***2021 Fall Semester AerE310 Course***
***Homework Problem Set #06:***

***<u>Due</u>: Midnight, Sunday, 12/12/2021***

**Problem#1:**

Consider an airplane that weighs 14,700 N and cruises in level flight at 300 km/h at an altitude of 3000 m. The wing has a surface area of 17.0 m$^2$ and an aspect ratio of 6.2. Assume that the lift coefficient is a linear function of the angle of attack and $\alpha_{L=0}$ =-1.2°. If the load distribution is elliptic, calculate the value of circulation in the plane of symmetry ($\Gamma$o), the down wash ($W_{y1}$), the induced drag coefficient ($C_{Di}$), the geometric angle of attack ($\alpha$), and the effective angle of attack ($\alpha_{eff}$).

**Solution:**

7.1) Since the airplane is flying at 3000 m, we can use Table 1.2 to find that: $\rho_\infty = 0.9092 \frac{kg}{m^3}$. The airspeed is:

$$U_\infty = 300 \frac{km}{h} \frac{(1000 \ m/km)}{(3600 \ s/h)} = 83.333 \frac{m}{s}$$

Thus, $q_\infty = \frac{1}{2}\rho_\infty U_\infty^2 = 3156.9 \frac{N}{m^2}$

Since $AR = \frac{b^2}{S}$ ; $b = \sqrt{(6.2)(17.0 m^2)} = 10.266 m$

$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S} = \frac{\pi b \Gamma_0}{2 U_\infty S} \qquad (7.12)$$

Thus,

$$\Gamma_0 = \frac{L}{q_\infty}\frac{2U_\infty}{\pi b} = \frac{(14,700)}{(3156.9)}\frac{2(83.333)}{\pi(10.266)} = 24.06 \frac{m^2}{s}$$

$$w_{y_1} = -\frac{\Gamma_0}{4s} = -\frac{\Gamma_0}{4(b/2)} = -1.172 \frac{m}{s}$$

Since $C_L = \frac{L}{q_\infty S} = 0.2739$ ; $C_{DV} = \frac{C_L^2}{\pi AR} = 0.00385$

$$\varepsilon = \frac{C_L}{\pi AR} = 0.01406 \text{ rad} = 0.8057°$$

Referring to Fig. 7.12:

$$C_L = 2\pi(\alpha_e - \alpha_{0L}) = 0.2739$$

Thus, $\alpha_e - \alpha_{0L} = 0.04359 \text{ rad} = 2.4977°$

$\underline{\alpha_e = 1.2977°}$ and $\alpha = \alpha_e + \varepsilon = 2.1034°$

**Problem#2:**

Consider the case where the spanwise circulation distribution for a wing is parabolic: $\Gamma(y) = \Gamma_0(1 - y^2/s^2)$. If the total lift generated by the wing with the parabolic circulation distribution is to be equal to the lift generated by a wing with an elliptic circulation distribution,

- What is the relation between $\Gamma_0$ values for the two distributions?
- What is the relation between the induced downwash velocities at the plane of symmetry for the two configurations?

**Solution:**

7.2 The lift is given by:

$$L = \int_{-s}^{+s} \rho_\infty U_\infty \Gamma(y)\, dy \qquad (7.6)$$

Let us designate $\Gamma_0$ for the parabolic load distribution as $\Gamma_{0,p}$ and that for the elliptic load distribution as $\Gamma_{0,e}$. Thus, the total lift for the parabolic load distribution is:

$$L_p = \rho_\infty U_\infty \Gamma_{0,p} \int_{-s}^{+s} \left(1 - \frac{y^2}{s^2}\right) dy$$

$$L_p = \rho_\infty U_\infty \Gamma_{0,p} \left[ y - \frac{y^3}{3s^2} \right|_{-s}^{+s} = \rho_\infty U_\infty \Gamma_{0,p} \frac{4s}{3}.$$

If the total lift generated by the wing with the parabolic circulation distribution is to be equal to the total lift generated by a wing with an elliptic circulation distribution, i.e., $L_p = L_e$, then:

$$\rho_\infty U_\infty \Gamma_{0,p} \frac{4s}{3} = \rho_\infty U_\infty \Gamma_{0,e} \frac{\pi}{2} s$$

Thus,
$$\Gamma_{0,p} = \frac{3\pi}{8} \Gamma_{0,e}$$

To calculate the downwash velocity at the plane of symmetry, we note that $y_1 = 0$ and:

$$w_{y_1} = \frac{1}{4\pi} \int_{-s}^{+s} \frac{\frac{d\Gamma}{dy}}{y - y_1} dy = \frac{1}{4\pi} \int_{-s}^{+s} \frac{1}{y} \frac{d\Gamma}{dy} dy$$

