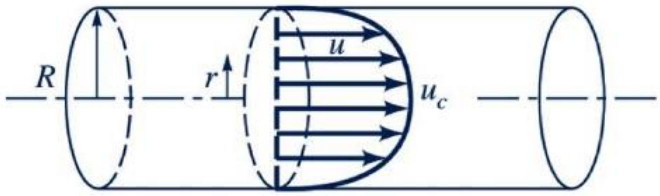
AerE310 Course - Homework Problem Set #03:

1. Consider the fully developed flow in a circular pipe, as shown in Fig. 1. The velocity u is a function of the radial coordinate only: $u = U_{CL}(1-\frac{r^2}{R^2})$, where U_{CL} is the magnitude of the velocity at the centerline (or axial) of the pipe.



Please use the integral form of the momentum equation to show how the pressure drop per unit length *dp/dx* changes if the radius of the pipe were to be doubled while the mass flux through the pipe held constant at the value of $\dot{m}$. neglect the weight of the fluid in the control volume and assume that the fluid properties are constant.

Solution:

$$\sum \vec{F} = \frac{\partial}{\partial t}\iiint \rho \vec{v} \mathrm{d}V + \oiint (\rho \vec{v}\cdot \hat{n}\mathrm{d}A)\vec{v}$$

$$= \int_0^R \rho \left[u_{CL}\left(1-\frac{\gamma^2}{R^2}\right)\right](2\pi r \mathrm{d}r)\left[u_{CL}\left(1-\frac{\gamma^2}{R^2}\right)\right]$$

$$- \int_0^R \rho \left[u_{CL}\left(1-\frac{\gamma^2}{R^2}\right)\right](2\pi r \mathrm{d}r)\left[u_{CL}\left(1-\frac{\gamma^2}{R^2}\right)\right]$$

$$= 0$$

$$\therefore \Sigma F_x = P_1A_1 - P_2A_2 + 2\pi R(\Delta x)\tau_w = 0$$

Based on definition of wall shear stress:

$$\tau_w = \mu(\frac{du}{dr})\bigg|_{r=R} = \mu U_{CL}(-\frac{2r}{R^2})\bigg|_{r=R} = \frac{-2\mu U_{CL}}{R}$$

$$Therefore: \frac{P_2 - P_1}{\Delta x} = -\frac{2}{R}(\frac{2\mu U_{CL}}{R})$$

$$\Rightarrow \frac{dp}{dx} = -\frac{4\mu U_{CL}}{R^2}$$

The mass flow rate through the pipe is:

$$
\begin{aligned}
\dot{m} &= \int_0^R \rho u_{cL}\left(1 - \frac{\gamma^2}{R^2}\right) 2\pi r dr \\
&= \rho U_{CL} 2\pi \int_0^R \left(r - \frac{r^3}{R^2}\right) dr \\
&= \frac{\rho U_{CL} \pi R^2}{2} \\
\therefore U_{CL} &= \frac{2\dot{m}}{\rho \pi R^2} \\
\therefore \frac{\mathrm{d}p}{\mathrm{d}x} &= -\frac{4\mu}{R^2}\left(\frac{2m}{p\pi R^2}\right)
\end{aligned}
$$

If we maintain the same mass flow while doubling the radius of the pipe then:

$$
\begin{aligned}
&\dot{m}_1 = \dot{m}_2, \quad R_2 = 2R_1 \\
&\left.\frac{\mathrm{d}p}{\mathrm{d}x}\right|_2 = -\frac{4\mu}{R_2^2}\left(\frac{2\dot{m}_2}{\rho \pi R_2^2}\right) = \left.\frac{1}{16}\frac{\mathrm{d}p}{\mathrm{d}x}\right|_1
\end{aligned}
$$

2. Simplify the Navier-Stokes equation in Cartesian coordinate for an incompressible, steady, 2-D flow between horizontal parallel plates. Assuming that $u = u(y), v = 0, w = 0$, write momentum equations in (x, y, z) all three directions.

