AerE310: Incompressible Aerodynamics

Homework Problem Set #04:

<u>Due</u>: 5:00 PM, Friday, 03/22/2024

Problem#1: Consider the incompressible, irrotational, 2-D flow where the potential functions is: $\phi = K\, ln(\sqrt{x^2 + y^2})$, where K is a constant.

a) What is the velocity field for this flow? Please verify the flow is irrotational. What is the magnitude and direction of the velocity vector at (2,0), at ($\sqrt{2}$, $\sqrt{2}$) and at (0, 2)?

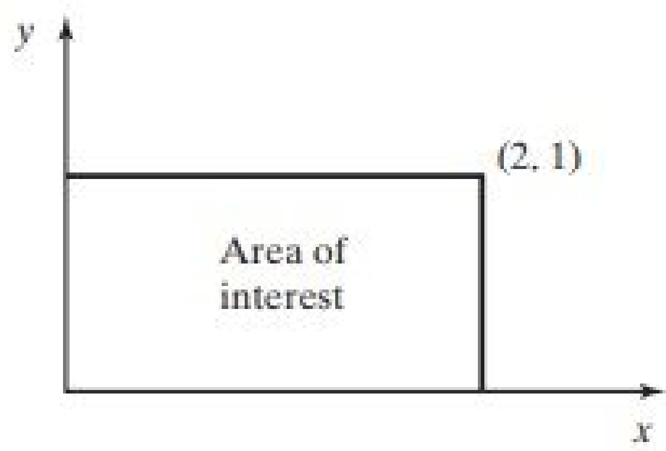
b) What is the stream function for this flow field? Sketch the streamline pattern.

c) Sketch the lines with constant potential values. How do the lines of equipotential relate to the streamlines?

Problem#2: The equation of the streamlines in a two-dimensional velocity field are given by the expression of $\psi = xy + y^2 + const$.

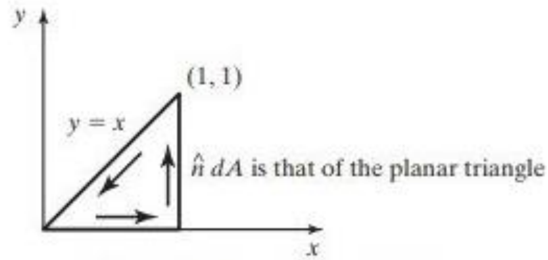
a) Pease find the expression of the flow velocity vector and the magnitude of the velocity vector.

b) Please find the integral over the surface shown in the figure for the normal component of vector of $\nabla \times \vec{V}$ by two methods.



Problem#3: Given an incompressible, steady flow, where the flow velocity is $\vec{V} = (x^2 y - xy^2)\hat{i} + (\frac{y^3}{3} - xy^2)\hat{j}$

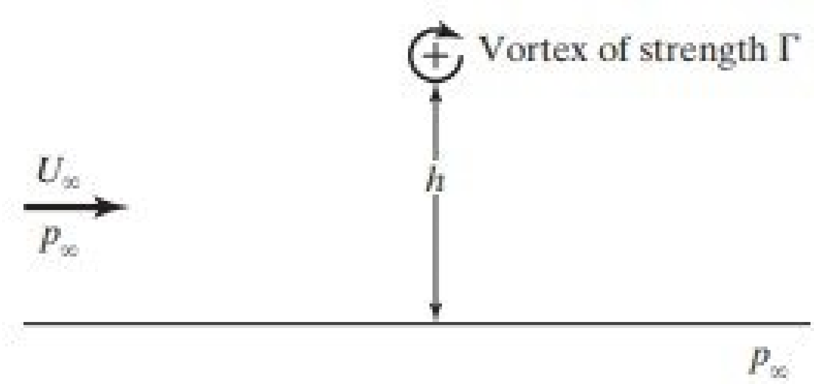
(a) Does the flow filed satisfy the continuity equation? Does a stream function exist? If a stream function exists, what is it?
(b) Does a potential function exist? If yes, what is it?
(c) For the region shown in the figure, evaluate $\iint (\nabla \times \vec{V}) \bullet d\vec{A} = ?$ and $\Gamma = \oint \vec{V} \bullet d\vec{S} = ?$ to demonstrate that Stokes's theorem is valid.



