

LECTURE 09: SHADOWGRAPH, SCHLIEREN & INTERFEROMETRY TECHNIQUES: PART - 02

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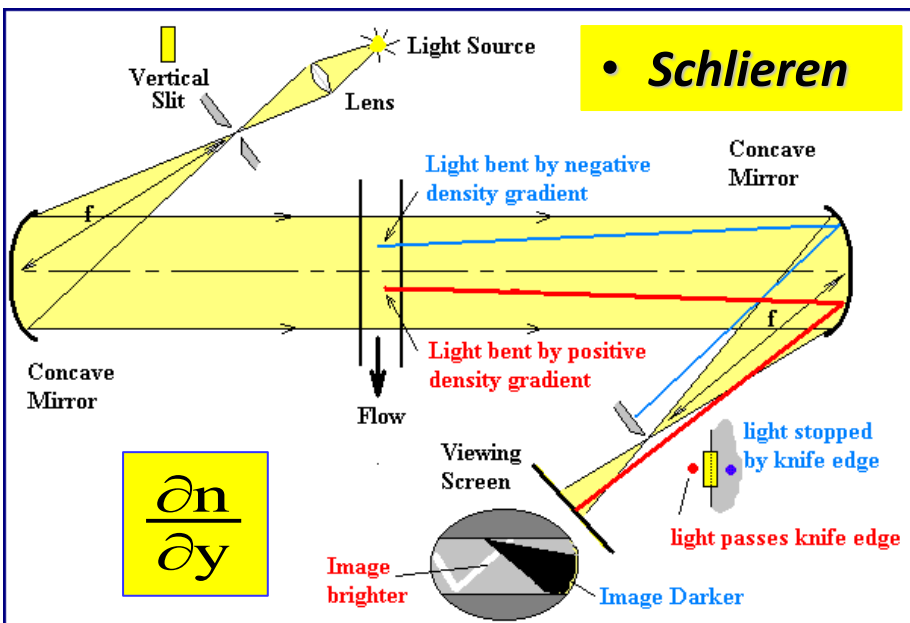
Tel: 515-294-0094 (O) / Fax: 515-294-3262 (O)

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❑ COMPARISON OF SCHLIEREN VS. SHADOWGRAPH

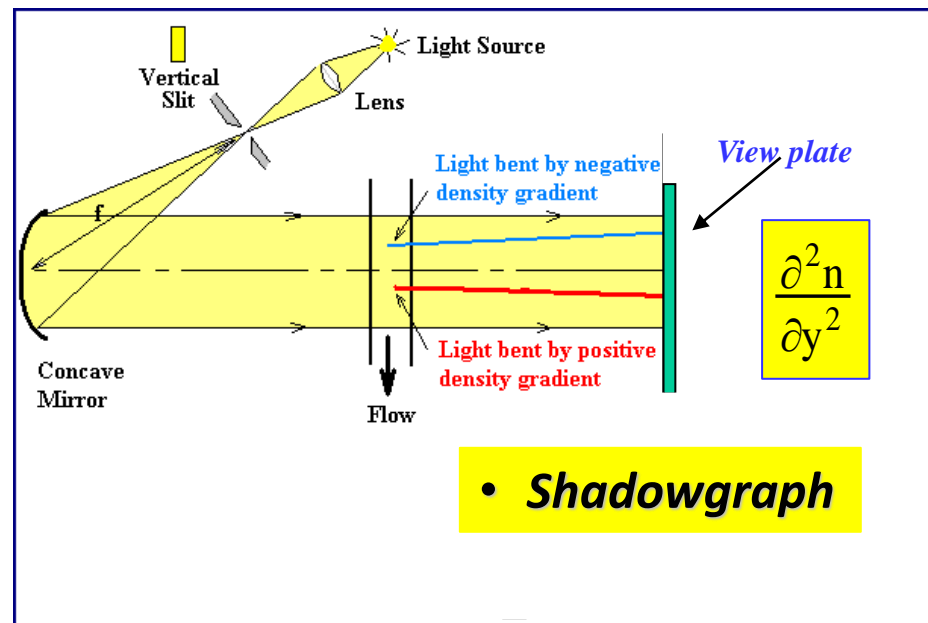
Schlieren:

- Displays a focused image
- Shows ray refraction angle, ε
- Contrast level responds to the 1st derivative of refractive index changes.
- Knife edge used for cutoff



Shadowgraph:

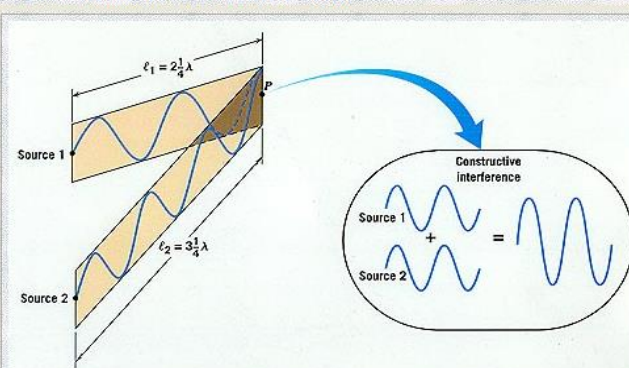
- Displays a mere shadow
- Shows light ray displacement
- Contrast level responds to the 2nd derivative of refractive index changes.
- No knife edge used



INTERFEROMETERS

- Unlike the Schlieren and shadowgraph systems, an interferometer does not depend upon the deflection of a light beam to determine density or index of refraction variation.
- Interferometers are often used for quantitative measurements

Constructive Interference



Waves from two sources start out *in phase*.

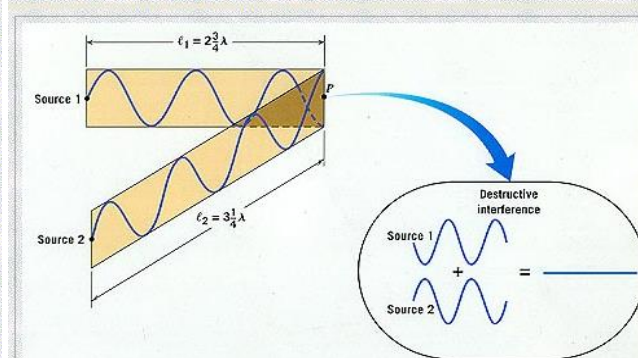
Constructive interference occurs if:

$$d_2 - d_1 = n\lambda$$

$$n = 0, 1, 2, 3, \dots$$

Difference must be an **integer** number of wavelengths

Destructive Interference



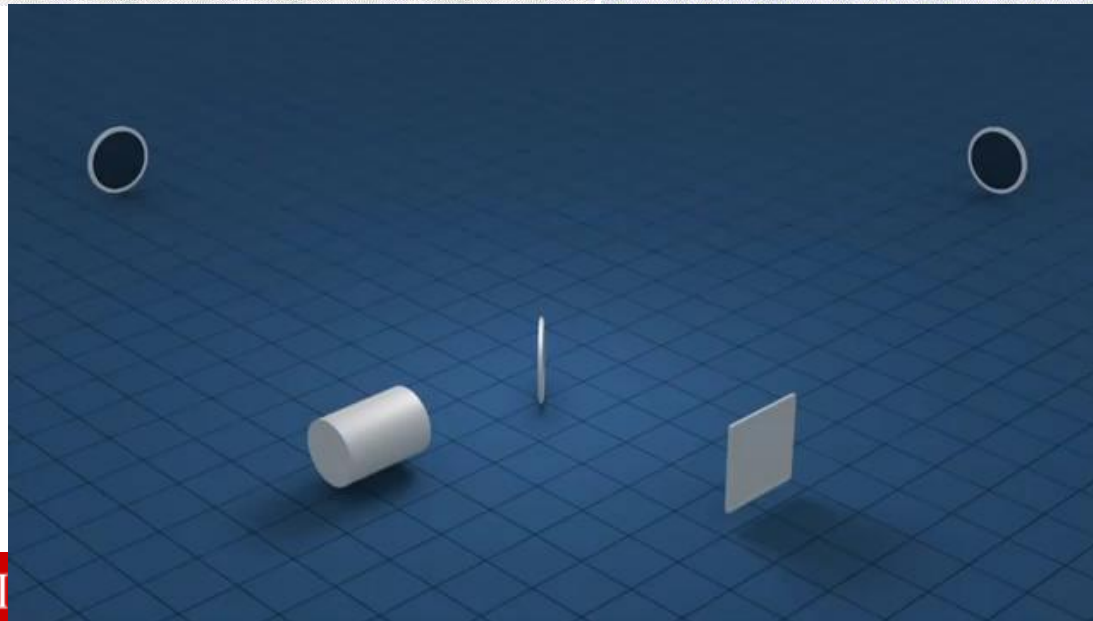
Waves from two sources start out *in phase*.

Destructive interference occurs if:

$$d_2 - d_1 = (2n + 1) (\lambda/2)$$

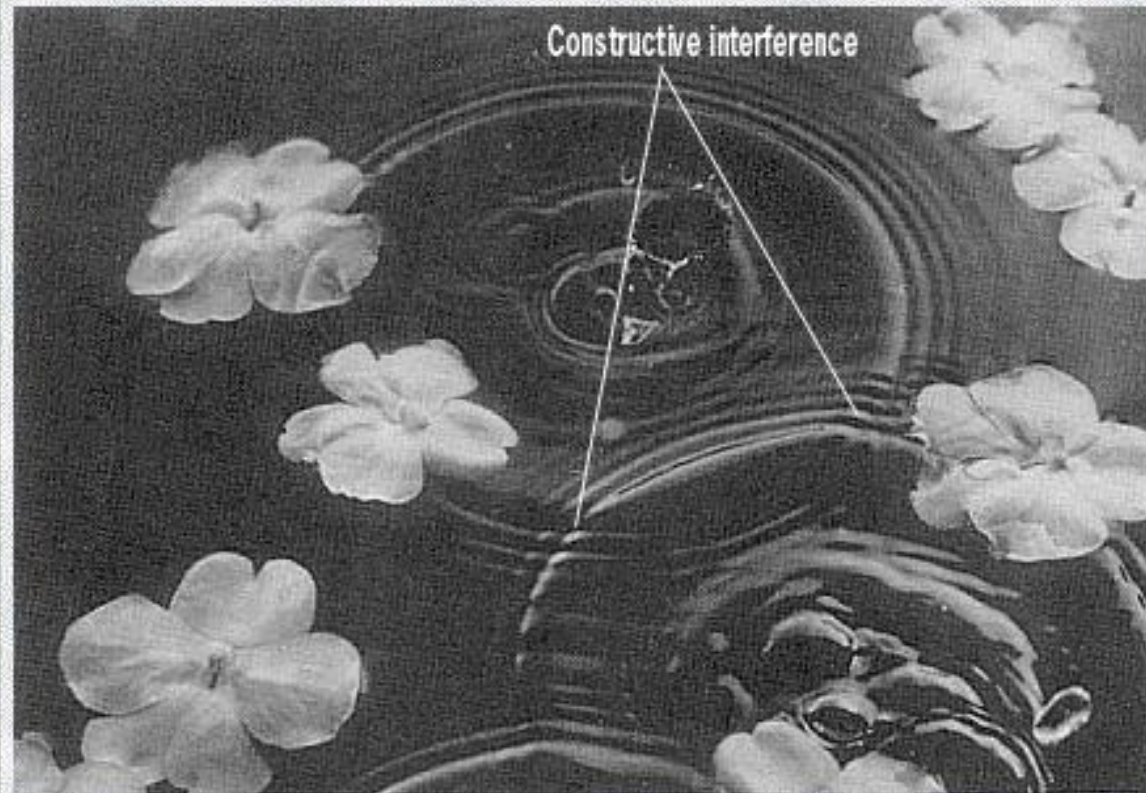
$$n = 0, 1, 2, 3, \dots$$

Difference must be an odd **half-integer** number of wavelengths.



