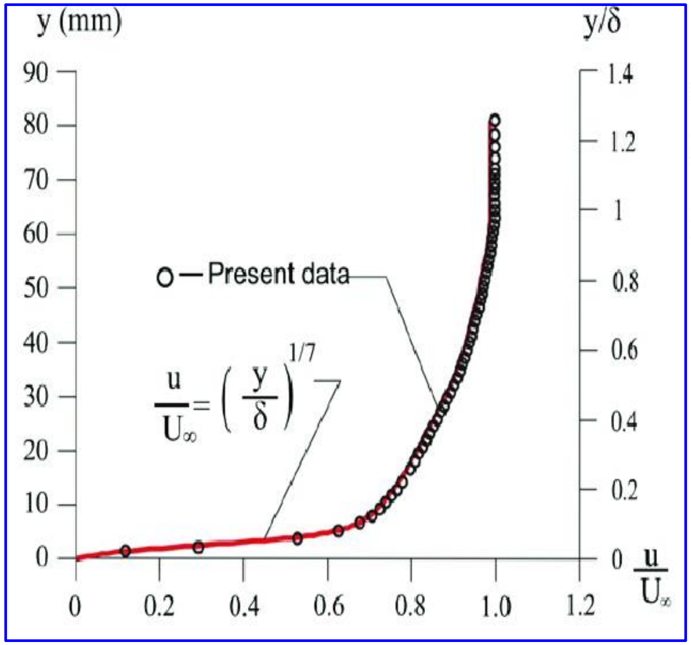
Velocity Profile in Turbulent Boundary Layer

- Power-Law Velocity Profile Models

Turbulent (exponential curve fit)

$$\frac{u_{(y)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

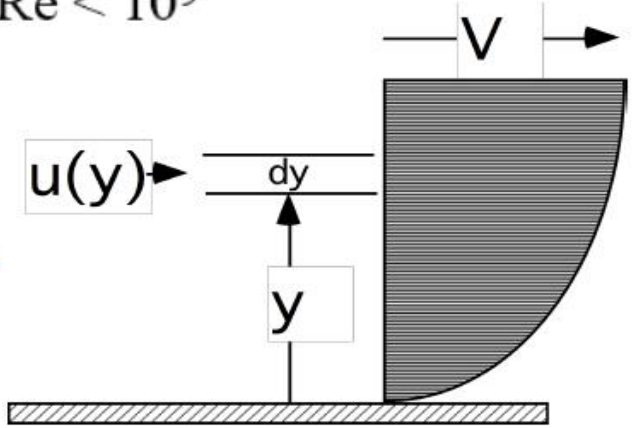
We'll find out what *n* is later



$n = 7 \qquad Re < 10^7$

$n = 8 \qquad 10^7 < Re < 10^8$

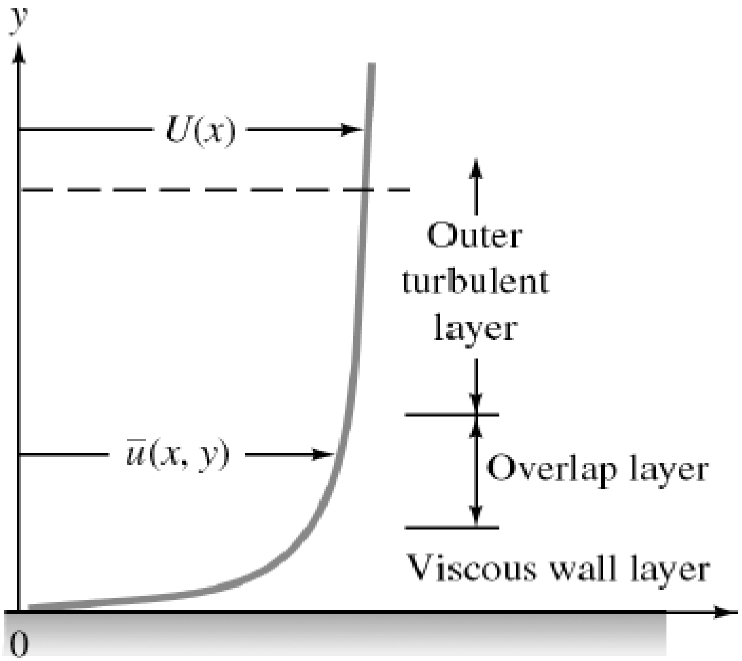
$n = 9 \qquad 10^8 < Re < 10^9$



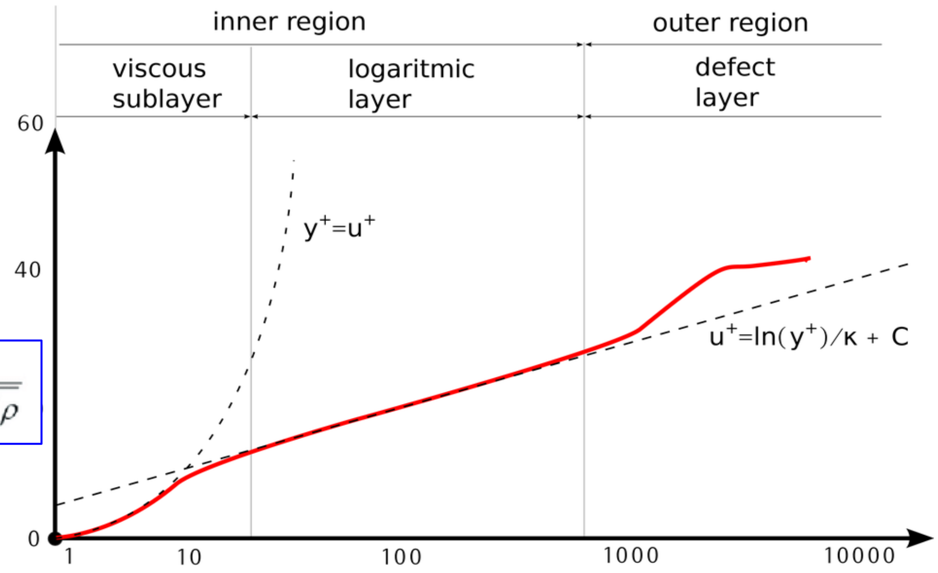
❑ Turbulent Boundary Layer Flows

$$Re_x = \frac{\rho U x}{\mu}$$

$$Re_{x,critical} \approx 5 * 10^5$$
for a flat plate



turbulent boundary layer



$$U^+ \equiv \frac{U}{\sqrt{\tau_0/\rho}}$$

$$y^+ \equiv \frac{y}{l_\nu} = \frac{y}{\nu}\sqrt{\tau_0/\rho}$$

❑ Turbulent Boundary Layer Flows

- *Some characteristics of turbulent boundary layer*
  - boundary layer thickness (smooth plate)

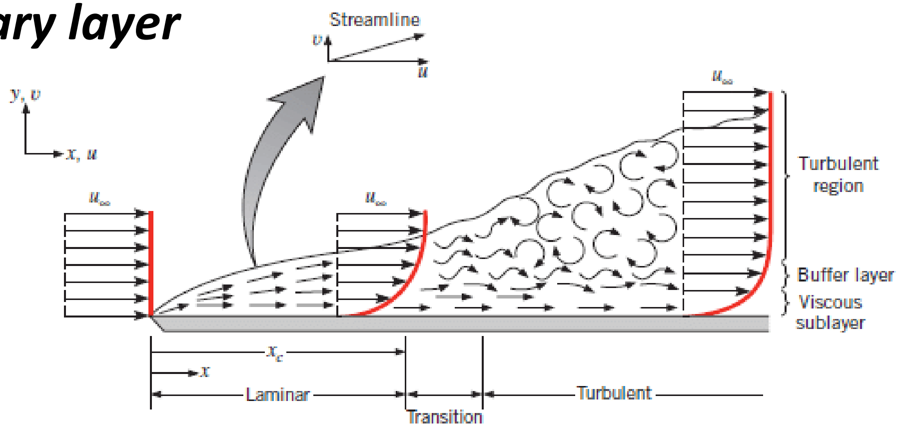
$$\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}}$$

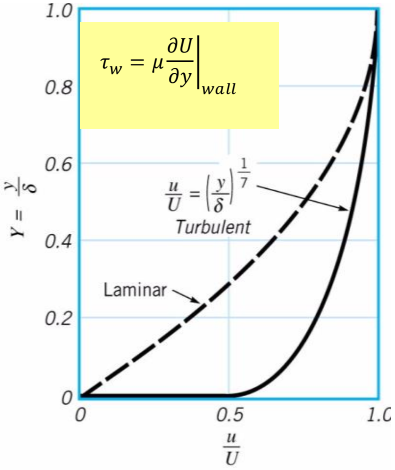
Recall for laminar flow $\delta/x \sim Re_x^{-1/2}$

  - Turbulent boundary layer grows faster! (instead of $x^{1/2}$)
  - Skin friction (Drag) for plate with length L

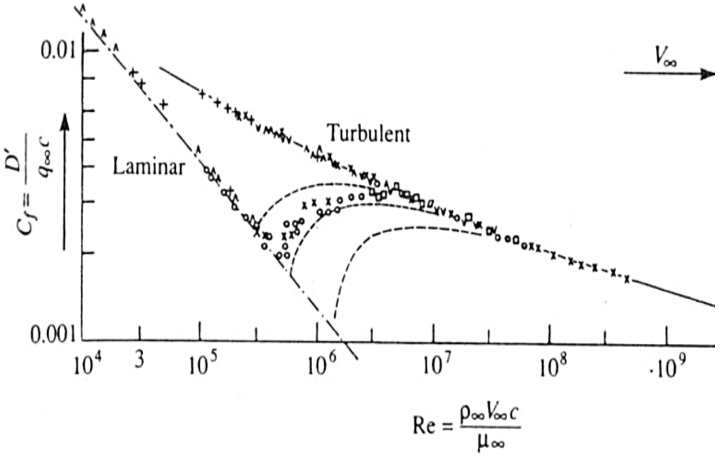
$$C_f = C_D = \frac{0.074}{Re_L^{1/5}}$$

