AerE310: Incompressible Aerodynamics

Homework Problem Set #1:

***Due*: 5:00 PM, Friday, 02/02/2024**

1. Expand following terms:

   a). $\frac{d}{dt}(\vec{A}\cdot(\vec{B}\times\vec{C}))$

   b). $\frac{d}{dt}(\vec{A}\times(\vec{B}\times\vec{C}))$

3. Find $\nabla\Phi$ if

   a). $\Phi = \ln|\vec{r}|$

   b). $\Phi = \frac{1}{r}$

   (Hint: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in Cartesian coordinate system)

4. Find directional derivative of $\Phi = x^2yz + 4xz^2$ at point (1, -2, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

5. If $\vec{R} = \vec{R}(t) = r\hat{e}_r + z\hat{e}_z$ is the position vector of a particle in cylindrical coordinates, Obtain expression for velocity vector, $\vec{V}$, and acceleration vector, $\vec{a}$, at that point.

6. Show that the directions of the isoline and the gradient line at any given points in a scalar field are orthogonal to each other. (Hint: use the concept of directional derivative)

7. Spherical coordinate (R, φ, θ) are defined by the following inverse transformation:

$$x = (R\sin\varphi)\cos\theta$$
$$y = (R\sin\varphi)\sin\theta$$
$$z = R\cos\varphi$$

Where

$$0 \le R \le \infty$$
$$0 \le \theta \le 2\pi$$
$$0 \le \varphi \le 2\pi$$

(a). Obtain the scale factors for the spherical coordinate system.
(b). Obtain the unit vectors in spherical system as the function of Cartesian unit vectors.
(c). Obtain the derivatives of the unit vectors with respect to spherical coordinate directions and simplify the results to be only functions of spherical coordinates.
(d). Using vector algebra to obtain the divergence of a general vector in spherical coordinates. Simplify the results to be in conservation form.

8. Find the acceleration of a fluid particle at (r, θ, z) in cylindrical coordinate system.