For the parabolic distribution: $\dfrac{d\Gamma}{dy} = \Gamma_{0,p}\left(-\dfrac{2y}{s^2}\right)$

Thus,
$$w_{y_1=0} = \frac{1}{4\pi} \int_{-s}^{+s} \left(-\frac{2\Gamma_{0,p}}{s^2}\right) dy = -\frac{\Gamma_{0,p}}{\pi s}$$

7.2 cont Thus, the relation between the induced downwash velocities at the plane of symmetry is:

$$\frac{(w_{y_1=0})_p}{(w_{y_1=0})_e} = \frac{-\frac{\Gamma_{0,p}}{\pi s}}{-\frac{\Gamma_{0,e}}{4s}} = \frac{4}{\pi}\frac{\Gamma_{0,p}}{\Gamma_{0,e}} = \frac{3}{2}$$

**Problem#3:**

For a wing with an aspect ratio of $AR_1$ at the geometric angle of attack of $\alpha_1$ generates the same amount of the lift as the other wing with an aspect ratio of $AR_2$, **then** at the geometric angle of attack of $\alpha_2$ can be expressed as,

$$\alpha_2 = \alpha_1 + (C_L/\pi)[(1/AR_2) - (1/AR_1)]$$

When a GA(W)-1 airfoil section (i.e., a wing of infinite span) is at an angle of attack of 4.0°, the lift coefficient is 1.0. using the above equation to determine:

- Calculate the angle of attack at which a wing whose aspect ratio is 7.5 would have to operate to generate the same lift coefficient.
- What would the angle of attack have to be to generate this lift coefficient for a wing whose aspect ratio is 6.0?

**Solution:**

$\boxed{7.3}$

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left\{\frac{1}{AR_2} - \frac{1}{AR_1}\right\}$$

If $AR_1 = \infty$ and $\alpha_1 = 4° = 0.0698$ radians ; $C_L = 1.0$.

The angle of attack required to generate the same lift coefficient when $AR_2 = 7.5$ is:

$$\alpha_2 = 0.0698 + \frac{1.0}{\pi}\left\{\frac{1}{7.5} - 0\right\} = 0.1123 \text{ rad}$$

$$\alpha_2 = 6.432°$$

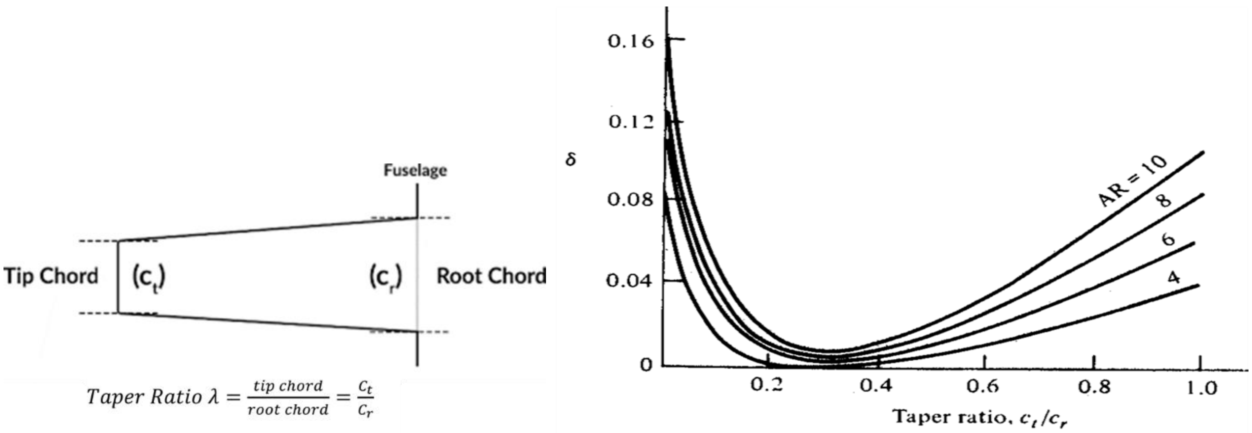
The angle of attack required to generate the same lift coefficient if the aspect ratio is 6, i.e. $AR_2 = 6.0$

$$\alpha_2 = 0.0698 + \frac{1.0}{\pi}\left\{\frac{1}{6.0} - 0\right\} = 0.1229 \text{ rad}$$

$$\alpha_2 = 7.040°$$

**Problem#4:**

For a finite wing with an aspect ratio of 8.0 and taper ration of 0.8. The airfoil section is thin and symmetrical. By using the data given in the following figure, please calculate the lift and induced drag coefficient for the wing when it is at an angle of attack $\alpha$=5.0°.