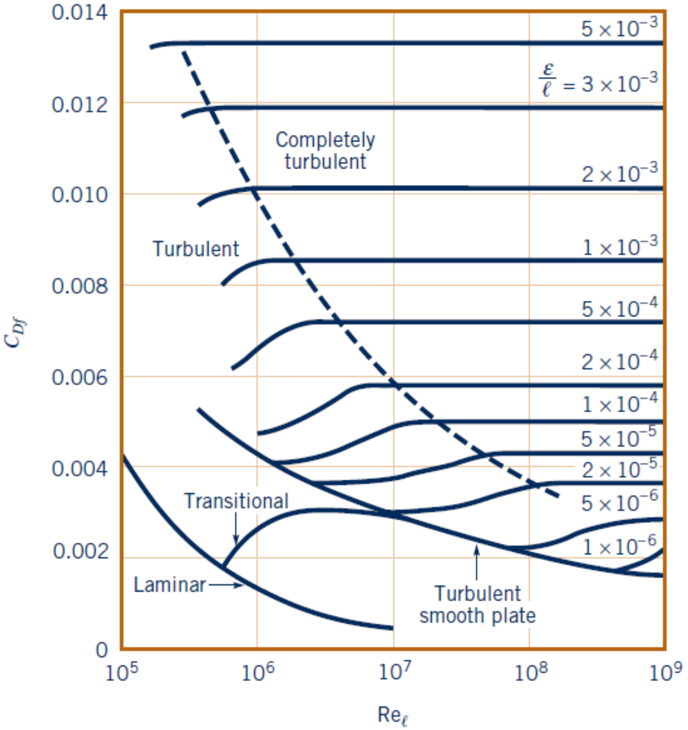
Larger friction drag for turbulent flow





$$\tau_w = \mu \frac{\partial U}{\partial y}\bigg|_{wall}$$





❑ Turbulent Boundary Layer Flows

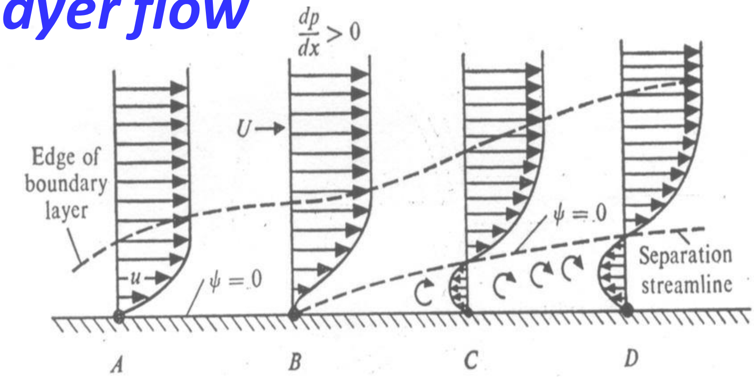
▪ *Effect of pressure gradient on boundary layer flow*

- X-momentum equation

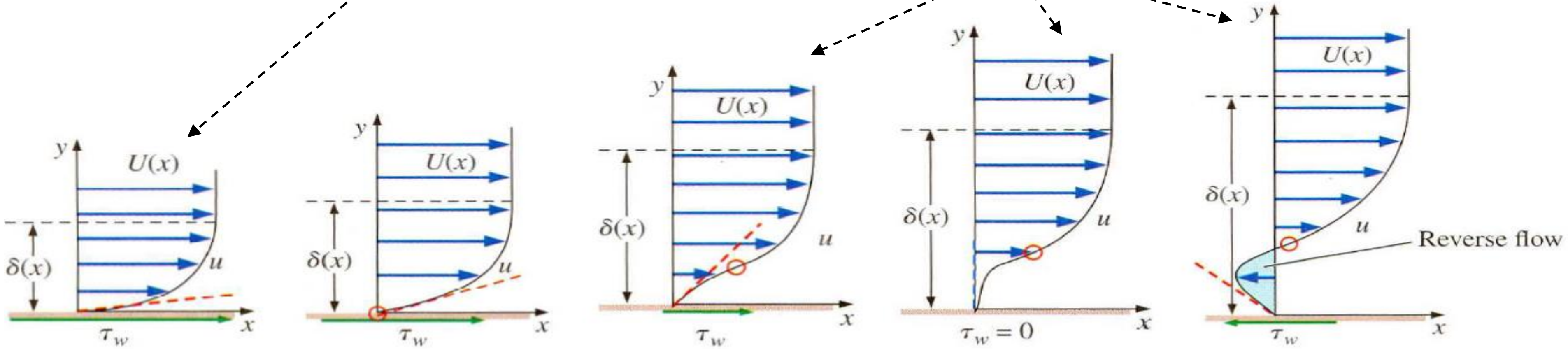
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

Evaluate at the wall ($u = 0, v = 0$)

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{wall} = \frac{1}{\mu}\frac{dp}{dx}$$



- ***Favorable pressure gradient:*** $\frac{\partial p}{\partial x} < 0$
- ***Adverse pressure gradient:*** $\frac{\partial p}{\partial x} > 0$



(a) FPG
$\frac{dp}{dx} < 0, \quad \frac{dU}{dx} > 0$
$\tau_w > 0$
No separation

(b) ZPG
$\frac{dp}{dx} = 0, \quad \frac{dU}{dx} = 0$
$\tau_w > 0$
No separation

(c) Mild APG
$\frac{dp}{dx} > 0, \quad \frac{dU}{dx} < 0$
$\tau_w > 0$
No separation

(d) Critical APG
$\frac{dp}{dx} > 0, \quad \frac{dU}{dx} < 0$
$\tau_w = 0$
Separation point

(e) Strong APG
$\frac{dp}{dx} > 0, \quad \frac{dU}{dx} < 0$
$\tau_w < 0$
Separated region

Fig. 15: BL velocity profiles under different pressure gradients $\frac{dp}{dx} = -\rho U\frac{dU}{dx}$ (source of figure [6]).

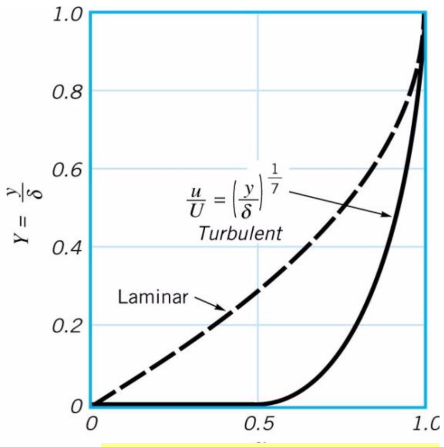
❑ Turbulent Boundary Layer Flows

▪ *Flow separation*

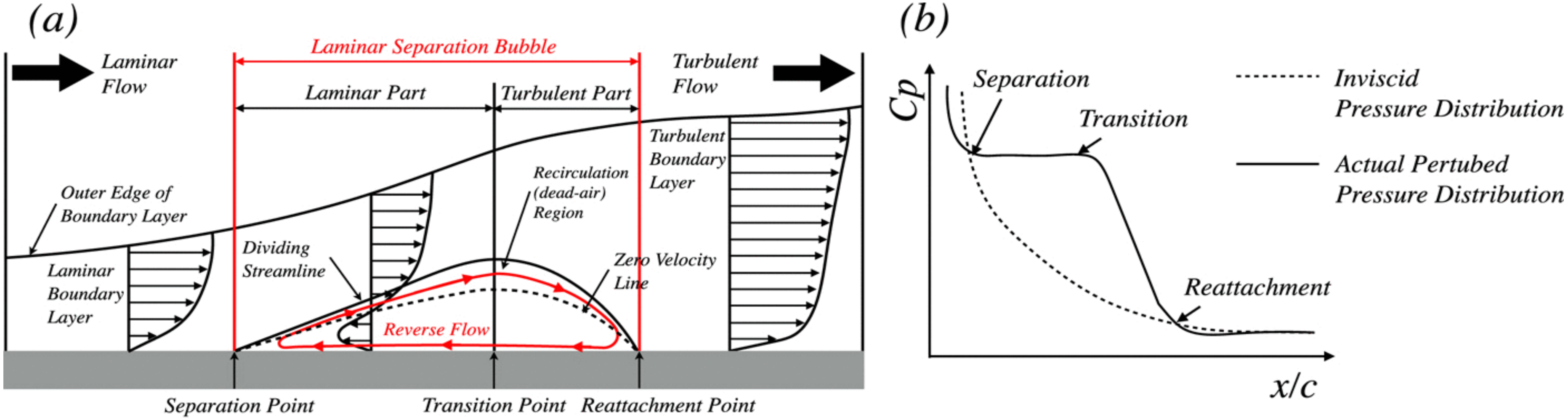
- Flow separation occurs when there is adverse pressure gradient $(\frac{dp}{dx} >$

