Lecture # 02: Measurement Uncertainties

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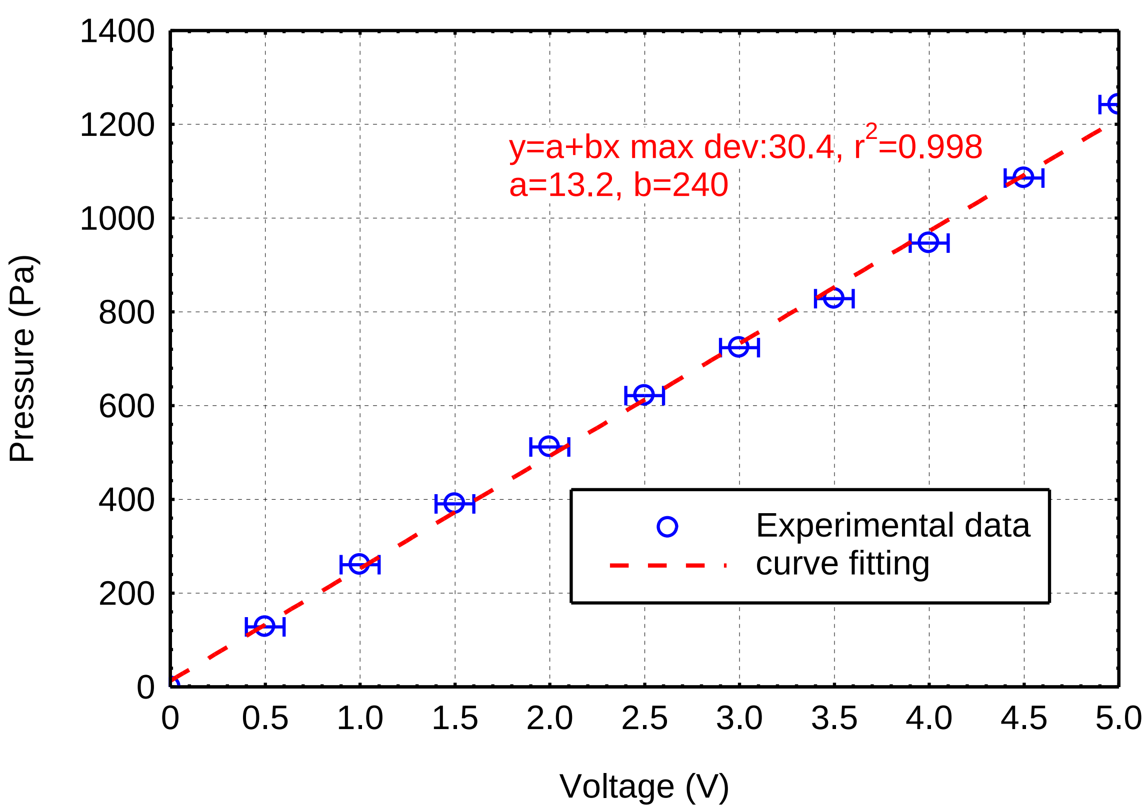
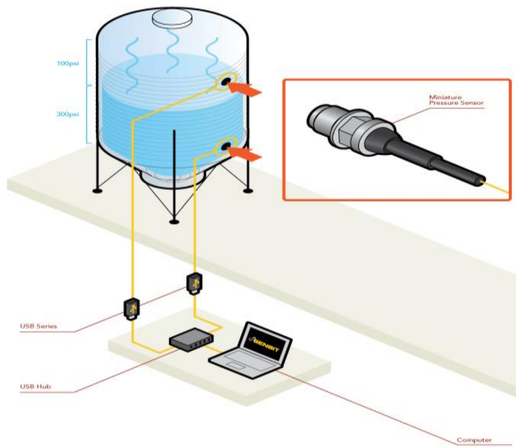
❑ CALIBRATION

- **Calibration**: *A calibration applies a known input value to a measurement system for the purpose of observing the system output value. It establishes the relationship between the input and output values.*
- *The known value used for the calibration is called standard.*





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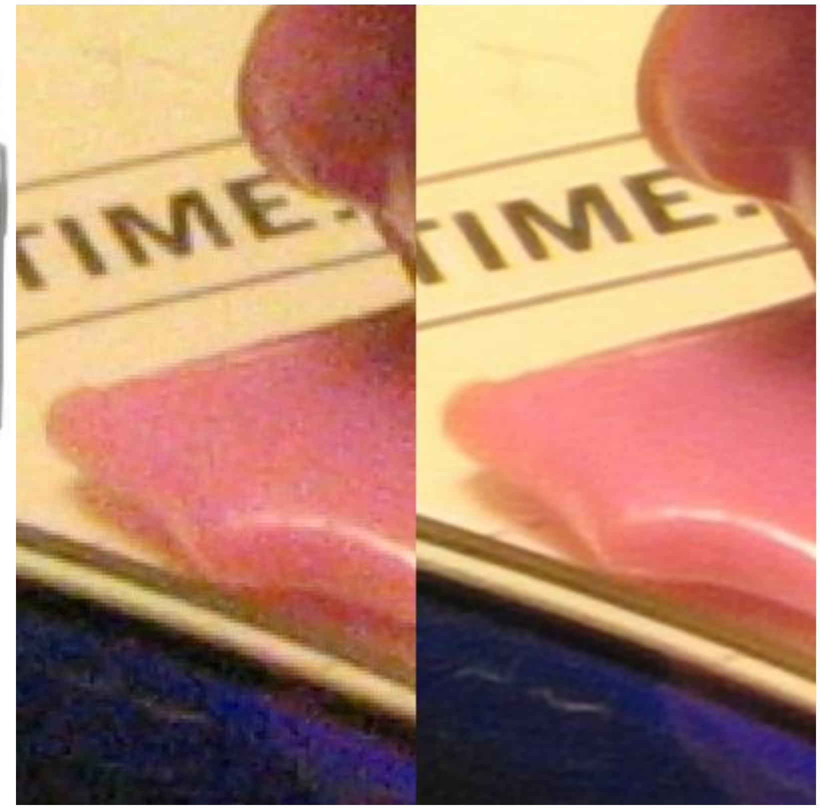
❑ Instrument Resolution

- *Instrument Resolution* ***represents the smallest increment in the measured value that can be discerned using the instrument. In terms of a measurement system, it is quantified by the smallest scale increscent of the least count.***



For measurements:

- $1 \neq 1.0 \neq 1.00 \neq 1.000$

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❑ Measurement Uncertainties

- ***"Accuracy" is generally used to indicate the relative closeness of agreement between an experimentally-determined quantity value and its true value.***
- ***"Error" is the difference between the experimentally-determined value and its true value; therefore, accuracy is said to increase as error decreases.***
- ***Since the true value is not known, it is necessary to estimate error, and that estimate is called an uncertainty, U.***
- ***Uncertainty estimates are made at some confidence level—a 95% confidence estimate, for example, means that the true value of the quantity is expected to be within the ±U interval about the experimentally-determined value 95 times out of 100.***

$$A_{error} = A_{measured} - A_{true} \quad \Rightarrow \quad E = A_m - A_{true}$$

Which Case is more accurate measurement?