**Solution:**

For a generaral wing, the slope of the lift coefficeint can be expressed as:

$$\boxed{a = \frac{a_0}{1 + (a_0 / \pi AR)(1+\delta)}}$$

Then, the induced drag coeffeicent can be expressed as:

$$C_{D_i} = \frac{C_L{}^2}{\pi AR}(1+\delta)$$

$$a = \frac{a_0}{1 + a_0/\pi \, \text{AR}(1+\delta)} = \frac{2\pi}{1 + 2\pi(1.055)/8\pi} = 4.97 \text{ rad}^{-1}$$

$$= 0.0867 \text{ degree}^{-1}$$

Since the airfoil is symmetric, $\alpha_{L=0} = 0°$. Thus,

$$C_L = a\alpha = (0.0867 \text{ degree}^{-1}(5°) = \boxed{0.4335}$$

From Equation (5.61),

$$C_{D,i} = \frac{C_L^2}{\pi \, \text{AR}}(1+\delta) = \frac{(0.4335)^2(1+0.055)}{8\pi} = \boxed{0.00789}$$

**<u>Problem#5:</u>** For a flat delta wing with sharp leading edge and an aspect ratio of 1.5.

- (a). Please calculate of the lift coefficient of the delta wing and then compare the solution with the experimental data given in the following figures.
- (b). Please calculate of the induced drag coefficient of the delta wing and then compare the solution with the experimental data given in the following figures.

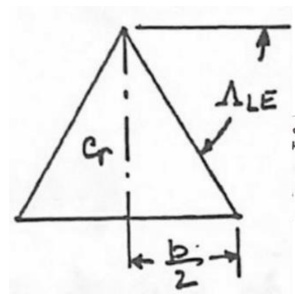
**Note:** For a triangle wing at the angle of attack of α, the lift and drag coefficients can be estimated as:

$$C_L = K_p \sin\alpha \cdot \cos^2\alpha + K_v \sin^2\alpha \cdot \cos\alpha$$

Where $C_{Dp}$ is drag coefficient of a flat plate perpendicular to the flow ($C_{Dp} \approx 1.95$).

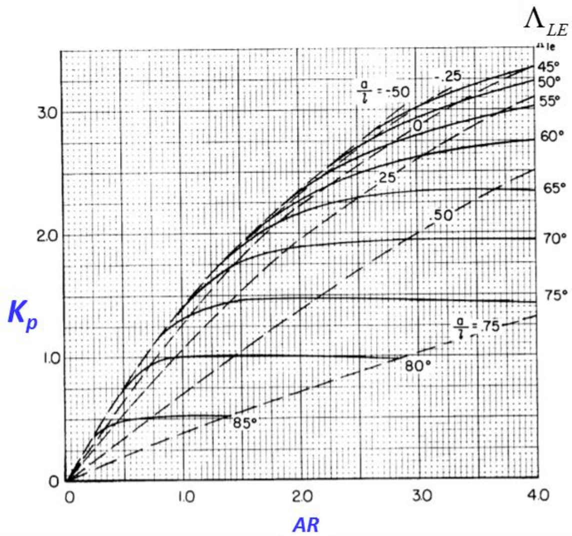
$$C_{D,i} = C_D - C_{D0} = C_L \tan\alpha$$

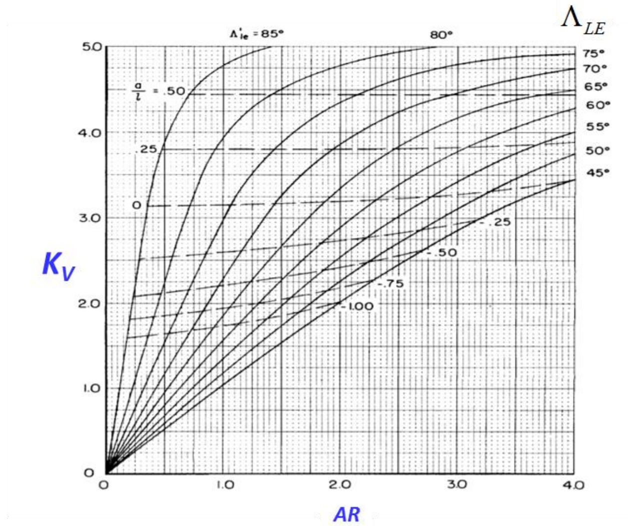
$$C_D = C_{D0} + C_{D,i} = C_{D0} + K_p \sin^2\alpha \cdot + K_v \sin^3\alpha$$

