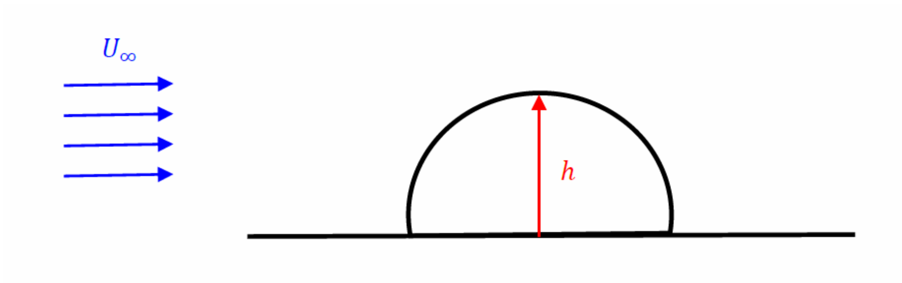
AerE310: Incompressible Aerodynamics

Homework Problem Set #05:

<u>Due</u>: 5:00pm, Friday, 04/05/2021

Problem#1: A hill with the height h has the shape of a half circle as shown in figure below. The wind approaching the hill has a constant velocity parallel to the ground, $U\infty$. Assume the stream function is in the form of $\psi(r,\theta) = r^n \sin\theta$.

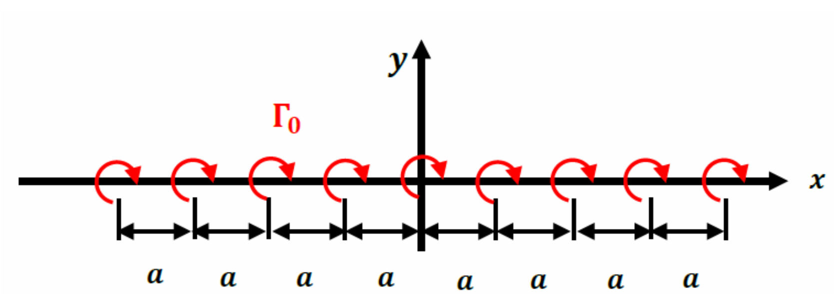
- Find the value of n such that the flow is irrotational.
- Use the surface (hill) streamline (ψ=0) and the fact that at far field ($r\to\infty$), the stream function approaches to that of a freestream ($U\infty r\sin\theta$) to formulate the final form of stream function in terms of the given parameters (h, $U\infty$).



Problem#2: Consider a row of vortices of equal strength, Γ_0 and equal spacing, a, as shown. The number of vortices is N+1.

- Write the stream function for the resulting flow field and determine velocity components u,v in the Cartesian system.
- Calculate u and v for a=1, Γ_0=10π at points (x, y) = (1,10) and (1, −10) for N+1=1, 101 and 1001. Assume these vortices are symmetrically placed with respect to y axis.

This problem illustrates the concept of a vortex sheet. For sufficiently large y values, the flow above a vortex sheet is essentially uniform. There is no velocity normal to the sheet at sheet surface and horizontal component of velocity changes sign across the sheet.



Problem#3: Based on the thin airfoil theory, the vorticity distribution along the mean camber line of a symmetrical airfoil can be expressed as:

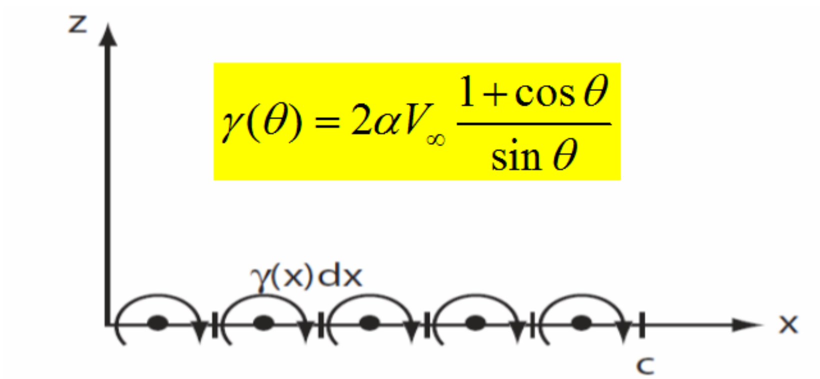
$$\gamma(\theta) = 2\alpha V_\infty \frac{1+\cos\theta}{\sin\theta}, \textit{where} \left| \begin{array}{l} \theta = 0 \;\; \textit{for leading edge} \\ \theta = \pi \;\; \textit{for leading edge} \end{array} \right. .$$

a) Prove that Kutta condition for airfoil trailing edge is satisfied

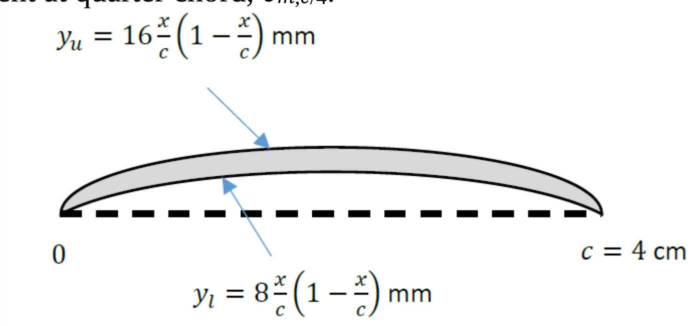
b) What is the physical significance of $2\gamma / V_\infty$?

c) What angle of attack is required for a symmetrical airfoil to develop a section of $C_l = 0.5$? Sketch the distribution $2\gamma / V_\infty$ as a function of x/c for a section lift coefficient of $C_l = 0.5$.

d) Using the vorticity distribution to calculate the pitch moment about 0.75 chord from the leading edge. Verify your answer using the fact that the center of pressure (Xcp) is at the quarter chord for all angles of attack and the definition for lift.



