AerE310: Incompressible Aerodynamics

Homework Problem Set #02:

<u>Due</u>: 5:00 PM, Friday, 02/16/2024

1. A ball is being inflated with an air supply of 0.6 m^3/s. Find the rate of growth of the radius at the instant when R=0.5 m. (Hint: the air flow is assumed to be incompressible)

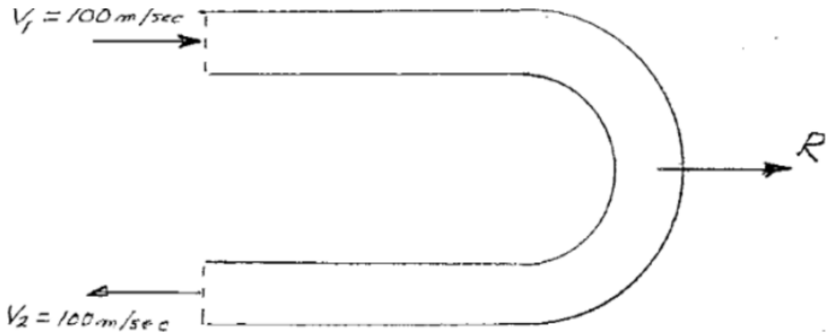
2. Which of the following flows are physically possible, that is, satisfy the continuity equation? Substitute the expression for density and for the velocity field into the continuity equation to substantiate your answer.

   (a). Water, which has a density of ρ =1.0 g/cm^3, is following radically outward from a source in a plan such that $\vec{V} = (K/2\pi r)\hat{e}_r$. Note that $u_\theta = u_z = 0$. Note also that, in cylindric coordinates,
   $$\nabla = \hat{e}_r \frac{\partial}{\partial r} + \frac{\hat{e}_\theta}{r}\frac{\partial}{\partial \theta} + \hat{e}_z \frac{\partial}{\partial z}$$

   (b). A gas is flowing at relatively low speeds (so that its density may be assumed constant) where the velocity can be expressed as:
   $$u = -\frac{2xyz}{(x^2+y^2)^2}U_\infty L$$
   $$v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}U_\infty L$$
   $$w = \frac{y}{(x^2+y^2)}U_\infty L$$

3. Consider the velocity field of $\vec{V} = -\frac{x}{2t}\hat{\imath}$ in a compressible flow where $\rho = \rho_0 x t$. What is the total acceleration of a fluid particle at (1, 1, 1) at the time of $t = 10$?

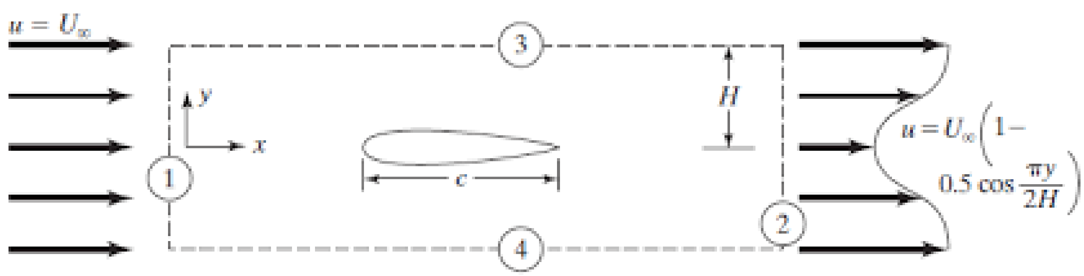
4. Consider a pipe is bent into a U-shape. The inside diameter of the pipe is 0.5m. Air enters on the leg of the pipe at a mean velocity of 100 m/s and exit the other leg at the same magnitude of the velocity but moving in the opposite direction. The pressure of the flow at the inlet and exit is the ambient pressure of the surroundings. Calculate the magnitude and direction of the force exerted on the pipe by the airflow. The air density is 1.23kg/m^3.



5. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of a rectangular control volume, as shown in the figure below. If the flow is incompressible, two dimensional, and steady, please show you work to answer following questions:

(a). What is the total volumetric flow rate (i.e., $\iint \vec{V} \cdot \hat{n} dA$) across the horizontal surfaces (i.e., surfaces 3 and 4).

(b). What is the drag coefficient for the airfoil? The vertical dimension *H* is 0.025c (i.e., *H* = 0.025c). The pressure is P_∞ (a constant) over the entire surface of the control volume.



6. As shown in the following figure, a rocket with an initial mass of 150 kg, burns fuel at the rate of 10 kg/s with a constant exhaust velocity of 700 m/s. Please show your work to determine.

   a). What is the initial acceleration of the rocket?

   b). What is the velocity after 1 s?

   Note: Neglect the drag on the rocket.