AerE545/AerE445: Experimental Fluid Mechanics and Heat Transfer

Lab # 02: Set up a Schlieren/Shadowgraph System to Visualize the Shockwave Structures of a Supersonic Jet Flow

Objectives:

1. To get "hands-on" experiences about Schlieren/shadowgraph techniques for flow visualization.
2. To learn how to do the optics alignment and experimental setup of a Schlieren/shadowgraph system.
3. To visualize the shockwave structures of a Supersonic Jet Flow by using a Schlieren/shadowgraph system.

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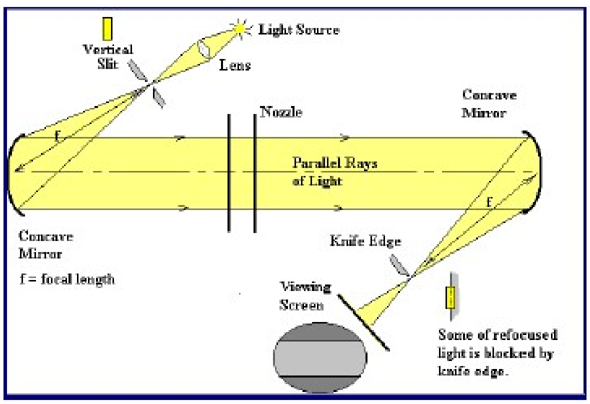
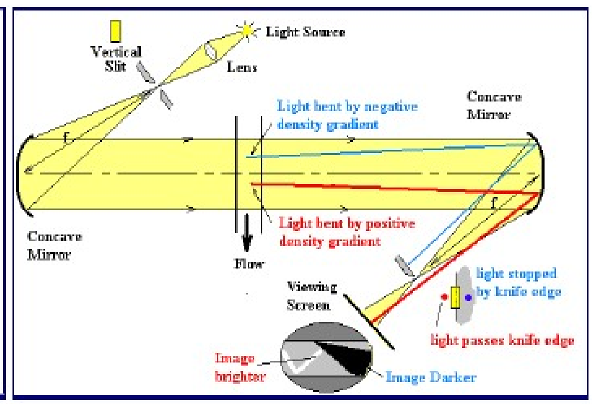
Technical Background:

- Schlieren technique is one of the most-commonly used flow diagnostic techniques for the flow visualization of shock waves and flame phenomena, in which the index of refraction would change due to the variations of the flow density, pressure or temperature in the measurement domain.
- While Schlieren technique is mostly used for qualitative flow visualization, it can be used to conduct quantitative pressure, density or temperature measurements theoretically.
- The contrast seen in the Schlieren images is closely related to the variation of the first directive of the index of refraction
- The Schlieren system used in the present laboratory is a z-type system which consists of a focused light source, two field mirrors, a display screen/board and a knife edge. The experimental setup of the system is shown in the following figures.





a). the light path before turning on wind tunnel b). the light path after turning on wind tunnel.

Figure 1. The optical setup of a Z-typed Schlieren system

Setup of a Schlieren System

In this lab, you will setup a Schlieren system to visualize various airflows. The system will be of the z-type consisting of a focused light source, two field mirrors, a display screen/board and a knife edge. Each group will have about twenty minutes to complete the task. The lab is complete when a focused image with some percentage of knife edge cutoff of a supersonic airflow is obtained. A secondary task which must be completed is determination of the focal length of the two field mirrors. The steps for completing the setup are listed below:

1. Construct a “test screen” by drawing a circle with the same diameter as the field mirrors onto a blank sheet of white paper. Tracing of the mirror covers is satisfactory.

2. Determine the focal length of the field mirrors (the focal length for both mirrors is the same). Shine the light source onto the first field mirror and adjust the mirror’s position until the collimated light beam just fills the circle on the test sheet, held some distance away. Measure the distance from the light source to the center of the field mirror and determine the focal length.

3. Setup the first field mirror so that the light source is at its focus and the collimated light beam shines through the test area. The angle between the illuminating beam and the collimated beam should be kept to a minimum.

