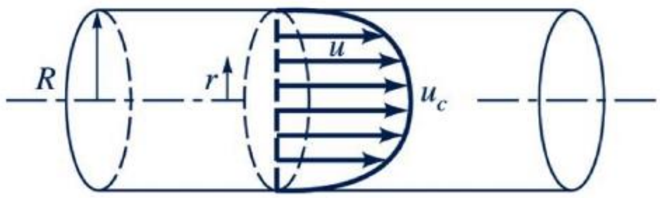
AerE310: Incompressible Aerodynamics

Homework Problem Set #03:

<u>Due</u>: 5:00 PM, Friday, 03/01/2024

1. Consider the fully developed flow in a circular pipe, as shown in Fig. 1. The velocity u is a function of the radial coordinate only: $u = U_{CL}(1 - \frac{r^2}{R^2})$, where U_{CL} is the magnitude of the velocity at the centerline (or axial) of the pipe.



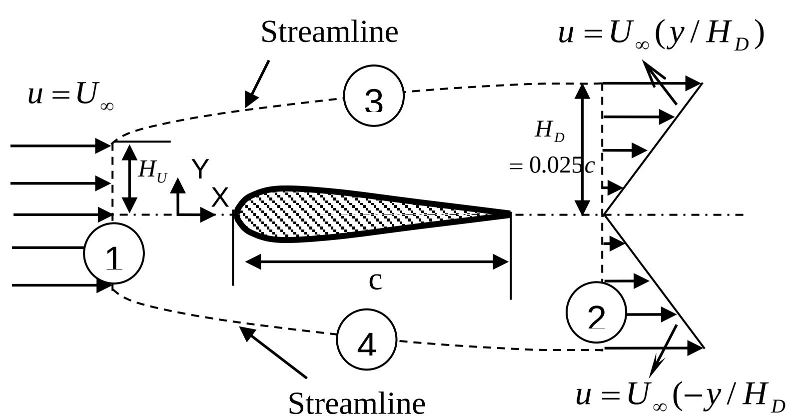
Please use the integral form of the momentum equation to show how the pressure drop per unit length *dp/dx* changes if the radius of the pipe were to be doubled while the mass flux through the pipe held constant at the value of $\dot{m}$. neglect the weight of the fluid in the control volume and assume that the fluid properties are constant.

2. Simplify the Navier-Stokes equation in Cartesian coordinate for an incompressible, steady, 2-D flow between horizontal parallel plates. Assuming that $u = u(y), v = 0, w = 0$, write momentum equations in (x, y, z) all three directions.

3. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of the control volume shown in the figure. The flow is incompressible, two-dimensional, and steady. The gauge pressure on the surfaces along the dashed line is equal to zero. Surface 3 and 4 are streamlines. [**Hint:** No fluid will flow across the streamlines]

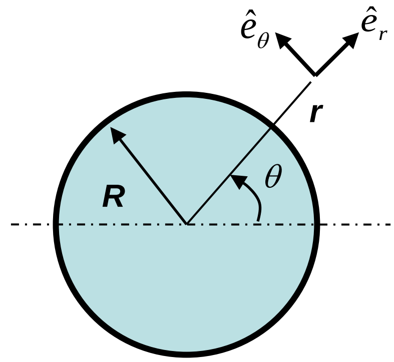
(a). What is the relationship between the H_U and H_D .

(b). If $H_D = 0.025c$, what is the drag coefficient of the airfoil? ($C_D = \frac{D}{\frac{1}{2}\rho U_\infty{}^2 c}$)



4. For a two-dimensional steady, inviscid, incompressible flow around a cylinder of radius of *R* as shown in the figure, the velocity field is given as :

$$\vec{V}(r,\theta) = U_\infty (1 - \frac{R^2}{r^2}) \cos\theta \cdot \hat{e}_r - U_\infty (1 + \frac{R^2}{r^2}) \sin\theta \cdot \hat{e}_\theta ;$$



Where U_∞ is the velocity of the undisturbined approaching stream (therefore, U_∞ is constant).

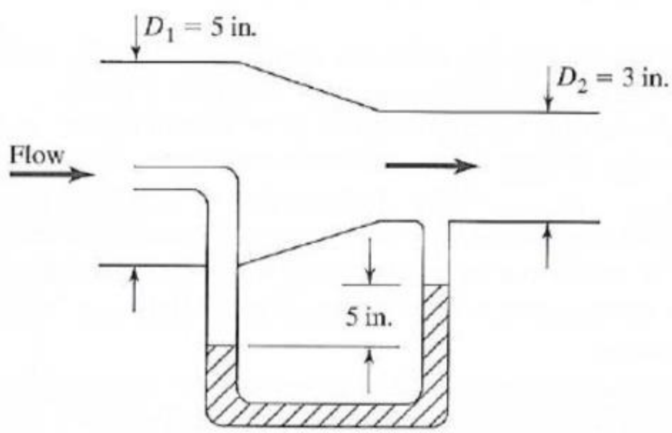
(a). Show your work to prove the flow with the velocity field given above is physically possible.

(b). Derive the expression for the acceleration of a fluid particle on the surface of the cylinder. Then, calculate the acceleration of a fluid particle at the points of $(r,\theta) = (R,0)$ and $(r,\theta) = (R,\pi)$.

5. Consider a velocity field where the *x* and *y* components of the flow velocity are expressed as $u = \frac{cy}{x^2 + y^2}$; $v = -\frac{cx}{x^2 + y^2}$, where C is a constant. Please obtain the equations for streamlines.

6. Consider a velocity field where the *radial* and *tangential* components of the flow velocity are expressed as $V_r = 0$; $V_\theta = cr$, where C is a constant. Please obtain the equations for streamlines.

7. Air flows through a converging pipe section as shown in the figure. Since the centerline of the duct is horizontal, the change in potential energy is zero. The Pitot probe at the upstream station provides a measure of total pressure (or stagnation pressure). The downstream end of the U-tube provides a measure of the static pressure at the second section. Assume the density of the air is 0.00238slug/ft^3,and neglecting the effect of the viscosity. Compute the volumetric flow rate in ft^3/s. The fluid in the manometer is unity weight oil with the density ρ_{oil} =1.9404 slug/ ft$^{3.}$



8. The stream function of 2-D, incompressible flow is given by : $\psi = \frac{\Gamma}{2\pi} \ln r$.

a) Graph the streamlines.

b) What is the velocity field represented by this strem function? Does the resultant velocity field satisfy the continuity equation?

c) Find the circulation about a path enclosing the origin. For the path of integration, use a circle of radius 3 with a center at the origin. Howe does the circulation dependent on the radius?