$V_t = 10m/s,$ Measurement error $\Delta V = 1m/s$

$V_t = 100m/s,$ Measurement error $\Delta V = 5m/s$

$$E_{relative} = \frac{A_{error}}{A_{true}}$$

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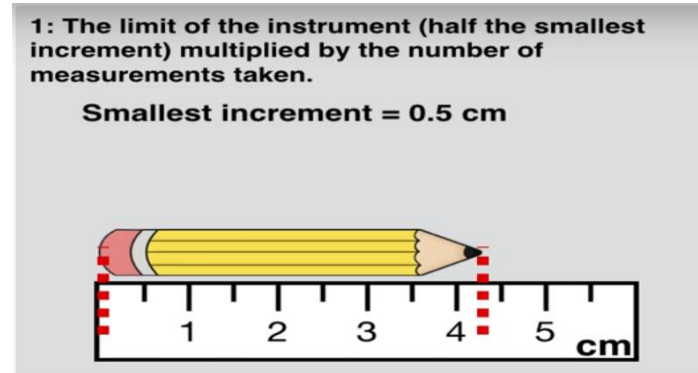
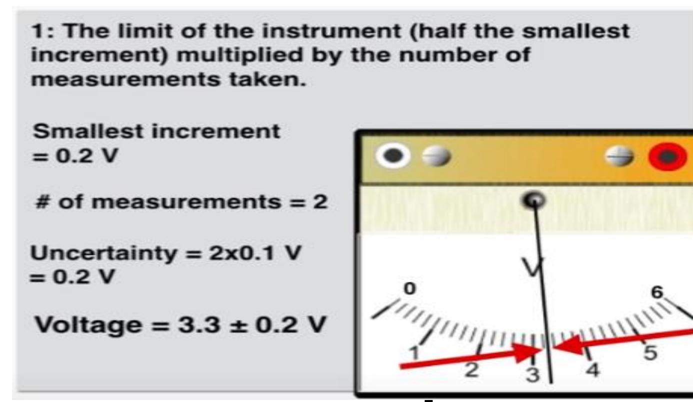
❑ Measurement Uncertainties

- *Total error, U, can be considered to be composed of two components:*
  - *a random (precision) component,*
  - *a systematic (bias) component,*
  - *We usually don't know these exactly, so we estimate them with P and B, respectively.*

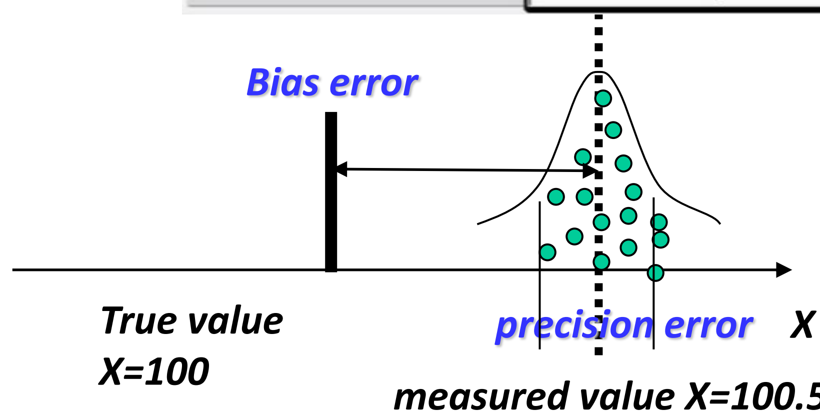




- *Precision Error: Random error*
  - *Normal Distribution or Gaussian Distribution*

- *Bias Error: Fixed Error, System Error*
  - *Constant Throughout the experiment*
  - *Can be positive or Negative*