□ INTERFEROMETERS

Inteference of Waves from Two Sources

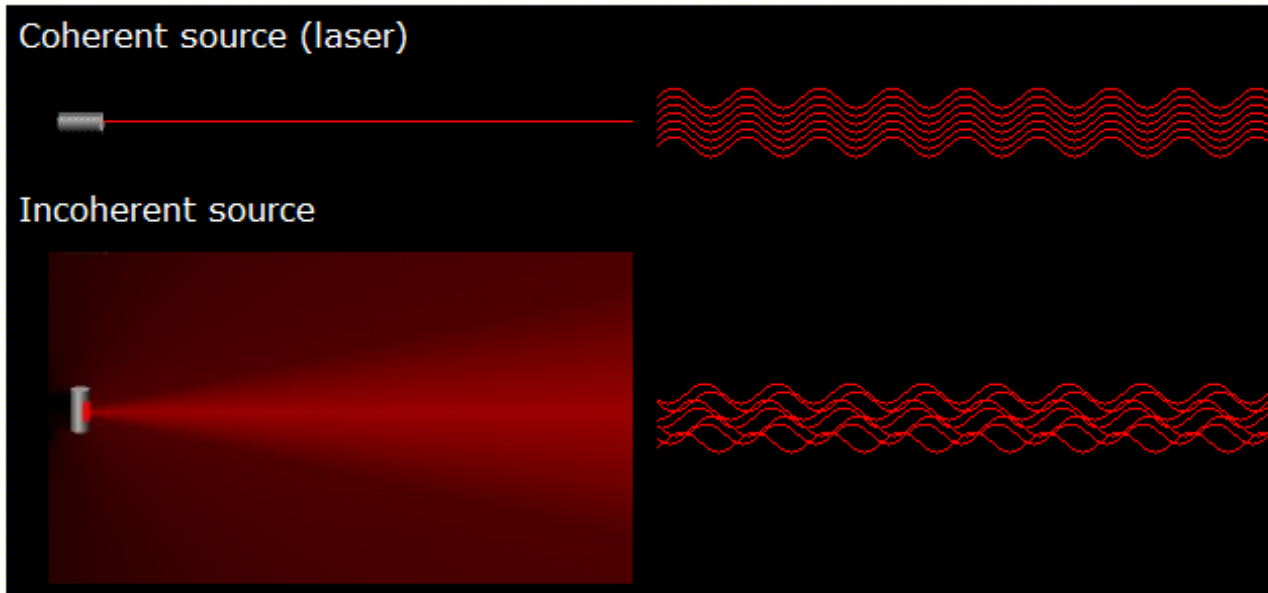


In some places the water wavefronts are in phase (bright spots).

In other places the fronts overlap with peak and valley and interfere destructively (darker spots).

☐ COHERENT LIGHT SOURCE

- *Coherent sources...*
- *Two sources of light are said to be coherent if the waves emitted from them have the same frequency and are 'phase-linked'; that is, they have a zero or constant phase difference.*



INTERFERENCE OF TWO COHERENCE LIGHT WAVES

Amplitude of a plane light wave in a homogeneous medium can be expressed as :

$$A = A_0 \sin \frac{2\pi}{\lambda} (ct - z)$$

therefore:

$$\text{wave 1: } A_1 = A_{01} \sin \left(\frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 \right)$$

$$\text{wave 2: } A_2 = A_{01} \sin \left(\frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \Delta \right)$$

$$\text{if } A_0 = A_{01} = A_{02}$$

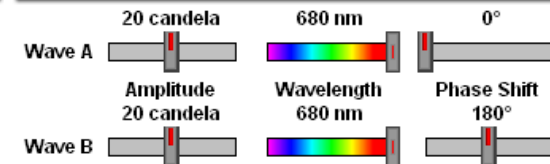
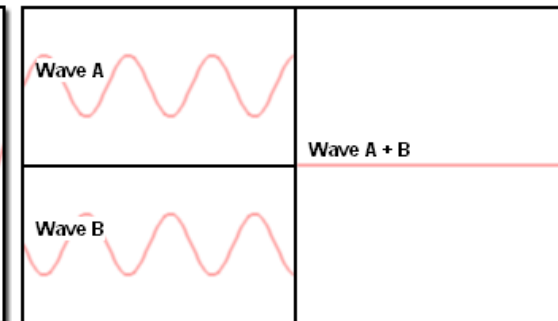
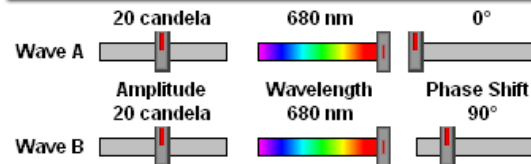
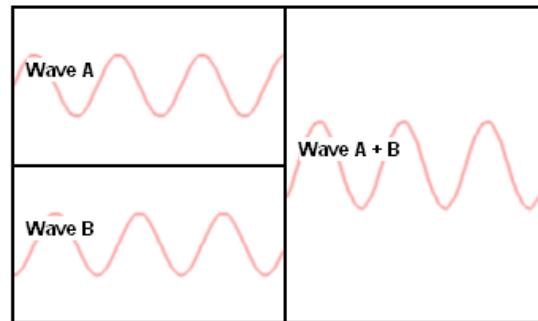
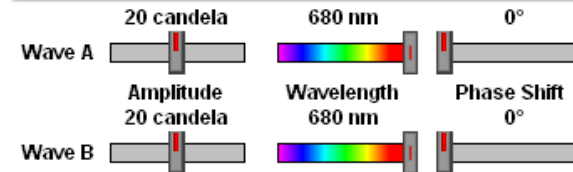
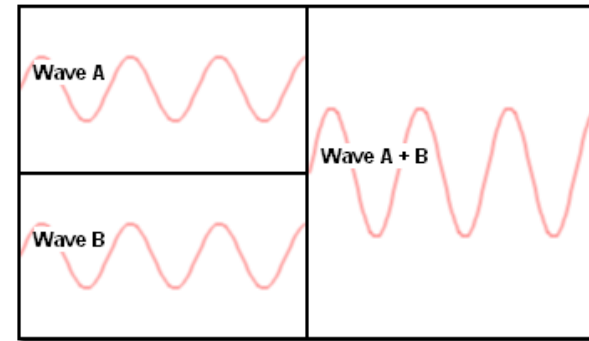
$$\text{then: } A_r = A_1 + A_2$$

$$= A_0 \left[\sin \left(\frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 \right) + \sin \left(\frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \Delta \right) \right]$$

$$= 2A_0 \cos \frac{\Delta}{2} \sin \left(\frac{2\pi}{\lambda} ct - \frac{2\pi}{\lambda} Z_0 - \frac{\Delta}{2} \right)$$

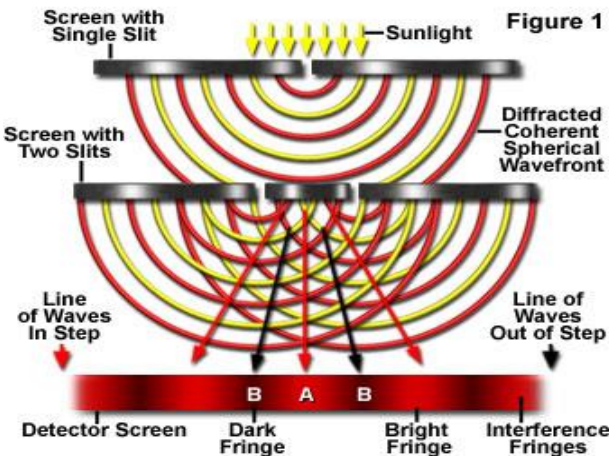
Therefore, the intensity of the combined wave (which is proportional to the square of the peak amplitude) will be :

$$I \sim 4A_0^2 \cos^2 \frac{\Delta}{2}$$



INTERFERENCE OF LIGHT WAVES

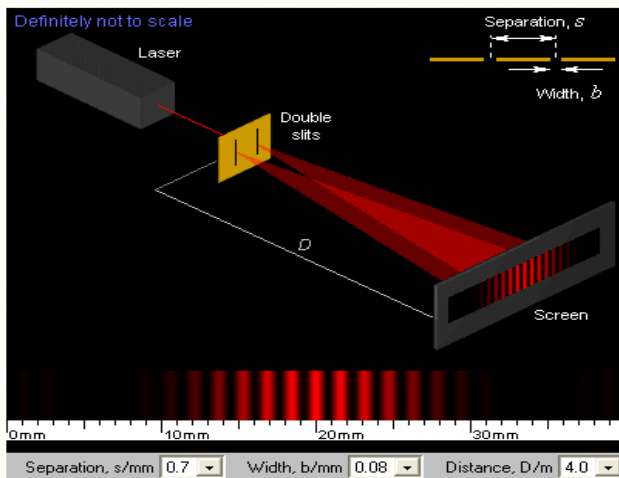
Thomas Young's Double Slit Experiment



<https://www.youtube.com/watch?v=ZQAvVgnreWk>

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Experimental of Thomas Young (1801)

□ INTERFEROMETERS

The optical path length along a light beam is defined as :

$$PL = \int n dz$$

or

$$PL = \int \frac{C_0}{C} dz = \frac{1}{\lambda_0} \int \frac{dz}{\lambda}$$

Therefore, the difference between path 1 and path 2 :

$$\overline{\Delta PL} = PL_1 - PL_2 = \int_{path-1} n dz - \int_{path-2} n dz$$

$$= \frac{1}{\lambda_0} \left(\int_{path-1} \frac{dz}{\lambda} - \int_{path-2} \frac{dz}{\lambda} \right)$$