4. Setup the second field mirror so that the collimated beam from the first mirror just fills the second mirror. The axis controls on the first mirror may need to be adjusted in order to do this. The second mirror should project a nearly circular light beam on to the viewing screen. Place a threaded bolt in the test section in order to confirm that the image on the screen is in focus. 5. Obtain a shadowgraph image of a convective airflow by placing a candle in the test section.

6. Setup the knife edge position. Begin by obtaining a focused image of the light source (you should see a coil shaped lamp) on the knife edge. The light source is mounted vertically, so the knife edge should be mounted vertically also. Move the knife edge slightly so that it is cutting the focused light source in half.
7. Next, move the knife edge either toward or away from the second field mirror until a uniform darkening of the source image is observed.
8. Obtain a Schlieren image of a convective airflow by placing a candle in the test section.
9.

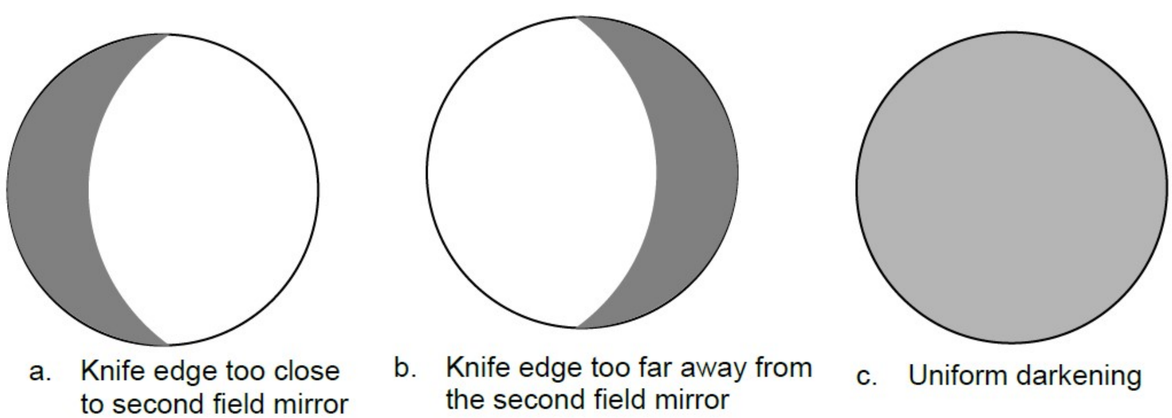


Fig. 2. Effect of the knife edge position on the Schlieren image

Experimental Setup:

In this laboratory, Schlieren technique is used to visualize the shock wave structures of s supersonic Jet as a function the pressure ratio between the upstream total pressure and the downstream exit pressure. Figure 3(a)(b) shows the schematic of the experimental setup. It is well known that the shock waves of a Supersonic Jet could be quite complicated as shown in Fig. 3(d), which depends on the pressure ratio between the upstream total pressure and the downstream exit pressure. The experimental setup and test model: The experiments will be performed in the Supersonic speed, open-circuit wind tunnel located at the Department of Aerospace of Iowa State University. With a relative large settling chamber located at the upstream of a converging nozzle (i.e., with a diameter of D = 25.4 mm), it has the capability of providing stable and high Mach number jet flow. The total upstream pressure (P0) can be adjusted through a series of the sophisticated pressure regulator. It is worth noted that the compressed air is supplied by using three big pressurized tanks, which is about 8.0 m^3 in volume and 150 psi in pressure at full capacity. Figure 3 shows the schematic of the experimental setup for the supersonic jet flow. A 14-bit (1600 pixel × 1200 pixel) charge-coupled device (CCD) camera (PCO2000, Cooke Corp.) with a 610 nm long-pass filter was used to records the Schlieren/Shadowgraph images.