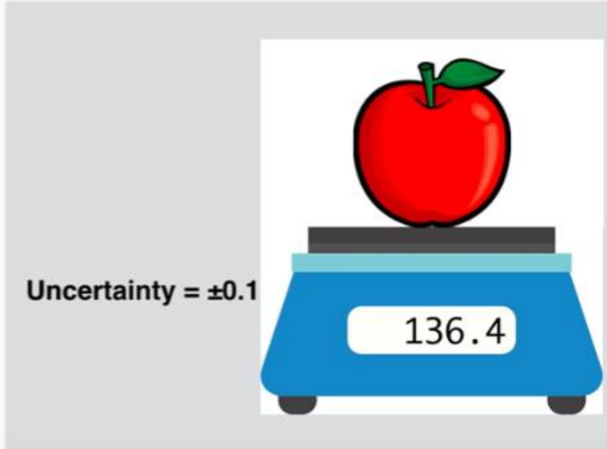
$$U^2 = B^2 + P^2$$



❑ Measurement Uncertainties

Measurement Uncertainty

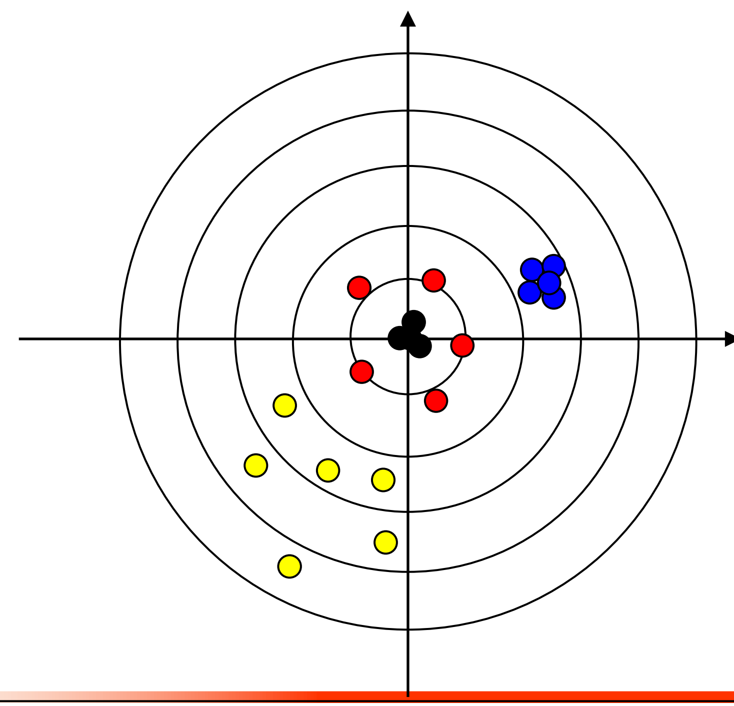
If you are working with a digital reading, the uncertainty will be at least ± 1 digit of the last significant figure of the reading.



For measurements:

- *136.4*
  ≠ 136.40
  ≠136.400

- ***Precise but biased***
- ***Unbiased but Imprecise***
- ***Biased and Imprecise***
- ***Precise and Unbiased***

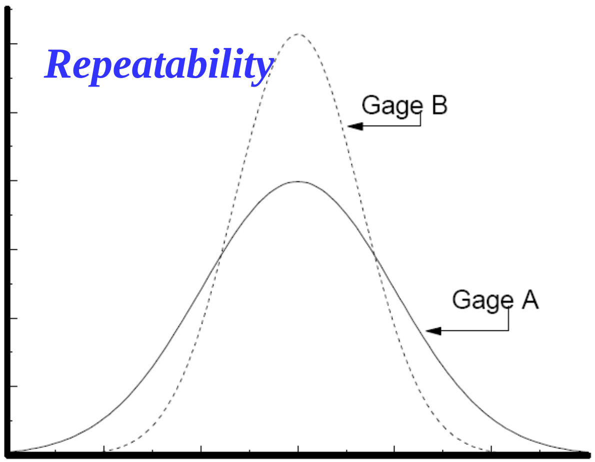


- ***Qualification of measurement error:***

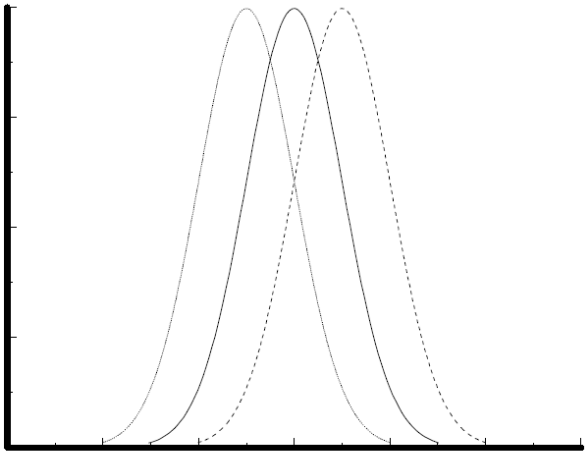
$$E^2 = B^2 + P^2$$

❑ Repeatability and Reproducibility

- *Repeatability* ***is the variability of the measurements obtained by one person while measuring the same item repeatedly. This is also known as the inherent precision of the measurement equipment.***
    - ***Consider the probability density functions shown in Figure 1. The density functions were constructed from measurements of the thickness of a piece of metal with Gage A and Gage B. The density functions demonstrate that Gage B is more repeatable than Gage A.***



- *Reproducibility* ***is the variability of the measurement system caused by differences in operator behavior. Mathematically, it is the variability of the average values obtained by several operators while measuring the same item.***
    - ***Figure 2 displays the probability density functions of the measurements for three operators. The variability of the individual operators are the same, but because each operator has a different bias, the total variability of the measurement system is higher when three operators are used than when one operator is used.***



Reproducibility

Repeatability	*Precision Error*
Reproducibility	*Both Bias and Precision Errors*

❑ MEASUREMENT UNCERTAINTIES

- ***We almost always are dealing with a data reduction equation to get to our final results.***
  - ***In this case, we must not only deal with uncertainty in the measured values but uncertainty in the final results.***
- ***A general form looks like this:***

$$R = R\left(X_1, X_2, X_3, \ldots, X_J\right)$$

  - ***R is the result determined from J independent variables.***

- ***https://www.youtube.com/watch?v=K347QyJGkvs***

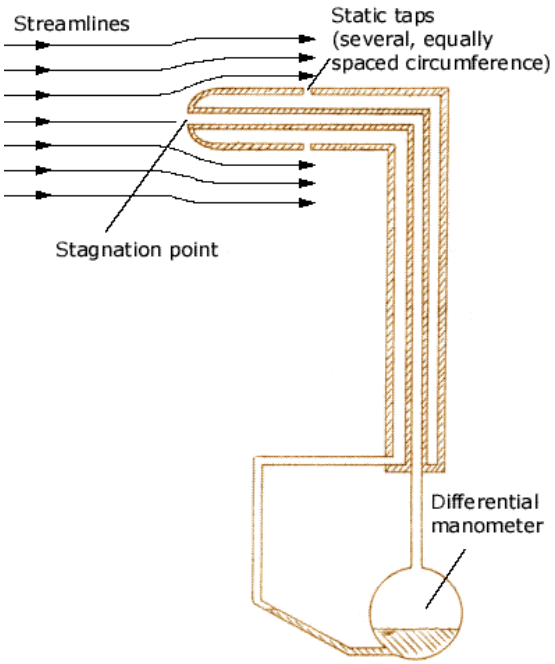
EXAMPLE

Uncertainty in velocity V:

$$U_R^2 = B_R^2 + P_R^2$$

$$B_R^2 = \sum_{i=1}^{J}\left[\frac{\partial R}{\partial X_i} B_i\right]^2; \qquad P_R^2 = \sum_{i=1}^{J}\left[\frac{\partial R}{\partial X_i} P_i\right]^2$$

$$B_i = \sqrt{\sum_{j=1}^{M} B_{i_j}^2}$$



For a large number of samples (N>10)

$$P_i = 2S_i$$

$$S_i = \left[\frac{1}{N-1}\sum_{k=1}^{N}\left[(X_i)_k - \bar{X}_i\right]^2\right]^{1/2}; \qquad \bar{X}_i = \frac{1}{N}\left[\sum_{k=1}^{N}(X_i)_k\right]$$

$$p_{total} = p_{static} + \frac{1}{2}\rho V^2, (Bernoulli)$$

$$V = \sqrt{\frac{2(p_{total} - p_{static})}{\rho}} = \sqrt{\frac{2\Delta p}{\rho}}$$