The phase difference between the two wave will be :

$$\Delta = 2\pi \left(\int_{path-1} \frac{dz}{\lambda} - \int_{path-2} \frac{dz}{\lambda} \right)$$

or

$$\frac{\Delta}{2\pi} = \frac{\overline{\Delta PL}}{\lambda_0}$$

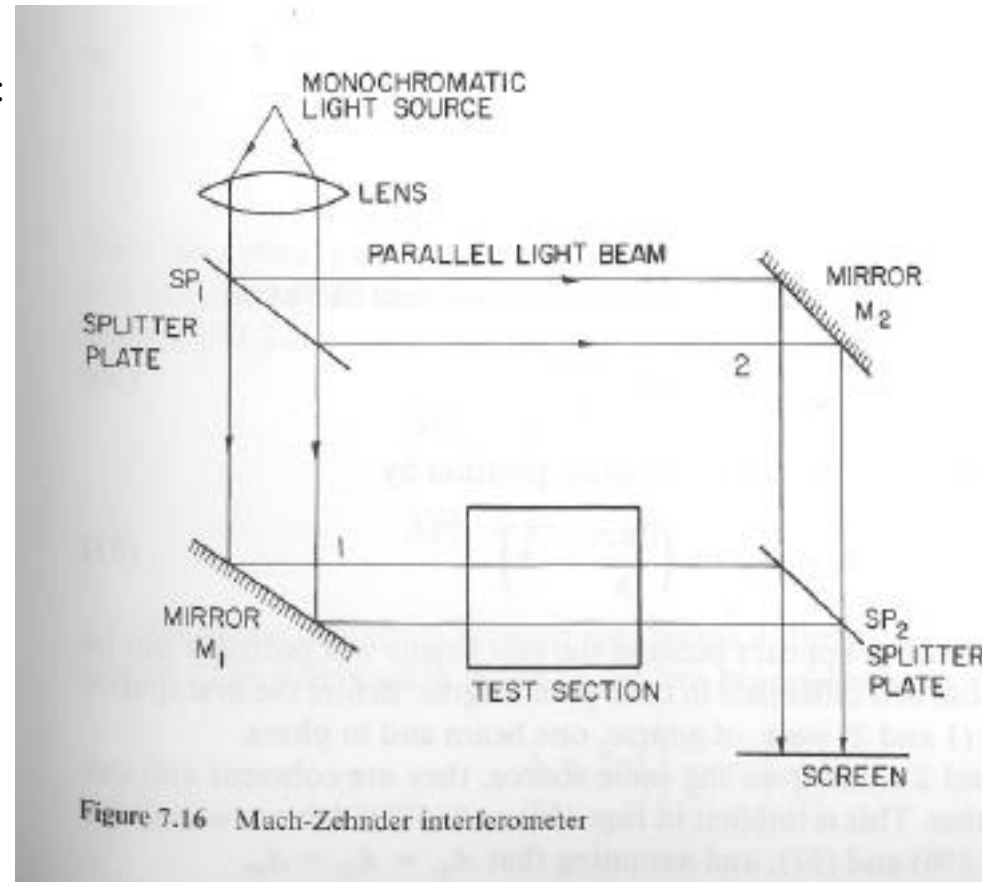


Figure 7.16 Mach-Zehnder interferometer

□ INTERFEROMETERS

$$\varepsilon = \frac{1}{\lambda_0} \int (n - n_{ref}) dz$$

According Gladstone - Dale equation : $\rho = \frac{n-1}{Const}$

$$\Rightarrow \varepsilon = \frac{const}{\lambda_0} \int (\rho - \rho_{ref}) dz$$

if only varies over a length L , then, the fringe shift will be :

$$\varepsilon = \frac{n - n_{ref}}{\lambda_0} L$$

for gaseous flows

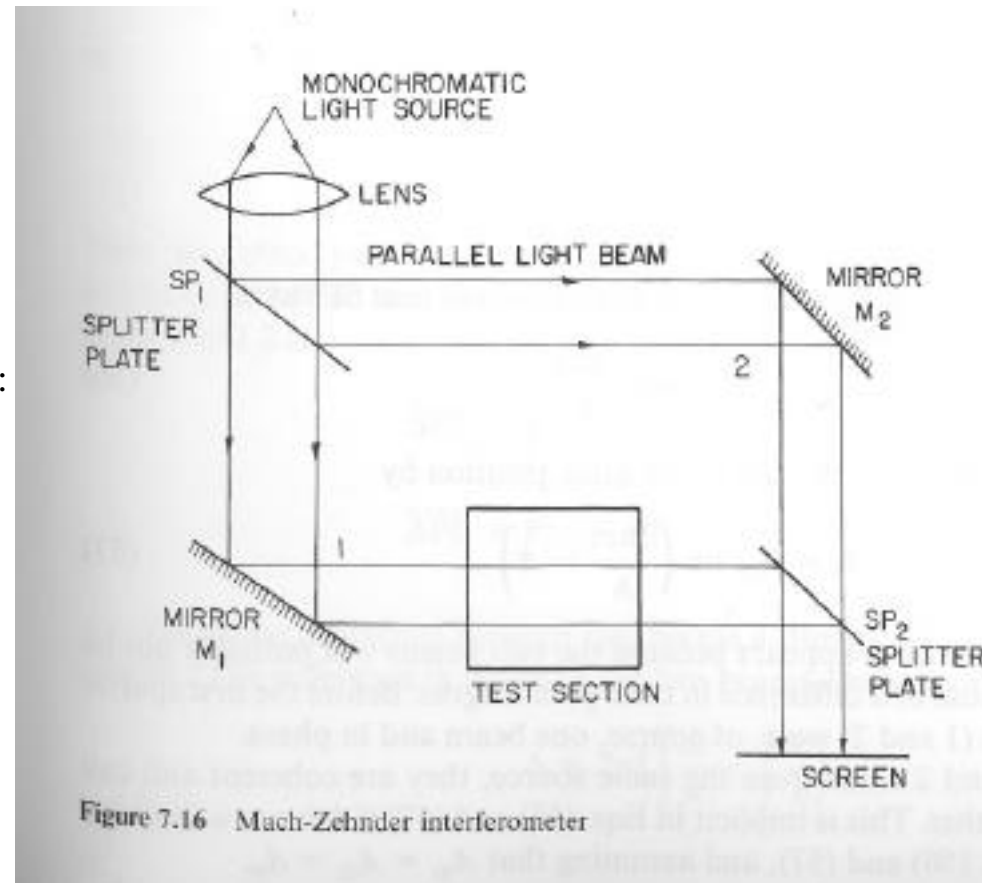
$$\varepsilon = \frac{const}{\lambda_0} (\rho - \rho_{ref}) L$$

or

$$\rho - \rho_{ref} = \frac{\lambda_0 \varepsilon}{const \cdot L} = \frac{\lambda_0 \varepsilon}{n_0 - 1} \frac{\rho_0}{L}$$

for temperature measurements in gaseous flows

$$T - T_{ref} = \frac{\lambda_0 \varepsilon}{L} \frac{1}{dn/dT}$$



APPLICATIONS OF INTERFEROMETRY FOR FLUID FLOW STUDIES

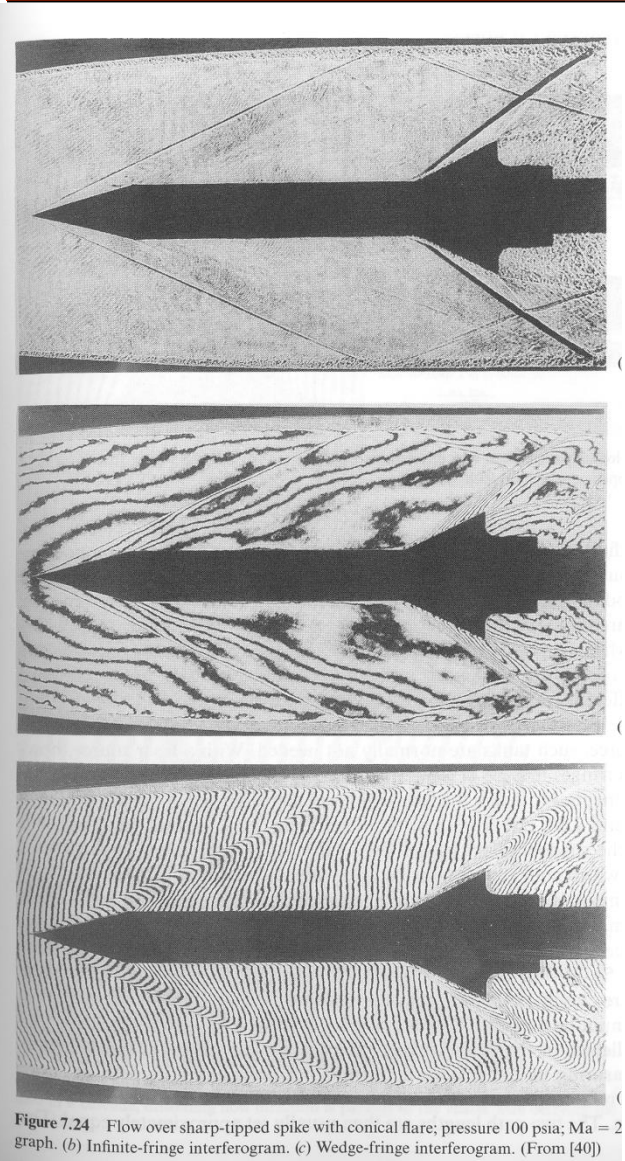


Figure 7.24 Flow over sharp-tipped spike with conical flare; pressure 100 psia; $Ma = 2$ graph. (b) Infinite-fringe interferogram. (c) Wedge-fringe interferogram. (From [40])

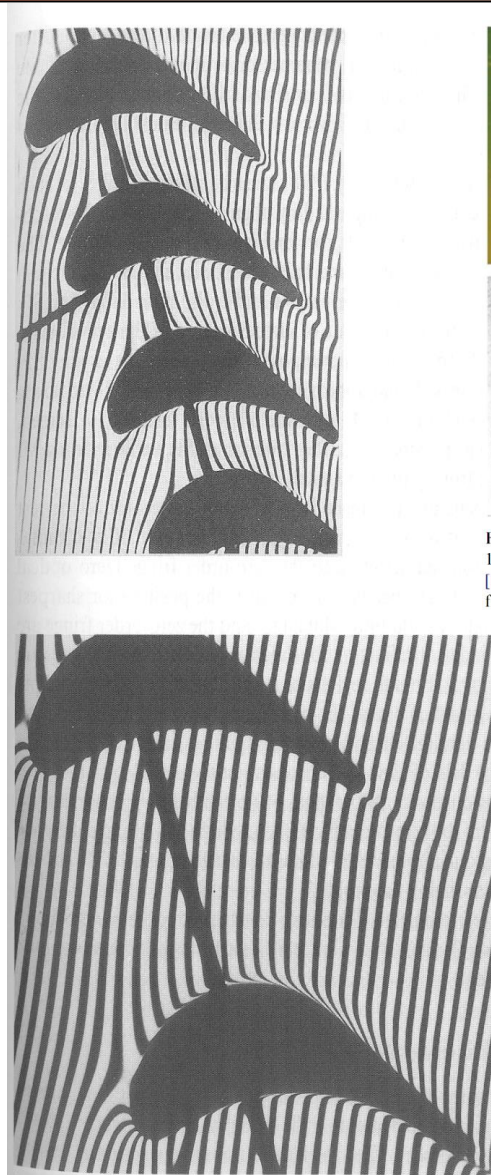


Figure 7.26 Wedge-fringe interferograms used for visualizing flow over held in a cascade; oncoming flow direction is parallel to the visible wire (From [42])

