Class Notes for the Course AerE 310

Aerodynamics 1

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Chapter 1
Review of Multivariable Calculus

1.1. Review of Partial Differentials and Chain Rule

1.1.1 Definition of Partial Differentials

$$f_x(x_0, y_0) = \lim_{\Delta x \to 0}[\frac{f(x_0 + \Delta x, y_0) - f(x_0 + \Delta x, y_0)}{\Delta x}]$$
$$f_y(x_0, y_0) = \lim_{\Delta y \to 0}[\frac{f(x_0 + \Delta y, y_0) - f(x_0 + \Delta x, y_0)}{\Delta y}]$$

1.1.2 Properties of Partial Derivatives

$$(f+g)_y = f_y + g_y;$$
$$(f-g)_y = f_y - g_y;$$
$$(f\,g)_y = f_y\,g + f\;g_y;$$
$$(f/g)_y = \frac{f_y\,g - f\;g_y}{g^2}$$
$$(f_x)_x = f_{xx} = \frac{\partial^2 f}{\partial x^2};$$
$$(f_x)_y = f_{xy} = \frac{\partial^2 f}{\partial y \partial x};$$
$$(f_y)_x = f_{yx} = \frac{\partial^2 f}{\partial x \partial y};$$
$$(f_y)_x = (f_x)_y = f_{yx} = f_{xy}$$

1.1.3 Chain Rule

1). In two- dimensional space:

$$\left.\begin{array}{l} z = f(x,y) \\ x = g_1(t) \\ y = g_2(t) \end{array}\right\} \Rightarrow \quad z = z(x,y) = f(g_1(t), g_2(t)) = z(t)$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\left.\begin{array}{l} w = f(x,y) \\ x = g_1(u,v) \\ y = g_2(u,v) \end{array}\right\} \Rightarrow \quad w = f(x,y) = f(g_1(u,v), g_2(u,v)) = w(u,v)$$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u}$$
$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v}$$

2). In three-dimensional space:

$$\left.\begin{array}{l} w = f(x,y,z) \\ x = g_1(t) \\ y = g_2(t) \\ y = g_2(t) \end{array}\right\} \Rightarrow \quad \frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$\left.\begin{array}{l} w = f(x,y,z) \\ x = g_1(u,v) \\ y = g_2(u,v) \\ z = g_3(u,v) \end{array}\right\} \Rightarrow \quad \left\{\begin{array}{l} \dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial u} \\ \dfrac{\partial w}{\partial v} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial u} \end{array}\right.$$