solution:

For 2-D compressible steady flow. The momentum equations can be written as:

$$x \text{ direction: } \rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + \rho f_x$$

$$y \text{ direction: } \rho\left(\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) + \rho f_y$$

$$z \text{ direction: } \rho\left(\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) + \rho f_z$$

At x direction:

$$\because \frac{\partial u_x}{\partial t} = \frac{\partial u_x}{\partial x} = \frac{\partial u_x}{\partial z} = u_y = u_z = f_x = 0$$

$$\therefore \frac{\partial p}{\partial x} = \mu\frac{\partial^2 u_x}{\partial y^2}$$

At y direction:

$$\because u_y = u_z = f_y = 0$$

$$\therefore \frac{\partial p}{\partial y} = 0$$

At z direction:

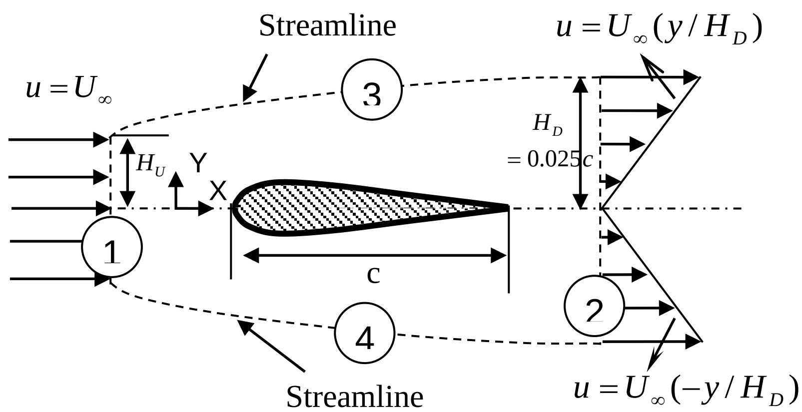
$$\because u_y = u_z = 0$$

$$\therefore \frac{\partial p}{\partial z} = \rho f_z$$

3. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of the control volume shown in the figure. The flow is incompressible, two-dimensional, and steady. The gauge pressure on the surfaces along the dashed line is equal to zero. Surface 3 and 4 are streamlines. [**Hint:** No fluid will flow across the streamlines]

(a). What is the relationship between the H_U and H_D .

(b). If $H_D = 0.025c$, what is the drag coefficient of the airfoil? ($C_D = \dfrac{D}{\frac{1}{2}\rho U_\infty{}^2 c}$)



(a). Chose the C.V. as the volume enclosed by the dashed lines in the figure.

Integral from of the mass conservation equation is:

$$\frac{\partial}{\partial t}\left(\int_{CV}\rho d\forall\right)+\int_{CS}\rho\vec{V}\cdot\hat{n}\,ds=0$$

Since steady flow $\Rightarrow$ $\dfrac{\partial}{\partial t}\left(\displaystyle\int_{CV}\rho d\forall\right)=0 \quad\Rightarrow\quad \displaystyle\int_{CS}\rho\vec{V}\cdot\hat{n}\,ds=0$

$$\Rightarrow \int_1\rho\vec{V}\cdot\hat{n}\,ds+\int_2\rho\vec{V}\cdot\hat{n}\,ds+\int_3\rho\vec{V}\cdot\hat{n}\,ds+\int_4\rho\vec{V}\cdot\hat{n}\,ds=0$$

Boundaries 3 and 4 are streamlines:

$$\Rightarrow\int_3\rho\vec{V}\cdot\hat{n}\,ds=0 \quad and \quad \int_4\rho\vec{V}\cdot\hat{n}\,ds=0.$$