Separation point: $\tau_w = 0 \Rightarrow \frac{\partial u}{\partial v} = 0$

- ***Separation can occur both in laminar and turbulent boundary layer.***
- ***Laminar boundary layer is more readily to sperate at relatively small adverse pressure gradient.***
- ***After turbulence transition, separated laminar boundary flow can re-attach to the surface after separation, creating a separation bubble.***



$$\tau_w = \mu \frac{\partial U}{\partial y}\bigg|_{wall}$$



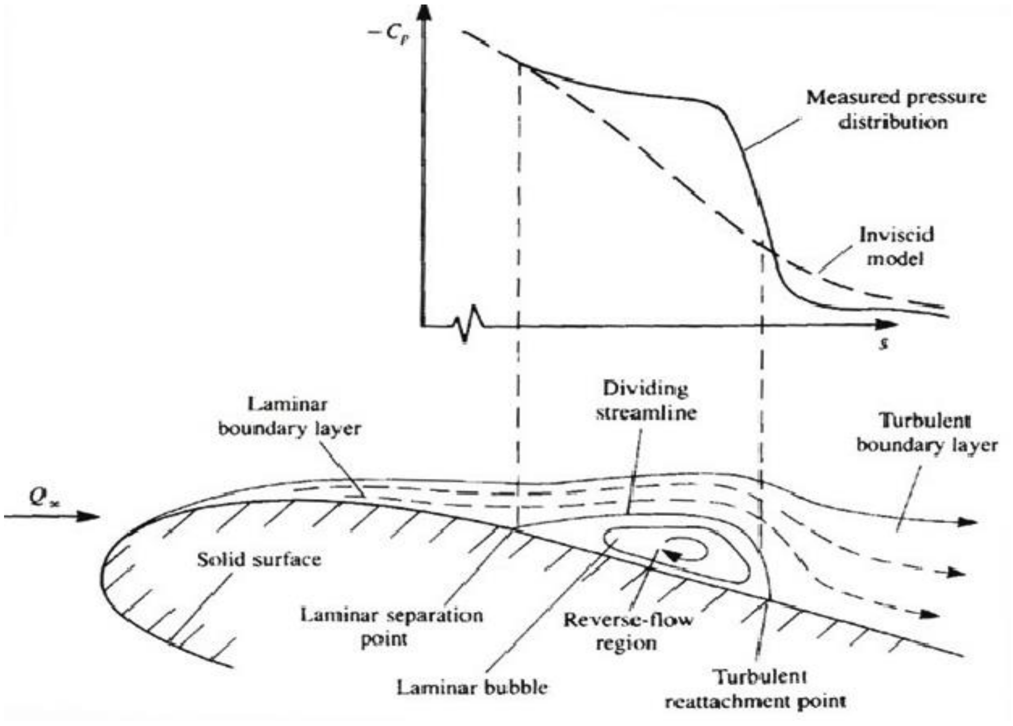
❑ Turbulent Boundary Layer Flows

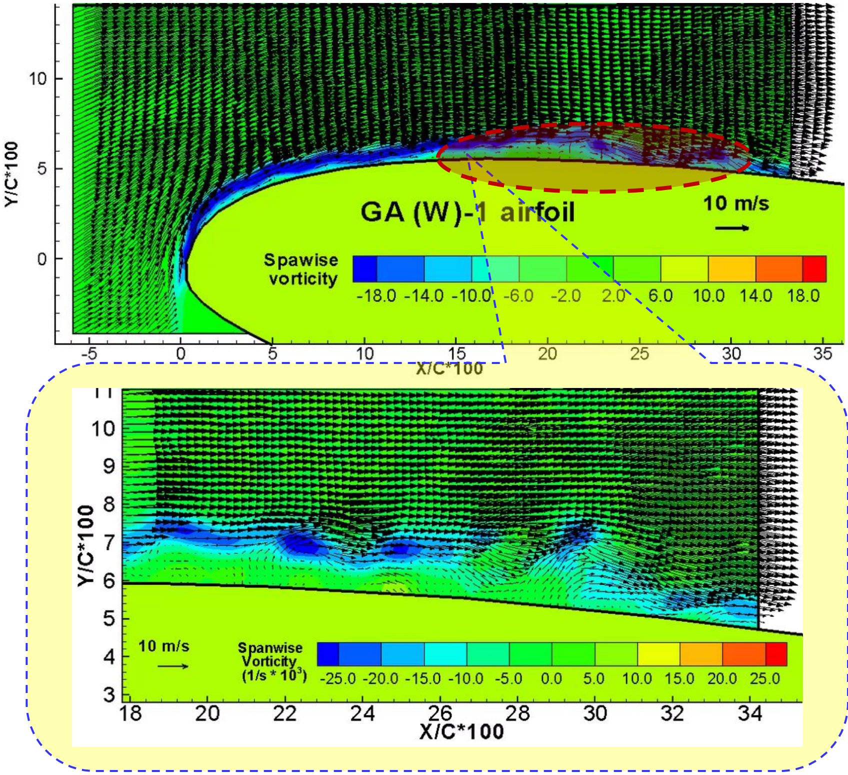
Flow separation over airfoil surfaces

- Flow separation occurs when there is adverse pressure gradient ($\frac{dp}{dx} > 0$)

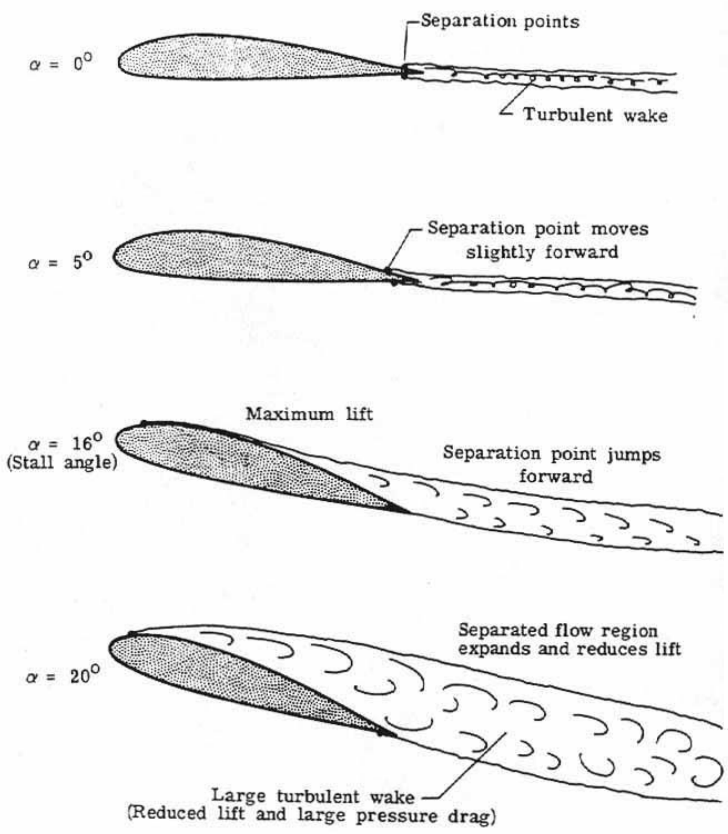
$$\text{Separation point: } \tau_w = 0 \Rightarrow \frac{\partial u}{\partial y} = 0$$

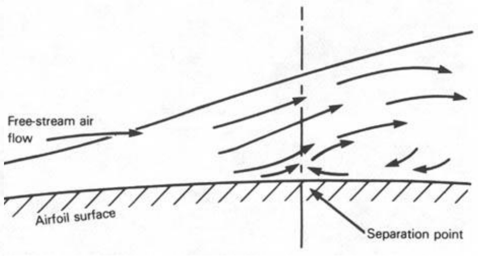
- Separation can occur both in laminar and turbulent boundary layer.
- Flow can re attach after separation, creating a separation bubble.

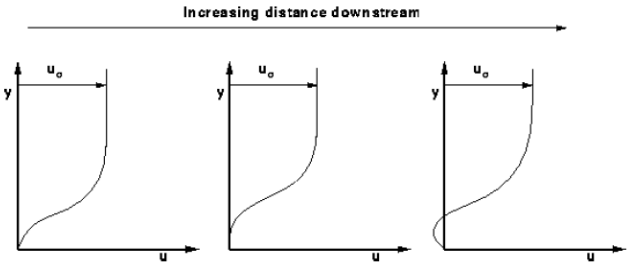


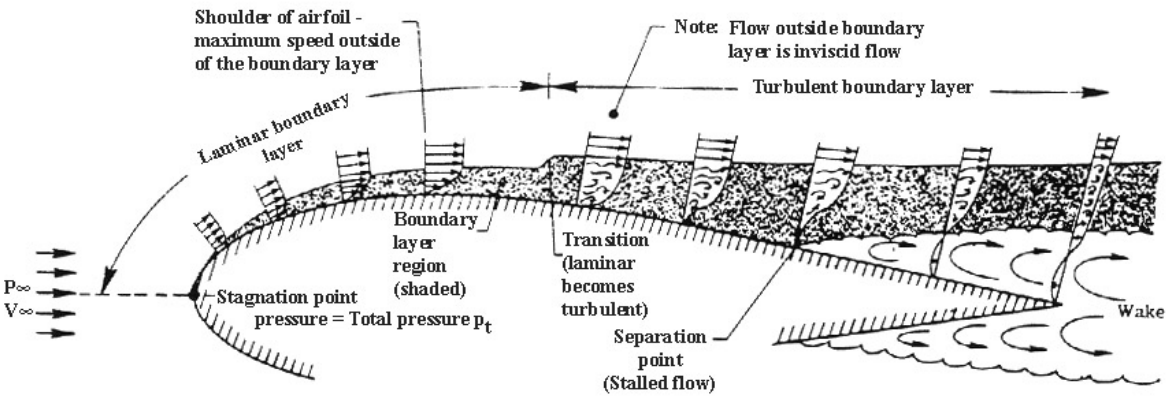


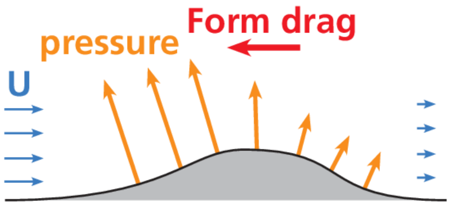
Flow Separation on an Airfoil





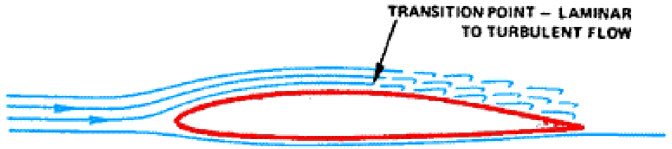




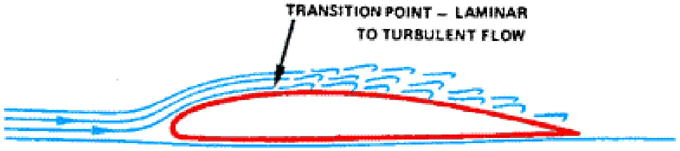


Conventional vs Laminar Airfoils

- Laminar flow airfoils are usually thinner than the conventional airfoil.
- The leading edge is more pointed and its upper and lower surfaces are nearly symmetrical.
- The major and most important difference between the two types of airfoil is this, the thickest part of a laminar wing occurs at 50% chord while in the conventional design the thickest part is at 25% chord.
- Drag is considerably reduced since the laminar airfoil takes less energy to slide through the air.
- Extensive laminar flow is usually only experienced over a very small range of angles-of-attack, on the order of 4 to 6 degrees.
- Once you break out of that optimal angle range, the drag increases by as much as 40% depending on the airfoil



