2021 Fall Semester AerE310 Course
Homework Problem Set #07:

<u>Due</u>: Midnight, Sunday, 12/12/2021

Problem#1:

A rectangular plate, whose streamwise dimension (or chord c) is 0.2 m and whose width (or span b) is 1.8 m, is mounted in a wind tunnel. The freestream velocity is 40 m/s. The density of the air is 1.2250 kg/m^3, and the absolute viscosity is 1.7894*10^{-5} kg/m·s.

- Graph the velocity profiles at x = 0.0 m, x = 0.05 m, x = 0.10 m, and x = 0.20 m.
- Calculate the chordwise distribution of the skin-friction coefficient and the displacement thickness.
- What is the drag coefficient for the plate?

Solution:

Since the span (or width) of the plate is 9.0 times the chord (or streamwise dimension), we will assume that the flow is two dimensional (i.e., it is independent of the spanwise coordinate). The maximum value of the local Reynolds number, which occurs when x = c, is

$$Re_c = \frac{\left(1.225kg/m^3\right)(4.m/s)(0.2m)}{(1.7894*10^{-5}kg/m{\cdot}s)} = 5.477 * 10^5$$

This Reynolds number is close enough to the transition criteria for a flat plate that we will assume that the boundary layer is laminar for its entire length. Therefore, we will use the relations developed in this section to calculate the required parameters. Noting that

$$y = \sqrt{\frac{2\nu x}{u_e}}\eta = 8.546 * 10^{-4}\sqrt{x}\eta$$

we can use the results presented in the Table from the Blasius solution given below to calculate the velocity profiles. The resultant profiles are presented in Fig. 4.6 . At the leading edge of the flat plate (i.e., at x = 0), the velocity is constant (independent of y). The profiles at the other stations illustrate the growth of the boundary layer with distance from the leading edge. Note that the scale of the y coordinate is greatly expanded relative to that for the x coordinate. Even though the streamwise velocity at the edge of the boundary layer(u_e) is the same at all stations, the velocity within the boundary layer is a function of x and y .However, if the dimensionless velocity (u/u_e) is presented as a function of *h*, the profile is the same at all stations. Since the dimensionless profiles are similar at all x stations, the solutions are termed similarity solutions .The displacement thickness in meters is:

$$\delta^* = \frac{1.72x}{\sqrt{Re_x}} = 1.0394 * 10^{-3}\sqrt{x}$$

TABLE 4.3 Solution for the Laminar Boundary Layer on a Flat Plate ($\beta = 0$)

η	f	f'	f''
0.0	0.0000	0.0000	0.4696
0.1	0.0023	0.0470	0.4696
0.2	0.0094	0.0939	0.4693
0.3	0.0211	0.1408	0.4686
0.4	0.0375	0.1876	0.4673
0.5	0.0586	0.2342	0.4650
0.6	0.0844	0.2806	0.4617
0.7	0.1147	0.3265	0.4572
0.8	0.1497	0.3720	0.4512
0.9	0.1891	0.4167	0.4436
1.0	0.2330	0.4606	0.4344
1.2	0.3336	0.5452	0.4106
1.4	0.4507	0.6244	0.3797
1.6	0.5829	0.6967	0.3425
1.8	0.7288	0.7610	0.3005
2.0	0.8868	0.8167	0.2557
2.2	1.0549	0.8633	0.2106
2.4	1.2315	0.9010	0.1676
2.6	1.4148	0.9306	0.1286
2.8	1.6032	0.9529	0.0951
3.0	1.7955	0.9691	0.0677
3.2	1.9905	0.9804	0.0464
3.4	2.1874	0.9880	0.0305
3.5	2.2863	0.9907	0.0244
4.0	2.7838	0.9978	0.0069
4.5	3.2832	0.9994	0.0015