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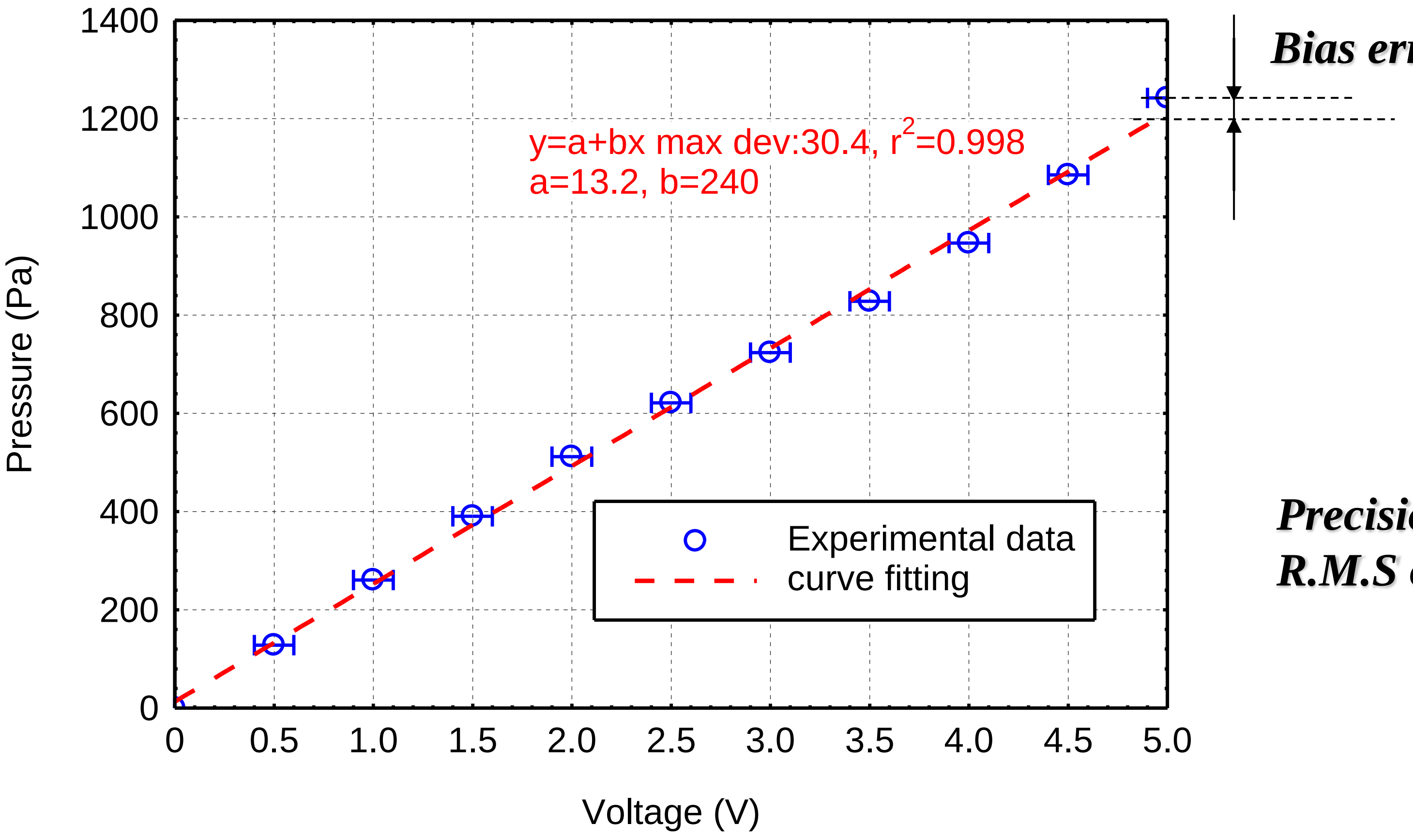
❑ Example of Measurement Results



Bias error

Precision error:
R.M.S of your data

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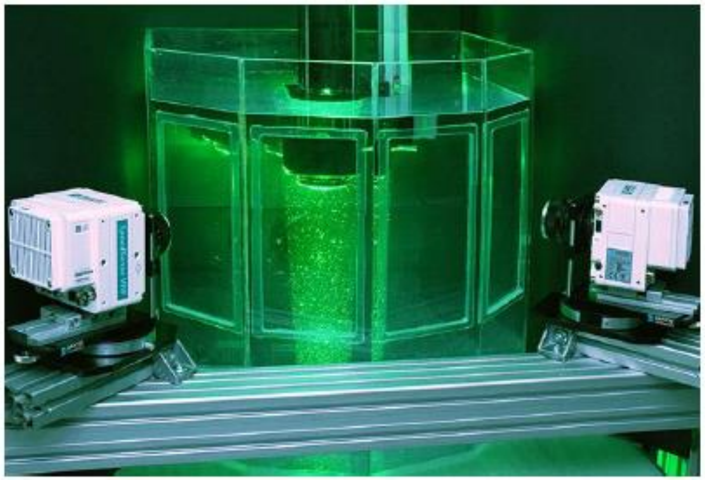
❑ Measurement Signal

Signal:
Signal is a function that conveys information about a phenomenon.