True Laminar Flow Wing



"Ordinary" Wing

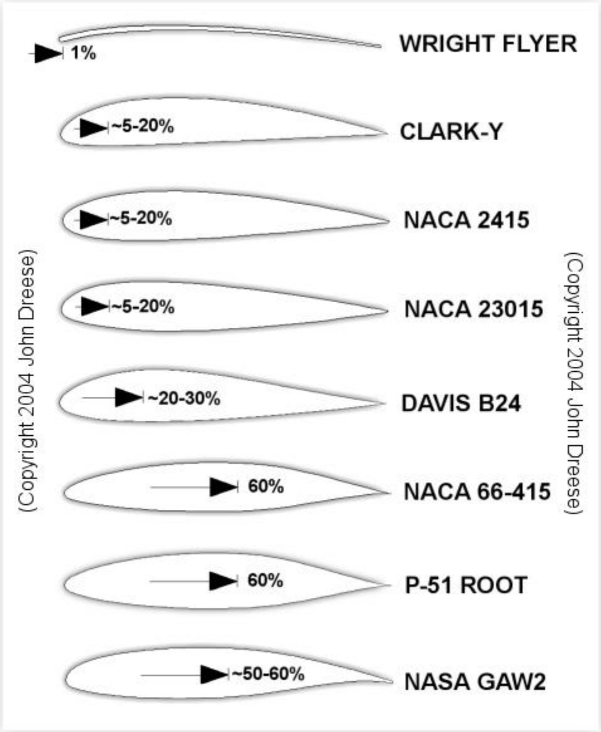


FIGURE 2: Extent of laminar flow on some famous airfoils.

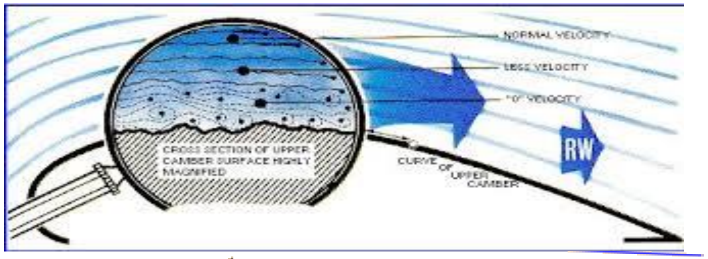
AERODYNAMIC DRAGS

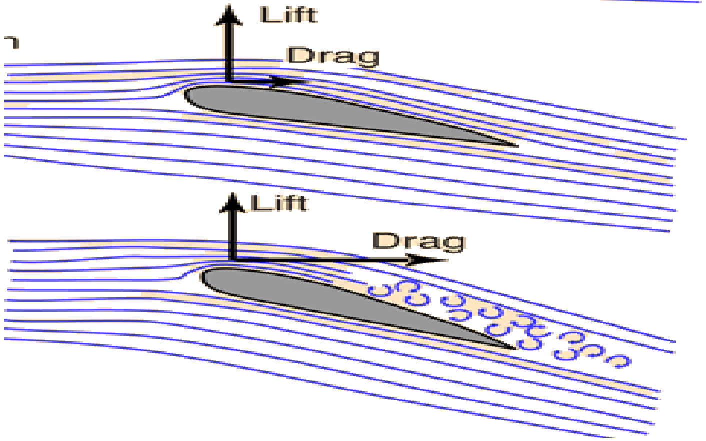
- ***Skin friction drag***, ***D_f– drag caused by skin friction.***
- ***Pressure drag or form drag***, ***D_p – drag due to flow separation, which causes pressure differences between front and back of the wing.***
- ***Induced drag***, ***D_i – drag due to lift force redirection caused by the induced flow or downwash.***
- ***Total drag coefficient:***

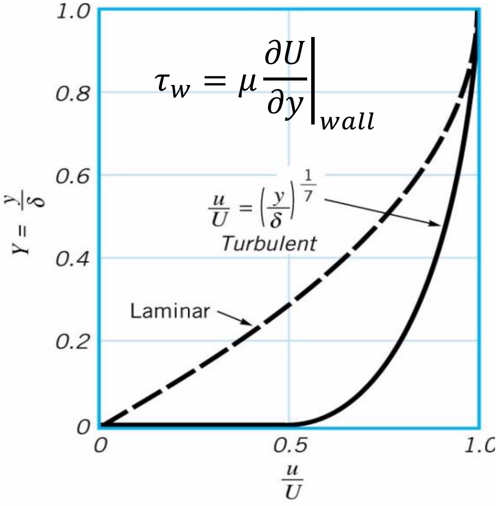
$$C_D = (D_f + D_p + D_i) / (1/2 \rho V_\infty^2 S)$$

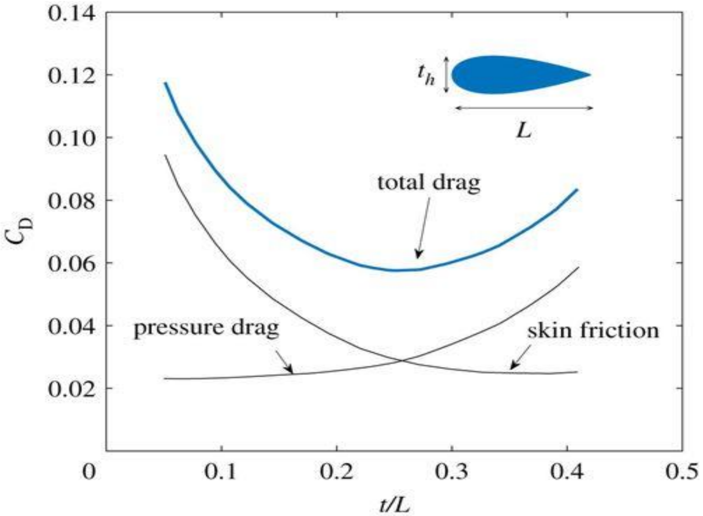
$$\underbrace{\frac{F_p}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}_{c_p} + \underbrace{\frac{F_f}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}_{c_f} = \underbrace{\frac{F_d}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}_{c_d}$$

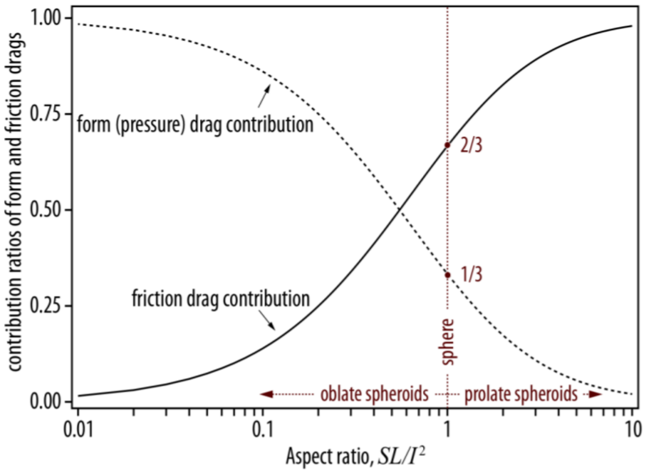
$$\boxed{c_d := \frac{F_d}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}$$