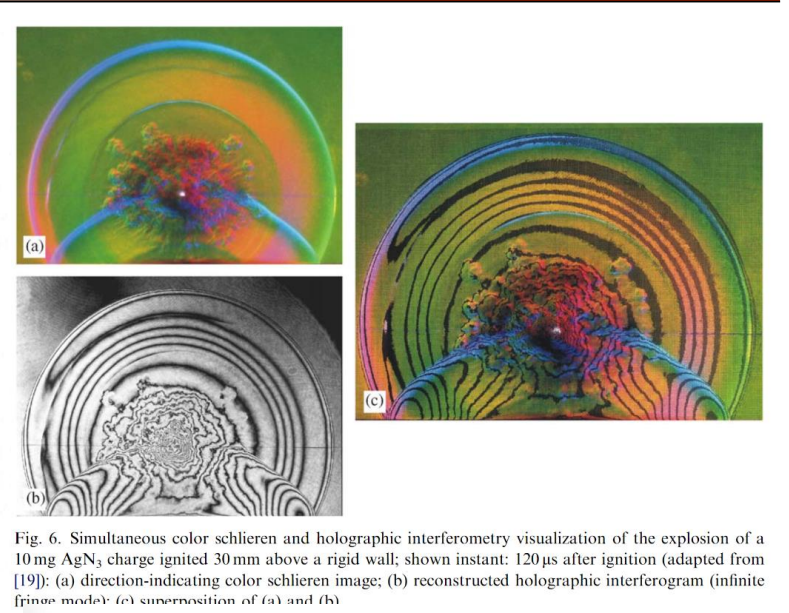


Fig. 6. Simultaneous color schlieren and holographic interferometry visualization of the explosion of a 10 mg AgN_3 charge ignited 30 mm above a rigid wall; shown instant: 120 μs after ignition (adapted from [19]): (a) direction-indicating color schlieren image; (b) reconstructed holographic interferogram (infinite fringe mode); (c) superposition of (a) and (b)

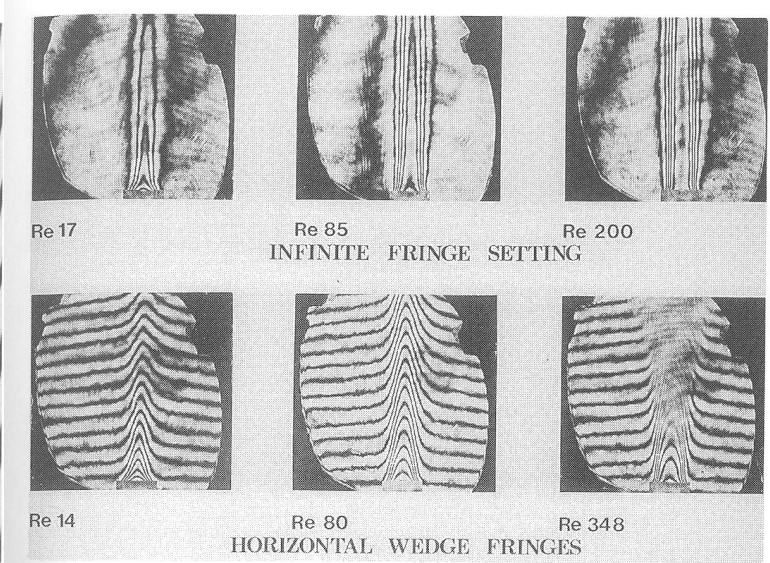


Figure 7.20 Interferograms of a low-Reynolds-number helium jet entering

□ APPLICATIONS OF INTERFEROMETRY FOR FLUID FLOW STUDIES

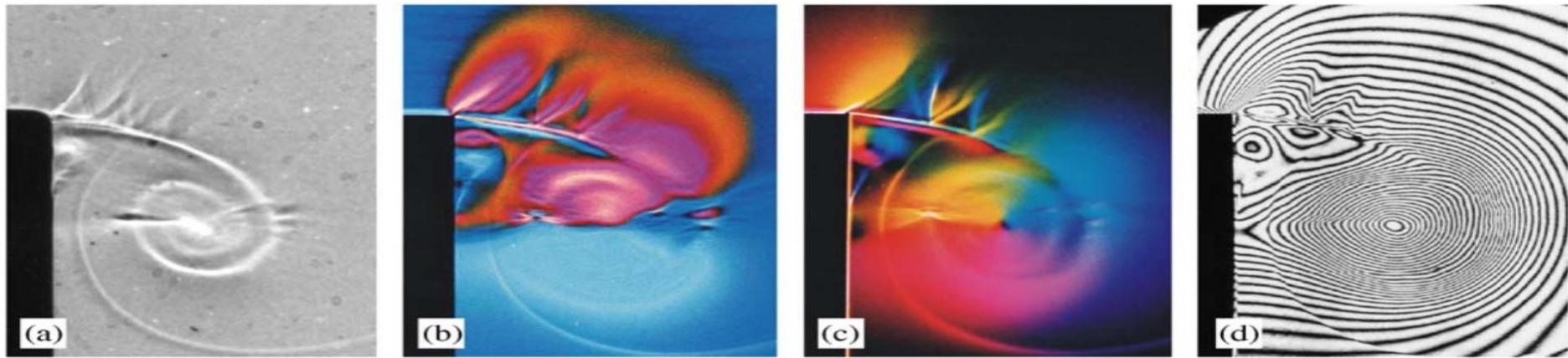


Fig. 2. Shock-generated vortex following the diffraction of a shock wave ($M_S = 1.6 \pm 0.02$ in N_2) at a sharp 90° corner (adapted from [15]): (a) shadowgram; (b) magnitude-indicating color schlieren image, horizontal cutoff; (c) direction-indicating color schlieren image; (d) reconstructed holographic interferogram (infinite fringe mode, $\Delta\rho = 0.054 \text{ kg/m}^3$).

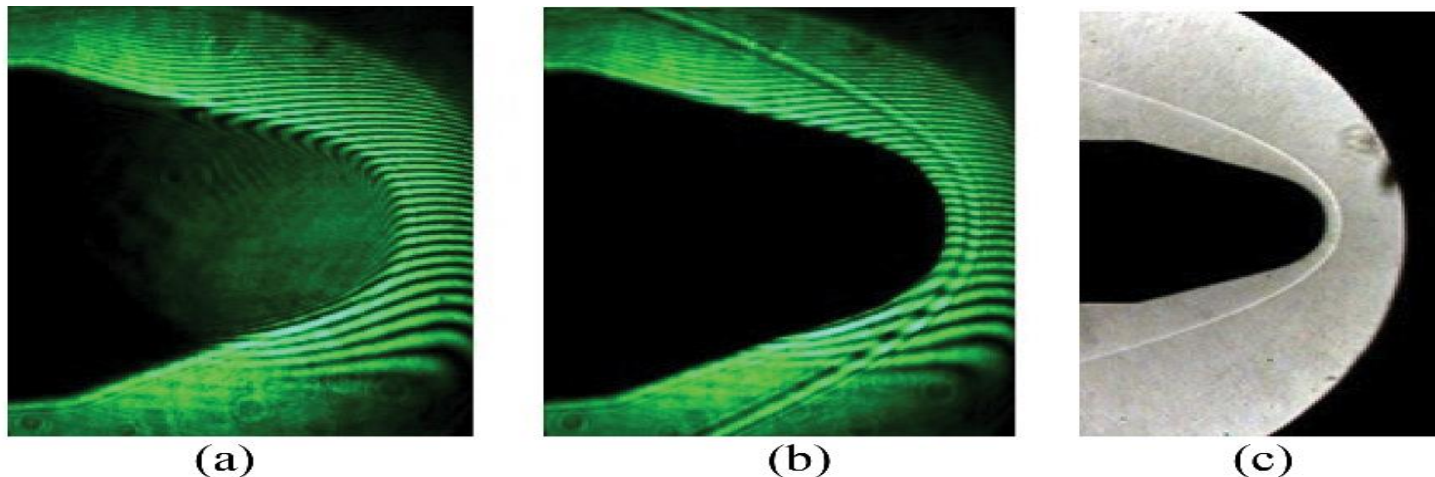
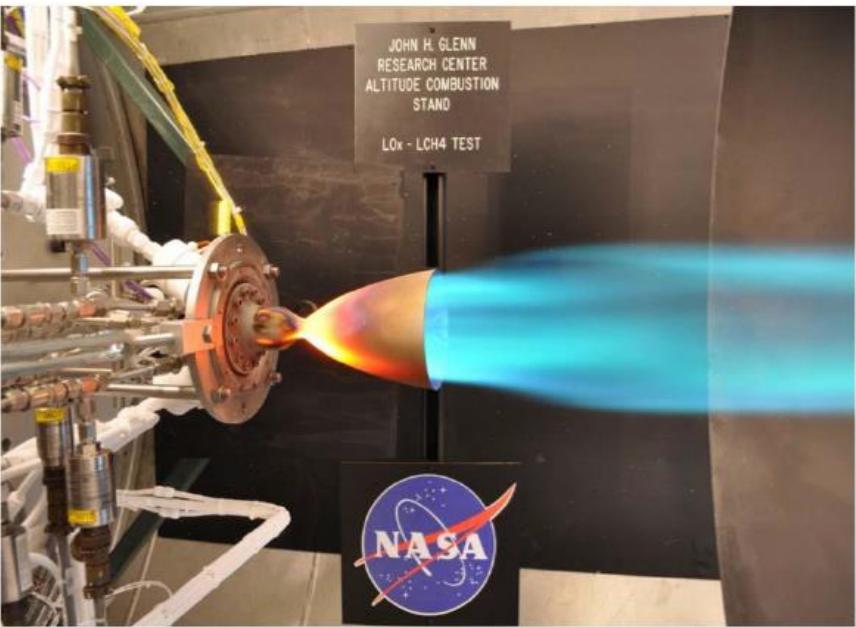
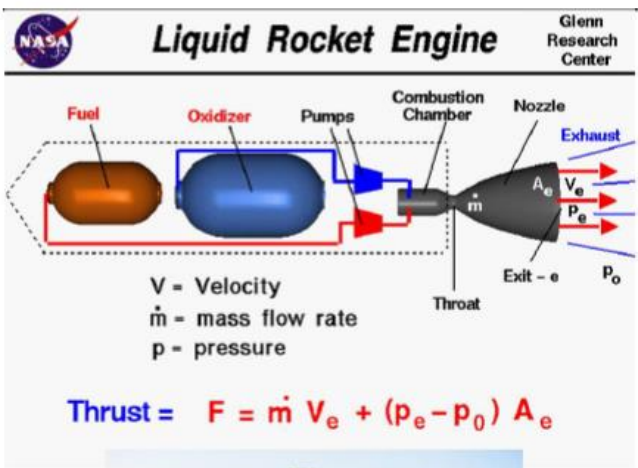


Figure 2: Digital interferometric images a) before flow b) during steady flow, and c) Schlieren image of the flow field around the blunt model at Mach 6 hypersonic flow in HST4 shock tunnel.