Therefore:

$$\Rightarrow \int_1\rho\vec{V}\cdot\hat{n}\,ds+\int_2\rho\vec{V}\cdot\hat{n}\,ds=0$$

$$\int_1\rho\vec{V}\cdot\hat{n}\,ds=\int_{-H_U}^{H_U}\rho(-U_\infty)\,dy=-2\,\rho\,U_\infty\,H_U$$

$$\int_2\rho\vec{V}\cdot\hat{n}\,ds=\int_{-H_u}^{H_u}\rho\,u\,dy=\int_0^{H_u}\rho[U_\infty\frac{y}{H_D}]dy+\int_{-H}^{0}\rho[U_\infty\frac{-y}{H_D}]dy=2\int_0^{H_D}\rho[U_\infty\frac{y}{H_D}]dy=2\rho\frac{U_\infty}{2H_D}y^2\Big|_0^{H_D}=\rho\,U_\infty\,H_D$$

$$\int_1\rho\vec{V}\cdot\hat{n}\,ds+\int_2\rho\vec{V}\cdot\hat{n}\,ds=0 \quad\Rightarrow\quad -2\rho U_\infty H_U+\rho U_\infty H_D=0 \quad\Rightarrow\quad H_D=2H_U$$

(b). Chose the C.V. as the dashed lines in the figure.

Integral from of the momentum conservation equation is:

$$\frac{\partial}{\partial t}\left(\int_{CV}\vec{V}\,\rho d\forall\right)+\int_{CS}\vec{V}\,\rho\,\vec{V}\cdot\hat{n}\,ds=\sum\vec{F}$$

Along the X-direction: $\sum\vec{F}_x=-D+\sum \begin{matrix} X\ \text{component of the force} \\ \text{due to pressure distributions at } C.S. \end{matrix}$

Since Gauge pressure on the surfaces along the dashed line is Zero

$$\sum pressure\quad at\quad C.S=0\quad .\quad\Rightarrow\qquad \sum\vec{F}_x=-D$$

Steady flow $\quad\Rightarrow\quad \frac{\partial}{\partial t}\left(\int_{CV}\vec{V}\,\rho d\forall\right)=0$

Along the X-direction:

$$\int_{C.S.}\vec{V}_x\,\rho\,\vec{V}\cdot\hat{n}\,ds=\int_1\vec{V}_x\,\rho\,\vec{V}\cdot\hat{n}\,ds+\int_2\vec{V}_x\,\rho\,\vec{V}\cdot\hat{n}\,ds+\int_3\vec{V}_x\,\rho\,\vec{V}\cdot\hat{n}\,ds+\int_4\vec{V}_x\,\rho\,\vec{V}\cdot\hat{n}\,ds$$

$$=\int_{-H_U}^{H_U}U_\infty\,\rho\,(-U_\infty)\cdot dy+\int_{-H_U}^{H_U}u\,\rho\,u\cdot dy+\int_3 U_\infty\,\rho\,\vec{V}\cdot\hat{n}\,ds+\int_4 U_\infty\,\rho\,\vec{V}\cdot\hat{n}\,ds$$

$$=-\int_{-H_U}^{H_U}U_\infty^{\ 2}\,\rho\,dy+\int_{-H_U}^{H_U}\rho\,u^2\cdot dy+U_\infty\int_3\rho\vec{V}\cdot\hat{n}\,ds+U_\infty\int_4\rho\vec{V}\cdot\hat{n}\,ds$$

Since C.S. 3 and 4 are streamlines, i. e. $\int_3\rho\vec{V}\cdot\hat{n}\,ds=\int_4\rho\vec{V}\cdot\hat{n}\,ds=0$

Therefore:

$$\sum\vec{F}_x=-D=\int_{C.S.}\vec{V}_x\,\rho\,\vec{V}\cdot\hat{n}\,ds=-\int_{-H_U}^{H_U}U_\infty^{\ 2}\,\rho\,dy+\int_{-H_U}^{H_U}\rho\,u^2\cdot dy=-2H_U\,\rho\,U_\infty^{\ 2}+2\rho U_\infty^{\ 2}\int_0^{H_U}\left(\frac{y}{H_D}\right)^2\cdot dy$$