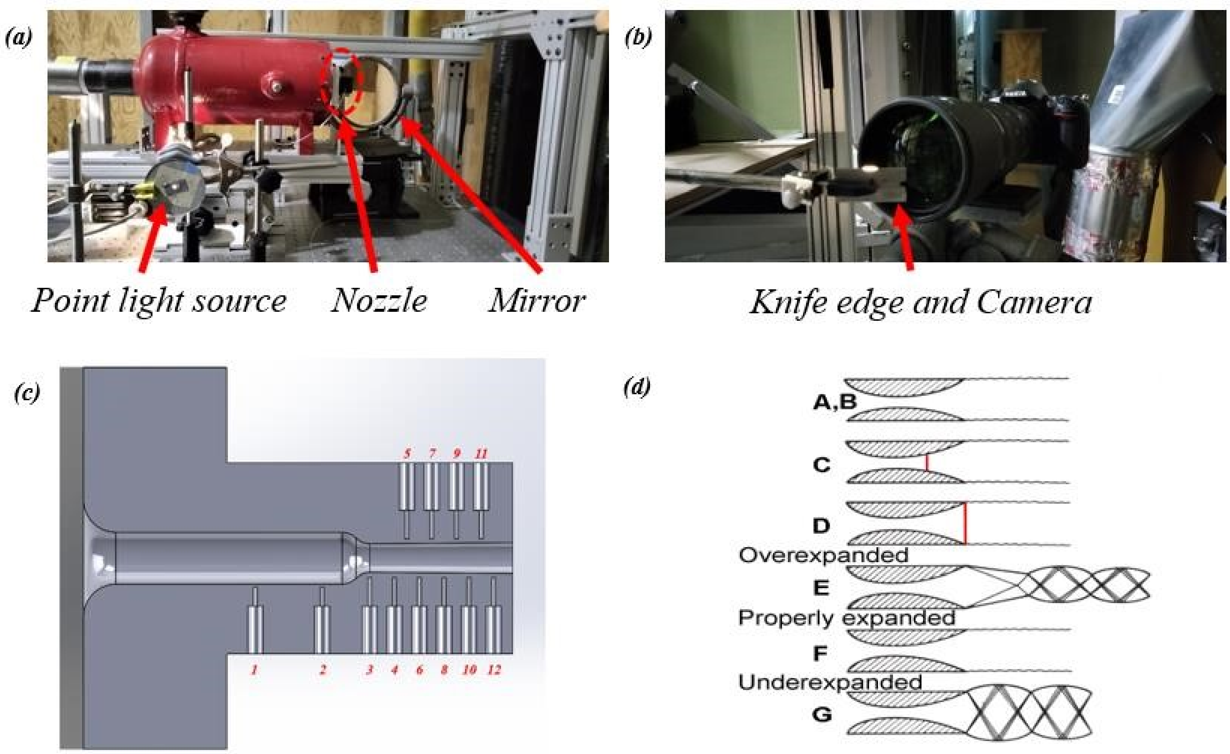


Figure 3. (a) and (b) are experimental setup for Schlieren Imaging of Supersonic Jet Flow, (c) shows the pressure tap configurations inside the nozzle, and (d) shows some typical shock patterns of supersonic jet nozzle.

Experimental procedure:

A. Supersonic Nozzle Operation

1) Remove Schlieren mirror covers
2) Turn on Schlieren light and check that the image is aligned; adjust knife edge cutoff as required
3) Turn on the camera
4) Verify nozzle inlet valve is closed
5) Close selected tank isolation valve
6) Open selected tank outlet valve
7) Open selected tank pressure gauge valve
8) Connect wiring for selected tank temperature probe
9) Check that selects instruments on instrument panel give proper readings
10) Apply ear protection
11) Open nozzle inlet valve when ready to test

B. Supersonic Nozzle Shutdown

1) Close nozzle inlet valve.
2) Close selected tank pressure gauge valve.
3) Close selected tank outlet valve. 4) Turn off Schlieren light.
5) Replace Schlieren mirror covers.

AerE 545/AerE445 Lab#02: Pressure Measurements in a Aupersonic Jet Nozzle

Many analyses in design problems assume 1D nozzle flow theory in cases of internal supersonic flow. One example of this might be in the prediction of the form drag of a rocket nozzle. This drag term is simply the difference between pressures on the inside and outside of the nozzle integrated over the area of the nozzle. In order to predict the wall pressure on the nozzle interior, 1D nozzle theory may be used. This theory obviously makes many assumptions about the flow and it is desired to know how much confidence can be placed in the resulting solutions.