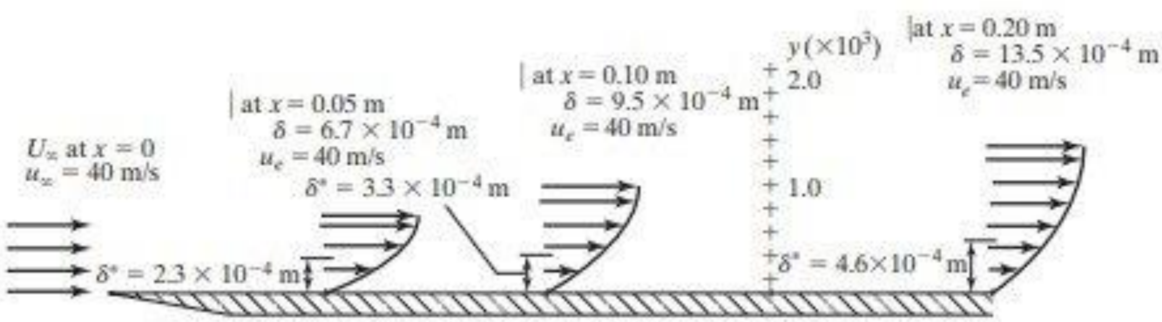

Figure 4.6 Velocity profile for the flat-plate laminar boundary layer $Re_c = 5.477 \times 10^5$.

The chordwise (or streamwise) distribution of the displacement thickness is presented in Fig. 4.6 . These calculations verify the validity of the common assumption that the boundary layer is thin. Therefore, the inviscid solution obtained neglecting the boundary layer altogether and that obtained for the effective geometry (the actual surface plus the displacement thickness) are essentially the same. The local skin-friction coefficient is given by:

$$c_f = \frac{0.664}{\sqrt{Re_x}} = \frac{4.013*10^{-4}}{\sqrt{x}}$$

Now we can calculate the drag coefficient for the plate. Obviously, the pressure contributes nothing to the drag since there is no dy dimension for an infinitely thin flat plate. Therefore, the drag force acting on the flat plate is due only to skin friction. Using general notation, we see that:

$$D = 2b \int_0^c \tau dx$$

We need integrate only in the x direction, since by assuming the flow to be two dimensional, we have assumed that there is no spanwise variation in the flow. In the above equation, the integral, which represents the drag per unit width (or span) of the plate, is multiplied by ***b*** (the span) and by 2 (since friction acts on both the top and bottom surfaces of the plate). Substituting the expression for the laminar shear forces given in equation of $\tau = 0.332\sqrt{\frac{\rho e^3}{x}}$,

$$D = 0.664b\sqrt{\rho\mu U_e^3} \int_o^c \frac{dx}{\sqrt{x}} = 1.328b\sqrt{c\rho\mu u_e^3}$$

Since the edge velocity (u_e) is equal to the free-stream velocity (U_∞), the drag coefficient for the plate is:

$$C_D = \frac{D}{q_\infty cb} = \frac{2.656}{\sqrt{Re_c}}$$

For the present problem C_D=3.589*10^{-3}

Alternatively, using the total skin-friction coefficient, equation of $\mu \cdot U_{cf} = \frac{1.328}{\sqrt{Re_l}}$, and computing drag on the top and bottom of the plate, we obtain:

$$C_D = C_f \frac{S_{wet}}{S_{ref}} = \frac{1.325}{\sqrt{Re_c}} \frac{2cb}{cb} = \frac{2.656}{\sqrt{5.477*10^5}} = 3.589 * 10^{-3}$$

Problem#2:

The streamwise velocity component for a laminar boundary layer is sometimes assumed to be roughly approximated by the linear relation $u = \frac{y}{\delta}u_e$ where $\delta = 1.25 * 10^{-2}\sqrt{x}$. Assume that we are trying to approximate the flow of air at standard sea-level conditions past a flat plate where u_e = 2.337 m/s.

- Calculate the streamwise distribution of the displacement thickness (δ^*), the velocity at the edge of the boundary layer\left (v_e), and the skin-friction coefficient C_f.
- Compare the values obtained assuming a linear velocity profile with the Blasius solutions.

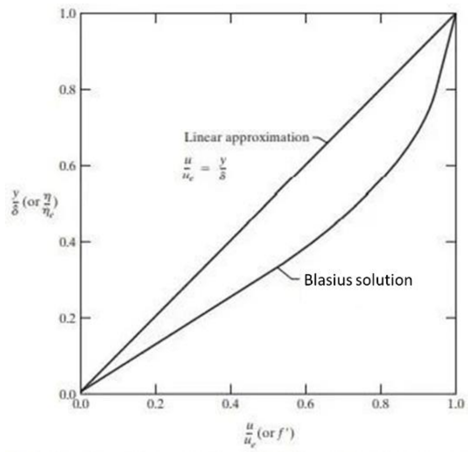

Figure 4.7 Comparison of velocity profiles for a laminar boundary layer on a flat plate.