- Voltage, current
- Sound strength, light intensity or other electromagnetic wave that carries information



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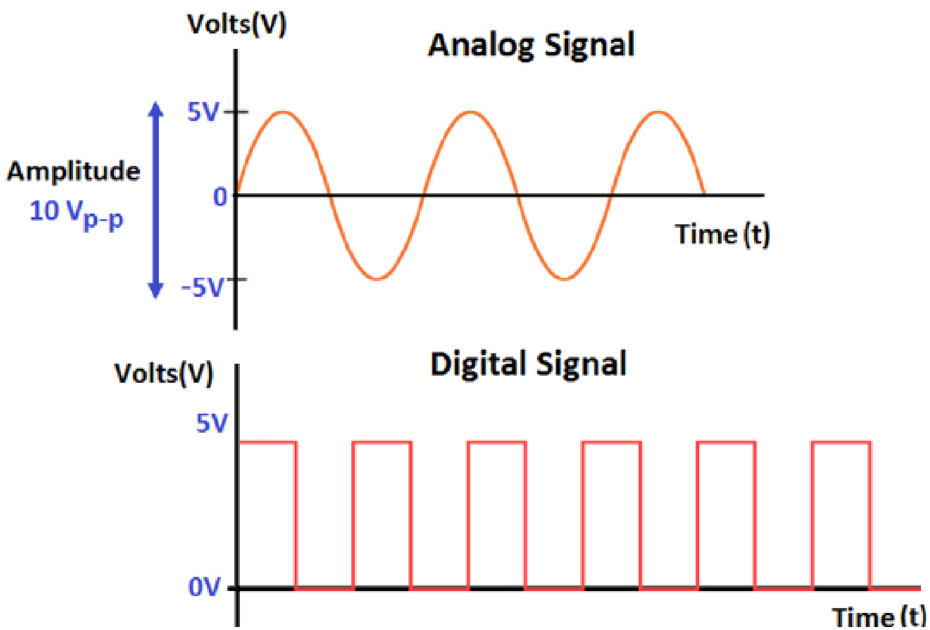
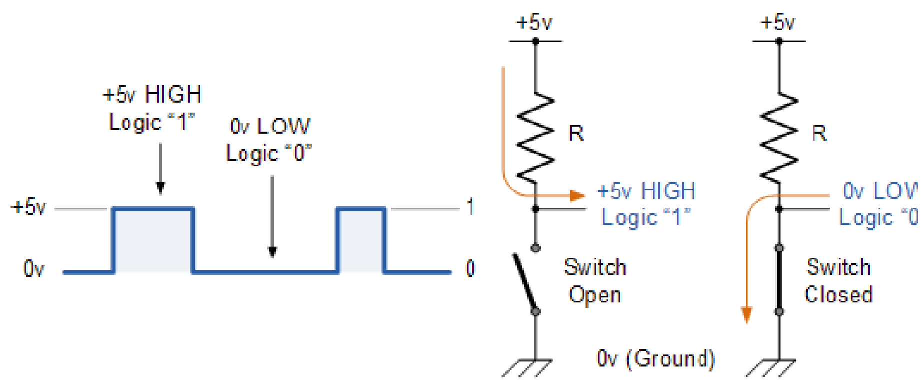
❑ Measurement Signal

Signal classifications:

- Analog signal: Continuous signal for which its variation represents some other varying quantity.

- Digital signal: a signal is constructed from a discrete set of values to represent a physical quantity.





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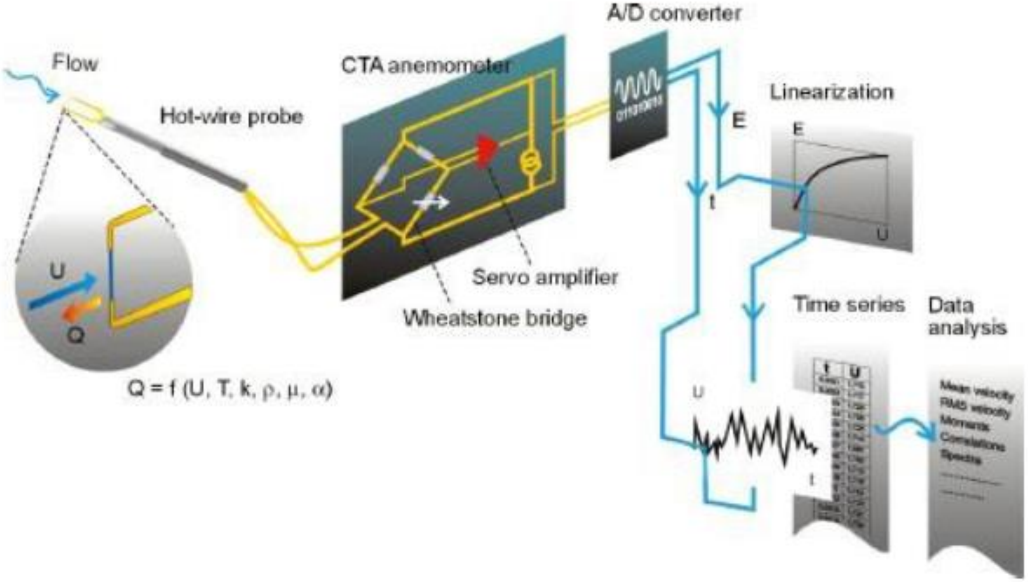
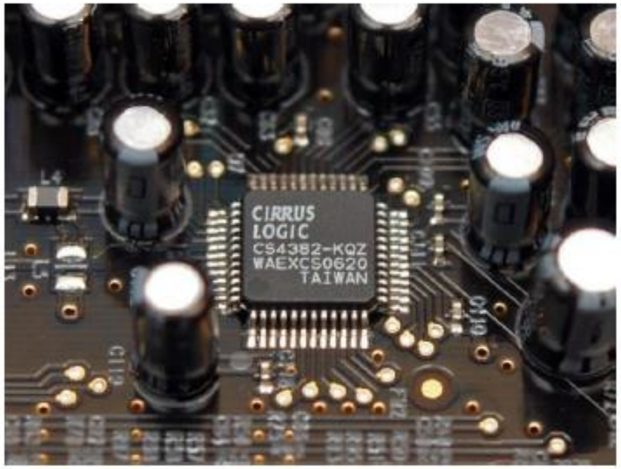
❑ MEASUREMENT SIGNAL

Analog-to-digital converter (ADC):
ADC is a system that converts an analog signal into a digital signal.