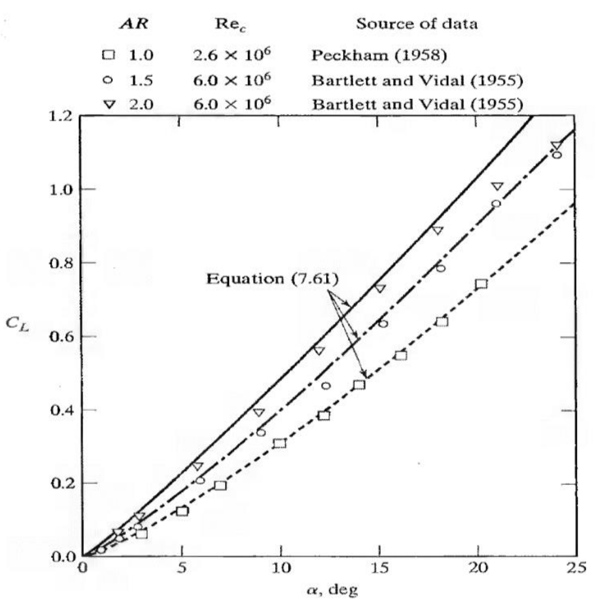


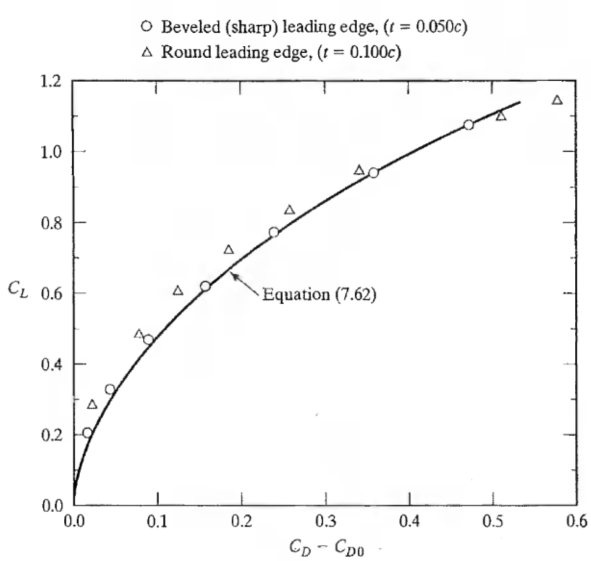

$S = \frac{c_r b}{2}$

$\tan\Lambda_{LE} = \frac{c_r}{b/2}$



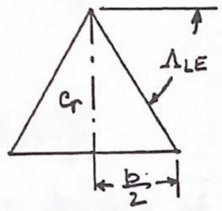



**Solution:**

7.13) For a delta wing




$$S = \frac{c_r b}{2}$$

and $\tan \Lambda_{LE} = \dfrac{c_r}{b/2}$

Combining these two relations: $S = \frac{b^2}{4} \tan \Lambda_{LE}$ ; $\tan \Lambda_{LE} = \frac{4}{AR}$

Since the AR (aspect ratio) is 1.5, $\Lambda_{LE} = 69.45°$.

Using Fig. 7.39, $K_p = 1.76$ and using Fig. 7.40, $K_v = 3.18$ as the "constants" in equation (7.61)

$$C_L = K_p \sin\alpha \cos^2\alpha + K_v \sin^2\alpha \cos\alpha$$

we have:

| $\alpha$ | 0° | 5° | 10° | 15° |
|---|---|---|---|---|
| $C_L$ | 0.000 | 0.176 | 0.391 | 0.631 |

These calculations reproduce those represented by the broken line of Fig. 7.41

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7.14) Taking the coefficients found in the previous problem, i.e., $K_p = 1.76$ and $K_v = 3.18$, and substituting them into equations (7.61) and (7.62)

$$\Delta C_D = C_D - C_{D0} = C_L \tan\alpha$$

and using the results from the previous problem:

| $\alpha$ | 0° | 5° | 10° | 15° |
|---|---|---|---|---|
| $C_L$ | 0.000 | 0.176 | 0.391 | 0.631 |
| $\Delta C_D$ | 0.000 | 0.015 | 0.069 | 0.169 |

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