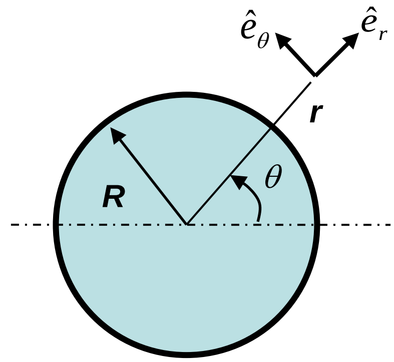
$$=-2H_U\,\rho\,U_\infty^{\ 2}+2\rho U_\infty^{\ 2}\left.\frac{y^3}{3H_D^{\ 2}}\right|_0^{H_D}=-2H_U\,\rho\,U_\infty^{\ 2}+2\rho U_\infty^{\ 2}\frac{H_D}{3}=(-1+\frac{2}{3})\rho U_\infty^{\ 2}H_D=-\frac{1}{3}\rho U_\infty^{\ 2}H_D$$

$$\Rightarrow\quad D=\frac{1}{3}\rho U_\infty^{\ 2}H_D$$

$$C_D=\frac{D}{\frac{1}{2}\rho U_\infty^{\ 2}\,c}=\frac{\frac{1}{3}\rho U_\infty^{\ 2}H_D}{\frac{1}{2}\rho U_\infty^{\ 2}\,c}=\frac{2H_D}{3c}=\frac{2*0.025c}{3c}=0.0167$$

4. For a two-dimensional steady, inviscid, incompressible flow around a cylinder of radius of R as shown in the figure, the velocity field is given as :

$$\vec{V}(r,\theta)=U_\infty(1-\frac{R^2}{r^2})\cos\theta\cdot\hat{e}_r-U_\infty(1+\frac{R^2}{r^2})\sin\theta\cdot\hat{e}_\theta;$$



Where U_∞ is the velocity of the undisturbined approaching stream (therefore, U_∞ is constant).

(a). Show your work to prove the flow with the velocity field given above is physically possible.

(b). Derive the expression for the acceleration of a fluid particle on the surface of the cylinder. Then, calculate the acceleration of a fluid particle at the points of $(r,\theta)=(R,0)$ and $(r,\theta)=(R,\pi)$.

Solution to Question

Part (a).

The continuity equation for a steady incompressible flow is:

$$\nabla\cdot\vec{V}=0$$

For the given velocity field in a cylindrical system:

$$\nabla\cdot\vec{V}=\frac{1}{r}(\frac{\partial(rV_r)}{\partial r}+\frac{\partial(V_\theta)}{\partial\theta}+\frac{\partial(rV_Z)}{\partial Z})=\frac{1}{r}(\frac{\partial(rV_\infty(1-\frac{R^2}{r^2})\cos\theta)}{\partial r}+\frac{\partial(-V_\infty(1+\frac{R^2}{r^2})\sin\theta)}{\partial\theta}+0)$$

$$=\frac{1}{r}(V_\infty(1-\frac{R^2}{r^2})\cos\theta+rV_\infty\cos\theta\cdot(-(-2)\cdot\frac{R^2}{r^3})-V_\infty(1+\frac{R^2}{r^2})\cos\theta)$$

$$=\frac{1}{r}(V_\infty(1-\frac{R^2}{r^2})\cos\theta+2V_\infty\cos\theta\cdot\frac{R^2}{r^2}-V_\infty(1+\frac{R^2}{r^2})\cos\theta)$$

$$=\frac{1}{r}(V_\infty(1+\frac{R^2}{r^2})\cos\theta-V_\infty(1+\frac{R^2}{r^2})\cos\theta)=0$$

The given velocity field satisfies the continuity equation; therefore, the two-dimensional flow is physical possible.