Problem#4: For the parabolic-arc airfoil shown below, find the equations for lift coefficient c_l and moment coefficient at quarter chord, $c_{m,c/4}$.

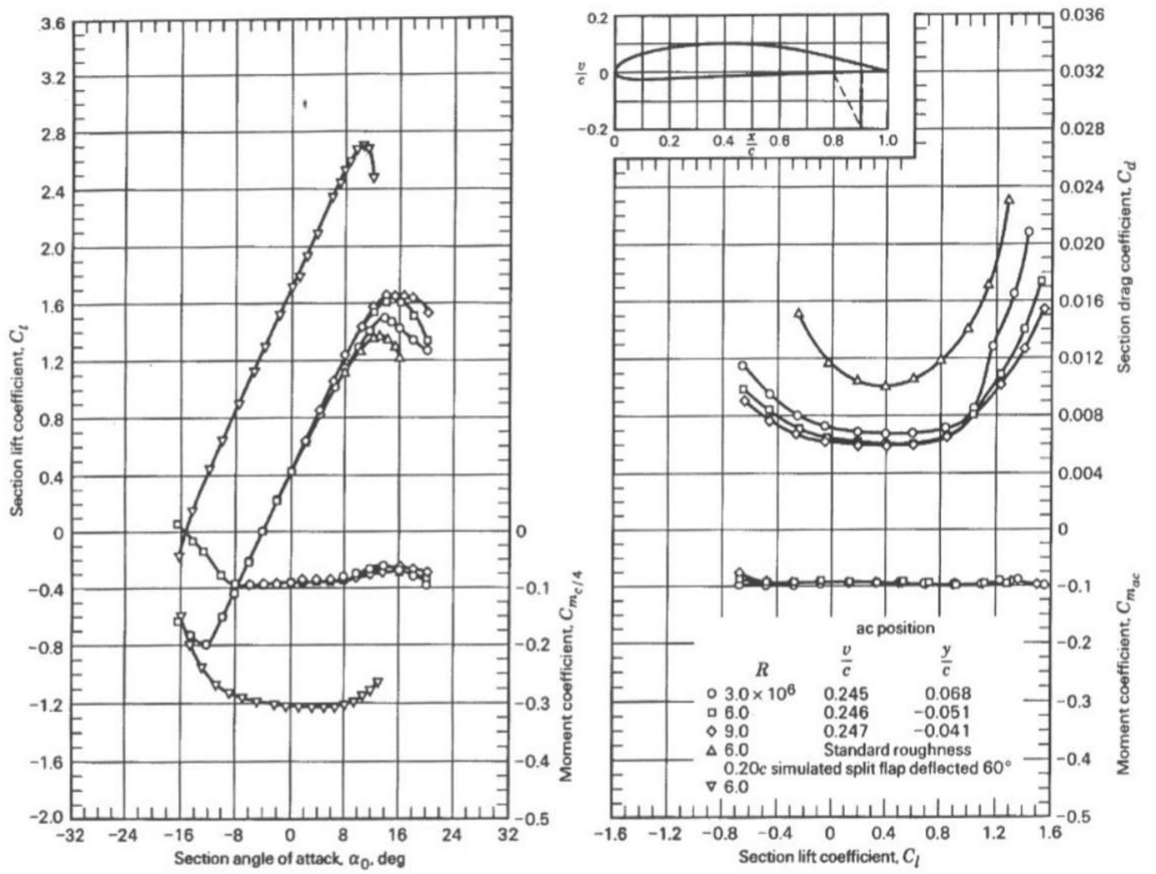


Problem#5: The NACA 4412 airfoil has a mean camber line given by:

$$\frac{\eta_c}{c} = \begin{cases} 0.25\left[0.8\frac{x}{c} - \left(\frac{x}{c}\right)^2\right] & \text{for } 0 \le \frac{x}{c} \le 0.4 \\ 0.111\left[0.2 + 0.8\frac{x}{c} - \left(\frac{x}{c}\right)^2\right] & \text{for } 0.4 \le \frac{x}{c} \le 1 \end{cases}$$

Using thin airfoil theory, calculate

- $\alpha_{L=0}$ and c_l when α=3°.
- $c_{m,c/4}$ and x_{cp}/c for α=3°.
- Compare the results of part (a) and (b) with experimental data of NACA 4412 airfoil given below.
- Lift per unit length of span and circulation for an airfoil with chord length of 2m flying at a standard altitude of 3 km and velocity of 60 m/s (same angle of attack of 3°).



Aerodynamic characteristics of the NACA 4412 airfoil.

Problem#6: The question is often asked: Can an airfoil fly upside-down? To answer this, make the following calculation. Consider a positively cambered airfoil with a zero-lift angle of -3°. The lift slope is about 0.1 per degree.

a) Calculate the lift coefficient at an angle of attack of 5°.
b) Now imagine the same airfoil turned upside-down, but at the same 5° angle of attack as part (a). Calculate its lift coefficient.
c) At what angle of attack must the upside-down airfoil be set to generate the same lift as that when it is right-side-up at a 5° angle of attack?