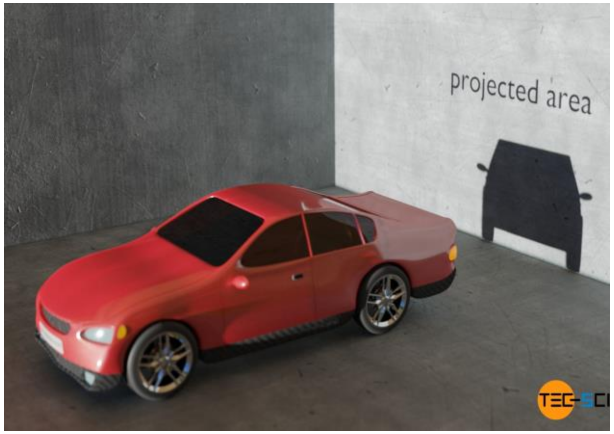


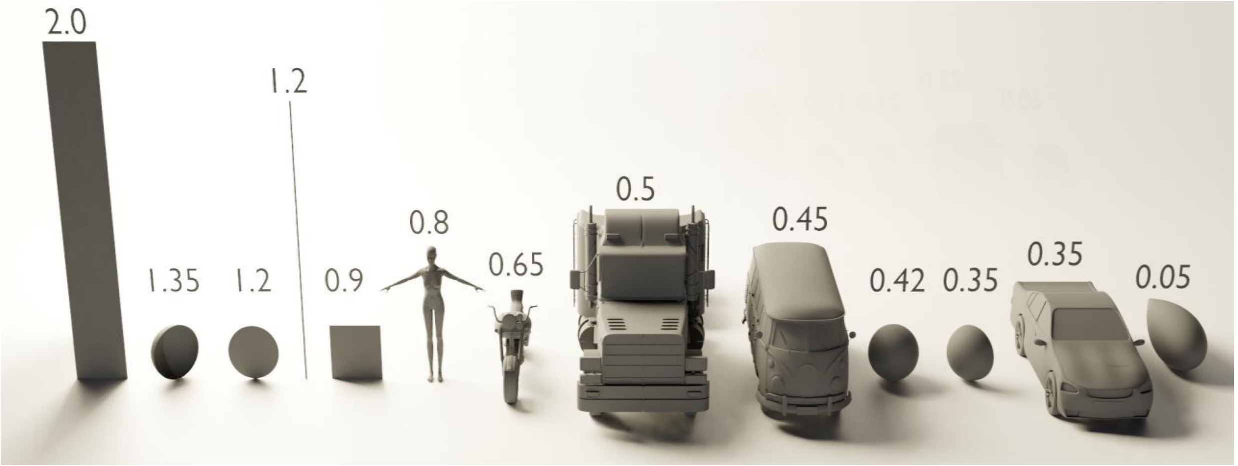


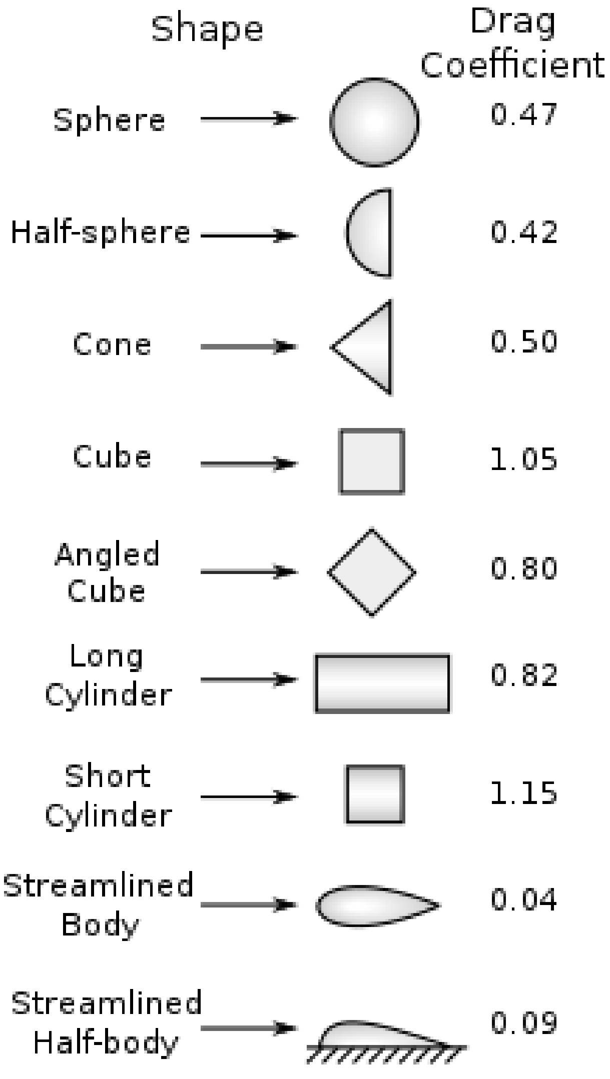
AERODYNAMICS DRAG

$$c_d := \frac{F_d}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}$$

- ***A: projected area***



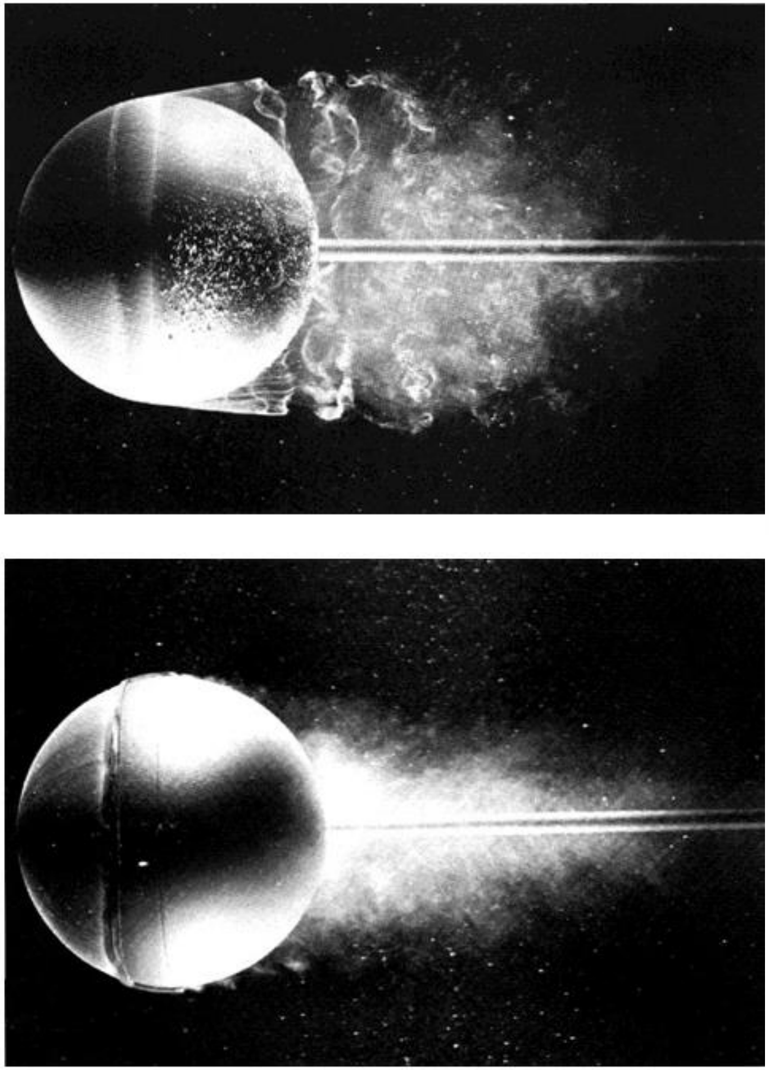


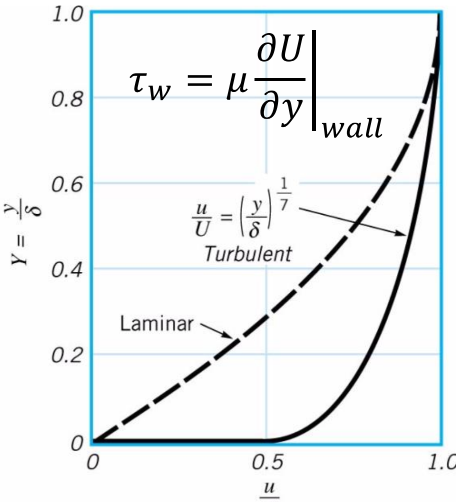


Shape	Drag Coefficient
Sphere	0.47
Half-sphere	0.42
Cone	0.50
Cube	1.05
Angled Cube	0.80
Long Cylinder	0.82
Short Cylinder	1.15
Streamlined Body	0.04
Streamlined Half-body	0.09