LAB 03: VISUALIZATION OF SHOCK WAVES BY USING SCHLIEREN TECHNIQUE

Quasi-1D nozzle flows:



LAB 03: VISUALIZATION OF SHOCK WAVES BY USING SCHLIEREN TECHNIQUE

Laval nozzle:

Mass conservation:

$$\frac{dA}{A} + \frac{d\rho}{\rho} + \frac{du}{u} = 0$$

Momentum conservation:

$$\frac{dA}{A} = (M^2 - 1) \frac{du}{u}$$

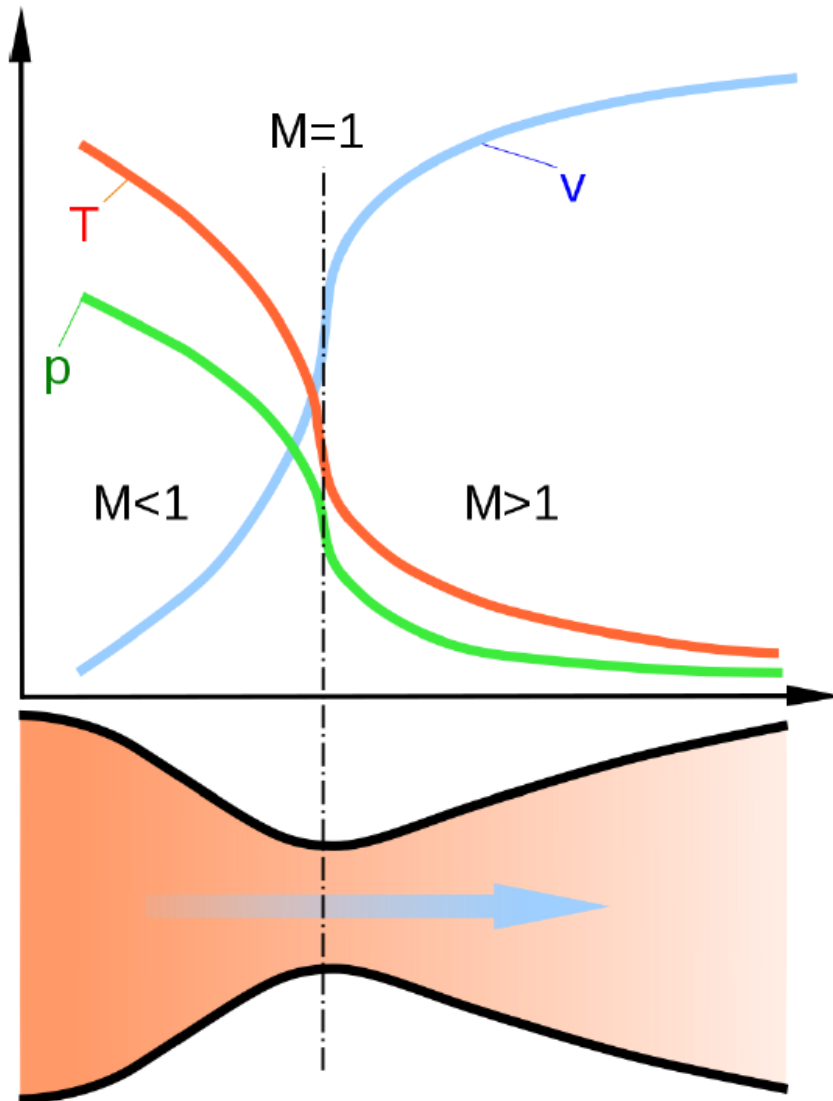
Energy conservation:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

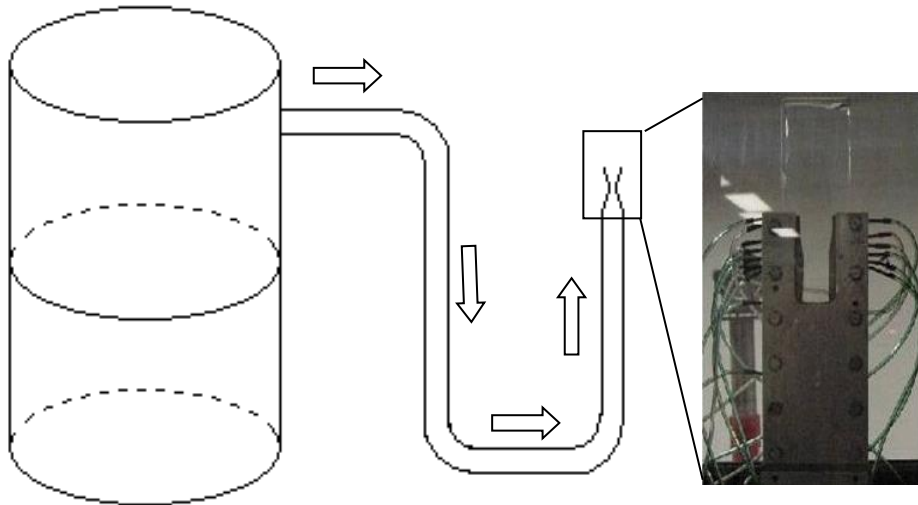
$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

Q. Zhang



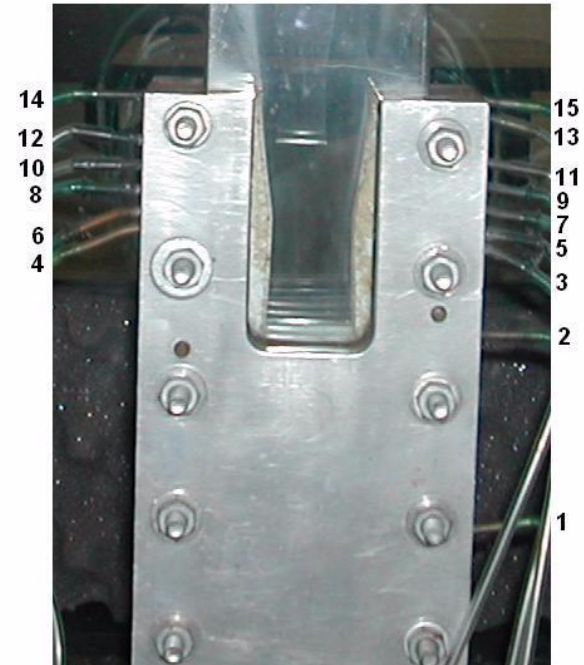
❏ LAB 03: VISUALIZATION OF SHOCK WAVES BY USING SCHLIEREN TECHNIQUE



Tank with compressed air

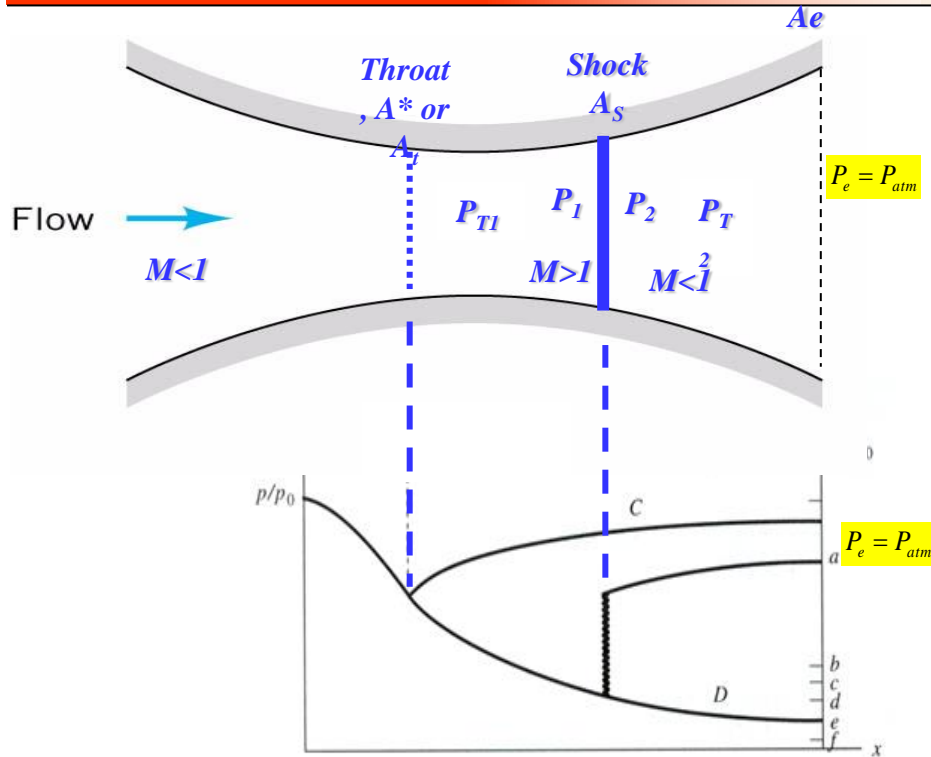
Test section

Nozzle Pressure Tap Numbering Diagram



Tap No.	Distance downstream of throat (inches)	Area (Sq. inches)
1	-4.00	0.800
2	-1.50	0.529
3	-0.30	0.480
4	-0.18	0.478
5	0.00	0.476
6	0.15	0.497
7	0.30	0.518
8	0.45	0.539
9	0.60	0.560
10	0.75	0.581
11	0.90	0.599
12	1.05	0.616
13	1.20	0.627
14	1.35	0.632
15	1.45	0.634

❏ PREDICTION OF THE PRESSURE DISTRIBUTION WITHIN A DE LAVAL



- Using the area ratio, the Mach number at any point up to the shock can be determined:

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}}$$

- After finding Mach number at front of shock, calculate Mach number after shock using:

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

- Then, calculate the A_2^*

$$(A_2^*)^2 = M_2^2 A_s^2 \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M_2^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}}$$

which allows us calculate the remaining Mach number distribution

$$\left(\frac{A}{A_2^*}\right)^2 = \frac{1}{M^2} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}}$$

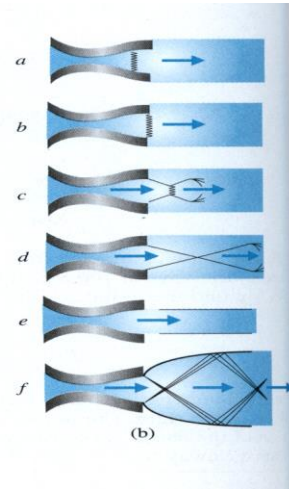
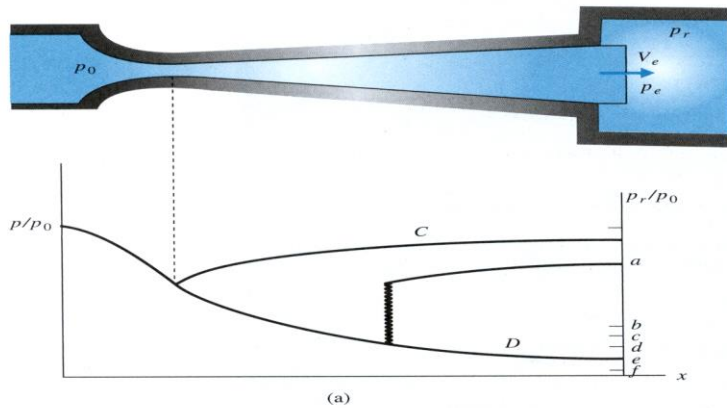
- d. To calculate Mach number given the Mach-Area relation, can use Newton iteration to find M

$$F = \left(\frac{A}{A^*}\right)^2 = M^2 \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}} \quad (2.8)$$