Solution:

the standard day atmospheric conditions at sea level are:

ρ_∞=1.2250kg/m^3 and μ_∞=1.7894*10^{-5}kg/s m. So, for constant-property flow past a flat plate,

$$Re_x = \frac{\rho_\infty u_e x}{\mu_\infty} = 1.60 * 10^5 x$$

Using the definition for the displacement thickness of an incompressible boundary layer,

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_e}\right) dy \; \delta^* = \int_0^\delta \left(1 - \frac{u}{u_e}\right) dy = \delta \int_0^1 \left(1 - \frac{u}{u_e}\right) d\left(\frac{y}{\delta}\right)$$

Notice that, since we have u/u_e in terms of y/δ, we have changed our independent variable from y to y/δ,We must also change the upper limit on our integral from δ to 1. Therefore, since:

$$\frac{u}{u_e} = \frac{y}{\delta} and \delta = 1.25 * 10^{-2}\sqrt{x}$$

Then

$$\delta^* = 1.25 * 10^{-2}\sqrt{x} \int_0^1 \left(1 - \frac{y}{\delta}\right) d\left(\frac{y}{\delta}\right) = 0.625 * 10^{-2}\sqrt{x}$$

for the linear profile.

Using the equation for the formulation for Blasius solution and noting that.

$$\frac{\delta^*}{x} = \frac{\sqrt{2}(\eta_e - f_e)}{\sqrt{Re_x}} = \frac{1.72}{\sqrt{Re_x}}$$

$Re_x = 1.60 * 10^5 x$, we find that

$$\delta^* = 0.430 * 10^{-2}\sqrt{x}$$

Using the continuity equation, we would find that the linear approximation gives a value for

$$\nu_e = \frac{3.125{*}10^{-3}}{\sqrt{x}} u_e$$

Using the formulation of Blasius's solution

$$\frac{\nu_e}{u_e} = \frac{0.84}{\sqrt{Re_x}}$$

$$\nu_e - \frac{0.84}{\sqrt{Re_x}} u_e - \frac{2.10{*}10^{-3}}{\sqrt{x}} u_e$$

Finally, we find that the skin friction for the linear velocity approximation is given by:

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu u_e}{\delta}$$

Therefore, the skin-friction coefficient is:

$$C_f = \frac{\pi}{\frac{1}{2}\rho_\infty u_e^2} = \frac{2\mu_\infty}{\rho_\infty u_e \delta} = \frac{2}{1.60{*}10^5\left(1.25{*}10^{-2}\sqrt{x}\right)} = \frac{1.00{*}10^{-3}}{\sqrt{x}}$$

For the formulation of Blasius's solution,

$$C_f = \frac{0.664}{\sqrt{Re_x}} = \frac{1.66{*}10^{-3}}{\sqrt{x}}$$

Summarizing these calculations provides the following comparison:

the formulation of Blasius's solution	linear approximation	
$0.430 * 10^{-2}\sqrt{x}$	$0.625 * 10^{-2}\sqrt{x}$	δ^*
$(2.10 * 10^{-3}u_e)/\sqrt{x}$	$(3.125 * 10^{-3}u_e)/\sqrt{x}$	ν_e
$(1.66 * 10^{-3})/\sqrt{x}$	$(1.00 * 10^{-3})/\sqrt{x}$	C_f

Problem#3:

A small airplane flies at speed of 90 m/s at 1000 m altitude. The airplane wing has rectangular shape with chord length of 1m and span of 11m. Assume boundary layer over the wing surface is fully laminar and model the wing airfoil as a flat plate.

- a) Estimate the boundary layer thickness at the trailing edge.
- b) Estimate the displacement thickness at the trailing edge.
- c) estimate the friction drag of the wing.

Solution:

At 1000 m using standard atmospheric table: $\rho_\infty = 1.112 \frac{kg}{m^3}$, $\mu_\infty = 1.78 \times 10^{-5}\ Pa.s$

a)

Reynolds number at trailing edge where $x = c = 1\ m$

$$Re_c = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{1.112 \times 90 \times 1}{1.78 \times 10^{-5}} = 5.62 \times 10^6$$

Boundary layer thickness at chord length then

$$\frac{\delta_c}{c} = \frac{5}{\sqrt{Re_c}} = \frac{5}{\sqrt{5.62 \times 10^6}} = 0.0021$$

$$\delta_c = 2.1\ mm$$

b)

displacement thickness at chord length

$$\frac{\delta_c^*}{c} = \frac{1.72}{\sqrt{Re_c}} = \frac{1.72}{\sqrt{5.62 \times 10^6}} = 7.25 \times 10^{-4}$$

$$\delta_c^* = 0.0725\ mm$$

c)

drag coefficient assuming a fully laminar boundary layer over the wing:

$$C_D = \frac{1.328}{\sqrt{Re_c}} = \frac{1.328}{\sqrt{5.62 \times 10^6}} = 5.60 \times 10^{-4}$$