Measured Drag Coefficients

❑ LAMINAR & TURBULENCE BOUNDARY LAYER OVER A BALL

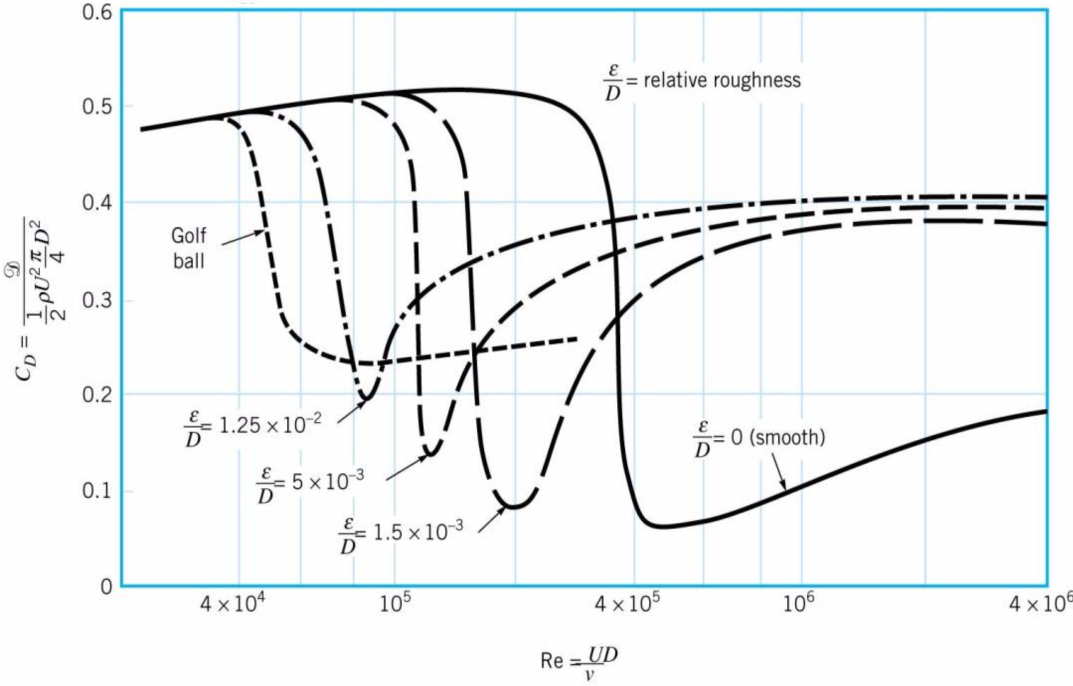




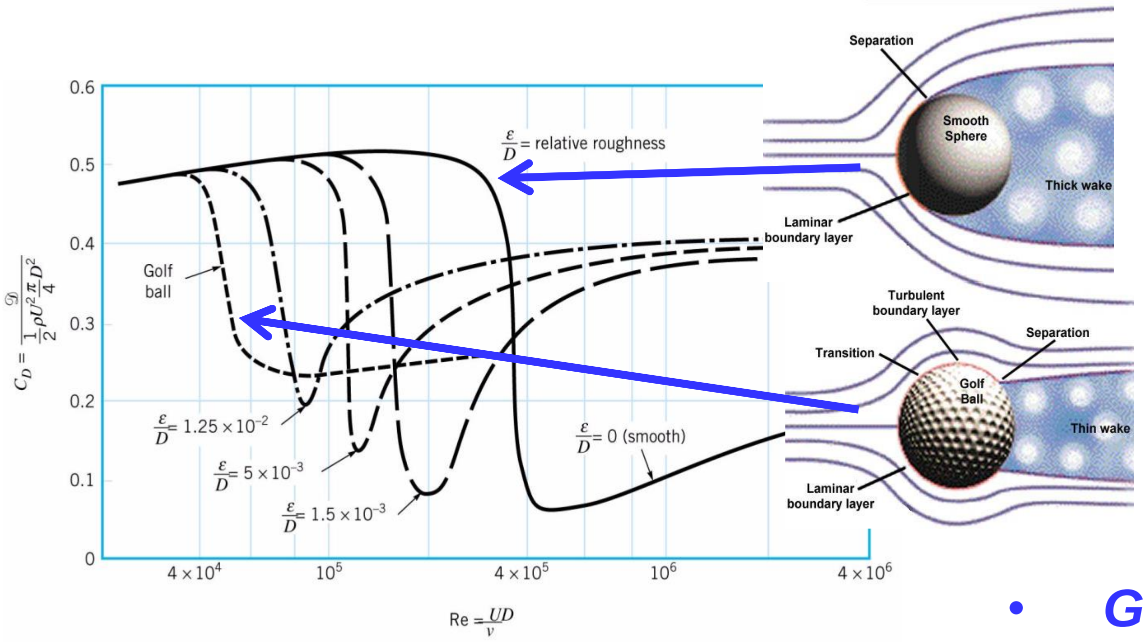
$$\tau_w = \mu \frac{\partial U}{\partial y}\bigg|_{wall}$$

$$\underbrace{\frac{F_p}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}_{c_p} + \underbrace{\frac{F_f}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}_{c_f} = \underbrace{\frac{F_d}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}_{c_d}$$

$$\boxed{c_d := \frac{F_d}{\frac{1}{2}\rho \cdot v_\infty^2 \cdot A}}$$



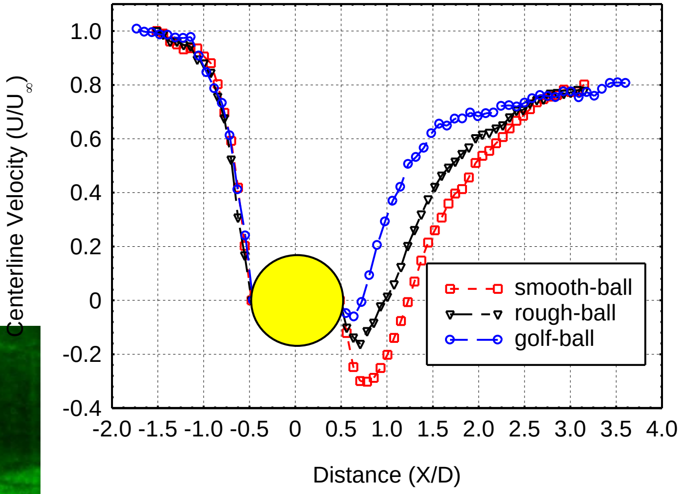
AERODYNAMICS OF GOLF BALL

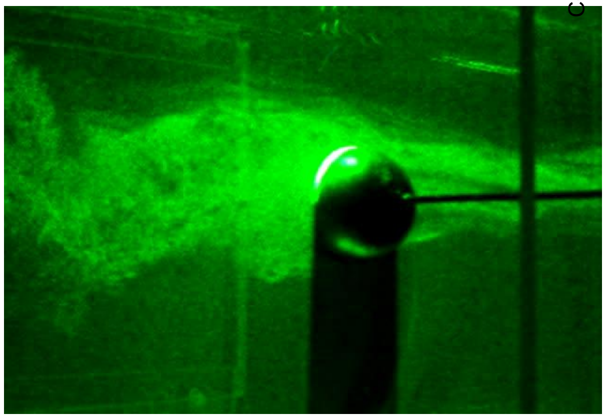


- ***Golf ball aerodynamics***

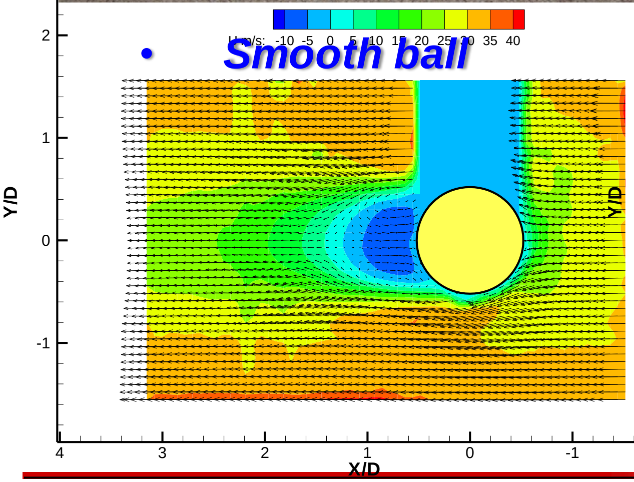
❑ AERODYNAMICS OF PAINTBALL : LAMINAR AND TURBULENT FLOWS