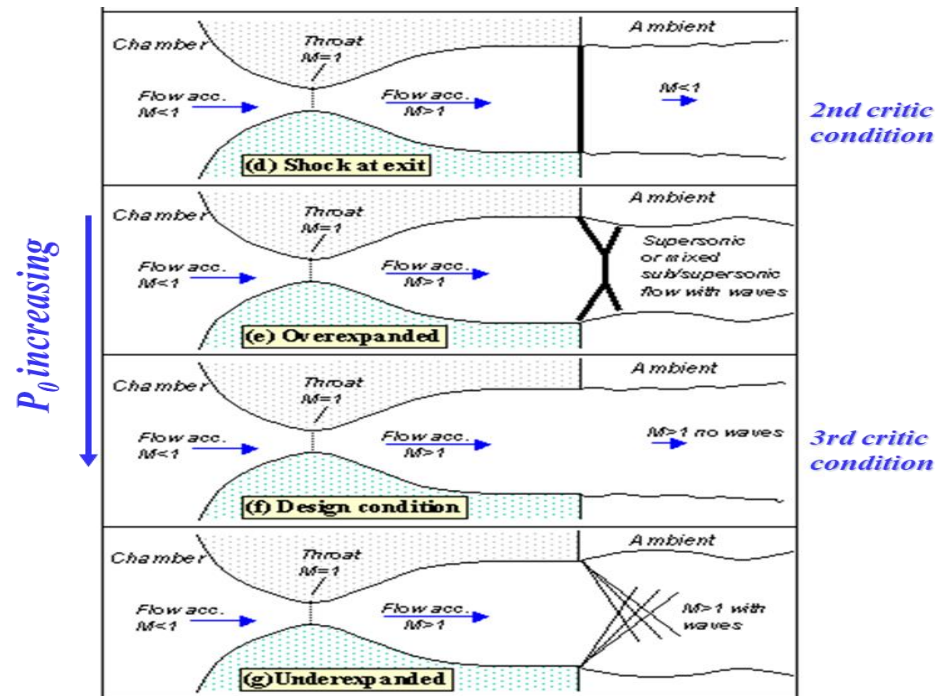
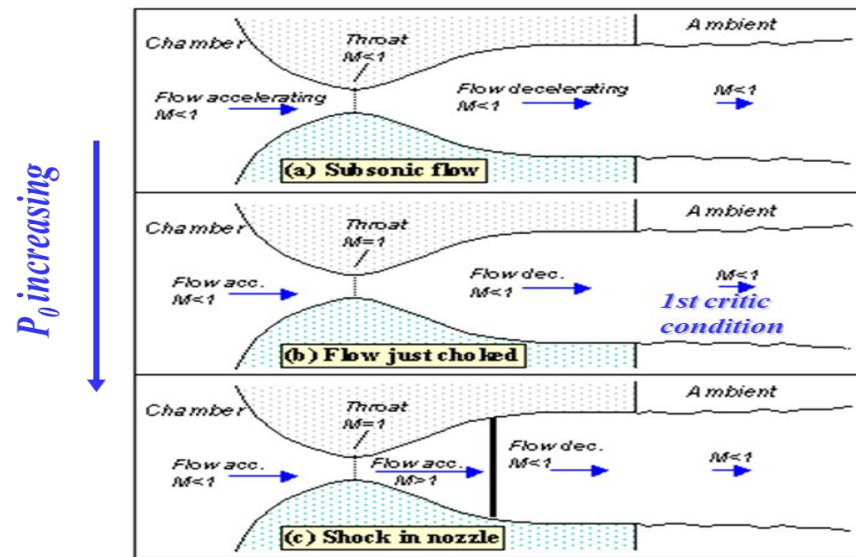
$$F' = \frac{dF}{dM} = \frac{2}{M^3} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}} - \frac{2}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{2}{\gamma-1}} \quad (2.9)$$

$$M^{n+1} = M^n - \frac{F}{F'} \quad (2.10)$$

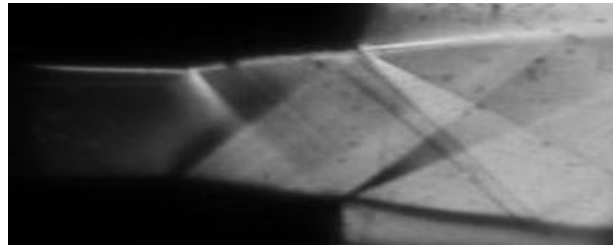
LAB02: PRESSURE MEASUREMENTS IN A DE LAVAL NOZZLE



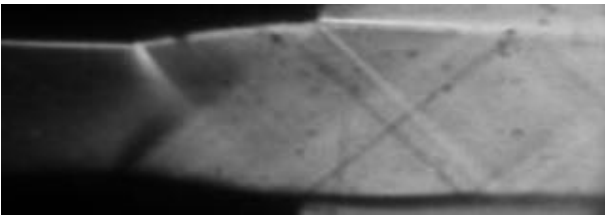
1. Under-expanded flow
2. 3rd critical
3. Over-expanded flow with oblique shocks
4. 2nd critical
5. Normal shock existing inside the nozzle
6. 1st critical



□ 1ST, 2ND AND 3RD CRITIC CONDITIONS



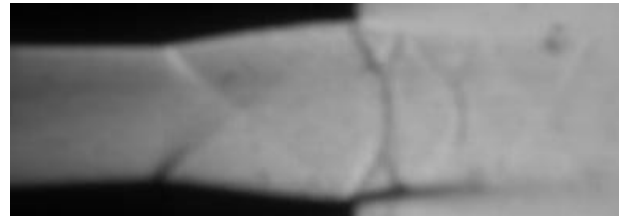
- Under-expanded flow



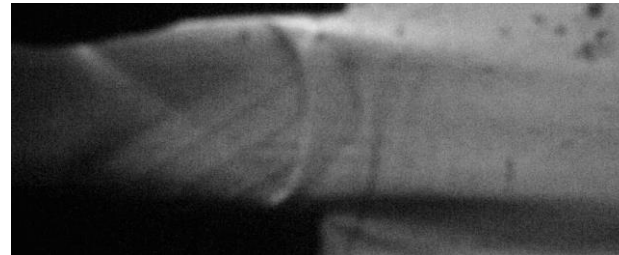
- Flow close to 3rd critical



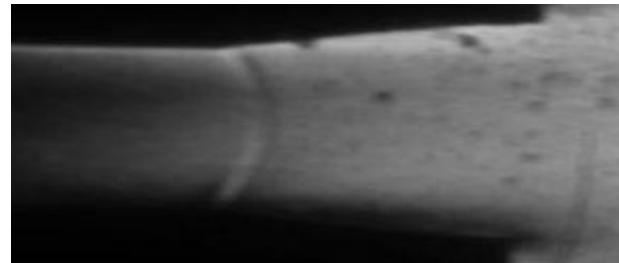
- Over-expanded flow



- 2nd critical – shock is at nozzle exit



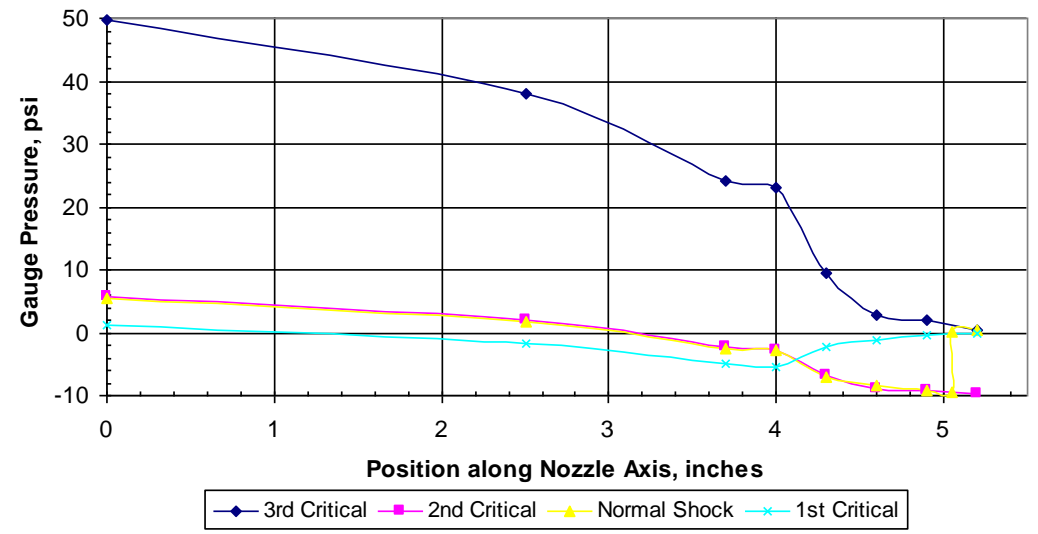
- Over-expanded flow with shock between nozzle exit and throat



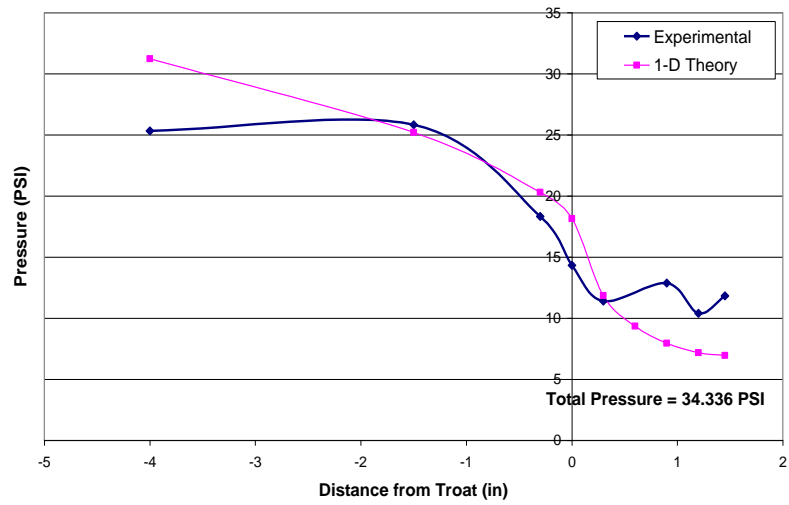
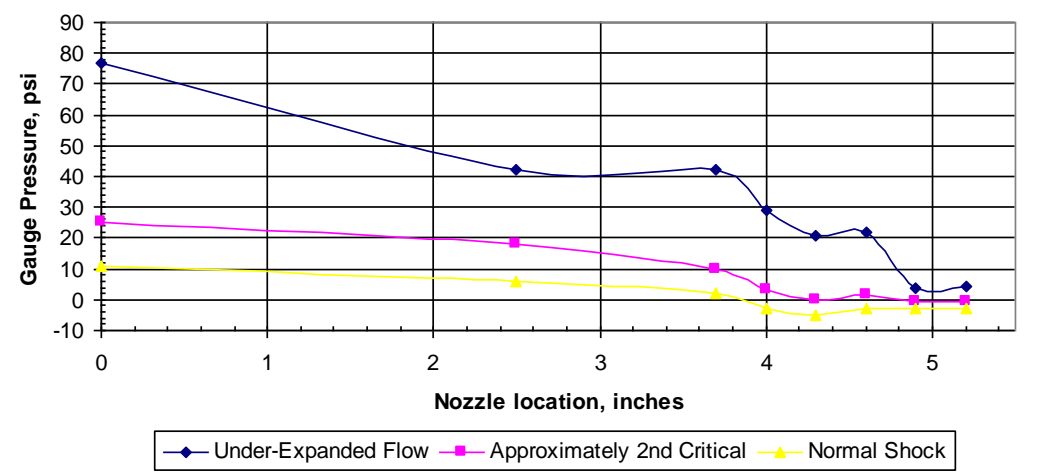
- 1st critical – shock is almost at the nozzle throat.