To calculate the friction drag force, note that there are boundary layers on both top and bottom surface of the wing, hence multiply the area by 2:

$$D_f = \frac{1}{2}\rho_\infty V_\infty^2 S C_D = \frac{1}{2} \times 1.112 \times 90^2 \times (11 \times 1 \times 2) \times 5.6 \times 10^{-4}$$

$$D_f = 55.5\ N$$

Problem#4:

A well-hit golf ball (diameter D=1.69 in., weight w=0.0992 lb) can travel at U=200 ft/s as it leaves the tee. A well-hit table tennis ball (diameter D=1.50 in., weight w= 0.00551 lb) can travel at U=60 ft/s as it leaves the paddle.

- Determine the drag on a standard golf ball, a smooth golf ball, and a table tennis ball for the conditions given.
- Also determine the deceleration of each ball for these conditions.

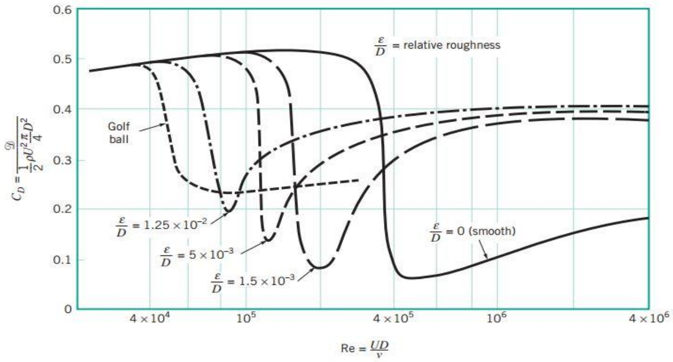

FIGURE 9.18 The effect of surface roughness on the drag coefficient of a sphere in the Reynolds number range for which the laminar boundary layer becomes turbulent (Ref. 4).

Solution:

For either ball, the drag can be obtained from

$$\mathscr{D} = \frac{1}{2}\rho U^2 \frac{\pi}{4}\ D^2 C_D \quad (1)$$

where the drag coefficient, C_D, is given in Fig. 9.18 as a function of the Reynolds number and surface roughness. For the golf ball in standard air

$$Re = \frac{UD}{\nu} = \frac{(200ft/s)(1.69/12ft)}{1.57\times10^{-4}ft^2/s} = 1.79 \times 10^5$$

while for the table tennis ball

$$Re = \frac{UD}{\nu} = \frac{(60ft/s)(1.50/12ft)}{1.57\times10^{-4}ft^2/s} = 4.78 \times 10^4$$

The corresponding drag coefficients are $C_D = 0.25$ for the standard golf ball, $C_D = 0.51$ for the smooth golf ball, and $C_D = 0.50$ for the table tennis ball. Hence, from Eq. 1 for the standard golf ball

$$\mathscr{D} = \frac{1}{2}(2.38 \times 10^{-3}slugs/ft^3)(200ft/s)^2\frac{\pi}{4}\ (\frac{1.69}{12}ft)^2(0.25) = 0.185lb$$

for the smooth golf ball

$$\mathscr{D} = \frac{1}{2}(2.38 \times 10^{-3}slugs/ft^3)(200ft/s)^2\frac{\pi}{4}\ (\frac{1.69}{12}ft)^2(0.51) = 0.378lb$$

and for the table tennis ball

$$\mathscr{D} = \frac{1}{2}(2.38 \times 10^{-3}slugs/ft^3)(60ft/s)^2\frac{\pi}{4}\ (\frac{1.50}{12}ft)^2(0.50) = 0.0263lb$$

The corresponding decelerations are $a = \mathscr{D}/m = g\mathscr{D}/W$, where m is the mass of the ball. Thus, the deceleration relative to the acceleration of gravity, a/g (i.e., the number of g's decel eration), is $a/g = \mathscr{D}/W$ or

$$\frac{a}{g} = \frac{0.185lb}{0.0992lb} = 1.86 \text{ for the standard golf ball}$$

$$\frac{a}{g} = \frac{0.378lb}{0.0992lb} = 3.81 \text{ for the smooth golf ball}$$

And

$$\frac{a}{g} = \frac{0.0263lb}{0.00551lb} = 4.77 \text{ for the table tennis ball}$$