Reverse ADC:
Digital-to-Analog converter (DAC)



Dantec CTA



ADC clip in a sound card

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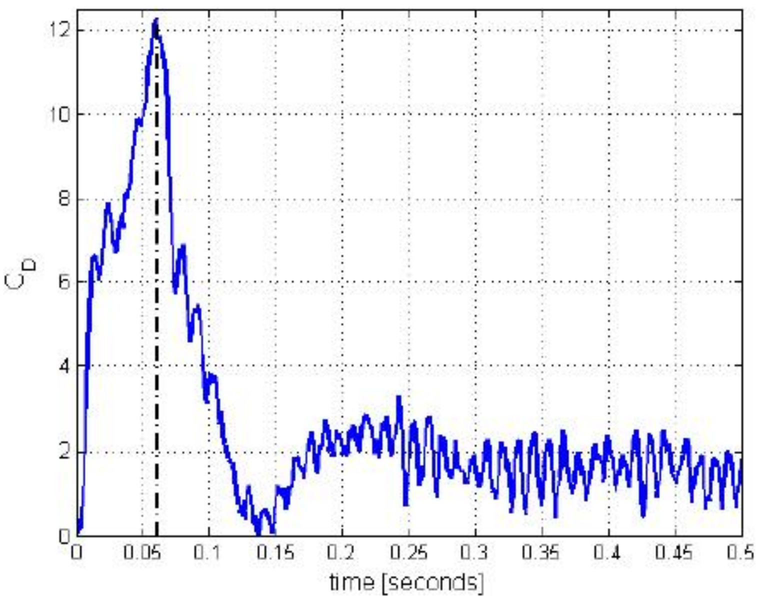
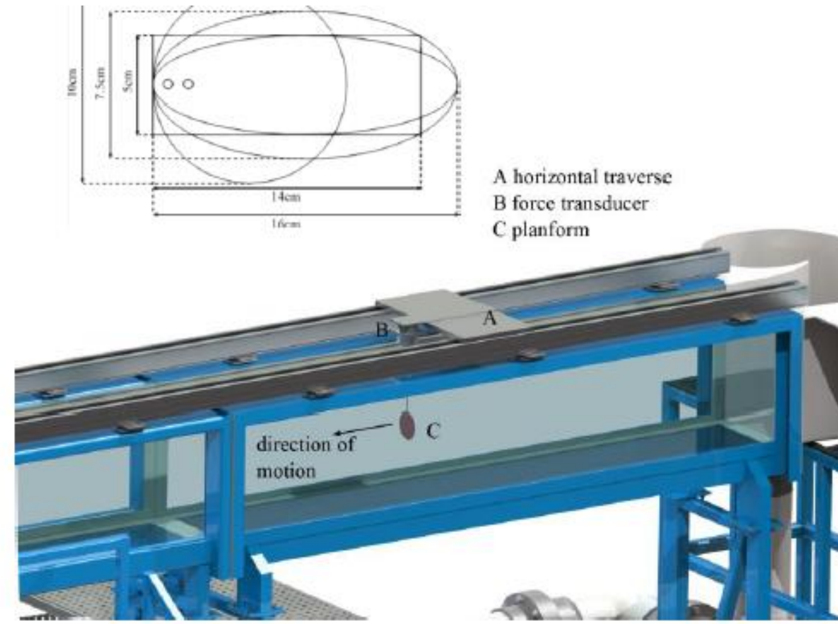
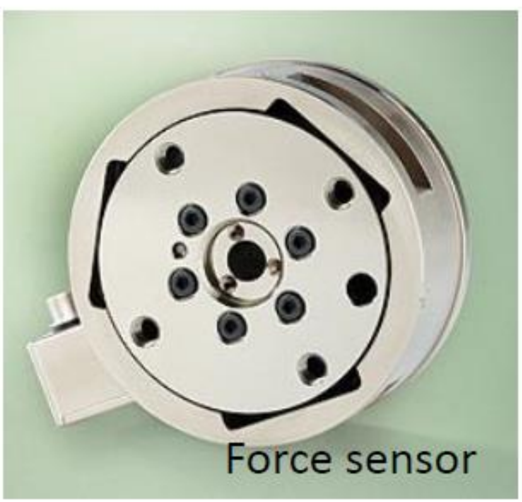
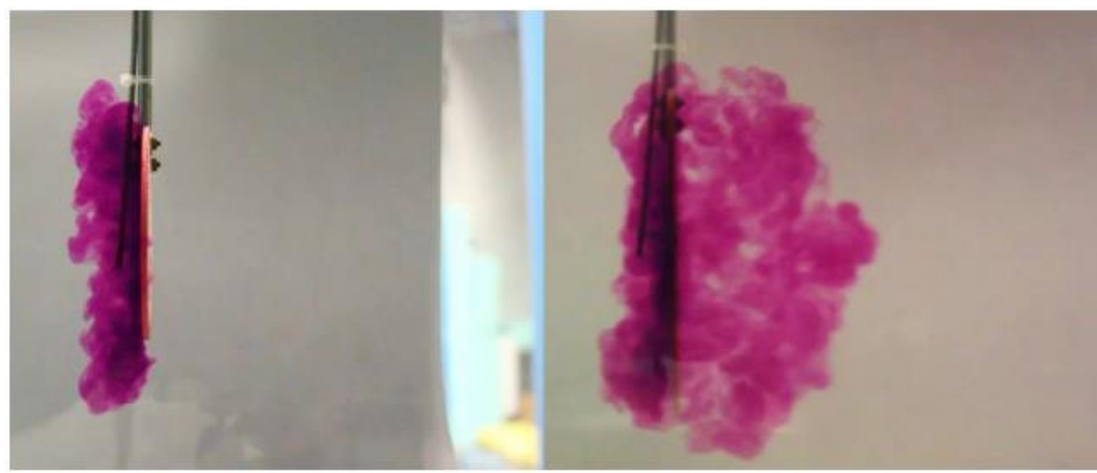
❑ Measurement Signal