Part B

(b). The acceleration of a fluid particle will be:

$$\vec{a}=\frac{D\vec{V}}{Dt}=\frac{\partial\vec{V}}{\partial t}+\vec{V}\cdot\nabla\vec{V}$$

Since it is steady flow, therefore $\frac{\partial\vec{V}}{\partial t}= \quad\Rightarrow\quad \vec{a}=\vec{V}\cdot\nabla\vec{V}$

In a cylindrical system,

$$\vec{a} = \vec{V}\cdot\nabla\vec{V} = [V_r\frac{\partial}{\partial r} + V_\theta\frac{\partial}{r\partial\theta}][V_r\cdot\hat{e}_r + V_\theta\,\hat{e}_\theta]$$
$$= V_r\frac{\partial}{\partial r}[V_r\hat{e}_r + V_\theta\,\hat{e}_\theta] + V_\theta\frac{\partial}{r\partial\theta}[V_r\hat{e}_r + V_\theta\,\hat{e}_\theta]$$
$$= V_r[\hat{e}_r\frac{\partial V_r}{\partial r} + V_r\frac{\partial\hat{e}_r}{\partial r} + \hat{e}_\theta\frac{\partial V_\theta}{\partial r} + V_\theta\frac{\partial\hat{e}_\theta}{\partial r}] + \frac{V_\theta}{r}[\hat{e}_r\frac{\partial V_r}{\partial\theta} + V_r\frac{\partial\hat{e}_r}{\partial\theta} + \hat{e}_\theta\frac{\partial V_\theta}{\partial\theta} + V_\theta\frac{\partial\hat{e}_\theta}{\partial\theta}]$$
$$= V_r[\hat{e}_r\frac{\partial V_r}{\partial r} + \hat{e}_\theta\frac{\partial V_\theta}{\partial r}] + \frac{V_\theta}{r}[\hat{e}_r\frac{\partial V_r}{\partial\theta} + V_r\hat{e}_\theta + \hat{e}_\theta\frac{\partial V_\theta}{\partial\theta} - V_\theta\hat{e}_r]$$
$$= \hat{e}_r[V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial\theta} - \frac{V_\theta^{\ 2}}{r}] + \hat{e}_\theta[V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta V_r}{r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial\theta}]$$
$$= a_r\hat{e}_r + a_\theta\hat{e}_\theta$$

since

$$V_r = U_\infty(1-\frac{R^2}{r^2})\cos\theta\cdot \qquad \frac{\partial V_r}{\partial r} = U_\infty(0+2\frac{R^2}{r^3})\cos\theta\cdot \qquad \frac{\partial V_r}{\partial\theta} = -U_\infty(1-\frac{R^2}{r^2})\sin\theta\cdot$$
$$V_\theta = -U_\infty(1+\frac{R^2}{r^2})\sin\theta \;; \qquad \frac{\partial V_\theta}{\partial r} = -U_\infty(0-2\frac{R^2}{r^2})\sin\theta \;; \qquad \frac{\partial V_\theta}{\partial\theta} = -U_\infty(1+\frac{R^2}{r^2})\cos\theta$$

At the surface of the cylinder: r=R. Then

$$V_r = 0\cdot \qquad \frac{\partial V_r}{\partial r} = \frac{2U_\infty}{R}\cos\theta\cdot \qquad \frac{\partial V_r}{\partial\theta} = 0\cdot$$
$$V_\theta = -2U_\infty\sin\theta \;; \qquad \frac{\partial V_\theta}{\partial r} = \frac{2U_\infty}{R}\sin\theta \;; \qquad \frac{\partial V_\theta}{\partial\theta} = -2U_\infty\cos\theta$$

$$a_r = V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial\theta} - \frac{V_\theta^{\ 2}}{r} = 0 + 0 - \frac{4U_\infty^{\ 2}\sin^2\theta}{R}$$
$$= -\frac{4U_\infty^{\ 2}\sin^2\theta}{R}$$

$$a_\theta = V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta V_r}{r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial\theta} = 0 + 0 + [-\frac{2U_\infty\sin\theta}{R}][-2U_\infty\cos\theta]$$
$$= \frac{4U_\infty^{\ 2}}{R}\sin\theta\cos\theta\cdot$$