EXAMPLES OF THE PREVIOUS MEASUREMENT RESULTS

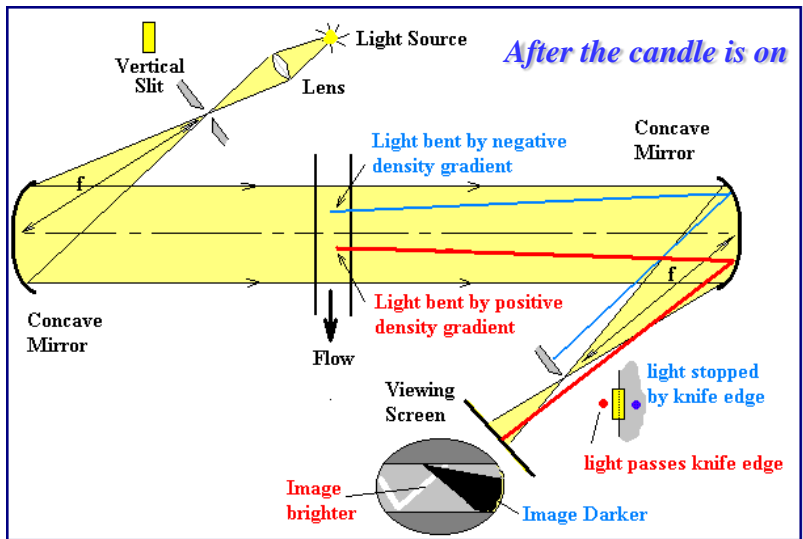
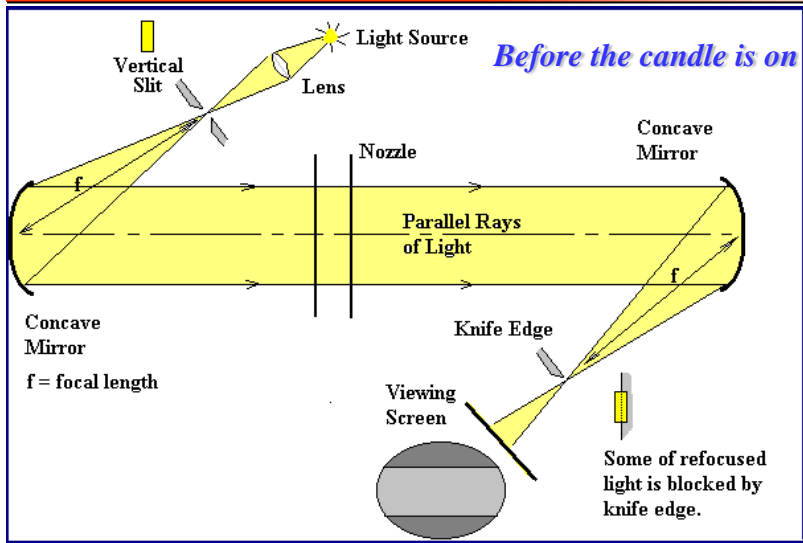
Theoretical Data - Gauge Pressure vs. Position