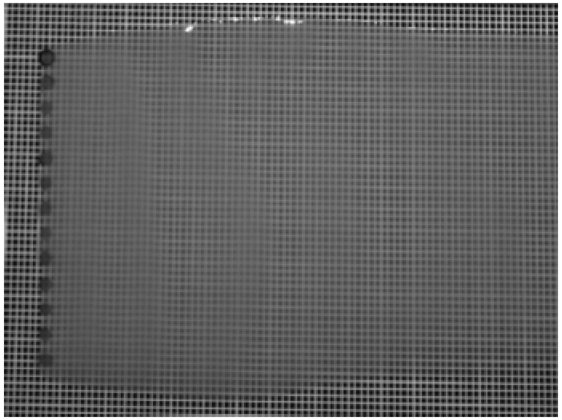


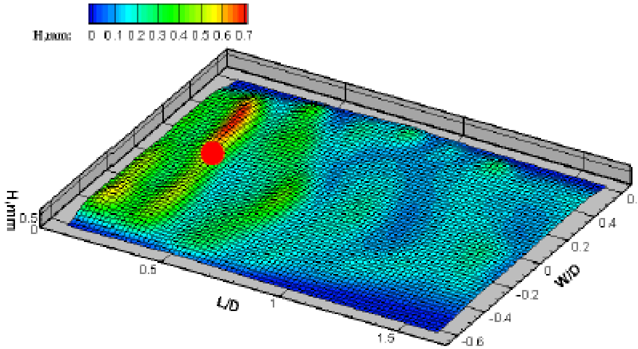


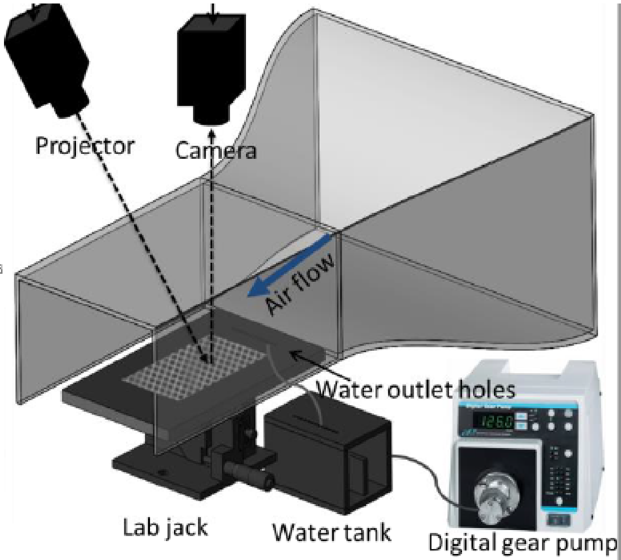


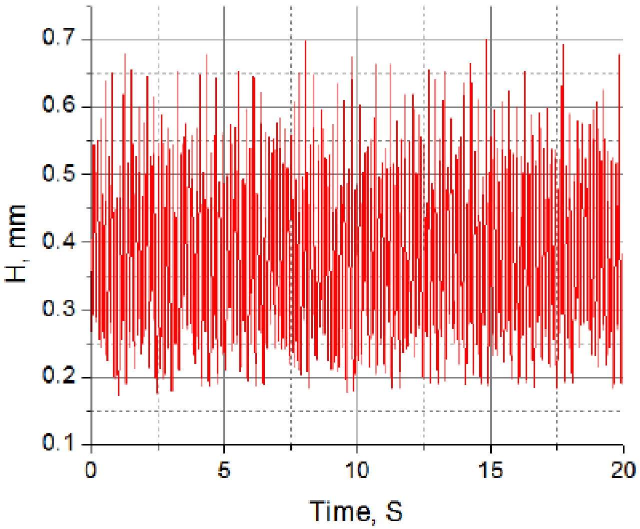
In fluence of shape on vortex development (drag)

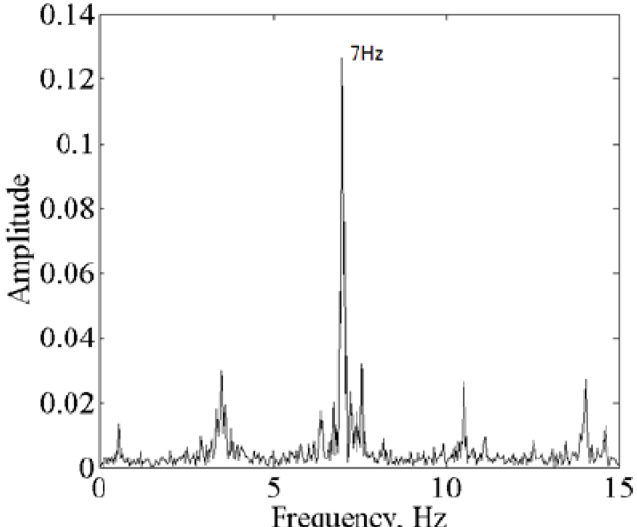
Measurement Signal











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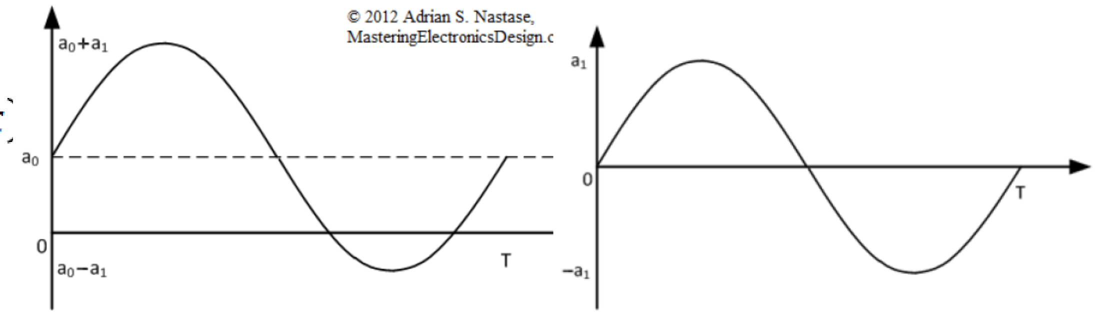
❑ FOURIER TRANSFORM AND SPECTRAL ANALYSIS

- ***Any complex signal can be broken into set of sine and cosine waves of different periods and amplitudes.***