In this experiment you will measure the wall pressure along a Converging-Diverging nozzle. The experiment should be run over a wide range of operating conditions. At the least, these conditions should be included:

1) Under-expanded flow
2) 3rd critical
3) Over-expanded flow with oblique shocks
4) 2nd critical
5) Normal shock existing inside the nozzle
6) 1st critical

The time at which each of the above cases occurs will be determined by observing the shock patterns in the Schlieren image. For each case, record the total pressure (tank pressure) and the wall pressure (see nozzle diagram). The wall pressures will be recorded by using a DSA system.

We will use the nozzle that is generally used in the laboratory at ISU which has the following properties shown in Table I. The tap number in the table corresponds to the configurations shown in Figure 3(c).

Table I. Tap locations and corresponding areas

Tap No.	Distance downstream of throat (mm)	Area (Sq. mm)
1	-12	22.90
2	-5	22.90
3	0	7.07
4	2.6	7.28
5	3.9	7.38
6	5.2	7.49
7	6.5	7.60
8	7.8	7.70
9	9.1	7.81
10	10.4	7.92
11	11.7	8.03
12	13	8.14

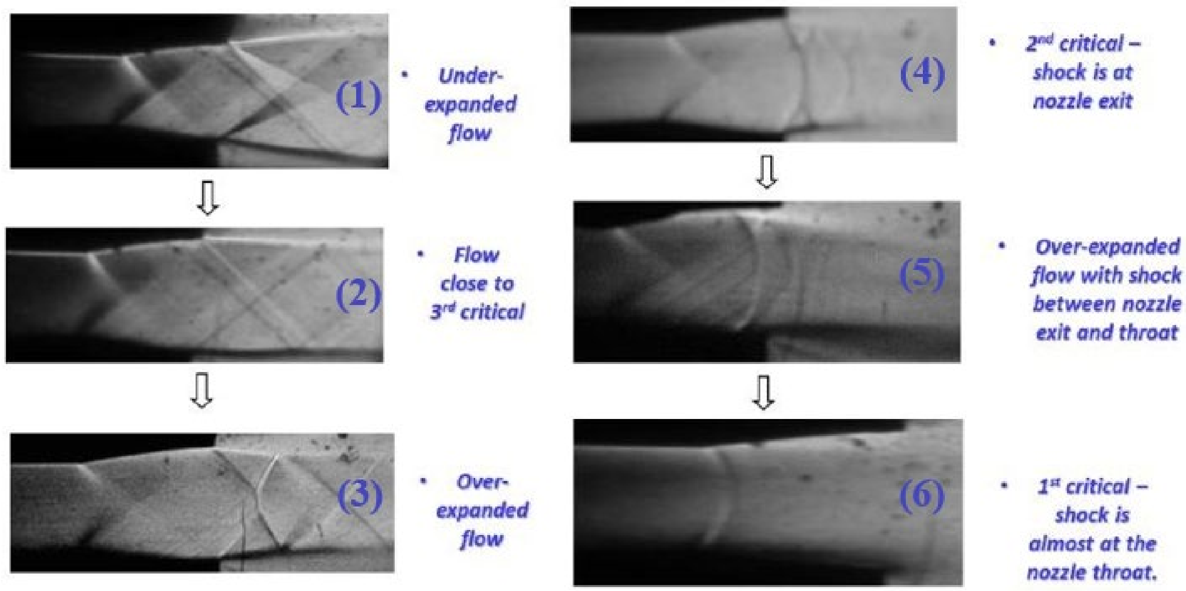


Figure 4. Typical shock patterns in a supersonic nozzle (Images are not taken from the experiments in this lab, they are served as a reference to help answer the questions)

Questions that should be answered:

1) The nozzle used in this experiment is NOT transparent and the shock wave structures inside nozzle cannot be seen. However, based on the pressure measurements, the schlieren video and theoretical calculations, it is possible to have an estimate of the time range/moment of each stage shown in Figure 4 for the supersonic jet flow done in this lab. Please list the time range of each stage based on your data and give the reasons and evidence.