At point $(r,\theta) = (R,0)$, $a_r = 0$; $a_\theta = 0$ therefore, $\vec{a} = 0$

At point $(r,\theta) = (R,\pi)$, $a_r = 0$; $a_\theta = 0$ therefore, $\vec{a} = 0$

5. Consider a velocity field where the x and y components of the flow velocity are expressed as $u = \frac{cy}{x^2+y^2}$; $v = -\frac{cx}{x^2+y^2}$, where C is a constant. Please obtain the equations for streamlines.

Solution:

Method#1

$$\frac{dy}{dx} = \frac{v}{u} = -\frac{x}{y}$$

$$\Rightarrow ydy = -xdx$$

$$\therefore x^2 + y^2 = C$$

The streamlines are concentric circles with centers at the origin.

Method#2:

Since $u = \frac{cy}{x^2+y^2}$; $v = -\frac{cx}{x^2+y^2}$, it may be easier to solve it in cylindric system.

$$V_r = u \cdot cos\theta + v \cdot sin\theta = \frac{c\,r\sin\theta}{r^2}\cos\theta - \frac{c\,r\cos\theta}{r^2}\sin\theta = 0$$

$$V_\theta = -u \cdot \sin\theta + v \cdot \cos\theta = -\frac{c\,r\sin\theta}{r^2}\sin\theta - \frac{c\,r\cos\theta}{r^2}\cos\theta = -\frac{c}{r}$$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

$$Since \quad V_\theta = -\frac{\partial\psi}{\partial r} = -\frac{c}{r} \Rightarrow \psi = c\ln(r) + f(\theta)$$

$$Since \quad V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 0 \Rightarrow 0 + \frac{\partial f}{\partial\theta} = 0 \Rightarrow f(\theta) = Const.$$

$$Therefore \quad \psi = c\ln(r) + Const.$$

$\Rightarrow$ The streamlines are concentric circles with the center at the origin.

, it may be easier to solve it in cylindric system.

6. Consider a velocity field where the *radial* and *tangential* components of the flow velocity are expressed as $V_r = 0;\quad V_\theta = c\,r$, where C is a constant. Please obtain the equations for streamlines.

Solution:

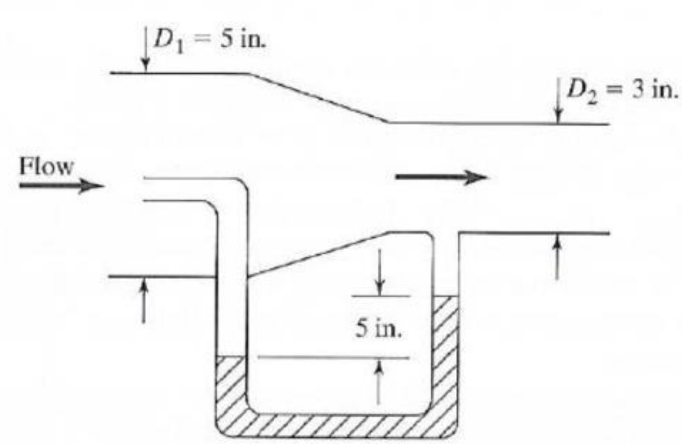
$$Since \quad V_\theta = -\frac{\partial \psi}{\partial r} = cr \Rightarrow \psi = -\frac{r^2}{2} + f(\theta)$$

$$Since \quad V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0 \Rightarrow 0 + \frac{\partial f}{\partial \theta} = 0 \Rightarrow f(\theta) = Const.$$

$$Therefore \quad \psi = -\frac{r^2}{2} + Const.$$

$\Rightarrow$ The streamlines are concentric circles with the center at the origin.