Experimental Results - Pressure vs. Nozzle Location

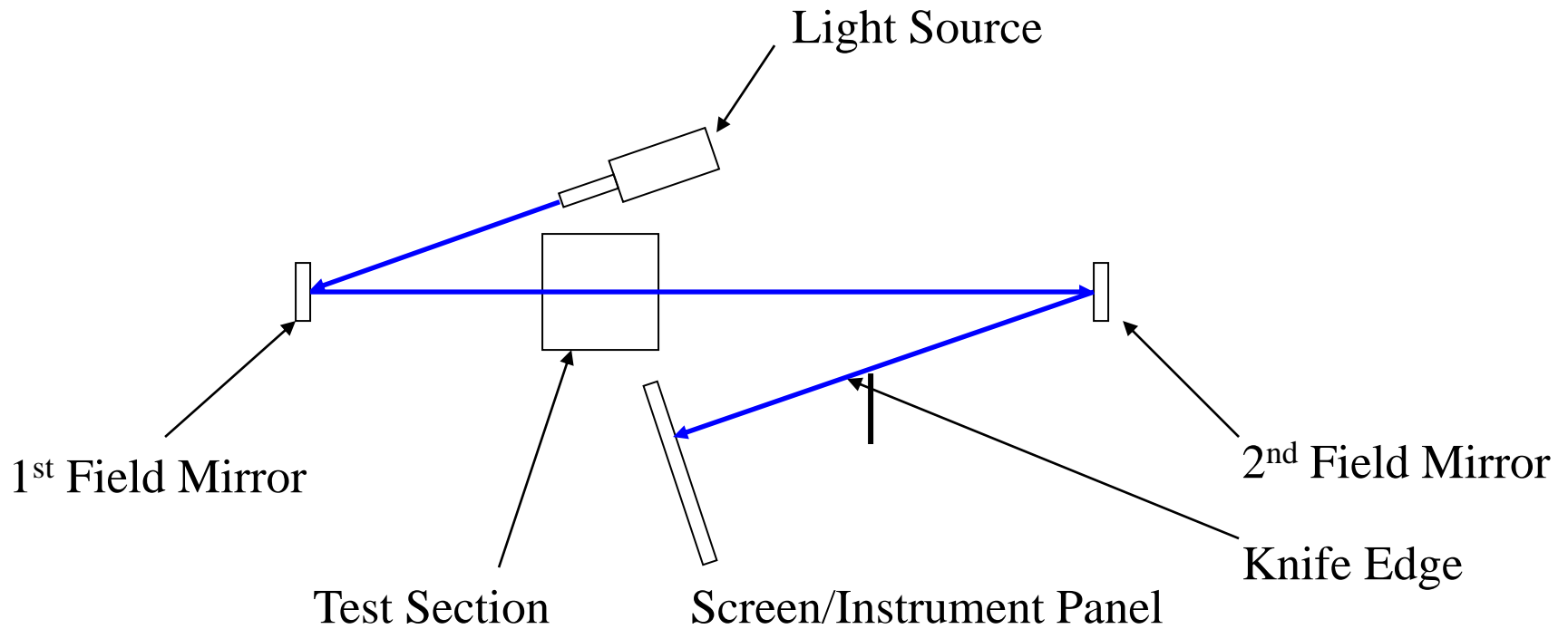


LAB 02: VISUALIZATION OF SHOCK WAVES BY USING SCHLIEREN TECHNIQUE

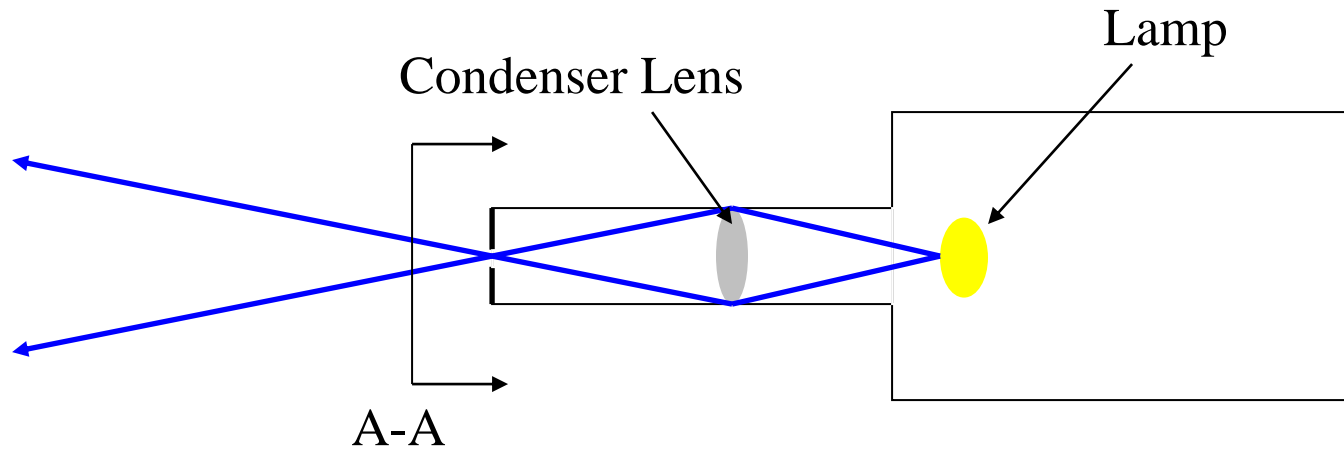


Schlieren imaging result of a thermal plume above a burning candle

ISU's Z-TYPE SCHLIEREN SYSTEM



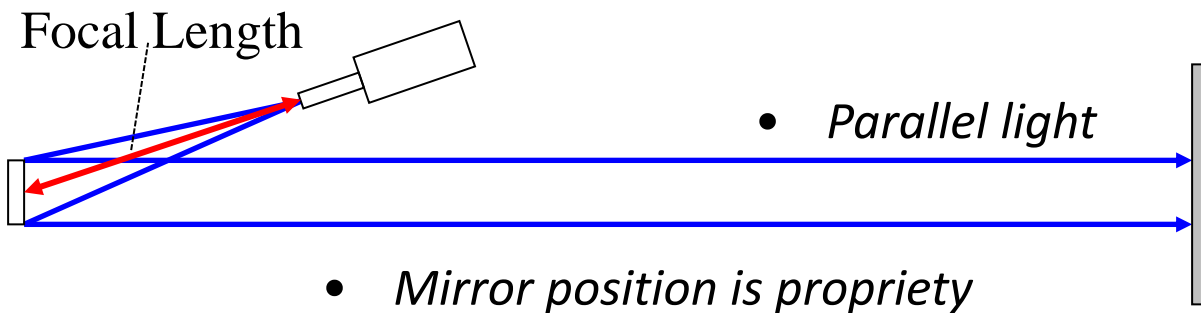
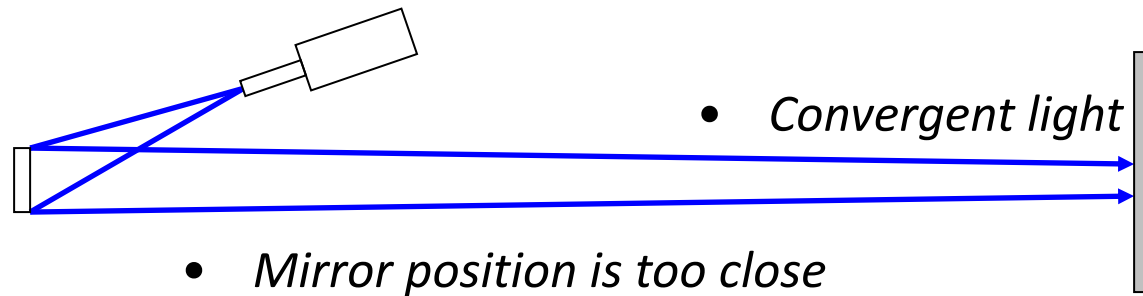
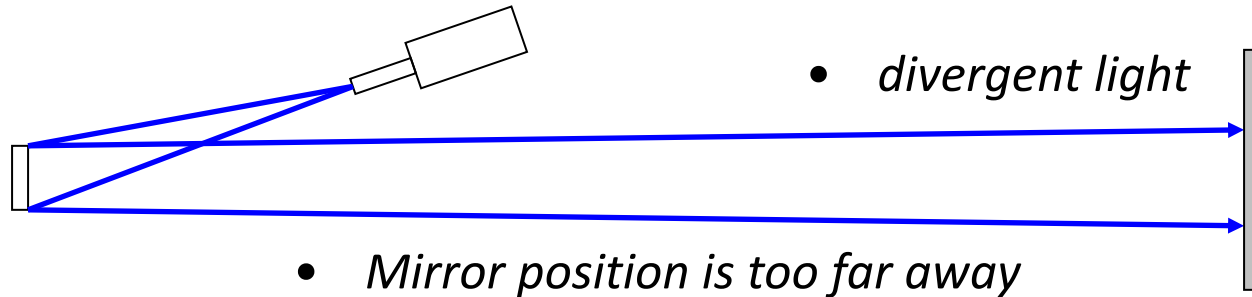
LIGHT SOURCE



Section A-A

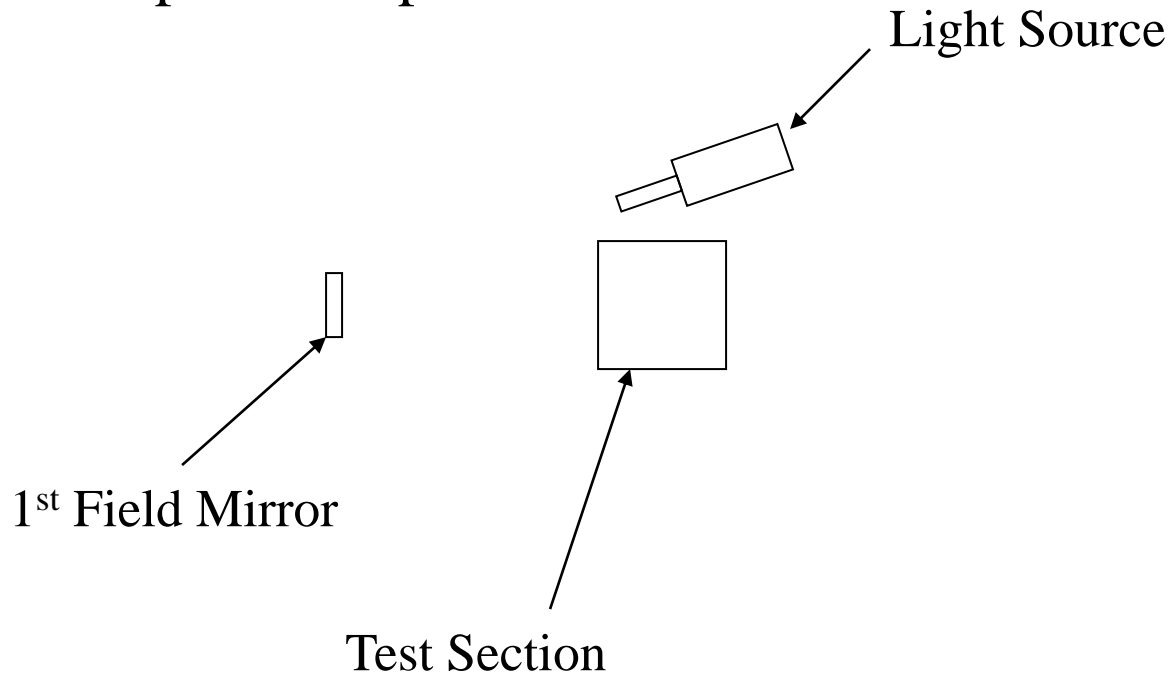
□ SETTING UP THE SCHLIEREN SYSTEM

Step 1: Find the focal length of the field mirror



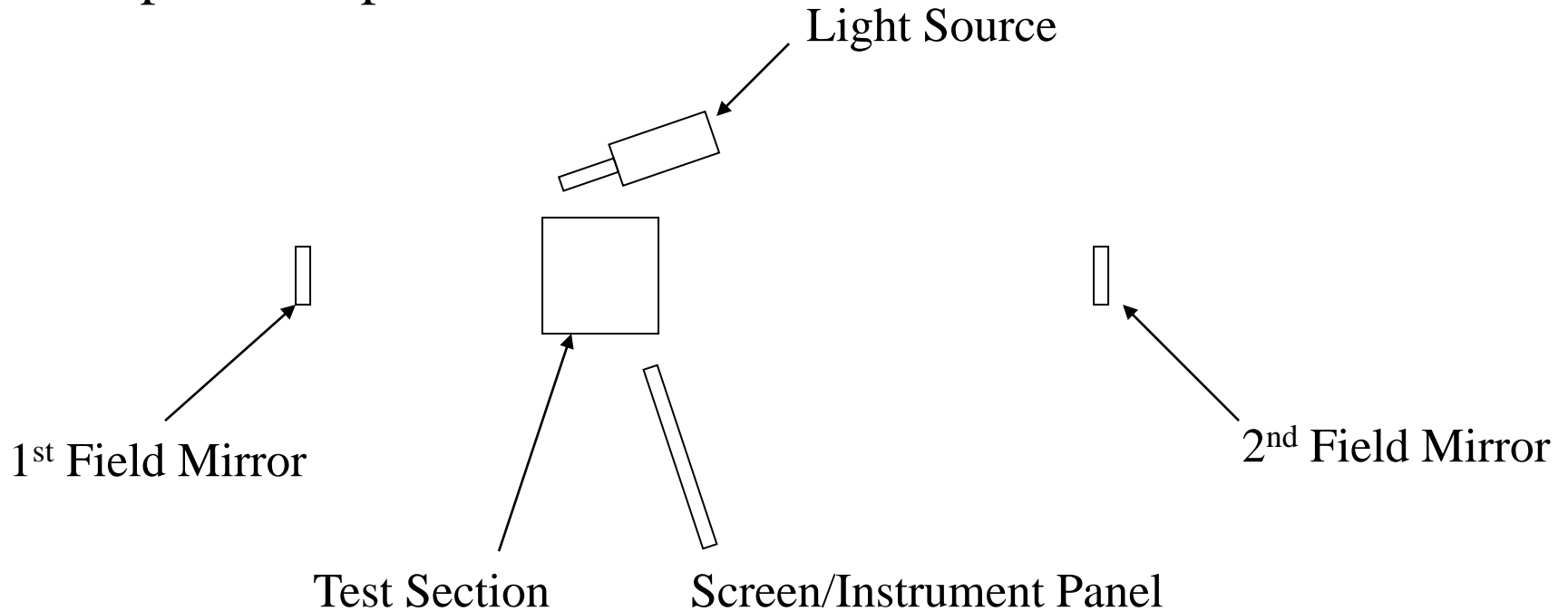
□ SETTING UP THE SCHLIEREN SYSTEM

Step 2: Set up the first field mirror



□ Setting Up The Schlieren System

Step 3: Set up the second field mirror



□ SETTING UP THE SCHLIEREN SYSTEM

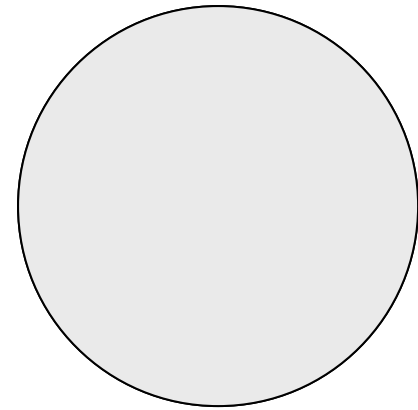
Step 4: Set up the knife edge



Focus the
source image
on the knife

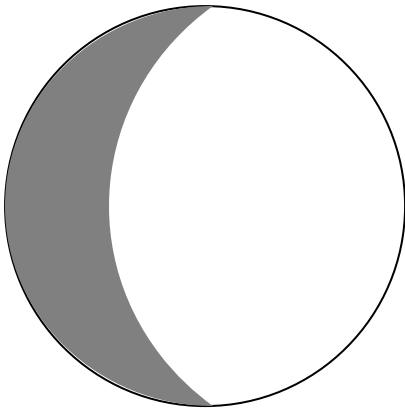


Adjust the cutoff

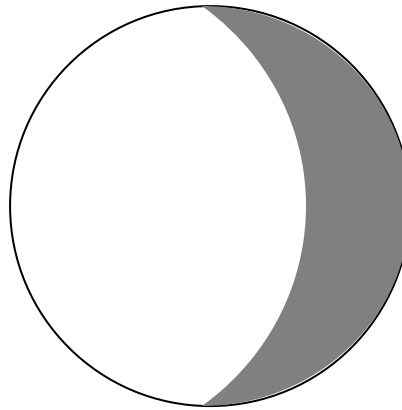


Obtain a uniform
darkening of the image

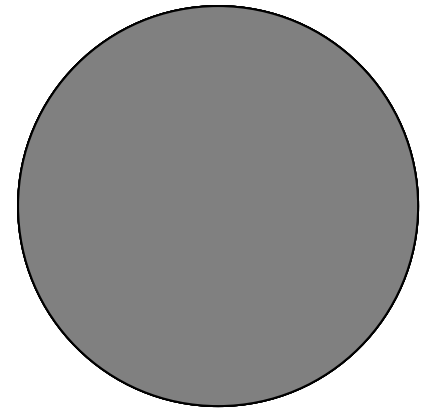
UNIFORM DARKENING



Knife edge too close
to second field mirror



Knife edge too far from
second field mirror



Uniform darkening