2) Based on the time series data of pressure tubes, showing the location evolution of normal shock wave after stage 4. (Hint: Plot shock location vs. time)

3) Compare the theoretical and experimental pressure (e.g., gauge pressure, pressure ratio, total pressure) and Mach number in nozzle for stage 2. (Hint: Plot the pressure/Mach number as a function of distance along the nozzle axis)

4) Compare the theoretical and experimental pressure (e.g., gauge pressure, pressure ratio, total pressure) and Mach number in nozzle for stage 4. (Hint: Plot the pressure/Mach number as a function of distance along the nozzle axis)

5) Compare the theoretical and experimental pressure (e.g., gauge pressure, pressure ratio, total pressure) and Mach number in nozzle for stage 5. The location of the shock can be got

from results of question 1 (Hint: Plot the pressure/Mach number as a function of distance along the nozzle axis at an arbitrary time moment in that stage)

6) What can you say about the predicted pressures in comparison to the measured pressures? That is, does the theory under- or over-predict the wall pressure? Give some possible reasons for the differences.

7) What might the differences in question (6) mean for the prediction of other flow quantities such as Mach number, temperature, etc.?

8) Point out any interesting anomalies you might see in the measured data.

9) Present your results in formal report format.

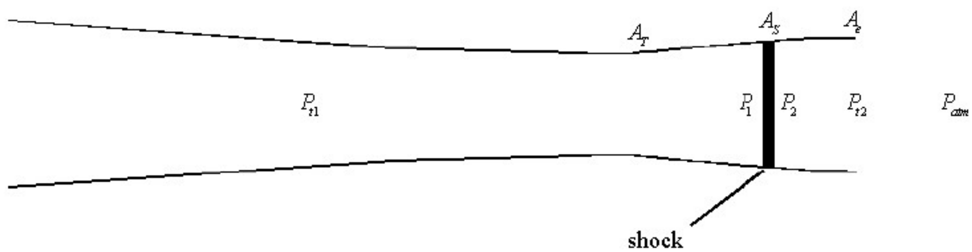
(Some useful formulas are attached in the APPENDIX)

APPENDIX

Pressure distribution in a Supersonic Nozzle

This experiment is concerned with the measurement of pressure in a converging/diverging (de Laval) nozzle. The goal is to compare the measurements with the values predicted by the theory of one-dimensional inviscid compressible flow. As the name implies, the two major assumptions in the theory is that the flow is inviscid and uniform across the cross-sectional area at any given nozzle location. It will be seen by comparing with experiments that these assumptions can have significant influence on the pressures being measured along the wall which in reality is covered by a boundary layer with its own unique shock system.

There are two possible ways to formulate this problem as far as the theory is concerned. One way is to specify a total pressure of the flow upstream of the throat and then proceed to calculate the resulting shock location (if any) within the nozzle's diverging section. The second approach is to specify a shock location within the nozzle and then calculate the corresponding total pressure upstream of the throat. In both cases, it is assumed that the exit (usually atmospheric) pressure is known. This document describes the latter approach. In either case, the goal is to find the pressure distribution throughout the nozzle. A typical de Laval nozzle is shown in the following figure along with the definition of several key variables.



We will outline two methods which can be used to calculate the pressure distribution throughout the nozzle. The first method will be a numerical method by which a continuous pressure distribution can be obtained. The second method will be used to calculate the pressure at a discrete number of points along the nozzle. This method will use the NACA 1135 Tables in order to calculate the pressure.

A). Numerical Approach

One can also calculate the pressure in the nozzle using the derived equations. This would allow you to calculate several shock locations using the same program or spreadsheet without consulting the shock tables for every value.