Circulation around the triangle

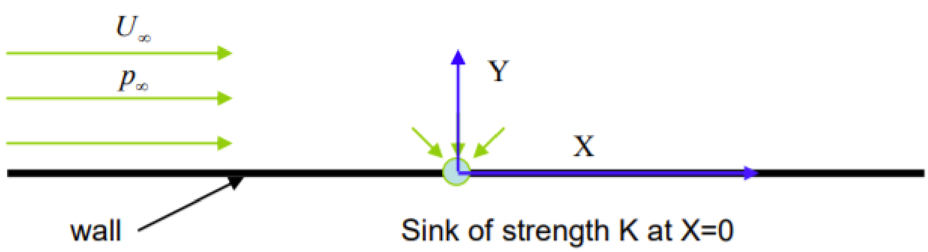
Problem#4: What is the stream function that represents the potential flow about a cylinder whose radius is 1.0m and which is located in an air stream where the free-stream velocity is 50m/s? What is the change in pressure from the free-stream value to the value at the top of the cylinder (i.e., $\theta = 90^o$)? What is the change in pressure from the free-stream value to the value at the stagnation point (i.e., $\theta = 180^o$)?

Problem#5: A two-dimensional free vortex is located near an infinite plane at a distance *h* above the plane as shown in the figure. The pressure at infinity is P_∞, and the velocity at infinity is U_∞, parallel to the plane. Please find the total force (per unit depth normal to the paper) acting on the plane if the pressure on the underside of the plane is P_∞. The strength of the vortex is Γ. The fluid is incompressible and perfect. To what expression does the force simplify if *h* becomes very large?



Problem#6: In an ideal, 2-D incompressible irrotational flow field, the fluid is flowing past a wall with a sink of strength K per unit length at the origin as shown in the Figure. The potential function of a 2-D sink is $\phi = -\frac{k}{2\pi}\ln\sqrt{x^2 + y^2}$. At infinity the flow is parallel to wall and of uniform velocity U_∞ .

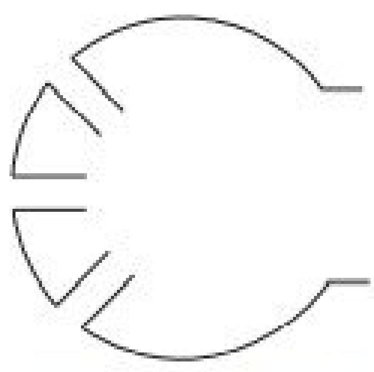
a. Determine the location of the stagnation point X0 at the wall in terms of U_∞ and K.
b. Find the pressure distribution along the wall as a function of X. Taking the free stream static pressure at infinity to be P_∞, express the pressure coefficient as a function of 0 X/X_0 .
c. Sketch the resulting pressure distribution.



Problem#7: A cylindrical tube with three radially drilled orifices, as shown in the figure below can be used as a flow-direction indicator. Whenever the pressure on the two side holes is equal, the pressure at the center hole is the stagnation pressure. The instrument is called a direction-finding Pitot tube, or a cylindrical yaw probe.

a) If the orifices of a direction-finding Pitot tube were to be used to measure the freestream static pressure, where would they have to be located if we use our solution for flow around a cylinder?

b) For a direction-finding Pitot tube with orifices located as calculated in part (a), what is the sensitivity? Let the sensitivity be defined as the pressure change per unit angular change (i.e., $\partial p / \partial \theta$)



Problem#8: Consider the flow around the Quonset shown in the figure to be represented by superimposing a uniform flow and 2-D a doublet. Assume steady, incompressible, potential flow. The ground plane is represented by the plane of symmetry and the hut by the upper half of the cylinder. The free-stream wind velocity is 30 m/s; the radius of the hunt is 10m. The door of the hut is not well sealed, and the leakage opening is very small compared with the radius of the hut R. Therefore, the static pressure inside the hunt is equal to that the outer surface of the hunt where the door is located. Density of air ρ=1.2kg/m^3.

a) If the door to the hunt is located at ground level (i.e., at the stagnation point), what is the net lift acting on the hunt?

b) Where should the door be located (i.e., at what angle θ 0 relative to the ground) so that the net force on the hut will vanish?