- *Paintball Game*

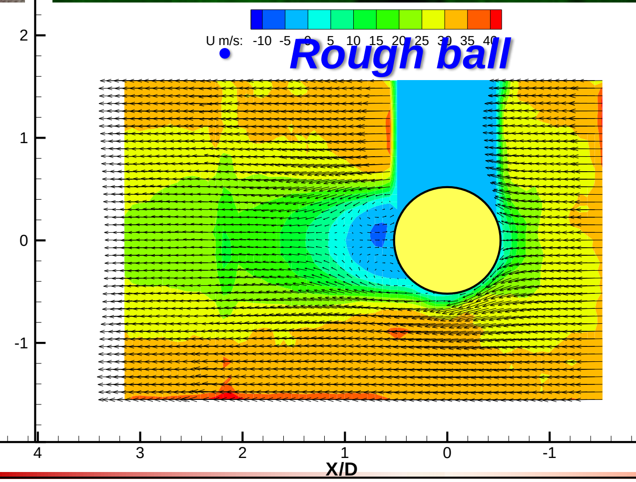




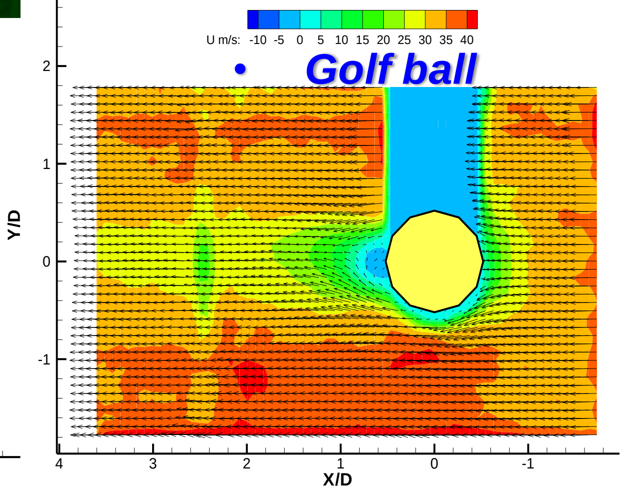
- ***Re=100,000***



- ***Smooth ball***

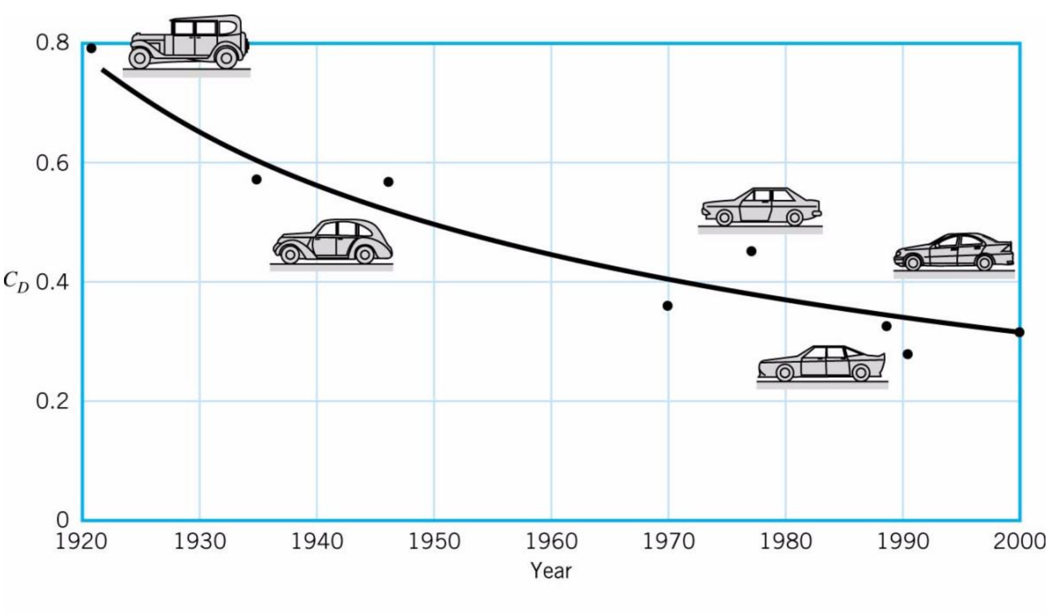


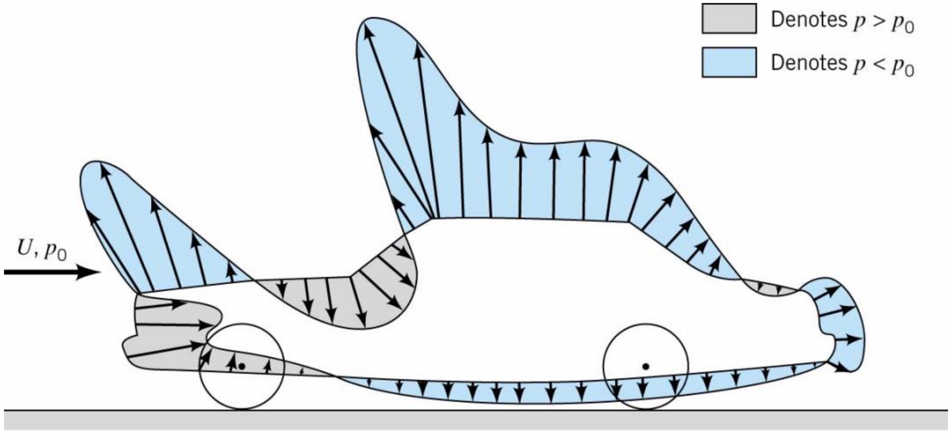
- ***Rough ball***

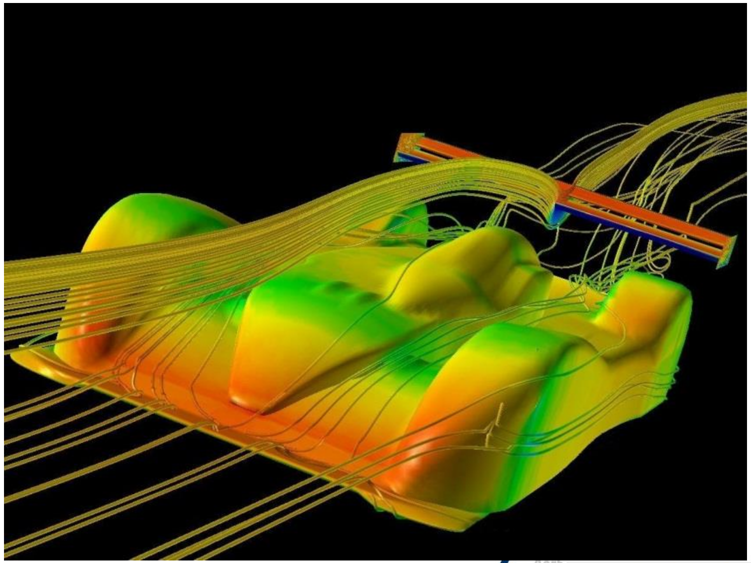


- ***Golf ball***

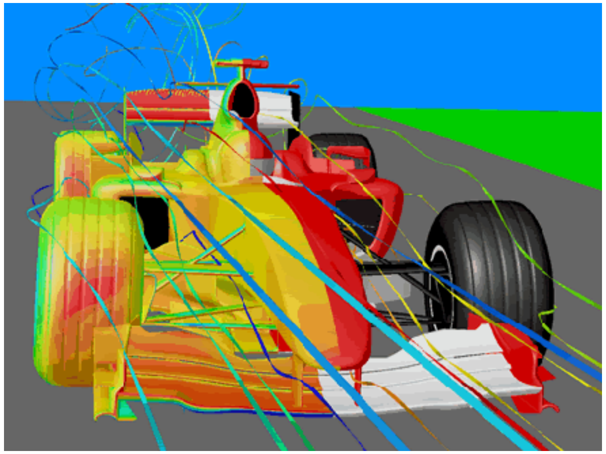
Automobile aerodynamics

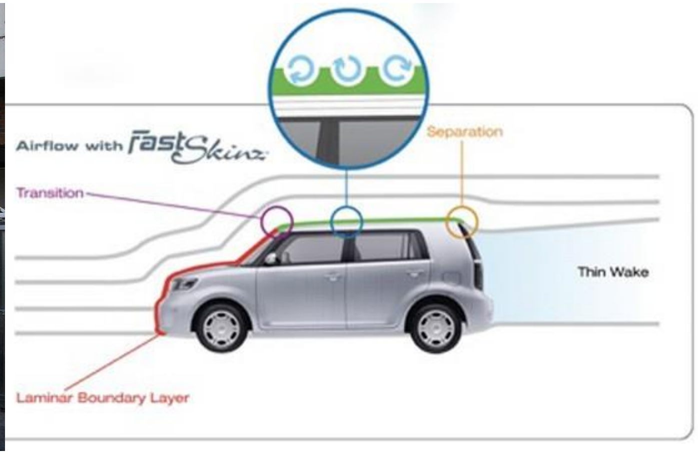






Automobile aerodynamics

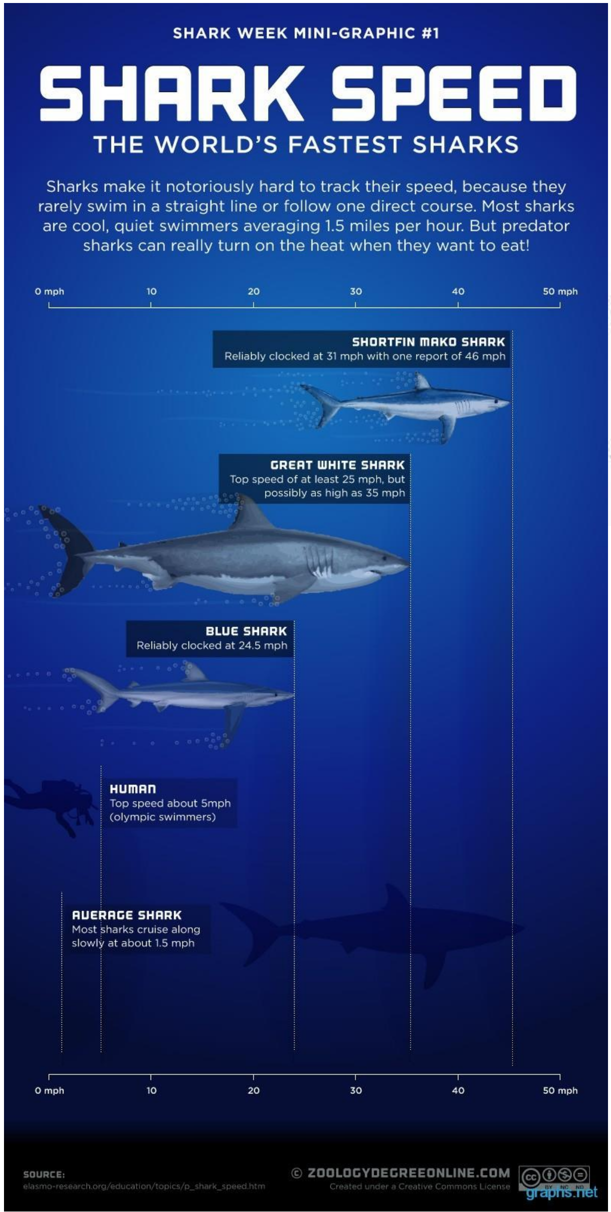




Mercedes Boxfish

Vortex generator above a Mitsubishi rear window

Passive Flow Control: Shark Skin



SHARK WEEK MINI-GRAPHIC #1

SHARK SPEED

THE WORLD'S FASTEST SHARKS

Sharks make it notoriously hard to track their speed, because they rarely swim in a straight line or follow one direct course. Most sharks are cool, quiet swimmers averaging 1.5 miles per hour. But predator sharks can really turn on the heat when they want to eat!

SHORTFIN MAKO SHARK
Reliably clocked at 31 mph with one report of 46 mph

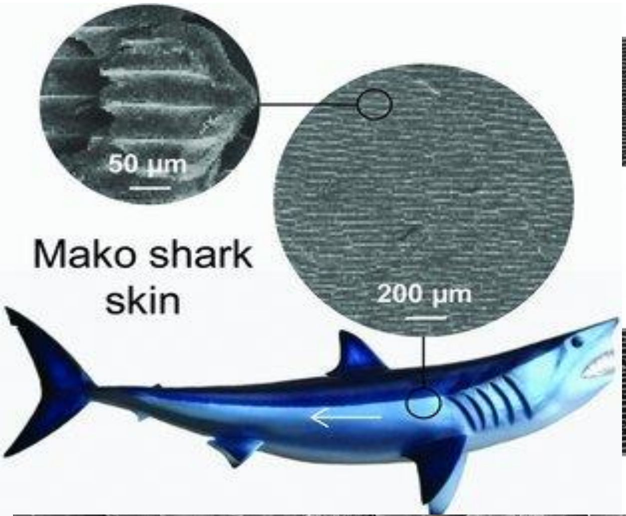
GREAT WHITE SHARK
Top speed of at least 25 mph, but possibly as high as 35 mph