1. Find M1 (assuming choked flow)
   a. Using the area ratio, can calculate the Mach number at any point up to the shock using:

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} \qquad (2.1)$$

   b. After finding Mach number at front of shock, calculate Mach number after shock using:

$$M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \qquad (2.2)$$

Can now calculate A_2^* using:

$$\left(A_2^*\right)^2 = M_2^2 A_s^2 \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right]^{-\frac{\gamma+1}{\gamma-1}} \qquad (2.3)$$

which allows us calculate the remaining Mach number distribution

Find pressure distribution

a). Pressure at exit is same as atmospheric pressure for shock inside nozzle ($P_e = P_{atm}$). For shock after lip of nozzle, total pressure is constant throughout the interior of the nozzle ($P_{t2} = P_{t1}$).

b). Find total pressure behind the shock:

$$P_{t2} = \frac{P_{t2}}{P_e}P_e \text{ where } \frac{P_{t2}}{P_e} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}} \qquad (2.4)$$

c). Any pressure behind the shock is therefore:

$$P = P_{t2}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}} \qquad (2.5)$$

d). Calculate P_{t1} ahead of shock:

$$P_{t1} = \frac{P_{t1}}{P_1}\frac{P_1}{P_2}\frac{P_2}{P_{t2}}P_{t2} \qquad (2.6)$$

where you can use Total-Static relation for 1st and 3rd ratios, and for the middle ratio:

$$\frac{P_1}{P_2} = \frac{1+\gamma M_2^2}{1+\gamma M_1^2}$$

$$or \qquad (2.7)$$

$$\frac{P_2}{P_1} = 1+\frac{2\gamma}{\gamma+1}\left(M_1^2-1\right)$$

e) Now that you have the total pressure upstream of the shock, as well as the Mach number calculated earlier you can calculate the pressure upstream of the shock.

a. Notes

a. For 3rd Critical

1. $P_1 = P_2 = P_e$

2. $M_1 = M_2 = M_e$ (supersonic)

b. For 1st Critical

1. Same as 3rd critical, but M_e is subsonic

c. For 2nd Critical

1. $M_2 = M_e$

2. $P_2 = P_e$

d. To calculate Mach number given the Mach-Area relation, can use Newton iteration to find M

$$F = \left(\frac{A}{A^*}\right)^2 = M^2\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} \tag{2.8}$$

$$F' = \frac{dF}{dM} = \frac{2}{M^3}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} - \frac{2}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{2}{\gamma-1}} \tag{2.9}$$

$$M^{n+1} = M^n - \frac{F}{F'} \tag{2.10}$$

B). Shock Tables Method and Example

In this approach, the geometry of the nozzle is known at a discrete number of points along its length. The method then consists of using the tables to systematically arrive at a Mach number distribution and ultimately a pressure distribution. This method is best illustrated by an example. We will use the nozzle that is generally used in the laboratory at ISU which has the following properties:

Tap No.	Distance downstream of throat (inches)	Area (Sq. inches)
1	-4.00	0.800
2	-1.50	0.529
3	-0.30	0.480
4	-0.18	0.478
5	0.00	0.476
6	0.15	0.497
7	0.30	0.518
8	0.45	0.539
9	0.60	0.560
10	0.75	0.581
11	0.90	0.599
12	1.05	0.616
13	1.20	0.627
14	1.35	0.632
15	1.45	0.634

We will start by setting up a table with columns that we will need to fill in as we go. We will evaluate flow properties at a limited number of tap points. Specifically, we will use taps 1,2,3,5,7,9,11,13 and 15. Also, we will prescribe the shock to be located at tap 12. It is useful to assume the shock is located at one of the taps because there is no need to interpolate the nozzle area. The table we want to fill out is

Tap No.	A/A*	Mach #	P/Pt	P	Pg
1	1.681				
2	1.111				
3	1.008				
5	1.000				
7	1.088				
9	1.176				
11	1.258				
p re-shock	1.294				
post-shock					
13					
15					

Note that we have two rows at the shock, one labeled pre-shock and the other post-shock. This is because properties such as A* change across the shock and must be evaluated on either side of it. We have already carried out the first step in the above table. That is, we have found A/A* from the tables using the value of the nozzle area at each tap location and the fact that A* is equal to the nozzle throat area in front of the shock. We next use the isentropic flow part of the tables to find the Mach number corresponding to each A/A*. We can also use the isentropic flow part of the tables to find the corresponding P/Pt.