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$



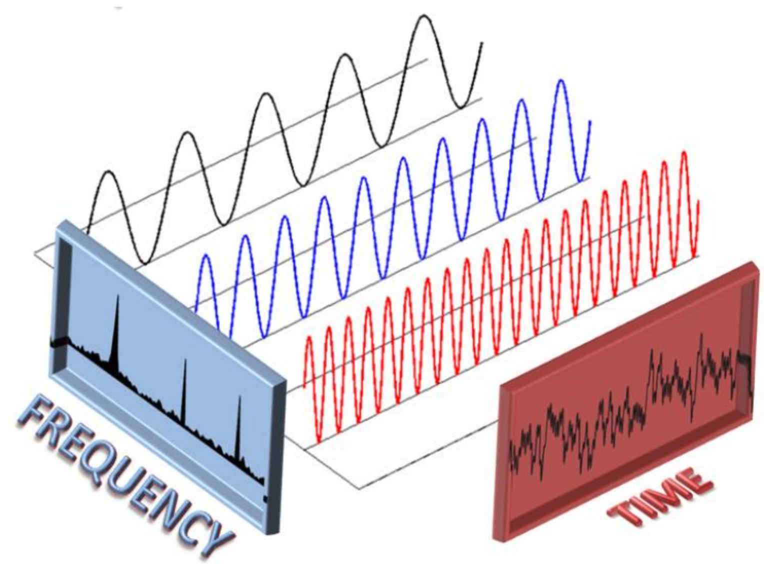
Fourier series for a periodic function with period of 2τ

$$f(x) = \sum_{n=0}^{\infty} \left(a_n \cos\frac{n\pi x}{\tau} + b_n \sin\frac{n\pi x}{\tau}\right)$$

$$a_0 = \frac{1}{2\tau}\int_{-\tau}^{\tau} f(x)dx\,, a_n = \frac{1}{\tau}\int_{-\tau}^{\tau} f(x)\cos\frac{n\pi x}{\tau}dx, b_n = \frac{1}{\tau}\int_{-\tau}^{\tau} f(x)\sin\frac{n\pi x}{\tau}dx$$

Let $\tau \to \infty$, then a nonperiodic function can be considered as periodic but with an infinite period $2\tau \to \infty$.

$\frac{n\pi}{\tau}$'s are the set of frequencies (spatial or temporal), called **frequency spectrum** $0, \frac{\pi}{\tau}, \frac{2\pi}{\tau}, \frac{3\pi}{\tau}, \ldots$



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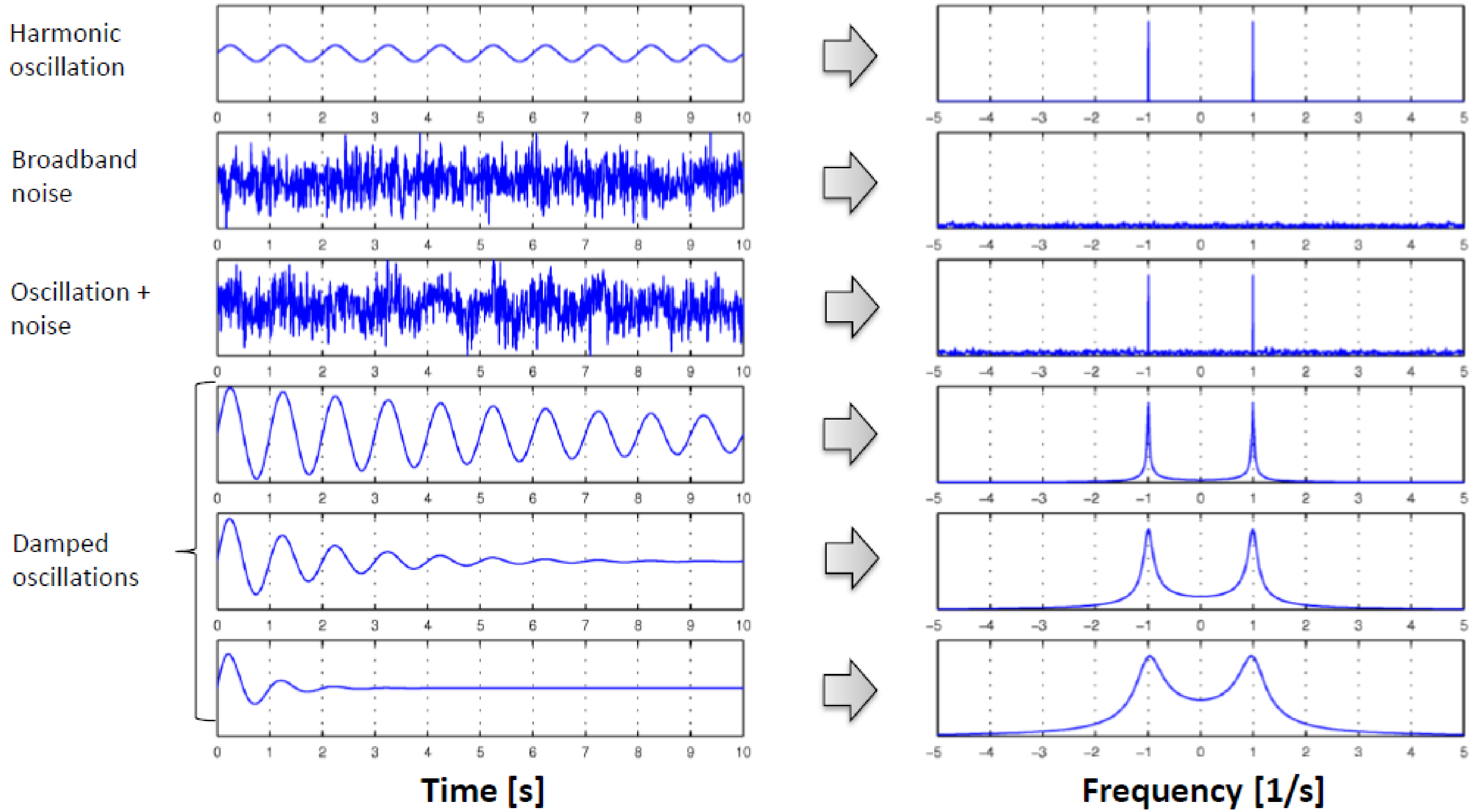
Fourier transform and Spectral Analysis

Examples of Fourier transform:



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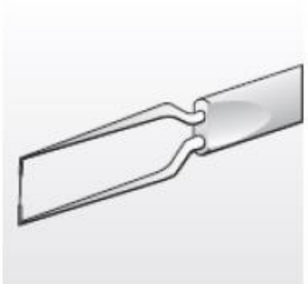
Fourier transform and Spectral Analysis



Hotwire