7. Air flow through a converging pipe section as shown in the figure. Since the centerline of the duct is horizontal, the change in potential energy is zero. The Pitot probe at the upstream station provides a measure of total pressure (or stagnation pressure). The downstream end of the U-tube provides a measure of the static pressure at the second section. Assume the density of the air is 0.00238slug/ft^3,and neglecting the effect of the viscosity. Compute the volumetric flow rate in ft^3/s. The fluid in the manometer is unity weight oil with the density ρ_{oil} =1.9404 slug/ ft$^{3.}$



solution:

$p_{t1} = p_{t2}$ We get

$$p_1 + \frac{1}{2}\rho U_1^2 = p_2 + \frac{1}{2}\rho U_2^2 \qquad (U_1 = 0)$$

From the principles of manometry:

$$p_{t1} = p_2 + \rho_{oil} g \Delta h$$

Equating the two expressions for p_{t1}:

$$\frac{1}{2}\rho U_2^2 = \rho_{oil} g \Delta h$$

$$\therefore \quad U_2 = 147.85 ft/s$$

$$\therefore \quad Q = V_2 A_2 = 7.258 ft^3/s$$

8. The stream function of 2-D, incompressible flow is given by : $\psi = \frac{\Gamma}{2\pi} \ln r$.

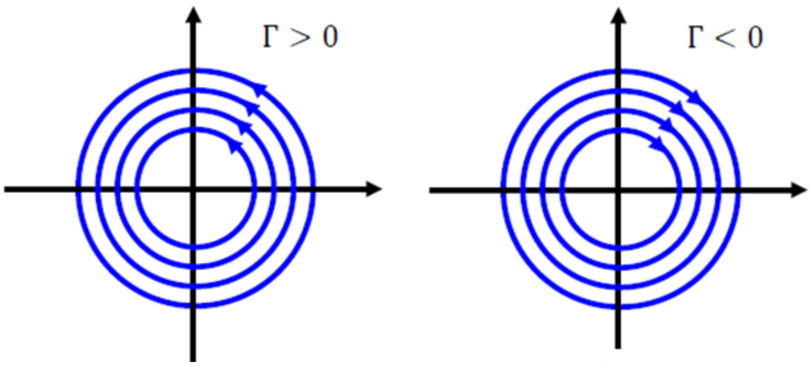
a) Graph the streamlines.

b) What is the velocity field represented by this stream function? Does the resultant velocity field satisfy the continuity equation?

c) Find the circulation about a path enclosing the origin. For the path of integration, use a circle of radius 3 with a center at the origin. Howe does the circulation dependent on the radius?

Solutions:

Part #(a)



Part #(b)

The velocity components in cylindrical coordinates may be written in terms of the derivatives of the stream function as: $y_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0$ and $V_\theta = -\frac{\partial \psi}{\partial r} = -\frac{\Gamma}{2\pi r}$

These velocity components are consistent with the seemliness, shown above:

$$\nabla \cdot \vec{v} = \frac{1}{r}\frac{\partial (rV_r)}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(-\frac{\Gamma}{2\pi r}\right) = 0$$

The velocity field satisfies continuity.

Part #(c)

co) To calculate the ciriulation $\oint \vec{v} \cdot d\bar{s}$, if the pith of integratim is a circle of unstant raclius $\mathrm{d}\vec{s} = R_1 \mathrm{d}\theta \hat{e}_\theta$

$$\oint \vec{v} \cdot d\vec{s} = \int_0^{2\pi} \left(-\frac{\Gamma}{2\pi R_1}\hat{e}_\theta\right) \cdot (R_1 d\theta \hat{e}_\theta) = -\frac{\Gamma}{2\pi}\int_0^{2\pi} d\theta = -\Gamma$$

Since the final integral does not contain the radius, it is clear that the circulation is independent of the radius. Indeed, the circulation is - Γ for any closed path that encloses the origin.