Tap No.	A/A*	Mach #	P/Pt	P	Pg
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
p re-shock	1.294	1.64	0.2217		
post-shock					
13					
15					

We don't yet know the total pressure in front of the shock, Pt1. We will now have to find some of the values behind the shock. We will start by calculating the Mach number immediately behind the shock. Using the normal shock tables with M1 = 1.64 we find that M2 = 0.686. Next, we find the sonic reference area behind the shock using the area-Mach relation:

$$\left(A_2^*\right)^2 = A_s^2 M_2^2 \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right]^{-\frac{\gamma+1}{\gamma-1}} \qquad (2.11)$$

where A_s is the nozzle area at the shock, in this case at tap 11. For the given problem, A_2^*=0.5574 sq. inches. The above table now becomes:

Tap No.	A/A*	Mach #	P/Pt	P	Pg
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post- shock	1.105	0.69	0.7274		
13					
15					

We can complete the first three columns by using the isentropic flow part of the tables to find pressure ratios and Mach numbers for the various area ratios. The first three columns are shown completed in the following table

Tap No.	A/A*	Mach #	P/Pt	P	Pg
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post- shock	1.105	0.69	0.7274		
13	1.125	0.66	0.7465		
15	1.137	0.65	0.7528	14.7	

In the previous table we have entered the exit pressure in pounds per square inch. In this case we take the exit pressure to be sea-level standard pressure. We now calculate the total pressure behind the shock using this value of exit pressure and the pressure ratio at the exit:

$$P_{t2} = \frac{P_t}{P}P = \left(\frac{1}{0.7528}\right)14.7 = 19.53 \qquad (2.12)$$

Using this value of total pressure, the other static pressures behind the shock can be calculated and tabulated using the pressure ratios in column 4 and P_{t2} = 19.53:

Tap No.	A/A*	Mach #	P/Pt	P	Pg
1	1.681	0.37	0.9098		
2	1.111	0.67	0.7401		
3	1.008	0.97	0.5469		
5	1.000	1.00	0.5283		
7	1.088	1.35	0.3370		
9	1.176	1.50	0.2724		
11	1.258	1.61	0.2318		
pre-shock	1.294	1.64	0.2217		
post- shock	1.105	0.69	0.7274	14.21	
13	1.125	0.66	0.7465	14.58	
15	1.137	0.65	0.7528	14.7	

Our last major task is to find the total pressure ahead of the shock, Pt1. These values can be calculated from:

$$P_{t1} = \frac{P_{t1}}{P_1}\frac{P_1}{P_2}\frac{P_2}{P_{t2}}P_{t2} \tag{2.13}$$

The underlined terms in (2.13) are known. The other pressure ratio can be found using the NACA tables and a value of M1 = 1.64. For the current case, the total pressure ahead of the shock was found to be Pt1 = 21.57 psi. With the total pressure known, the only task remaining is to finish filling out column 5 of the above table. This can be done using the equation

$$P = \frac{P}{P_t}P_t \tag{2.14}$$

where everything on the right side of (2.14) is known. Finally, since we will be measuring gauge pressure in pounds per square inch in the lab, we convert absolute pressure in psi using

$$P_g = P - P_{atm} \tag{2.15}$$

where Pg is the gauge pressure. The following table completes the procedure:

Tap No.	A/A*	Mach #	P/Pt	P	Pg
1	1.681	0.37	0.9098	19.6	4.9
2	1.111	0.67	0.7401	16	1.3
3	1.008	0.97	0.5469	11.8	-2.9
5	1.000	1.00	0.5283	11.4	-3.3
7	1.088	1.35	0.3370	7.27	-7.43
9	1.176	1.50	0.2724	5.88	-8.82
11	1.258	1.61	0.2318	5	-9.7
p re-shock	1.294	1.64	0.2217	4.78	-9.92
post-shock	1.105	0.69	0.7274	14.21	-0.49
13	1.125	0.66	0.7465	14.58	-0.12
15	1.137	0.65	0.7528	14.7	0

We have now found the pressure distribution at discrete points throughout the nozzle. Note that when analyzing a large variety of nozzles or exit conditions, this is a very time consuming process. In general, a better method is to solve the equations numerically using a computer program. It should be clear from the above example how this method can be used to calculate pressure for shocks at different locations or for first, second or third critical.