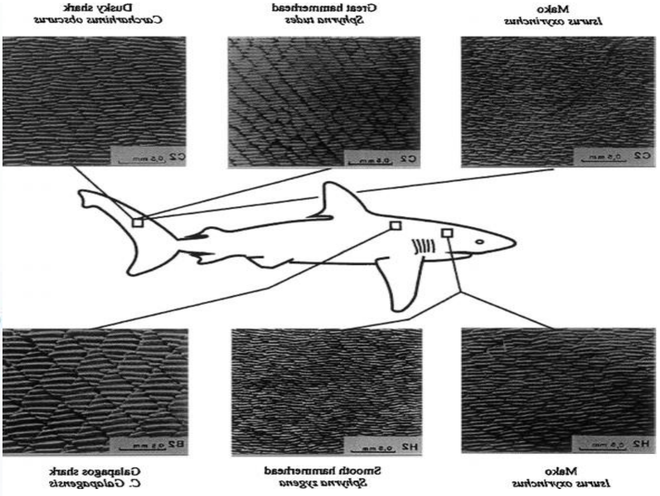
BLUE SHARK
Reliably clocked at 24.5 mph

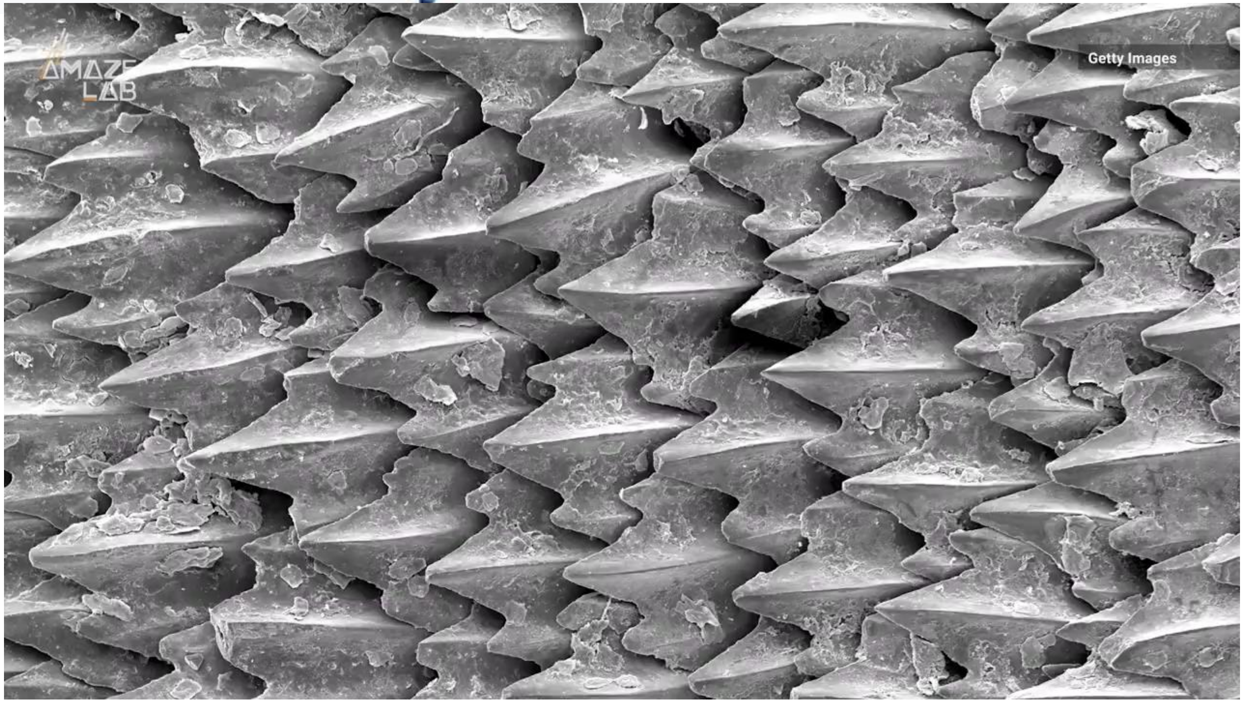
HUMAN
Top speed about 5mph
(olympic swimmers)

AVERAGE SHARK
Most sharks cruise along slowly at about 1.5 mph

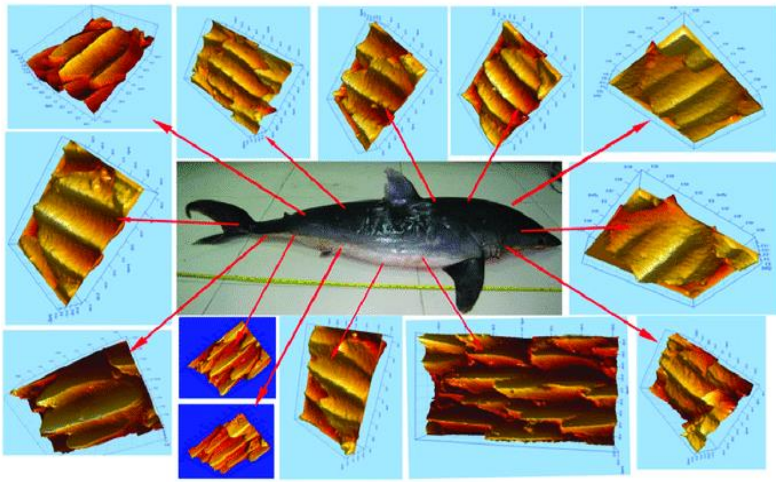
SOURCE:
elasmo-research.org/education/topics/p_shark_speed.htm



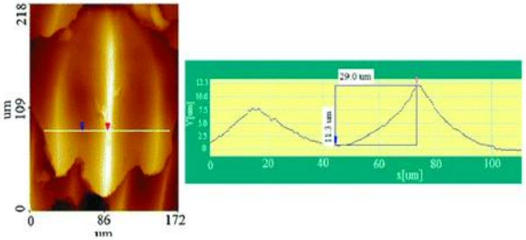




Shark Skin Structures for Drag Reduction



(a)



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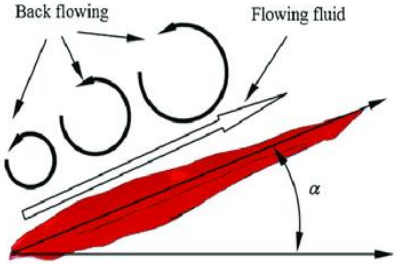

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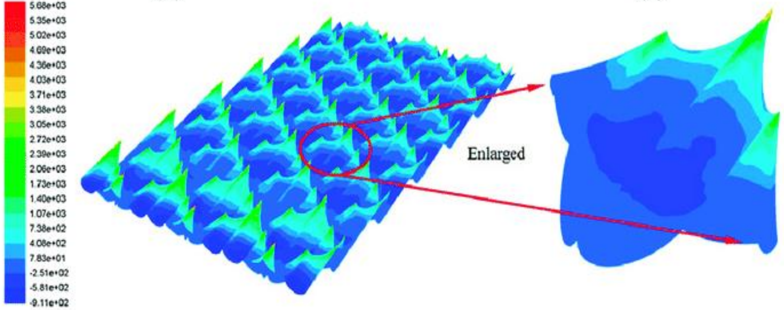

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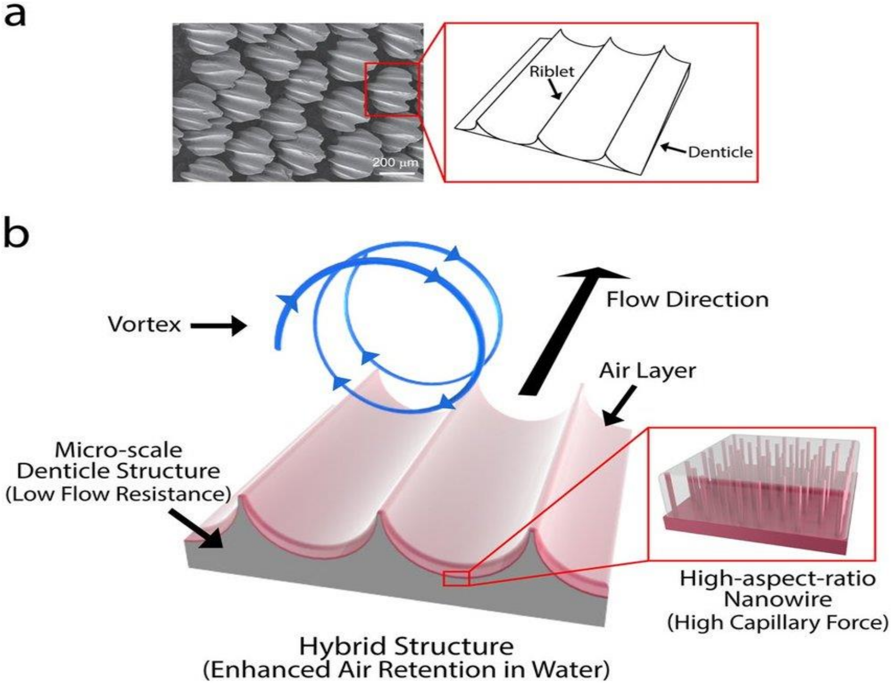
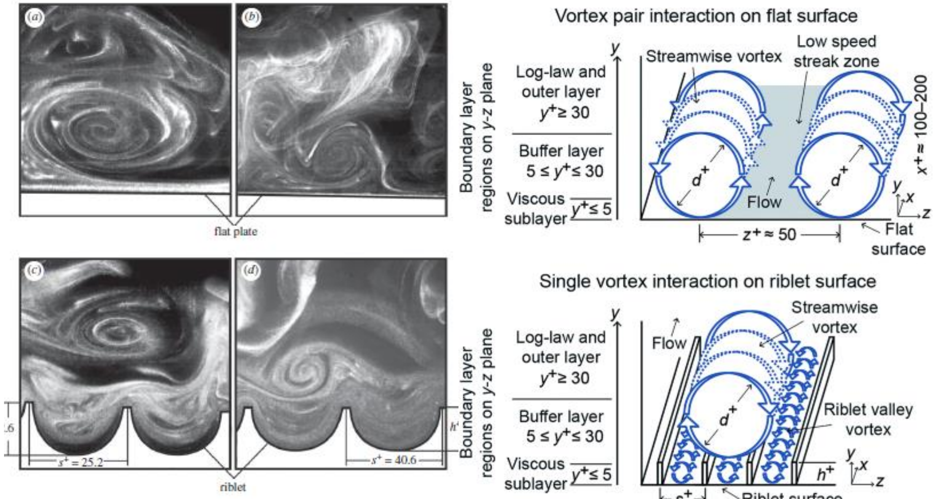


Hybrid Structure
(Enhanced Air Retention in Water)



Shark Skin Inspired Engineering